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AIR TECHNICAL INTELLIGENCE TRANSLATION

(Title Unclassified)
INTERIOR BALLISTICS

by

M. E. Serevryakov

State Printing House of the Defense Industry

Moscow, 1949, 2nd Edition

672 Pages

(Part 1 of 10 Parts, Pages 1-82)

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The manual contains the theoretical principles of interior ballistics and the contemporary methods of solution of its main problems. The course includes investigations and studies accomplished in recent years in various branches of this science. Also, a short historical description is given of the development of interior ballistics. The latter emphasizes the leading part played by Russian scientists prior to the October revolution, and particularly after it.

Considerable attention is given to the practical aspects of a series of problems. Reference is made to a sizable quantity of test data, examples and problems helpful in mastering the method of basic ballistic calculations.

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The domestic interior ballistics, comparably to all other related branches of artillery sciences, progressed and developed along outlines peculiar to themselves. Artillery sciences and techniques, among them interior ballistics, constantly expanded and developed during the years of Stalin's Five Year Plans. This created types of artillery systems which were markedly superior, as respects quality, to those of foreign armies, as proven by the practice of the Great Homeland War (World War II).

Artillery technology continued to develop after [illegible], and the [illegible] on it are increased. Thus, the tasks [illegible] problems which interior ballistics has to cope with [illegible] widened.

In order to solve these tasks [illegible], it [illegible] to have an [illegible] theory [illegible] [illegible]. [illegible], we need [illegible] [illegible] by [illegible] [illegible] "the basic [illegible] [illegible] [illegible]

publication of a new study [illegible] interior ballistics, which would be adequate for the [illegible] demands of modern artillery technology and which would [illegible] materials and methods for the further development of this [illegible] science.

The proposed manual [illegible] from the textbook published in 1939. It contains [illegible] of newly written sections and chapters, which present solutions [illegible] of problems in complex cases resulting from the development of [illegible] technology.

The manual reflects an exte[illegible] methodical study, carried out in the Interior Ballistics [illegible] of the Artillery Academy during the years of the Great [illegible] (World War II) and subsequently to it.

Basically, the course was [illegible] from studies and investigations of our Soviet sc[illegible] including lecturers of the Interior Ballistics Departm[illegible] Artillery Academy.

Studies of foreign authors [illegible] only for the purposes of clarification of the history [illegible] or another. Those relate mainly to studies on in[illegible] of powder combustion, and on some others conducted [illegible]

The course is designated [illegible] manual for students of the engineering departments [illegible] Academy and other institutions of higher educat[illegible] the guidance of employees of design offices, [illegible] laboratories active in processing, investi[illegible] types of artillery weapons and ammuni[illegible] also be of use to a wide circle of scientifi[illegible] students in advanced courses of instituti[illegible]

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The course is composed of the introduction and three sections.

For the most part, the introduction is newly written. Special emphasis is placed on the relation of interior ballistics to the design of artillery types and ammunition. A new chapter entitled "From the History of the Development of Interior Ballistics" was written, stressing the leading part of Russian scientists prior to and after the October revolution.

The first section, entitled "Physical Principles of Interior Ballistics", [illegible] an account of the physical principles of interior ballistics and of the phenomena taking place during firing, from the start of powder combustion to the end of the period of the after-effects of gases on the projectile and on the gun barrel.

The basic [illegible] is preceded here by the first chapter, containing information on powders and their basic characteristics.

Disregarding the opinions of some specialists, to the [illegible] that the terms [illegible] and pyro[illegible] are outdated at [illegible] present time, the author continues to utilize these expressions [illegible] of [illegible] and brevity.

The [illegible] of the material contained in [illegible] II, "[illegible]," was changed considerably.

[illegible]

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on the basis of the physical principle of combustion, in accordance with the method of Professor M.E. Serebriakov.

Chapter VII, "Numerical Methods of Solution," was kept without much change.

Chapter VIII, discussing empirical methods and tables, was considerably abbreviated, because the former lost their importance with the existence of precise tables composed on the basis of analytical formulas. At the same time, the correction tables of Professor V.E. Slukhotski were added.

In Chapter IX, "Tabular Methods for the Solution of Interior Ballistics Problems," the fundamentals for the compila[illegible] were re-written, and material was added on the new [illegible]

The idea of the method of relative variables and a [illegible] number of parameters, evolved during recent years by [illegible] B.N. Okunev, Professor M.F. Drozdov, Lecturer M.S. [illegible] L.I. Sviridov, is introduced in the course for the fi[illegible].

The theory of similarity is so arranged that its [illegible] follow inherently from formulas used as a basis for th[illegible] of ballistic tables.

Chapter X, "Ballistic Design of Weapons," is [illegible] written, offering new methods for solving this [illegible] the theoretical substantiation and new criteria [illegible] and technical requirements. [illegible] developed by the author, [illegible] number of variants in [illegible]. In this [illegible] evaluation of gun life by the method of [illegible] introduced.

The third section, "Solution of Interior Ballistics Problems in [illegible]lex Cases," gives solutions of interior ballistics problems for [illegible]tain special cases of great interest in practical applications. [illegible]r instance, Chapter XI includes:

[illegible] The solution of problems for combination projectiles; which are [illegible]ytical and apply the GAU 1942 table.

[illegible] The solution of problems for mortars, with consideration of th[illegible]l escape of gases through the clearance, and with reference to [illegible]led example of calculation.

[illegible] solution of interior ballistics problems relating to [illegible] acceleration, treated by Professor G.V. Oppokov.

[illegible] and last chapter clarifies peculiarities of ballistic [illegible]ing a conical bore, and offers ideas on the design of [illegible].

[illegible] way, the course covers the greater part of the basic [illegible] modern interior ballistics.

[illegible]r portion of the study was written by Professor M.E. [illegible] Doctor of Technical Sciences, Active Member of the [illegible]tillery Sciences, Major-General of Artillery Engineering [illegible]ut six pages of print were written by Professor [illegible] Doctor of Technical Sciences, Major-General of [illegible]neering Services.

[illegible]s express deep gratitude to the corps of lecturers of [illegible] for criticism in reviewing the manual, and to [illegible] Ventsel for a review of the study and for a series of [illegible]nts.

[illegible]s also express gratitude to the junior scientific

collaborator P.I. Lizorkin, who made a series of basic calculations and provided examples; and to the editor, Colonel-Engineer B.V. Smirenskii, for his great services in the preparation of the manual for printing.

M. Serebraikov
G. Oppokov

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[illegible]

Art[illegible]

The [illegible]

[illegible] initial [illegible] located at [illegible] distance [illegible] point [illegible] at aerial [illegible], in [illegible] angle of [illegible] a given [illegible] (i.e., in [illegible] of [illegible]).

The [illegible] in [illegible] the target by [illegible] a weapon. [illegible] fire [illegible] is the basic [illegible] artillery [illegible].

The [illegible] imparts motion to the pistol bullet, as well as to the heavy projectile of [illegible] guns or to the [illegible] piercing shell of an anti-tank gun.

[illegible], one of the principal specialized, [illegible] branches of artillery science, is concerned with the study of the laws governing the projectile's motion in the bore of the weapon and in the air.

Two principal periods may be distinguished in the motion of a projectile:

1) [illegible] within the bore of the weapon during [illegible] when the projectile moves at a constantly increasing [illegible] as a result of the pressure of powder gases, and leaves the bore of the weapon with a given, so-called muzzle velocity v_0.

2) Motion or flight in the air of a projectile discharged from a weapon with a muzzle (maximum) velocity, and undergoing the effects of gravity and of air resistance until the moment of impact with the target.

In connection with those two periods of motion, ballistics are divided into two basic sections: interior ballistics, and exterior ballistics. This is done on the basis of the characteristics of the phenomena and processes studied.

Exterior ballistics are a study of the flight of a projectile from the moment of its departure from the bore, or from the end of the period of after-effect, when it has its highest velocity, to the moment of impact with the target. By determining the principle of air resistance to the motion of the projectile, exterior ballistics permit determination of the angle to the horizon, and velocity, with which a projectile of a given caliber, weight and form should be fired in order for it to strike a target at a given distance, at a given angle of fall and with a given velocity, or to pass through a given point of space (firing at aerial targets).

Interior ballistics are a study of phenomena and processes taking place during the discharge, and, particularly, the motion of the projectile in the bore, the characteristics of its acceleration, and the principles of the powder gases' pressure growth (see figs. 1 and 2). The discharge itself represents a process of the very rapid conversion of the initial chemical energy of the powder into thermal, and then into a kinetic energy of motion of the projectile - charge - barrel - gun carriage system.

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uniformity of discharge, interior ballistics permits the solution of the

following fundamental practical problems.

Fig. 1 - Gas Pressure and Projectile Velocity Curves as a Function of Time
1) period of after-effect

Fig. 2 - [illegible]

The [illegible] fundamental [illegible] ballistics [illegible] determining [illegible] method of measuring [illegible] powder gases pressure and [illegible]

projectile velocity in a given weapon, for given charge conditions and in particular to determine the maximum pressure p_m and the muzzle velocity v_0 of the projectile.

The second fundamental problem of interior ballistics is the problem of the ballistic design of weapons. This problem consists of a determination of the design factors of the barrel of a gun and of the loading conditions (weight of charge, volume and form of powder) which will give a projectile of a given caliber and weight with a given muzzle velocity, at a certain maximum gas pressure.

The first problem is incorporated in this more general task, as the final step.

In addition to these fundamental problems, interior ballistics serves to solve a considerable number of related problems of an experimental and theoretical nature, which permit more accurate definition of our concept of the discharge phenomenon.

The scope of interior ballistics covers the investigation and analysis of conditions and factors controlling the characteristics of gas pressure and projectile velocity variations in the bore, the determination of general rules controlling the discharge phenomenon and processes occurring in this phenomenon, the treatment of [illegible] theoretical and experimental problems arising in the investigation process, the treatment of special equipment for investigating phenomena occurring during a discharge and the methods of utilizing such equipment; and the exploration of ways leading to the further development of interior ballistics.

With respect to its scope, interior ballistics offers an

unusually wide and variable field. In the process of investigating its numerous interdependent processes and phenomena, one will have to deal with a large number of parameters, variable values and characteristics of weapons, projectile and charge.

Therefore, in a determination of relations between various values characterizing a discharge, as well as in solving problems of interior ballistics, it is necessary to approach the phenomenon initially through its basic characteristics, to simplify it and give a schematic solution for some not quite precise assumptions; then to proceed to a clarification of the effects of secondary factors; and having found these, to include them into the elementary schematic functions, thus expanding the latter and making them more complex. Of course, this type of complex arrangement of the various processes, when expressed mathematically, results at times in quite complex functions representing relations between the basic values.

The following basic processes are distinguishable in the discharge phenomenon:

1) The process of powder combustion and the production of high-temperature gases, strongly compressed and containing a large reserve of energy. The rate of powder combustion, or the rate of its explosive conversion, depends basically on the pressure and temperature of the gases, and on the temperature and characteristics of the powder.

2) The process of the conversion of the thermal energy, contained in the heated and strongly compressed gases, into kinetic energy of motion of the projectile - charge - barrel system.

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3) The processes of projectile motion, barrel recoil, and motion of gases of the charge, all overcoming a number of different resistances.

All these processes are interrelated, take place simultaneously and exert complementary effects.

For the purpose of studying the first series of processes, it is necessary to be familiar with the principles of physics, physical chemistry, thermo-chemistry and the theory of explosive substances, because powder is a propulsive explosive substance. General physical principles for gases are also applicable to powder gases, while the rules of chemical kinetics apply to the combustion of powder.

For the purpose of investigation and calculation of the energy conversion process on the basis of thermodynamics, a balance of energy in a discharge is compiled, with a calculation of heat accumulation and of its expenditure for the performance of various external functions and the heating up of the gun bore wall. In this connection, the first principle of thermodynamics is utilized.

Principles of theoretical and applied mechanics and of gas dynamics are applicable, and are utilized for an investigation of the projectile, gas and barrel motion, and for the calculation of resistance forces.

All these processes are expressed by definite mathematical functions and formulas, which permit interrelating the elements of the discharge and conditions of charging, and which yield solutions to a whole series of problems arising in an analysis of the phenomena of a discharge.

It is clear from the above statements that interior ballistics utilizes the following branches of science in the molding of its fundamentals: physics, physical chemistry, theory of explosives, thermodynamics, theoretical and applied mechanics, and mathematics.

Because the discharge of weapons is the object of its investigations, interior ballistics shows their specilized technical artillery character, in conformance to their tasks, on the basis of a complex application of all those general technical branches of science.

An artillery ballistic specialist should detect conditions which permit the most advantageous exploitation of the weapon and its charge, and the best possible perfection of discharge control. He can exert influence on the type, volume and form of the powder, the design and weight of the projectile, the design of the weapon and the relation between chamber volume and bore. Combining all these factors, he should attempt to modify the results of the discharge process to conform to practical requirements.

BREAKDOWN OF INTERIOR BALLISTICS INTO BRANCHES

Interior ballistics investigates the most complex artillery phenomena, the discharge, and teaches how to control it. That is, how to calculate the design of the bore, and to regulate the efflux of gases, in a combustion of powder, in a manner insuring the attainment of a given initial velocity of the projectile at a given value of maximum gas pressure.

The experimental investigation of the phenomena of a discharge and the combustion of powder considers the simultaneous effects of

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the following factors, distinguishing the phenomena of discharge from the common physico-chemical processes:

1) Higher value of pressure (2000-3000 atm. and more).

2) High temperature of powder gases (2500°-3500°C).

3) Short duration of the phenomenon (0.001-0.060 sec).

4) Combustion of the powder in a varying space, with the performance of various types of functions by the gases.

Powder plays a decisive part in the phenomena of discharge. Therefore special consideration should be given to the investigation of the principle of gas formation in a combustion of powder in the bore at the time of discharge.

The principles of gas formation are first studied under simpler conditions, in an invariable space, by igniting charges of powder - in special, so-called manometric bombs. The latter permit bringing the pressure up to 3000 atm. and more. The increase of pressure in this type of bomb during the ignition of a given charge of powder is registered by means of special devices.

Because the volume of a manometric bomb, in which the combustion of powder occurs, remains constant and the gases do not perform any work, it is easier to investigate the principles of gas formation. Knowing the principles of powder gas formation in a constant space, it is possible to calculate the changes for a variable space where gases propelling the projectile perform work and cool off.

In connection with this method of investigation, interior ballistics is usually divided into two basic branches: pyrodynamics and pyrostatics.

Pyrostatics investigates principles of powder combustion, gas formation and pressure development in simpler cases, in a constant space, as for instance with an immovable projectile (statics). Having determined those rules, we utilize them to control the process of powder combustion during a discharge from a weapon.

Pyrodynamics, using pyrostatic data on the principles of gas formation, investigates the phenomenon of discharge in all its complexity, where a conversion of energy occurs together with the combustion of the powder and the inception of the motion of the projectile (dynamics). At the same time, the gases perform a series of mechanical functions and cool off.

Gas dynamics investigates phenomena connected with the motion and escape of gases, such as the escape of gases from the bore during the period of after-effects, their escape through openings in muzzle brakes, through the clearance in mortars, through nozzles of reactive projectiles, etc.

The theoretical assumptions of pyrostatics and pyrodynamics are based on and verified by experiments conducted in special laboratories, as well as on firing ranges, by firing conventional or specially adapted weapons.

Ballistic equipment for the investigation of phenomena occurring in a discharge is very extensive and varied. Its design, principles of functioning and methods of utilization are contained in a special course, entitled "Experimental Ballistics."

THE IMPORTANCE OF INTERIOR BALLISTICS IN THE DESIGN OF ARTILLERY

The theoretical and experimental investigation of the phenomena

of discharge facilitates the solution of problems of the ballistic design of weapons. In a ballistic design, the design factors of the bore, the weight of the charge and quantity of powder are determined for a given caliber and weight of the projectile and for its muzzle velocity. Subsequently, the principles of gas pressure variation inside of the bore and the principle of projectile acceleration in its motion along the bore are calculated. In addition, calculations are made on the gas pressure variations and velocity of projectile during the period of the after-effects of gases on the projectile and the gun carriage.

The results of the calculations are represented in the form of curves p, ℓ and v, ℓ being a function of the trajectory, the curves ρ, t and v, t as well as V, t (where V is the recoil speed) as a function of time (figs. 1 and 2).

These data, obtained by a solution of interior ballistics problems for a selected variant of the ballistic design of the weapon, are elementary, and basic for the subsequent calculations of the barrel, carriage, projectile, charge, fuse and shell case.

On the basis of these data, obtained in a solution of interior ballistics problems, the gun designer calculates the barrel (thickness of wall, weight of barrel, design of breech block assembly, location of center of gravity). He calculates the form, depth and width of lands and grooves in the bore, and treats the design of the counter-recoil facilities, as well as the gun mount in general. The ammunition designer calculates the body of the projectile and its drive collar for strength, calculates the charge of explosive substance,

the shell case and the primer cup; he designs the fuze mechanism and time fuzes. The technologist at a powder factory calculates and designs pressing dies and determines the technological process of powder preparation on the basis of a given form and grain size of powder.

In this manner, a series of branches of artillery sciences are used for the design and building of new weapons and ammunition for them. These sciences are interior and exterior ballistics, strength of weapons, the theory of gun mounts, the theory of fuze and projectile design, the technology of powder and explosive substances, and metal working. In this connection, interior ballistics provide the principal and fundamental information.

The design of a rather complex aggregate, such as the modern artillery weapon with its attached fire control devices, and of its ammunition, is a product of the results of prolonged calculations Each of the component parts of that aggregate requires for its manufacture a complex and prolonged technological process.

THE HISTORY OF THE DEVELOPMENT OF INTERIOR BALLISTICS

The history of the development of interior ballistics is inseparably connected with the general development of artillery.

The origin of firearms and the history of the development of artillery up to 1860 is presented in the well known article of Engels entitled "Artillery" [1]. Without reference to the earlier stages of this development and the conversion from a home industry to the factory methods of weapons and ammunition production, we quote an extract from that article on the development of ballistics

as a science:

"The end of the 17th and the beginning of the 18th centuries comprised the period when artillery was finally incorporated into the military organizations of a majority of countries, the elimination of its medieval guild character, and its recognition as a special military branch, promoted its adaptation to normal and rapid development. This resulted in an almost instantaneous and quite appreciable progress. The diversity and irregularity of calibers and types became apparent, along with the unreliability of all existing empirical rules and the complete lack of precisely determined principles. It became impossible to endure these conditions any longer. Therefore large-scale tests were conducted everywhere, in order to clarify the problems of caliber, the relation of caliber to charge, as well as of the length and weight of the gun, the distribution of metal in the gun, the range of fire, the effects of recoil on the gun carriage, etc.

The result of this was a significant simplification of calibers, better distribution of metal in the gun, and a very considerable reduction of the charge, which then amounted to from one-third to one-half of the weight of the projectile."

In Russia, Peter I exhibited a great deal of interest in the development of artillery. He personally wrote the "Guide to the Utilization of Artillery." During the reign of Peter I, the Russian artillery became one of the best in Europe.

The progress of artillery science, mainly in the field of investigation of projectile flight and air resistance (Galileo,

Bernouilli, Euler and others), ran parallel to the organizational and tactical improvements of artillery.

In his classical study entitled "Hydrodynamica," Daniel Bernouilli gave the basic knowledge about gases, introduced to science the conception of an expansion of gases in consequence of their buoyancy, and showed how on the basis of this expansion to calculate the motion of a projectile in the bore of gun.

The famous mathematician Euler, member of the Russian Academy of Sciences, gave considerable attention in his studies to the investigation of processes occurring in the bore of a weapon. However, as a result of the lack of means for experimental investigation at that time, his studies were limited to the setting up of problems.

In the middle of the 18th century, Robins submitted the first instrument for determinations of projectile velocity. Called the "ballistic pendulum," it was used up to the 1860's. In Robins' study, entitled "New Principles of Artillery Science" and written in 1742, ballistics were first divided into exterior and interior ballistics. In this connection, the scope of interior ballistics was defined as follows: "Knowing the length and caliber of the gun, the weight of the cannon ball, the powder charge and the elastic force at the first moment of ignition, to determine the velocity with which the projectile will depart from the gun."

The technical reorganization of the artillery proceeded parallel to the theoretical and experimental investigations. This embraced the reduction of the number of calibers, improvement in charging and mechanisms, and increasing combat qualities.

The period 1750-1760 witnessed a great step in the development of the Russian artillery. At that time, a number of new artillery types ("unicorns") were introduced under the leadership of Count P.I. Shuvalov. Also, the loading of the guns was modified by the introduction of powder bags for the charges; and new organization of the artillery was effected. Shuvalov's "unicorns" exhibited superior combat properties not only during the Seven Year War (1756-1763, when the Russian armies occupied Berlin), but also during the Homeland war of 1812, particularly in battle of Borodino. These artillery types lasted for nearly one hundred years, up to the introduction of rifled guns. The basic personalities active in the reorganization of artillery in other countries (Friedrich II of Prussia, Gribeval in France) to all intents followed in Shuvalov's footsteps.

Fundamental theoretical and experimental ballistic investigations, which produced proper assumptions about the phenomenon of discharge and its uniformity, were conducted beginning with the second half of the 19th century, on the basis of the general development of technicology and a series of related branches of science.

The first theory of powder combustion, published abroad in 1857, was written by the Russian chemist Shipkov and the German chemist Bunsen. In 1860, Captain A.P. Gorlov wrote an article on the motion of the projectile in the bore of a rifled gun. An abstract of this article was contained in the reports of the Paris Academy of Sciences in 1862. In 1868, Colonel N.P. Fedorov determined the effect of powder combustion conditions on the composition of the

products, by firing a pistol and a four-pound cannon. These studies laid the foundation for the development of proper hypotheses on the combustion of powder in a discharge, and were used in much later studies by numerous researchers.

Significant progress of experimental ballistics, expressed in the appearance of two basic instruments which are still widely used in our times (the chronograph of Le Boulanger for measurements of projectile velocity and Nobel's crusher gage for measurements of powder gas pressure), occurred in the 1860's.

The crusher gage, which permits estimating gas pressure on the basis of the compression of a copper column, laid the foundation for the development of a special branch of experimental ballistics, "manometry," and promoted the production of manometric bombs. The latter facilitated investigations of the principles of powder combustion at high pressures.

From 1868 to 1875, Nobel and Abel conducted experiments on the ignition of black powder in a manometric bomb. They determined the quantitative and qualitative composition of combustion products, their thermal capacity, the amounts of emissible heat, combustion temperature, and also the dependence of the maximum pressure on the power of the powder and uniformity of charging.

These investigations were based on studies of the properties and characteristics of powder combustion products in a discharge, made earlier by Shipkov, Bunsen and Fedorov.

During the second half of the 19th century, the general principles of a knowledge of heat and of the kinetic theory of

gases, as well as the fundamentals of chemistry and other branches of science, developed parallel to the above investigations. They permitted obtaining a scientific theoretical basis for the process of powder combustion, and for the conversion of the energy of powder gases into the kinetic energy of the projectile and of the gases.

The differential equation was introduced in 1864. On the basis of the first law of thermodynamics, this equation served to determine the equilibrium of the energy emitted during the combustion of the powder charge and the energy expended for the performance of numerous functions (Resal).

The French scientist Cappo used this equation in 1876 for solutions of problems occurring in artillery practice. Cappo's formulas were the basic functional ballistic formulas, in almost all countries. In our country they were replaced by the formulas of A.F. Brink, and later by the formulas evolved by Professor N.F. Drozdov from 1903 through 1910. (See details farther on).

A great contribution to the development of not only interior ballistics, but also of artillery in general, was made by the invention of smokeless powder: the pyroxylin powder of Vel (French scientist, chemist, powder specialist, 1884), the nitroglycerin powder of Nobel and Abel in England (1888-1889), and the pyrocolloid powder of D.I. Mendeleyev in this country (1890).

After improving the manometric bomb and equipping it with registration of the pressure increase as a function of time, Vel determined the difference between the practically instantaneous

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combustion of black powders and the uniform gradual combustion of smokeless colloidal powders. This permitted regulation of the gas supply by varying the dimensions of powder elements.

A new type of powder was prepared as a result of numerous theoretical and laboratory studies. It was smokeless, colloidal on a pyroxylin basis, and was obtained by an ethyl alcohol compound treatment of pyroxylin explosive substance. Vel projected and prepared a strip-type, pyroxylin powder for a 65 mm gun. By firing, he obtained results fully verifying calculated data. The new powder proved to be almost three times as powerful as black powder, and produced a significant increase of projectile velocity, with a lower pressure of powder gases in the bore.

Aside from the elimination of smoke on battlefields and a considerable increase of the range of fire, the introduction of smokeless powders also caused a modification of battle tactics.

In Russia, a specimen of the French pyroxylin powder was obtained. Experiments toward its production began in 1887 at the Okhtensk gunpowder factory; while firing tests with it were conducted by the research committee of the same factory.

The famous Russian chemist D.I. Mendeleev in the 1890's developed a special pyroxylin powder, which offered numerous advantages in a comparison with Vel's powder. However, the Artillery Committee rejected the powder of D.I. Mendeleev for armament purposes, under the influence of the at that time customary neglect of the prominent personalities of Russian science, and the worship of all that was foreign. The value of this powder was

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properly recognized in the U.S.A. where it was adapted for armament purposes. During the period of the first world war, 1914-1918, the Russian army obtained considerable quantities of the pyrocolloid powder from the U.S.A.

Disregarding the fact that the technology and industry of Tsarist Russia were at a lower level than the foreign standards, our ballistics scientistics frequently surpassed foreign researchers from the theoretical point of view, and played a leading part in the treatment of numerous problems. Many of their studies were immediately sent abroad and utilized. We have already mentioned the outstanding studies of Shipkov, Gorlov and Fedorov from 1857 to 1868.

The first course of interior ballistics in Russia was written by Colonel P.M. Albitski in 1870, and read at the Artillery Academy.

In 1879, Colonel Kalakutskii, a pupil of the Artillery Academy, published a study on tests conducted to determine conditions of the development of abnormal pressures in firearm bores, which long before Vel, touched on the problem of the propagation of undulatory gas motion. His studies were transferred the following year to France.

Colonel V.A. Pashkevich, a very skilled and talented artillery man, became successor to Albitskii. From 1885 to 1891, he wrote a course in interior ballistics: Part 1, theoretical; Part 2, experimental. In 1893, these books were translated into English in the U.S.A. His instructions on experimental ballistics were used at the Artillery Academy, for many years afterwards.

From among the most distinguished scientists active during the second half of the 19th century and at the beginning of the 20th

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century in the development of theoretical and experimental ballistics in Russia, it is proper to mention the founder of world-wide interior ballistics, Professor of the Artillery Academy N.V. Manevskii (born in 1823, active from 1850 to 1892,) and his pupil and follower N.A. Zabudskii (born in 1853, active from 1880 to 1917).

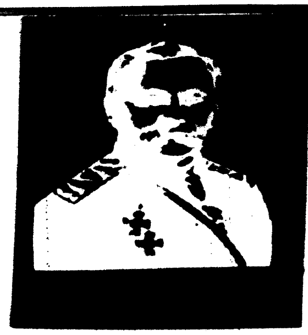
Although these two scientists acquired their fame through studies in the field of exterior ballistics, they have made great contributions to the development of interior ballistics.

For example, prior to the design of a 60-pounder smooth-bore gun, and before the investigations of Nobel, Manevskii submitted in 1856 an original method of determining the powder gas pressure at various cross sections of the bore of artillery weapons.

The gun calculated by Manevskii was built, and, when tested, showed considerably better results than the guns competing with it and built from the designs of other people, including English scientists.

In 1867, N.V. Manevskii organized special tests for the experimental determination of the projectile travel in the bore of a four-pounder gun as a function of time. From this information, curves of powder gas pressure in relation to projectile travel and time were plotted by means of calculations.

This study was of great importance to the development of interior ballistics and the design of guns.



N.V. Manevskii

In 1878, N.V. Manevskii was elected corresponding member of the Academy of Sciences, ("People of Russian Science," published by the Academy of Science of the USSR, 1944, volume 11).

Professor of the Artillery Academy N.A. Zabudskii, a pupil and successor of N.V. Manevskii, was greatly and successfully active in the theoretical direction, as well as in the field of the development of artillery technology.

In 1911, the French Academy of Sciences elected N.A. Zabudskii to corresponding membership of its department of mechanics, in recognition of his scientific accomplishments in ballistics. In the field of interior ballistics, N.A. Zabudskii completed in 1904 a study on investigations of pressure in the bore of several guns, and gave numerous empirical formulas for muzzle velocity and maximum pressure.

Later, in 1914, he published his main study on the experimental determination of pressure and velocity curves as a function of projectile travel in the bore of 300 mm field gun, applying for the first time to this purpose the original method of progressive shortening of the gun barrel.

On the basis of these investigations, he gave empirical formulas

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for the dependence of muzzle velocity of the projectile and of the maximum pressure of powder gases on the variation of numerous conditions of charging (weight of charge, weight of projectile, volume of chamber, size of powder).

This fundamental study of N.A. Zabudskii had a major share in the evolution of proper assumptions on the dependencies in the bore of a weapon during a discharge, and is used in part up to the present time.

In 1892, Colonel A.F. Brink began to lecture in the course of interior ballistics at the Artillery Academy. In 1901, he wrote a complete course of interior ballistics. For this purpose, he used the formulas of Cappo, and new expressions for coefficients and exponents for pyroxylin powders. These appeared for the first time in literature. In addition, he submitted his own empirical formula for the pressure curve.

N.A. Zabudskii

At the time, this course was the most complete among all systems of instruction known abroad, as then noted by Professor N.F. Drozdov.

This course of instruction was transferred to Germany and the U.S.A.

In 1903, Colonel N.F. Drozdov, lecturer at the Artillery Academy, submitted an article, contained in the Artillery Journal. It was the first mathematically precise solution in world literature of a basic problem of interior ballistics. It had none of the simplifications used prior to that time. In 1910, this study was considerably enlarged, and published in a separate edition. It was offered by him as a dissertation toward the attainment of a degree.

The tables compiled by him in 1920, on the basis of his solution, contributed greatly to the simplification and speeding up of ballistic calculations, and to the solution of a series of varied problems relating to the ballistic design of guns. Also, they served as a prototype for a whole series of detailed tables compiled later on. (Science and Research, Institute of Artillery, GAU).

The first dissertation study on interior ballistics in Russia was written and defended in 1904 by Captain of the Guard I.P. Grave, instructor at the Artillery Academy. This study dealt with the experimental and theoretical investigation of the principle of powder combustion rate and pressure development in the combustion of powder in a constant space. The study was of considerable scientific interest, and was translated in France in a somewhat abbreviated form.

In 1908 appeared the study by Charbonne (France) entitled

"Interior Ballistics." He proposed the division of interior ballistics into the fundamental branches: pyrostatics and pyrodynamics. He defined more precisely the conceptions of pressure propulsion, and the calculation of secondary functions in a discharge. While criticizing the geometrical principle of combustion evolved by Vel, he presented his own method of determining the principle of powder combustion on the basis of tests in a manometric bomb. This principle was also used by him in the solution of the basic problem of pyrodynamics.

The French ballistics specialist Siugo, who wrote a course of instruction for interior ballistics in 1926, should be counted among the circle of followers of Charbonne who developed his solution and rendered it more precise.

Parallel to the theoretical school of thought of Charbonne and Siugo, there existed in France an empirical approach by Gosse and Liubill who submitted solutions to the problem of interior ballistics on the basis of utilizing the results of a very large number of discharges.(1922).

The studies of the German ballistics specialist Kranz can be mentioned among the experimental activities of the beginning of the 20th century. He published a "Course of Ballistics" in three volumes (exterior, interior and experimental. 1920 through [illegible]). Kranz organized a special ballistic laboratory at the Technical Military Academy in Berlin, and invented a number of new instruments for the investigation of discharge phenomena,

From among Italian authors, mention should be made of [illegible],

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who gave the solution to a problem of interior ballistics (1917). With certain modifications introduced by Professor I.P. Grave, this solution was accepted and taught at the Academy of Artillery for many years afterwards.

Having given due recognition to the studies of several foreign scientists, it would be improper to pass up in silence the disregard of the works of our national scientists by the foreign literature on ballistics. For example, one of the recent theoretical courses of interior ballistics of the French ballistics specialist Vinter (1939) discusses widely and in detail the French school of interior ballistics as fundamental in character. It discusses the accomplishments of the Italian school of interior ballistics as a branch of the French school of ballistics, and refers to the names of English, German and American scientists. However, it entirely fails to mention representatives of our national Soviet school of interior ballistics. Also, nothing is said of the studies of the distinguished scientist, Professor N.F. Drozdov, who had between 1903 and 1910 already given the first, mathematically precise solution in world literature, of the fundamental problem of interior ballistics. Meanwhile, it is well known that Professor N.F. Drozdov is one of the founders of the Soviet school of ballistics, which contributed quite a few valuable and pioneer studies to many fields of interior ballistics. This disregard of Soviet scientists undoubtedly has a political character, in spite of the statements of burgeois scientists as to the non-political nature of science. Even during the pre-revolution period, Russian

artillery scientists, using development tests of national artillery and artillery technology, as well as the ideas of leading foreign artillery scientists, successfully treated and independently solved the main theoretical and practical problems of artillery science in general and of ballistics in part. The research of N.V. Manevskii, N.A. Zabudskii, V.M. Trofimov, N.F. Drozdov, I.P. Grave and others are outstanding contributions to the science of artillery, and have retained their value even to the present time. The major part of these investigations were printed in the older and widely known "Artillery Journal," which appeared for the first time in 1808. Artillery and interior ballistics underwent a still greater development after the Great October Revolution.

A new era in the development of artillery and artillery science in the new Soviet state began with the victory in October of 1917. The Bolshevik Party and the Soviet Government headed by Lenin and Stalin have enthusiastically supported artillery scientists from the very beginning of the existence of the Red Army, and have revised their task and goals to the present mission of universal strengthening of the armed forces of the Soviet Union for the purpose of preserving it from capitalist encirclement.

In response to the appeal to the Party and the government, to selflessly prevent the conquest of the Soviet Republic, patriotic artillery scientists urgently undertook the task of organization and improvement of Soviet artillery, using the best traditions of Russian artillery science for the purpose of fullfilling this mission.

There were many difficulties at the beginning of this course:

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collapse of the economy, a poor material and technical foundation for the development of artillery science and technology. However these difficulties were surmounted, and artillery science continued to work on the further development and the increase of the power of the artillery of the Red Army.

In this connection, a significant part was played by the activities of the Commission for Special Artillery Research, 1919 through 1926, under the leadership of the famous Russian artillery scientist V.M. Trofimov.

V.M. Trofimov

V.M. Trofimov was born in 1864. He graduated with top honors in 1892 from the Academy of Artillery and gave 25 years of his life to scientific and practical study at the Main Artillery Range. He was director of this range from 1910 to 1917, and has done much for the development and improvement of its equipment and organization, particularly during the time of the war from 1914 to 1918.

During his stay at the range, V.M. Trofimov conducted a large number of scientific investigations. Many of his studies were

incorporated in firing manuals, while some were translated into foreign languages.

For his study entitled "The Effects of Shrapnel from the 300 mm Gun" (1903), V.M. Trofimov was awarded the Rasskazov and the greater Mikhailov medals.

The period of particularly intensive activities of V.M. Trofimov is connected with the studies of the Commission for Special Artillery Research (KOSARTOP), which was formed through his initiative in 1919 for the purpose of working on problems connected with firing over very long distances.

After obtaining data on the shelling of Paris by the Germans over a distance of about 120 km, V.M. Trofimov undertook to obtain equal results in Russia, and began his efforts in 1918, in disregard of the difficult materials conditions.

He recruited the aid of several young employees of the range, and conducted a number of preliminary investigations in order to find ways of attaining the designated goal.

Using recent material, V.M. Trofimov submitted his principle of air density variation with altitude. Because firing over very long ranges involves variations of air density over a very wide range, Trofimov utilized, for the first time in ballistics, the method of numerical integration of the differential equations of exterior ballistics to calculate the trajectory (similarly to the system of Euler: breakdown into small portions and solution of each portion). For this purpose, he used the study of Academician A.N. Krylov, entitled "On an Approximate Numerical Solution of Common Differential Equations" (1918).

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Af[illegible] ...mber of investigations, he determined
condit[illegible] ... type of firing is possible, and
dem[illegible] ... of firing over ranges up to 140 km.
[illegible] ... studies had to be made in order to
obtai[illegible] ... to a problem of this type. It was
namely[illegible] ... the most advantageous design of the
bore [illegible] ..., to make the projectile streamlined
and [illegible] ... a fast and uniformly burning gun
powd[illegible] ... of the bore, etc. The KOSARTOP (Commission
for [illegible] ...) was formed under the leadership
of V[illegible] ... of dealing with all these problems.
[illegible] Commission for Special Artillery

Bec[illegible] ... period of blockade and full isolation
fr[illegible] ... Nevertheless, the work of this commission
be[illegible] ... year of death of V.M. Trofimov) progressed
[illegible] ... major part in the development of Russian
art[illegible] ... a certain period of time, the Commission
for[illegible] ...search became the central point of the
pr[illegible]

[illegible] ...vities and studies of the Commission for
[illegible] ..., from 1919 to 1926, is a period of
[illegible] ...tivity which cannot be compared with any
[illegible] ... the years preceding the war of 1914 to
19[illegible] ... of scientific research studies conducted
at[illegible] ... practically all important and pressing
ar[illegible] ... theoretical character, were of decisive
i[illegible] ..., News of the Artillery Academy, volume

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In addition to problems directly connected with super-long range firing, the Commission for Special Artillery Research processed the technical and tactical requirements for new artillery types, as well as for self-propelled artillery, mortars, problems of gas dynamics, etc. These activities provided the foundation for the modernization of the artillery of the Red Army, which was accomplished several years later.

V.M. Trofimov recruited not only all leading artillery scientists of the Artillery Committee (*) for work with the Commission, but also all professors from the Artillery Academy (**) and a number of leading civilian scientists (***) active in fields related to artillery.

At the same time, V.M. Trofimov used the freshman workers in science, who, under the guidance of leading scientists, attended the School of Science at the Commission for Special Artillery Research. Subsequently, many of them became leading specialists

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(*) Professor G.A. Zabudskii (gun powder); V.I. Rdultovskii (projectiles, fuzes); M.F. Rozenberg and A.G. Matiunin (technology, manufacture of guns); G.P. Kisnemskii (gunpowder); V.V. Makeladze (firing and tactics); V.A. Pashkevich (ballistics, mathematics); P.A. Durliakhov (gun carriages) and others.

(**) Director of the Academy, Professor S.G. Petrovich (mechanics and exterior ballistics); Professors: N.F. Drozdov (strength of guns, interior ballistics); I.P. Grave (interior ballistics); V.V. Mechnikov (exterior ballistics); A.V. Sapozhnikov (chemistry, gunpowder and explosives); I.A. Krylov (metallurgy); lecturers F.F. Lender (gun carriages); O.G. Fillippov (gunpowder, interior ballistics) and others.

(***) Academician A.N. Krylov (mathematics, mechanics); Professor N.E. Zhukovskii (mechanics, aerodynamics); S.A. Chaplygin (hydromechanics); N.N. Bakhgolts and V.P. Vetchinkin (gas dynamics); N.P. Molchanov (meteorology) and others.

in various fields of artillery activities (D.A. Venttsel, B.N. Okunev, V.E. Slukhotskii, M.E. Serebriakov).

In addition to the personal recruiting of leading scientists and specialists, the Commission for Special Artillery Research maintained close relations with a number of national scientific and technical institutions.

About 150 monographs relating to scientific and research studies conducted by the personnel of the Commission for Special Artillery Research, as well as about 80 design treatments, were published during the period of its activities.

The studies of the Commission for Special Artillery Research were of great importance for the assembling of a number of outstanding research people around problems of artillery materiel. After the death of V.M. Trofimov, the direction of the Commission was transferred to the hands of the outstanding specialist in the field of super-long range firing, Professor E.A. Berkalov, who has also obtained a great deal of experience in this field.

Due to the attention given by the party, the government, and particularly by Comrade Stalin to this problem, the efforts to improve the artillery of the Red Army, and to scientifically insure its development, continued to expand during the following period of time.

During the years of Stalin's Five-Year Plans, our country created the material and technical foundation necessary to provide the armed forces with modern combat materiel and, among those, artillery. Material conditions necessary to a fruitful development

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of scientific and technical artillery thought were created, parallel with the development of our industry and economy.

A number of designing offices were established under the direction of Heroes of Socialist Labor V.G. Grabin, I.I. Ivanov, F.F. Petrov and other designers, which produced many samples of artillery types. These showed superior combat and technical qualities during the period of the Great Homeland War. All these activities were personally supervised by Comrade Stalin, who examined test specimens of new artillery types and issued instructions on the course of their further development.

Comrade Stalin's anxiety in connection with the development of scientific artillery thought found its expression in the establishment of the Academy of Artillery Sciences in 1946, for the purpose of processing basic scientific problems confronting the artillery.

A new generation of Soviet artillery scientists and ballistics specialists grew up during the 30 years of existence of the Soviet government. The first to be counted among those are: Academician A.A. Blagonravov, president of the Academy of Artillery Sciences; M.F. Vasiliev, member of the governing body of the Academy of Artillery Sciences; K.K. Snitko, A.A. Tolochkov, D.A. Venttsel, M.E. Serebriakov, V.E. Slukhotzkii, Ia. M. Shapiro, active members of the Academy of Artillery Sciences; and Professors B.N. Okunev, G.V. Oppokov. These are followed by the younger generation of science workers: M.S. Gorokhov, M.A. Mamontov and others.

It can be definitely stated that, in our Union, we have established a leading Soviet scientific school of artillery designers

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and ballistic specialists, who are making their way in the field of artillery science and technology.

Like V.M. Trofimov, two leading scientists of our country, who became the instructors and educators of numerous generations of Russian ballistic specialists and designers, had a major share in the formation of this scientific school. These were the distinguished worker in science and technology of RSFSR (Russian Soviet Federative Socialist Republic), Colonel General of Artillery Nikolai Fedorovich Drozdov, laureate of Stalin's medal, member of the Presidium of the Academy of Artillery Sciences; and Major General of Artillery Engineering Services Ivan Platonovich Grave, laureate of Stalin's medal and active member of the Academy of Artillery Sciences.

As already stated, N.F. Drozdov submitted the first mathematically precise solution in world literature of the fundamental problem of interior ballistics, without any of the approximations utilized by foreign authors.

Since 1911, he has lectured for many years at the Artillery Academy in the course of weappn design, and from 1920 in the course of interior ballistics of the Naval Academy.

Professor N.F. Drozdov has written a number of studies toward the expansion of his method, and has compiled special tables for solution of problems in interior ballistics.

These tables were of great importance for the efforts on acceleration of ballistic design and improvement of our artillery systems, since the calculations of the latter were considerably simplified by the existence of these tables.

During very recent times (1947-1948), N.F. Drozdov has written two more studies. One pertains to the properties of highest power artillery weapons. The other presents solutions of the interior ballistics problem in relative variables for simple and combination charges, with appended tables which considerably expedite computations.

Professor N.F. Drozdov is the founder of the Russian School for the Ballistic Design of Guns, which created a number of outstanding artillery types.

Professor I.P. Grave lectured for many years (from 1911 till 1934) on interior ballistics at the Artillery Academy, and wrote the most complete course of theoretical interior ballistics in world literature. With respect to its variety of included material and the completeness of exposition, this study may be justly named an encyclopedia of theoretical interior ballistics. This course is composed of four volumes of pyro-dynamics (1932-to 1937) and pyrostatics (1938). The course contains extensive material, and presents a criticism of Russian and foreign articles and studies. All those are analyzed, and cite reference literature. For the first time in our literature, problems of gas dynamics and ballistics of an incompletely enclosed space are submitted to particular consideration in this study.

Aside from this, I.P. Grave had contributed largely to the development of an experimental base, at the Artillery Academy by organizing a ballistics laboratory in 1926.

After 1938 and during the period of the Great Homeland War, Professor I.P. Grave held the chair of interior ballistics at the Artillery Academy, conducted a series of investigations, and wrote several studies on current problems of interior ballistics.

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The Soviet School of Ballistics, whose cadre grows constantly, successfully solves all of the more complicated practical problems that occur, and lays out new paths of development for interior ballistics.

In the past, as well as particularly during the period of Soviet development, our national science of ballistics kept ahead of foreign thought with respect to many more important problems.

The following facts may be quoted by way of examples:

A mathematically precise solution of the fundamental equation of interior ballistics was offered, for the first time in world literature, by Professor N.F. Drozdov in 1903 in our country.

The problem of solving a series of questions relating to combination charges is completely untouched in the world literature. In our country, these problems were solved by N.F. Drozdov, I.P. Grave, V.E. Slukhotzkii and others, who also presented tables for the solution of problems for the case of combined charges.

An analytic solution of problems for mortars, with calculation of partial escape of gases through the clearance, was submitted in this country in 1940 by M.E. Serebriakov, K.K. Greten, and in more detail by G.V. Oppokov.

Ballistic design and its methodology has been treated most completely, thoroughly and rationally, as a result of the studies of our scientists.

The problem of highest power weapons, or weapons with smallest volume, is solved in this country differently than in French literature. A solution is given which is more economical and advantageous from

the standpoint of design.

A new method of ballistic analysis of gunpowder, which permits the test determination of the actual principle of powder combustion and recognition of the influence of a whole series of factors previously disregarded (physical principle of combustion), was developed by M.E. Serebriakov in this country between 1923 and 1937.

Also, the solution of ballistic problems through the methods of numerical integration was developed very thoroughly in this country. This method was used for the first time in ballistics by V.M. Trofimov in 1918, in exploitation of the studies of Academician A.N. Krylov.

The methods of numerical solution of problems were developed and expanded in particular details by Professor G.V. Oppokov in a series of his studies.

This incomplete account of the accomplishments of our scientists already permits recognition of the fact that interior ballistics has reached a high theoretical level in our country and progresses along the proper paths. In order to fulfill the mission assigned by Comrade Stalin, "to exceed the accomplishments of foreign science within a short period of time," it is necessary, by means of continued and persistent work, to raise still further the scientific level of our investigations.

Artillery technology develops with each passing year; while the problems confronting interior ballistics widen and become more complex. New methods of solution come into existence. Outdated hypotheses are eliminated, and are replaced by new ones. New experimental methods and more precise equipment are introduced, providing research

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scientists with new material and new methods of investigation.

Interior ballistics will progress as a result of the expansion of our knowledge of the discharge and of phenomena accompanying it, the establishment of new rules, the replacement of outdated hypotheses by new ones, the improvement of our ability to direct the discharge along the desired course.

The mission confronting the students of a course in interior ballistics is to become familiar with the modern status of this branch of science and with the theoretical fundamentals of interior ballistics, and to learn to apply them to solution of numerous practical problems arising in the design of various types of artillery and the ammunition for them.

LISTING OF NOMENCLATURE, SYMBOLS AND DEFINITIONS IN THE FIELD OF INTERIOR BALLISTICS

A. BASIC PROPOSITIONS

1) The following listing specifies only the most characteristic values used in interior ballistics as one of the branches of artillery science.

2) Individual terms relating to variables associated with certain characteristic instants are properly designated by adding the following subscripts to the symbol of the variable value:

0 - for the instant of commencement of projectile motion.

m - for the instant of maximum pressure of gases.

s - for the instant of decomposition of the powder grains.

k - for the instant of the end of powder combustion.

d - for the instant of the projectile leaving the bore.

3) As is proper, interior ballistics utilizes the fol
system of units: decimeter-kilogram (force)-second.

4) The term "powder grain" is construed as a separat
of the powder charge (strip, fluted cylinder, rod, cube,

5) The initial dimensions of powder grain are the
the grains prior to the commencement of combustion (expl
conversion).

6) Numbers in parentheses contained in the text
the item number in section B of this listing.

7) Nomenclature standardized as technical terms is
in heavy type.

B. NOMENCLATURE, SYMBOLS AND DEFINITIONS

Item No.	Nomenclature	Symbol	Defini
	I. Characteristics of the Barrel, Projecti		
1	Caliber of barrel (cylindrical)	d	Diameter of bore "lands"
1a	Caliber of barrel (tapered)	d_0 d_d	Entrance caliber Exit caliber
2	Cross section of bore (grooved)	s	Area of cros in the part grooves have (grooves incl
3	Length of bore	L_{kn}	Distance from the bore to th the barrel
4	Length of the rifled portion of the bore	L_{nr}	Distance from of grooves in muzzle face
5	Gun chamber		Initial air chamber port with a proper projectile

Item No.	Nomenclature	Symbol	Definition
6	Volume of gun chamber (5)	W_0	
7	Weight of projectile	q	
8	Weight of gunpowder charge	ω	
	II. Properties of Powder and Powder Gases		
9	Heat of formation	Q_w	The quantity of heat emitted by one kilogram of powder burning in a constant space and when cooling the gases down to the temperature of 18°C (water vapor)
10	Specific volume of powder gases	w_1	Volume occupied by gases of one kilogram of powder at a temperature of 0°C and a pressure of 760 mm of mercury column (water vapor)
11	Temperature of powder at the time of firing	T_1	Temperature of powder combustion (heat of formation) measured from 0°K (absolute scale)
12	Energy of powder charge	f	$f = \frac{p_a w_1 T_1}{273}$ (10;11), where p_a - one physical atmosphere
13	Covolume of powder gases	α	A coefficient representing the effect of the volume of gas molecules on the pressure of gases
14	Rate of burning (a variable value)	u	Linear velocity of propagation of combustion reaction of powder towards the center of the powder grain
15	Rate of burning under pressure equal to unity.	u_1	

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Item No.	Nomenclature	Symbol	Definition
	III. Dimensions of Gunpowder Grain		
16	Depth of the burnt layer of a gunpowder grain (variable value)	e	
17	Initial thickness of powder grain	$2e_1$	
18	Surface of powder grain (variable value)	S	
19	Initial surface of powder grain	S_1	
20	Volume of powder grain (variable value)	Λ	
21	Initial volume of powder grain	Λ_1	
22	Relative thickness of burned layer of powder grain (variable value)	$z(z)$	$z = \frac{e}{e_1}$ or $z = \frac{\ell}{\ell_k}$ (16, 17, 32, 33, 14, 15)
23	Relative surface of powder grain	σ	$\sigma = \frac{S}{S_1}$ (18, 19)
24	Specific volume of burned powder grain (variable value)	ψ	$\psi = \frac{\Lambda_1 - \Lambda}{\Lambda_1}$ (21, 20)
	IV. Travel, Velocities and Pressures		
25	Instant of departure		The instant of passage of base of projectile past muzzle face of barrel.
26	Relative travel of projectile.	ℓ	Displacement of projectile in relation to the bore, measured from the location of projectile base at the commencement of motion.
27	Total travel of projectile in the bore	ℓ_d	Travel of projectile in relation to the bore (26) at the instant of departure (25)
28	Relative speed of the	$v(v)$	Speed of projectile in its

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Item No.	Nomenclature	Symbol	Definition
	projectile (variable value)	$v(v)$	movement relative to the bore (26)
29	Muzzle velocity	v_d (v_d)	Relative velocity of projectile (28) at the instant of departure (25)
30	Pressure of powder gases (variable value)	p	Mean value of partial pressures of powder gases in the initial air space, at the given position of the base of projectile (at a given instant)
31	Mean pressure of powder gases	p_{sr}	$p_{sr} = \frac{\varphi q v^2}{2gs\ell}$ (37, 7, 28, 2, 26), where g - acceleration of gravity
32	Impulse of pressure of powder gases (variable value)	I	$I = \int_0^t pdt$ (24, 30), where t - time
33	Impulse of pressure powder gases at the end of powder combustion	I_k	$I_k = \int_0^{t_k} pdt$ (24,30), where t - time
	V. Special Values and Coefficients		
34	Density of loading	Δ	$\Delta = \frac{\omega}{W_0}$ (8:6), where ω is in kg, and W_0 in dm^3
35	Gravimetric density of powder		Ratio of weight of powder placed freely in a container of a given shape and volume to the weight of water at 4°C (density equal 1) occupying a container of the same volume Remark: The gravimetric density of powder depends on the volume and shape of the

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Item No.	Nomenclature	Symbol	Definition
			container, and on the method of placing the powder in it. These factors are specific in each given case.
36	Reduced length of chamber	ℓ_0	The length of a straight cylinder whose volume is equal to the volume of the gun chamber (6), and whose base area corresponds to the area of the cross section of the bore (2)
36a	Actual length of chamber	ℓ_{km}	Distance from the base of the bore to the base of the projectile
37	Factor determining secondary functions	φ	A coefficient for evaluating the secondary functions of powder gases (rotation of projectile, gun recoil, friction, etc.)
38	Coefficient of weight of projectile	c_q	$c_q = \frac{q}{d^3}$ (7;1), where q in kg and d in dm.
39	Coefficient of charge utilization	η_ω	$\eta_\omega = \frac{qv_0^2}{2g} \frac{1}{\omega}$ (7; 29; 8) where g- acceleration of gravity
40	Coefficient of projectile location at the instant of total combustion of charge	η_k	$\eta_k = \ell_k/\ell_d$ (27) where ℓ_k is travel of projectile (26) at the instant of total combustion of the powder
41	Relative weight of charge	$\frac{\omega}{q}$	
42	Power factor of weapon	C_E	$C_E = \frac{E_d}{d^3} = c_q \frac{v_d^2}{2g}$
43	Efficiency of the charge	r_d	$r_d = \frac{E_d\theta}{f\omega} = \eta_\omega \frac{\theta}{f}$

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Item No.	Nomenclature	Symbol	Definition
44	Charging parameter of Professor N.F. Drozdov	B	$B = \frac{s^2 I_k^2 g}{f \omega \varphi m}$
45	Coefficient of chamber expansion	X	$X = \frac{\ell_0}{\ell_{km}}$ (36; 36a)

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PART ONE
PHYSICAL PRINCIPLES OF INTERIOR BALLISTICS

SECTION ONE
GUNPOWDER AS THE SOURCE OF ENERGY

CHAPTER I - GENERAL INFORMATION ON GUNPOWDERS

I. TYPES OF POWDERS

Modern gunpowders belong to a group of smokeless colloidal powders. Black powders, used at the time of the invention of gunpowder, are now used in artillery only in the capacity of igniters, in primer cups, in rings of time fuses, and also in shrapnel.

Smokeless powders, which appeared almost simultaneously in France (Vel) and England (Nobel) during the 1880's and 1890's, were rapidly adopted in all countries. Their introduction greatly modified all artillery materiel and combat tactics.

The basic properties of smokeless powders are: considerably greater energy, and an ability to burn in parallel layers, which permits regulation of the influx of gases forming during the combustion of powder.

The main base of all smokeless powders is pyroxylin, or nitrated cellulose. In this connection, a division is made, as respects the degree of nitration, into highly nitric pyroxylin or No. 1 (nitrogen content 12.9 to 13.3%); lower nitric pyroxylin or No. 2 (nitrogen content 11.9 to 12.3%), and collodion ($\sim$11%).

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Pyroxylin No. 1 is also called insoluble, because it is practically insoluble in a mixture of alcohol and ethyl ether. Pyroxylin No. 2 is called soluble, because it dissolves almost completely in the same mixture.

In many countries, a mixture of pyroxylins No. 1 and No. 2 is used for the production of gunpowder (for example, in our country and in France).

In the U.S.A., powder is manufactured with a so-called pyrocollodion base. The latter ranks between No. 1 and No. 2 with respect to nitrogen content (12.5 to 12.75%). It is however entirely soluble in an alcohol-ethyl ether mixture.

Developed by D.I. Mendeleev as early as 1890, pyrocollodion gelatinizes very well, and provides a more homogenous powder substance than the powder containing insoluble pyroxylin.

When subjected to the action of an alcohol and ethyl ether mixture of a given ratio, the pyroxylin will gelatinize under pressure and become a colloid.

A mixture of pyroxylin with a solvent, so as to form a paste, can be given any form (strip, tube, rod, etc.) through extrusion.

Pure pyroxylin powders are prepared:

a) From a mixture of pyroxylin No. 1 and No. 2 (mixed pyroxylin);

b) From a pyrocollodion;

c) From one pyroxylin No. 2 (for special purposes).

In addition to pure pyroxylin powders, there are the so-called nitroglycerin powders. The latter contain from 25 to 60% of nitroglycerin, the remainder consisting of pyroxylin and a small

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quantity of various admixtures.

Up to the first imperialistic world war of 1914 to 1918, the nitroglycerin powders were divided into two basic groups: the ballistites and the cordites. They differed in their contents of the elements, the quality of the pyroxylin, as well as the solvent gelatinizing the powder.

Ballistites are prepared with a soluble pyroxylin, mainly a colloid with a small nitrogen content. Nitroglycerin is used as the gelatinizing agent. In the preparation of the powder, the substance is flattened out under hot rollers and cut into cubes or rectangular strips.

Cordites are prepared with an insoluble (highly nitric) pyroxylin, with acetone serving as the solvent. It is extruded in the form of cords or tubes.

The first specimens of cordite contained up to 58% of nitroglycerin (cordite M-1); while later specimens contained from 25 to 30% (cordite MD - modified).

Smokeless powders have a significantly higher energy as compared with black powders. At the same time they have one substantial disadvantage. Being prepared with an ethyl ether-alcohol solvent or with acetone, they contain some quantity of this free solvent. In this connection, depending upon atmospheric conditions, the solvent can evaporate from the powder, or, vice versa, the powder can absorb moisture from the air. Such variations in the content of volatile substances are reflected quite sharply in the ballistic qualities. These properties, volatility and hygroscopicity, of common pyroxylines prepared from a volatile solvent and, to a lesser degree, nitro-

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glycerin powders, make it necessary to store the powder in waterproof packing and, whenever possible, at a constant temperature.

Beginning with the first world war and subsequent to it, there appeared a powder prepared without a solvent or with a non-volatile solvent. Among powders of this type we count a powder prepared from a mixture of pyroxylin and trotyl. This powder, when heated and subjected to high pressure, will gelatinize and can be well pressed. A powder consisting of pyroxylin, nitroglycerin and an admixture of nitro derivatives of the aromatic series (di-nitro-toluol, di-nitro-benzene, centralite and others) also belongs to this type.

These powders are non-hygroscopic, non-volatile, and have a comparatively low ignition temperature. They are much simpler to produce, and therefore find increasing utilization in numerous countries.

Di-nitro-glycolic and nitro-guanidine powders appeared in Germany during the period of the Great Homeland War (World War II), because of the existing shortages of raw materials.

Insofar as pyroxylin is obtained by the nitration of cotton with a mixture of nitric and sulfuric acids, and the free acid remaining in the pyroxylin gradually decomposes it, a complete refining of the latter, for the purpose of eliminating the acid, comprises one of the main operations in the production of pyroxylin. However because traces of acids will remain after the preparation of the powder, and will affect its keeping qualities, about 1 to 2% of a stabilizer is suitably mixed into the powder for the purpose of neutralizing the action of the acids. This stabilizer combines with nitric oxides and neutralizes them. The most commonly used

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stabilizers are di-phenyl-amine and centralite (di-ethyl-di-phenyl-urea).

2. GENERAL PROPERTIES OF POWDERS, THEIR FORM, DIMENSIONS AND TYPES

Smokeless powder is a colloidal substance, a gel, and is similar in its external appearance to celluloid. It is semi-transparent or opaque, depending upon the composition of the powder and the thickness of the material. The usual color of pyroxylin powders is grayish green. The color of the nitroglycerin powders is brown. Stabilizing admixtures stain them into various colors (yellow, red, black). Pyroxylin powder is harder than nitroglycerin powders, the latter being more soft and elastic as a result of the presence of liquid nitroglycerin.

The surface of a powder may be rough, dull or polished. Fine-grain powders for small arms are for the most part coated with graphite to increase compacting and to reduce electrification of the powder as a result of friction. In this way, their surface takes on a shining black color resembling by its appearance black gunpowder.

The form of powders is usually varied: strips, rectangular sheets, blocks, cubes, short and long tubes, grooved grains, etc.

Powder in the form of thin square flakes, or beads with a hole through them are used for small arms. The ratio of a side of the square of a flake to the thickness varies from 5 to 10. The length of a bead with a hole through it is 5 to 10 times its wall thickness, while its inner diameter is from half to the entire wall thickness. Powder for weapons of small or medium calibers with cartridge loading have the form of long tubes (macaroni), with a ratio of the length

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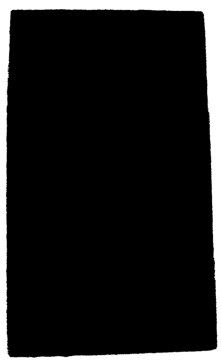
to wall thickness from 100 to 300, or they may be in the form of short cylinders with either one or seven holes through them (see further for details). Both of the two latter forms are called granular powders. Their length is 8 to 15 times greater than the wall thickness. (Grains of rifle powders are appropriately shorter.) Powders in the form of long tubes, either for the entire length of the gun chamber or in two semi-charges for half of the gun chamber, are used almost exclusively for weapons of larger caliber with individual loading. Since the loading of those weapons is performed individually and automatically, and the weights and volumes of the charges are larger, it is important to have a sturdy, inflexible charge. This requirement is fully satisfied by a bundle of tightly bound tubes.

The quantity of gases formed during burning of the powder, and the rate of their formation, depend on the weight of the charge and the numerical value for the surface of the powder. The latter depends on the thickness of the powder and its form. The smaller the grains of the powder, the larger their surface is in a given weight of the charge; the larger the quantity of gases forming in unit of time, the higher is the rate of powder combustion. The larger the caliber of the weapon and its length, the longer should the action of gases on the base of the projectile last in order to provide it with a given velocity, and the coarser should the powder be. The wall thickness varies from 0.1 mm for pistol powders to 6 mm for powders for the 354 mm (14 inches) guns. A porous fine powder is used for pistols.

Processing powder. In order to obtain a given form of powder,

the substance is forced through perforations (of a die) by means of a press. Strip powders are made either by flattening under rollers and subsequent cutting, or by forcing through a flat slot. In order to illustrate the preparation of powders with channels, a schematic drawing is shown below of a die, through which tubular powder is pressed (figs. 3 and 4).

The powder mass is contained in the space between the plunger A and the plate die BB. Under the pressure of the plunger, the mass is forced through the openings in the die BB and surrounds the attached pin C. The holes in the die are symmetrical relatively to the pin, and are designed in a manner such that their total area is larger than the cross-sectional area of the cylindrical portion d-d.



Figs. 3 and 4 - Diagram of Die for Pressing Powder

1) Bottom view at d-d; 2) top view of the die plate.

In view of the fact that the plunger A moves downward, the mass is extruded in the shape of tubes, which are broken off from time to time. Later, the latter are dried in the open air to eliminate the

[illegible] of the solvent and cut into sections [illegible]

Tubular powder is thus obtained. [illegible]

[illegible]

It was [illegible]

[illegible] the [illegible] of [illegible]

[illegible] it [illegible]

Powders for [illegible] are [illegible]

flake, (2) [illegible] powder, [illegible]

strip powder for small caliber bullets [illegible]

for large caliber [illegible]. For instance, [illegible]

[illegible], 2nd lot of [illegible] factory.

[illegible] powder is [illegible] by a fraction [illegible]

[illegible] the [illegible] in the [illegible]

the thickness of the wall in tenths of [illegible]

[illegible] – a grain with one hole and wall thickness [illegible] 0.7 mm; [illegible]

[illegible] with seven holes and a wall thickness [illegible] 1.5 mm.

Tubular and strip powders are usually [illegible] by the [illegible]

of the weapon for which they are used. [illegible]: 75/50 – [illegible]

[illegible] a 75-mm weapon with a 50-caliber length.

[illegible]

1. [illegible]

Powders used in our country are [illegible] pyroxylin.

The composition of these powders [illegible] determined [illegible]

characteristics, on which depend all [illegible] and

fundamental ballistic properties. These [illegible] are the

nitrogen content in the pyroxylin powder and the content of volatile

substances.

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Nitrogen Content

To characterize a powder with respect to nitration, it is necessary to know the nitrogen content in 1 gram of pyroxylin powder. This is usually expressed in percent or in cm^3 NO (nitric oxide) corresponding to 1 gram of pyroxylin powder.(*) The nitrogen content affects the energy of the powder, as well as its rate of combustion. The greater the content of nitrogen is, the stronger is the powder and the more intensively it will burn. On the average, the nitrogen content in pyroxylin fluctuates within the range of 11.8 to 13% (K = 188.5–208 cm^3 NO/g of powder).

Content of Volatile Substance in the Powder, Expressed in Percent

In a physical chemical analysis of powder, not only the total content H of volatile substances is determined, but also its component parts. Namely: volatile substances removable by means of 6 hours of drying at a temperature of 95°C (h%), which are usually considered to be the humidity contained in the powder, and then these inseparable by six hours of drying (h'%) which are attributed to the alcohol-ethyl ether solvent remaining in the powder mass and gelatinizing the powder.

The value H is usually related to the thickness of the powder, and the thicker the powder the higher will H be. In thin powders H

(*) Relation between the nitrogen content N% and the volume NO is given by the formula N% = K x 0.0006257 x 100, where K is the number of cm^3 in 1 gram of powder at 0° and a pressure of 760 mm.

$$(0.6257 \sim \frac{5}{8}), \text{ or } N \sim \frac{5}{80} K = \frac{K}{16}$$

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equals 2.0 to 2.5%; in powders with a strip thickness of about 1 mm, $H \sim 4.0\%$; in very thick powders, with a thickness up to 6 mm, H reaches up to 7%.

The value H mainly affects the rate of combustion of a powder. The higher H is, the slower the powder burns. The variation of moisture content in a powder, because of atmospheric conditions, is one of the main defects of pyroxylin powders having a volatile solvent.

2. PHYSICAL-CHEMICAL PROPERTIES OF POWDERS

Powder is a low explosive; therefore, all physical-chemical properties of explosives and their characteristics are also applicable to powders. These characteristics are:

Quantity of Heat (Q, Cal/kg) emitted in the combustion of 1 kg of powder, and in cooling the gases to the temperature of 15°C. This characteristic is the most essential one, insofar as at the instant of discharge the chemical energy is converted into thermal energy, and the latter into mechanical energy. Also, the larger Q is, the higher is the temperature of powder gases, and the greater is the mechanical work which they can perform.

As a rule, Q is determined by a test in a calorimetric bomb. In this connection, the following must be taken into consideration.

The calorimetric bomb is immersed in water at a temperature of 15°C. The temperature of water rises at the instant of ignition by only a few degrees, under the effect of heat emitted in the bomb, and after that it begins to drop.

The test is conducted for a period of 5 to 10 minutes. Consequently, if water vapors are present in the products of the explosion, then

they will proceed to condense and, in this manner, the quantity of heat determined in the test will relate to water in the liquid form rather than vapor form. Actually, the water is in a vapor state at the instant of ignition or explosion, and the equation

$$Q_{w\,H_2O\ \text{vapor}} = Q_{w\,H_2O\ \text{liquid}} + 620\frac{n}{100}$$

is used for its determination (where n represents the percentage content of water in the decomposition products of the powder, 620 is the quantity of kilocalories absorbed in the condensation of 1 kg of water vapors and reducing their temperature to 15°C).

Because the water is in a vapor state at the instant of explosion or discharge, the actual quantity of heat emitted in this connection is expressed in this way:

$$Q_{w\,H_2O\ \text{vapor}} = Q_{w\,H_2O\ \text{liquid}} - 620\frac{n}{100}.$$

If we convert all the quantity of heat Q_w', emitted in a combustion of 1 kg of powder, into mechanical energy by multiplying by the mechanical equivalent of heat E = 4270 kgdm/cal, then the resultant value $P = EQ_w$ will represent the potential energy of the powder, or the work it could perform if all its emitted heat would convert into mechanical work. This value is called the <u>potential of the powder</u>.

<u>Volume of Gases</u> w_1, dm^3/kg. formed in the combustion of 1 kg

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of powder, and occupied by them at a pressure of 760 mm and temperature of 0°C.

After the combustion of powder in a calorimetric or manometric bomb, gases may be conducted into the gasometer, and their volume may be measured at atmospheric pressure and a temperature of 15°C. The letter may then be reduced to 0°C. Then the water present in a vaporous state will condense into a liquid and the volume of gases measured in the gasometer will be smaller than the actual volume, if the water was in the form of a vapor. Therefore, the volume of gases determined in the gasometer refers to liquid water

$$\left(w_{1\ H_2O\ \text{liquid}} \right)$$

For conversion to the gas volume which they would occupy if the water were in the form of a vapor, the formula

$$w_{1\ H_2O\ \text{vapor}} = w_{1\ H_2O\ \text{liquid}} + 1240 \frac{n}{100}$$

is used (where n is the percent of water vapor content in the gaseous mixture, 1240 dm^3 is the volume which would be occupied by one kg of water vapor at atmospheric pressure and 0°C). The volume of gases w_1 has great significance, because the greater it is, the greater is the amount of work which can be performed by the gases in the gun.

Temperature of explosive decomposition, or the temperature of the powder at the time T_1, of firing, i.e., temperature possessed by powder gases forming during the combustion at the instant of their formation. It is measured on the absolute scale. The higher the temperature of the gases is, the greater is the amount of mechanical work which they can perform in a discharge.

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The value T is usually not determined directly during a test, in view of its large magnitude and the short duration of powder combustion. It is determined indirectly. This requires knowledge of the quantity of heat

Q_w

H_2O vapor,

the composition of the gases, their heat capacity, and their variation with the temperature.

Composition of gases and their heat capacity. An analysis of the gases after powder is ignited in a calorimetric bomb shows that the main bulk of gases from pyroxylin powders is composed of the diatomic gases CO, H_2, N_2, triatomic CO_2 and H_2O (in the form of vapor), and also a small percentage of metane CH_4 and ammonia NH_3. The ratio of these component parts varies somewhat, depending upon the compactness of loading. It is necessary to state that an analysis of gases is not made at the moment of combustion, but later, when the gases cool off. Therefore, the composition of the gases also depends on secondary reactions between the basic gases, while these secondary reactions may themselves depend on the compactness of loading and the conditions of cooling.

Knowing the composition of the gases, it is also necessary to know the heat capacity of gases c_p and c_v, as well as their variation with the temperature(*), in order to make computations of the temperature.

It was determined on the [illegible] of experiments conducted by Malliar and [illegible]tel, as [illegible]ent investigations, that

(*) c_p [illegible] heat [illegible] at a constant pressure and a constant [illegible] c_p

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the dependence of the heat capacity of gases c_v on temperature t may in a first approximation be expressed by a linear function

$$c_v = a + bt,$$

where a and b are constant values. These have the same fixed values for all diatomic gases, and others for triatomic gases.

At the present time, a series of very new and accurate formulas are available for the heat capacity. They are based on the quantum theory of heat capacity and take into consideration the oscillatory motion of atoms. In this connection, numerical characteristics in the formulas are determined spectroscopically ([illegible]'s formulas).

Professor A.N. [illegible] submitted a logarithmic dependence for the molecular heat capacity in the form of

$$\mu c_v = z \cdot 6.9225 \left(\ln \frac{T1}{98.1} + 1 \right)$$

All these formulas show abrupt variations of heat capacity at low temperatures, which cannot be expressed by linear factors. However, under the conditions of a discharge, gases as a rule cool off to a temperature of about 1800-2000°K, between the instant of gas formation at a temperature $T_1 \sim 2800$-3000°K to the instant of the projectile's passage through the muzzle face. In this range of temperature variations, all empirical and theoretical dependencies can be expressed to a satisfactory degree of accuracy by the common linear equation

$$c_v = a + bt = A + bT,$$

which we also shall adopt farther on in the manual. (For more details, see the energy equilibrium equation)

To determine the temperature, we utilize the equality

$$dQ = c_w \cdot dt$$

Substituting $c_w = a + bt$, we obtain

$$dQ = (a + bt)\, dt.$$

By integrating we obtain

$$Q = \int_0^t (a + bt)\, dt = at + \frac{bt^2}{2}.$$

From this quadratic equation, we determine t:

$$\frac{b}{2}t^2 + at - Q = 0;$$

$$t = \frac{-a \pm \sqrt{a^2 + 2bQ}}{b}.$$

We select the plus sign before the radical, since minus results in a negative temperature.

Here we have exemplified the method of determining the temperature for given values of a and b, pertaining to any one gas. However, for a mixture of gases the values a and b will be individual for each gas, and may be determined as follows.

Assume that the values of coefficients a and b for each gas will be:

$$a_1, a_2, a_3, \ldots\ldots, a_i \ldots\ldots\ldots$$

$$b_1, b_2, b_3, \ldots\ldots, b_i \ldots\ldots\ldots$$

The relative parts by weight for each gas are

$$g_1, g_2, g_3, \ldots\ldots, g_i \ldots\ldots,$$

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while

$$n_1 + n_2 + n_3 + \ldots + n_i + \ldots = 1.$$

Then

$$a = n_1a_1 + n_2a_2 + n_3a_3 + \ldots + n_ia_i + \ldots = \sum_1^k n_ia_i,$$

$$b = n_1b_1 + n_2b_2 + n_3b_3 + \ldots + n_ib_i + \ldots = \sum_1^k n_ib_i,$$

and these values must be substituted in the equation for computing t.

Values of the mean molecular heat capacity from 0 to t are listed in Table 1(*).

Table 1

t, °C	N_2, O_2, CO	H_2	H_2O	CO_2
100	6.96	6.95	8.04	9.68
500	7.07	7.02	8.32	10.34
1000	7.30	7.15	8.83	11.[illegible]
1500	7.52	7.28	9.46	11.[illegible]
2000	7.70	7.56	10.27	12.[illegible]
2500	7.84	7.70	11.28	13.[illegible]
3000	7.96	7.83	12.[illegible]	13.74

(*) Professor D.V. Alekseev, "Fizicheskaya Khimiya" (Physical Chemistry), page 60.

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Since various authors submit different values for a and b, the temperatures computed by the aid of the listed formula vary up to 10%.

More accurate equations are obtainable by an application of the quantum theory.

In view of the fact that interior ballistics equations also include the weight of powder charges, as well as their volumes, one of their physical characteristics, is the specific weight or <u>density of powder</u> δ. The density of powder varies within very narrow limits, from 1.63 to 1.56; and on the average it is assumed in approximate computations to be equal to 1.6. The density of powder depends little on the type of powder. It is equal to 1.6 for both the pyroxylin and nitroglycerin powders. For pyroxylin powders, the density depends on the content of volatile substances H (the higher H, the smaller δ). Powders with a non-volatile solvent have $\delta \approx 1.62$, with δ dependent on the conditions of pressing: the greater the pressure of pressing, the greater is the δ.

For black powders, δ varies between 1.50 and 1.80. In extreme cases it reaches 1.90.

Table 2 - Values for Several Physical Chemical Characteristics of Powders

Characteristic	For Pyroxylin Powders	For Nitroglycerin Powders
Calorific value Q_w cal/kg (water in vaporous state)	900-800	1100-1200
Volume of gases W_1, dm^3/kg (water in vaporous state)	900-970	[illegible]
Temperature of combustion T_1, °K	2800-2500	3000-2500

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Table 2 - (Cont'd.)

Characteristics	For Pyroxylin Powders	For [illegible]
Content of volatile substances H, %	2.0-[illegible].0	[illegible]
Density of powder δ, [illegible]	1.62-1.66	1.62-1.[illegible]

The famous Russian gunpowder specialist G.P. [illegible] (died in [illegible]) submitted the following empirical formulas for the fundamental physical-chemical and ballistic characteristics of pyroxylin powders (N - nitrogen content in percent):

$$W_1 = 1816 - [illegible]N; \quad T_1^2 = [illegible] + 24.7N^{5/3}$$

Calculations based on these formulas give the following values for W_1 and T_1 (table 3):

Table 3

N, %	11	12	13	14
W_1	979	931	882	833
T_1°	2166	2458	2750	3006

During recent years investigations were conducted to determine the experimental dependence of the fundamental physical-chemical characteristics [illegible], W_1 and T_1 on the composition of the powder.

For our powders, these dependencies were submitted by Lecturer V.G. Shoklein (19[illegible]).

If we designate the percentage content of nitrogen in pyroxylin as N, and correspondingly designate the content of nitroglycerin in gunpowder as n, while designating centralite (di-ethyl-di-phenyl-urea)

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as c, di-butyl-phthalate as d, vaseline as v, separable volatiles as h, inseparable volatiles as h', di-phenyl-amine as s, camphor as ϕ and graphite as g, then the formulas of Sheklein will yield:

$$Q_w = 730 + 148.5\ (N - 11.8) + 9.41\ n - 28.5\ c - 24.3\ d - 37.5v -$$

$$13.6\ h - 26.7\ h' - 31.0\ s - 32.5\ \phi - 42.0\ g,$$

where 730 represents the heat of explosive decomposition of pyroxylin having a nitrogen content of 11.8%.

$$w_1 = 944 - 47.3\ (N - 11.8) - 2.45n + 14c + 12d + 23v + 3.4h +$$

$$+ 16.9h' + 14.6s + 17.4\phi + 10g.$$

where 944 represents the volume of gases forming during the combustion of pyroxylin having a nitrogen content of 11.8%.

$$T_1{}^{\circ}K = 2790^{\circ} + 375\ (N - 11.8) + 22\ n - 71\ c - 59\ d - 100\ v - 54\ h -$$

$$- 82\ h' - 88\ s - 92\ \phi - 125g.$$

where 2790° corresponds to the temperature of the explosive decomposition of pyroxylin having a nitrogen content of 11.8%.

3. BALLISTIC PROPERTIES OF GUNPOWDER

Ballistic properties of gunpowder is the term applied to values governing the maximum pressure p_m of powder gases and to the rate of pressure increase $\frac{dp}{dt}$ during combustion of the powder inside a constant space.

One of them depends on the nature of the powder, and is related by a definite pattern to the latter's physical-chemical characteristics. The others are governed by the geometrical data of the powder grains

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composing the charge.

Ballistic properties dependent on the nature of the [illegible] are determined experimentally through combustion inside a manometric bomb.

In the combustion of powder in the bore during a discharge, the pressure of the gases and the rate of its variation depend not only on the characteristics of the powder, but also on other values and parameters related to the design of the bore of the gun and projectile, and characterizing the entire artillery system (for example p_m, v_0). These values may be termed the ballistic characteristics of the gun and projectile.

For the time being, only a short definition of ballistic properties will be presented at this place. A more detailed discussion of these properties will be given later, in the chapter on [illegible].

The "Force" f of the powder represents the work which the gases can perform during the combustion of 1 kg of powder, if we heat these gases up to the temperature $T_1°$ and permit them to expand at constant atmospheric pressure.

This work depends on the specific volume of gases and the temperature of the explosive decomposition of powder. It is expressed in kg-m/kg

$$f = RT_1 = \frac{p_a w_1}{273} T_1,$$

where $p_a = 1.033$ kg/cm^2 and represents atmospheric pressure;

w_1 is the specific volume of gases at 0°C and atmospheric pressure; and

T_1 is the temperature of explosive decomposition ([illegible]) of

powder.

By modifying the properties of the powder in a manner increasing w_1 and T_1, it is possible to also increase the energy of the powder.

The term "energy" appears to be a sort of historical survival, and does not truly define the amount of work. However, since it has been maintained in ballistics, we shall continue to use it in our treatise.

Covolume α in dm^3/kg. In the presence of great pressures, such as develop in the combustion of powder in bombs and weapons, gas densities become so great that the gaseous molecules by themselves occupy a liberally significant part of the space in which the combustion occurs. In physics, this is explained by the introduction of a value, proportional to the volume of gas molecules and equal to the sum of volumes of spheres of influence of each molecule, in the equation for the state of the aggregation of gases. Van-der-Vaals assumed that the volume of these spheres of influence is equal to the quadruplicated volume of the molecules themselves.

This value is called the "covolume." It is specific for a given type of powder, proportional to the volume of gas molecules, and exerts influence on the value for pressure.

We will assume that covolume is a volume proportional to the volume of molecules of gases forming during the combustion of 1 kg of powder. (It is expressed in dm^3/kg.)

The ratio of the covolume of a given gas to its volume at $0^\circ C$ and at a pressure of 760 mm, $\alpha : w_1$, varies within narrow limits for various gases. Namely;

Nitrogen	0.001350
Methane	0.001801
Oxygen	0.000890
Hydrogen	0.000887
Carbon dioxide	0.000836

Usually it is assumed that $\alpha \sim 0.001\ w_1$.

Kisnemskii gave for pyroxylin powders the formula $\alpha = 5.7/N^{0.7}$, i.e., $\alpha = 0.00108\ w_1$.

Krants writes in his study [2]: "In all probability, the adjustment from volume to covolume is not a constant value, but a function of the volume. The commonly accepted proposition that $\alpha = 0.001\ w_1$ constitutes an approximation, with the error increasing for an increase of pressure. It is most expedient and accurate to accomplish a determination of the covolume on the basis of pressure measurements."

Information gained in recent years provides the foundation for the assumption that the covolume becomes smaller with an increase of the pressure of gases above 10,000 kg/cm^2. The covolume may be considered as a constant value, under normal conditions and pressures up to 4000 kg/cm^2.

Rate of powder combustion u_1 at a pressure $p = 1$. Similarly to f and α, this value is a derivative of the physical-chemical properties of powders. Variations of the chemical composition of the powder are reflected very strongly in the value for the rate of combustion. For instance, the rate of combustion u_1 for nitroglycerin powders possesses values from 0.070 to 0.150 mm/sec at $p = 1$ kg/cm^2, depending mainly on the content of nitroglycerin.

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The rate of combustion u_1 for pyroxylin powders possesses values from 0.060 to 0.090 mm/sec at $p = 1$ kg/cm^2, depending on the content of volatile substances.

In the combustion of powder within a constant space, the energy f and the covolume α exert influence on the value of the pressure and on the rate of its intensification. The rate of combustion u_1 influences only the rate of pressure increase.

The value u_1 of the rate of combustion, when related to the pressure $p = 1$ has a compound magnitude dm/sec: kg/dm^2.

All these characteristic f, α and u_1 depend upon the nature of the powder.

Table 4 - Values f, α and u_1 for Various Powders

Powder	f in kgdm/kg	α in dm^3/kg	u_1 in dm/sec: kg/dm^2
Pyroxylin powders	770,000-950,000	0.90-1.1	0.0000060-0.0000090
Nitroglycerin powders	900,000-1,200,000	0.75-0.85	0.0000070-0.0000150
Black powders	280,000-300,000	~0.5	---

The last ballistic characteristic depends upon the geometrical data of the powder. This characteristic is the "Dimensions and form" of powder grains, and the related "specific surface of the powder," the ratio of the initial surface of the powder to its volume. The principle of gas formation and the rate of pressure increase in the combustion of powder depend upon these values.

Chief importance is attributed to the smallest dimension, the thickness of the strip or the wall. Because the combustion of the

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powder grain (strip, tube) occurs from two sides, the thickness is usually designated by $2e_1$ (e_1 corresponding to the half of the thickness which burns in one direction).

Apart from the ballistic characteristics of the powder, the density of loading Δ also affects the value and character of the pressure increase. It is a characteristic of the conditions of charging. The density of loading represents the ratio of weight ω of charge to the volume W_0, in which the combustion of the powder takes place:

$$\Delta = \frac{\omega}{W_0} \quad kg/dm^3.$$

In this sense, if we fill the entire space W_0 with powder, then the density of loading will become a gravimetric density.

The gravimetric density characterizes the degree of compactness of the charge. At a given density δ of powder, it will be greater for a fine powder with rounded edges and less for a rectangular grain with protruding edges. In this connection, for instance, a granular powder with seven holes proved to be more "suitable for packaging" than the strip type powder. By the same token, a shell for a field gun will house 1100 kg of strip powder, and up to 1300 kg of the powder grains having seven holes.

In this manner, we have four ballistic characteristics: the energy f, the covolume α, the rate of combustion u_1 at $p = 1$, the dimensions and form of the powder, and the characteristic of the charging conditions, the density Δ of loading.

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With a given composition of the powder, we can regulate the process of pressure increase and the magnitude of the pressure by varying Δ, the dimensions and the form of powder. The dimensions and form of powders are varied because it is necessary in each case to select the dimensions of the powder and the weight of the charge for the gun, in order to obtain the required muzzle velocity of the projectile, under conditions in which the pressure will not exceed a given fixed value governed by the strength of the barrel wall.

The ballistic characteristics will be discussed in greater detail in Section II.

SECTION TWO
GENERAL PYROSTATICS

BASIC RELATIONS AND PRINCIPLES OF GAS FORMATION DURING THE COMBUSTION OF POWDER IN A CONSTANT SPACE

CHAPTER I - COMBUSTION OF POWDERS

1. THE MANOMETRIC BOMB

Pyrostatics investigate the combustion of gunpowder in a constant space. It is one of the fundamental branches of interior ballistics. A familiarity with it is necessary for a clear comprehension of phenomena occurring during a gun discharge.

The combustion of powder is investigated in this connection under simplified (static) conditions, where the motion of the projectile is disregarded, the variations of volume do not exist, and gases do not perform exterior mechanical work. The mechanical work of the gases consists of a certain pressure, which the walls of the space in which the combustion takes place are subjected to from within.

On the basis of the experimental investigation of the development of the pressure of the gases during a combustion of powder within a constant space, pyrostatics form the theory of powder combustion and establish principles of formation for gases containing the energy which is expended for the performance of various exterior functions under the conditions of a gun discharge.

In this connection, investigation is made of the influence of physical-chemical properties of powder, of ballistic characteristics

and conditions of charging on the development and process of gas pressure. The latter is in itself a very important factor, affecting the rate of formation of gases.

Pyrostatics presents the methodology of the ballistic analysis of gunpowders, i.e., the methodology for determining the ballistic characteristics of the powder.

Knowing the ballistic characteristics of a powder and the principles of its combustion in a constant space at a given process of pressure, it is possible to account for the principles of gas formation and pressure development under the even more complicated conditions of a discharge, with an occurrence of projectile motion (pyrodynamics), and a variation of the volume and performance of exterior functions by the gases.

In this manner, pyrostatics offers a source of knowledge and fundamental data necessary for a comprehension and study of the more complicated phenomena occurring during a discharge.

Instruments in which the principles of gas formation during a combustion of powder in a constant space are investigated, are called manometric bombs. In view of the fact that in the course of the subsequent exposition of pyrostatics it will be necessary to take experimental materials as a basis, a description of the design of the most typical manometric bomb is included here for the purpose of lending clarity to statements on obtaining such material.

The manometric bomb is a laboratory instrument serving for the purpose of determining the magnitude and character of the increase of pressure developed by gases formed during the combustion of gunpowder or explosives inside a constant enclosed

space. Making it possible to determine all ballistic characteristics of gunpowder, (energy of the powder, covolume of the powder gases, duration of combustion and its rate, as well as several other characteristics), the manometric bomb at the present time constitutes one of the fundamental instruments of an interior ballistics and powder laboratory.

Fig. 5 - Manometric Bomb for the Investigation of Powder Combustion

Up to the present time, the Vel bomb, designed in the 1880's appears to be the most widely used. In this bomb, the pressure is determined from the compression figure of a copper cylinder, called "crusher." (English word "crush" means to crush or compress.)

The bomb (fig. 5) consists of a cylinder A, made of high-grade steel, which has a screw thread in each end of its inner surface. The igniter cap B is screwed in at one end, while the cap C with the crusher manometer is screwed into the other end. In the igniter cap, there is an insulated rod which conducts the electric current igniting the fuse, while a second conductor is connected directly to the body of the bomb. A thin wire connects the points c and c', and passes through a shell made of tissue paper which contains a given amount of igniter (pyroxylin, black powder). This wire is ignited by the current.

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The rod p moves within the duct of the crusher cap C, and transmits the pressure of the powder gases to the crusher column k. The latter is made of electrolytic copper. Its other end is placed against the head of a screwed-in plug which serves as an anvil. A small centering rubber ring n is placed on the crusher to obtain coincidence of its axis with that of the rod. The head of the rod r, adjacent to the crusher, has an outwardly protruding extension r' which moves in a lateral guide slot in the head of the rod cap. A light steel pen point s is fastened to the extension. It registers the travel of the rod as a function of time on smoked paper glued to the drum of the chronograph.

The copper obturating rings, d, d, serve to prevent the escape of gases between the walls of the bomb and the screwed-in caps, while the portion of the duct e which borders with the rod is filled up with a mastic to protect the rod from the immediate effects of high temperature gases.

The bomb 6 is fastened in a special clamp 5 near the drum 2 (fig. 6), in order for the pen point to touch the smoked paper only lightly, and when the drum revolves, to mark on the latter a thin straight line parallel to the base of the cylinder (as on fig. 7).

The tuning fork K is located on the other side of the drum 2. It is caused to vibrate by the opening and closing of electromagnets e,e, which attract the arms of the tuning fork. A thin pen point ℓ is fastened to one of the latter's arms, recording its oscillations on the same smoked strip of paper.

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Prior to the experiment, the drum of the chronograph is put into rapid rotation by means of an electric motor M or a time mechanism. When the rotation becomes uniform, the current is turned on and burns the wire going through the fuse. The fuse itself ignites the charge of powder located in the bomb.

Fig. 6

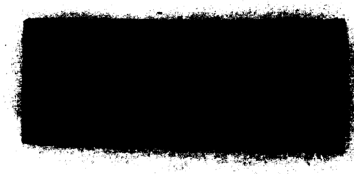

Fig. 7 - Curve of the Compression of the Crusher in the Manometric Bomb

The pressure produced during the combustion of the powder gases is transmitted through the rod to the crusher, compresses it, and the pen point of the rod plots the compression curve mm (fig. 7) in accordance with the process of pressure increase. After the end of the combustion and the attainment of maximum pressure, the rod stops, and the pen point draws a straight line bb parallel to the initial line. Here, the distance between both of these straight lines is equal to the full compression of the crusher.

Simultaneously with the turning on of the current, the vibrating tuning fork is placed near the drum for a short period of time by the aid of an automatic mechanism, and its pen point draws a wave-like curve "sine curve" about the center line. Because the number of tuning fork oscillations per second is already known, and having measured the length of one wave by means of a comparator microscope,

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we can determine on the circumference of the drum the length corresponding to 0.001 second at a given speed of drum rotation. In this way we obtain a time scale for measuring the crusher compression curve.

After completion of the test, the paper with the sinusoidal curve and the crusher compression curve is taken off and coated with lacquer. After the drying process, the curve is measured under a comparator microscope.

The pattern of the curve and of the sinusoid is shown in fig. 7.

After measuring the compression curve obtained at suitable time intervals, the compression of the crusher is determined as a function of time.

Then, in a given test, the dependence of powder gas pressure increase on time (curve p as a function of t) is obtained on the basis of the dependence of crusher compression on the magnitude of the pressure, the latter having been previously determined experimentally in a press.

In this way we can determine not only the maximum pressure developed by the gases of a charge of given weight (for a given density of loading), but also the pressure increase relatively to time. That is, just those values which depend on the ballistic characteristics of a gunpowder.

Consequently, if we knew the equations relating the ballistic characteristics to the law of gas pressure increase, and had the bomb test data for any given powder, we could determine the numerical values of its ballistic characteristics.

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In pyrostatics, relationships are also known between the ballistic characteristics of a powder, the condition of the test and test data obtained.

Even now, the manometric bomb forms the basic apparatus of pyrostatics. Sometimes an elastic manometer is used in place of the crusher cap.

The previously mentioned cylindrical crusher began to compress and registered the increase of pressure at approximately 300 kg/cm^2. The initial phase of powder combustion remained unexplored.

In order to obtain the entire law of pressure increase from the beginning of ignition to the end of the total combustion of the powder, one can use a crusher shaped so that, having a low resistance at low pressures in the first phases of the combustion process, it gradually increases in resistance as the pressure increases.

The conical crusher developed by the author in 1923, and obtained by machining a part of a cylindrical crusher into a cone, possessed that property. This type of crusher registers pressures from 5 to 7 up to [illegible] kg/cm^2. In this way, it permits not only the investigation of powder combustion from the very beginning to its end, but also a study of the burning of the igniter itself and the process of powder ignition.

The conical crusher was widely applied in our country in laboratory tests of powder combustion, as well as in determinations of the low pressures of powder gases in tests conducted on the firing range.

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At the present time, piezoelectric manometers based on modern achievements in electrotechnology and radiotechnology are also used. Still, the main aspect of the business remains thus: the process of pressure increase is registered by one or another method. Then, knowing what type of powder we deal with and the conditions of its combustion, on the basis of the pressure increase, we can determine the rate of gas formation and its dependence on various factors.

2. PRINCIPAL PHASES OF COMBUSTION

In the combustion of powder we can distinguish the following three phases.

Ignition of the powder. In order for the powder to become ignited, it must be heated up in such a manner as to obtain, at any given point of the charge, a temperature higher than its ignition temperature. The ignition of smokeless powder occurs at a temperature of about 200°C. Black powder with an ignition temperature of the order of 300°C, however, ignites and burns more vigorously. After the powder is ignited, even though only at one point, the combustion reaction proceeds by itself as a result of the heat emitted in the combustion of the powder. Two processes take place simultaneously: ignition of the powder, and the combustion proper.

Ignition is the process of the propagation of the combustion reaction over the surface of the powder grain.

Combustion is the process of the propagation of the reaction into the interior of the powder grain.

The above two processes differ one from the other. The figures characterizing them and relating to the rates of ignition and

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combustion of smokeless and black powders are listed below, in Table 5.

2. COMBUSTION OF POWDERS AT ATMOSPHERIC PRESSURE

Black Powders. After the ignition of black powders in the open air and at any given point, the flame spreads very rapidly in all directions over the surface. Then the grain continues to burn from all sides, by concentric layers toward the center of the grain. In accordance with the experiments of Piobert, ignition in identical grains will occur the slower, the more carefully the grains are polished, the greater the density of the powder, and the less burned is the charcoal used for the manufacture of the powder.

In the open air, the speed of ignition of black powder placed in the form of a path over corrugated iron varies from 1 to 3 m/sec, depending upon the circumstances.

The rate of combustion of black powders in open air was determined in experiments by igniting powder demolition blocks, with the lateral surface covered by grease. It proved that the rate of combustion, or the speed of the transmission of the flame from layer to layer, does not depend upon the cross-sectional area of the burning block, but that the speeds of combustion are inversely proportional to the density of the blocks. It was proved that for an identical composition of the powder mass, brown charcoal reduces the rate of combustion, and the rate of combustion is lowered with an increase of powder density.

The rate of combustion of black powder in open air is on the average around 0.01 m/sec = 10 mm/sec., i.e., many times less than

the rate of ignition.

Smokeless Powders ignite and burn in the open air considerably slower than the black powders.

If we fasten a strip of pyroxylin powder (or a stick of nitroglycerin powder) vertically and ignite an upper corner of it, the strip will burn calmly with a yellowish flame, while the propagation of flames over the surface of the strip will proceed comparatively slowly.

In addition to this, the burning grains will form an angle some time after the beginning of the combustion. This angle will remain constant to the end of the combustion (figs. 8a and b). The magnitude of this angle depends on the ratio of the combustion rate of powder to the rate of ignition.

Assume that the rate of combustion equals u, the rate of ignition u', while $u' > u$.

If we ignite a strip of powder at one corner (at the point a), then it will burn inside at the rate u, and on the surface at the rate u', for various intervals of time. Assume $u' - 2u$. Then the strip will have sequential burning surfaces of the type shown in fig. 8, i.e., 1-1-1; 2-2-2;...;5-5-5; 6-6-6. Beginning with the surface 5-5-5, the angle at the top maintains its magnitude and equals two times $\angle ode = \angle \varphi$.

Now, the ratio of rates is $u/u' - \sin \varphi$. In this manner, having measured the angle φ, we can determine how many times the rate of ignition is larger than the rate of combustion. It is of interest to note that after reaching the constant angle (surface 5-5-5), the further shortening of the strip proceeds in unit time not at u (rate of combustion), but at u' (rate of ignition). Therefore, noting

[illegible] for the total combustion of a stick of a given length, we [illegible] the rate of ignition but not of combustion.

[illegible] rate of combustion of smokeless powder in open air is [illegible] cm/sec., i.e., significantly less than in the case of black [illegible]

[illegible] rate of ignition is two to three times greater. Therefore, [illegible] burns in air for a sufficiently long time. However, if [illegible] the entire surface of the strip at one time, so that [illegible] by igniting a strip of [illegible] the entire surface [illegible] simultaneously into flames, and the [illegible] velocity, however, [illegible] small.

Table 5 - Rate of Ignition and Combustion of [illegible]

[illegible]	[illegible]	[illegible]
Rate of ignition u' cm/sec	[illegible]	2-5
Rate of combustion u cm/sec	[illegible]	1-3

[illegible] larger quantities of smokeless powder are ignited, individual grains are [illegible] as a result of the formation of a [illegible] quantity of gases and may be projected a certain distance, [illegible] their interior [illegible] become comparable [illegible] and fly at a higher velocity in various directions, [illegible] in the development of a [illegible]

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Fig. 8 - Combustion of a Strip of Powder in the Open Air

1) Front view; 2) side view.

4. COMBUSTION AT A PRESSURE LOWER THAN ATMOSPHERIC PRESSURE

The first observations and experiments were conducted with black powder. They have shown that powder burns less vigorously in rarefied atmosphere on high mountains than at the foot of the same mountains. Experiments on the combustion of powder fuzes at various altitudes and at various barometric pressures verified this conception. These experiments retain their significance even at the present time, because the combustion of powder in time fuzes, during antiaircraft firing at aerial targets, takes place much slower than in flat trajectory firing.

5. COMBUSTION IN A VACUUM

The significance of pressure becomes even more clearly evident in the event of great rarefaction. It was proven by experiments that if the powder is placed in a vessel under the bell of an air pump, it will completely fail to ignite, regardless of bringing it into contact with a red hot wire.

A platinum container filled with black powder was heated until

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red, in a greatly rarefied atmosphere under the bell of an air pump. After some time passed, the powder began to burn slowly. However it did not explode, as in the open air. If a platinum wire is conducted through the powder and heated up, the grains surrounding it will begin to melt; however, combustion will not commence immediately. If the heating up is [illegible], then grains in contact with the wire will burn only after the passage of some time, while the others will remain intact.

And, combustion occurs [illegible] only when pressure is increased to 1/10 atm.

In this manner, the heating up of the powder, even by a red-hot body, is by itself insufficient. It is necessary that a certain minimum of atmospheric pressure exist. The higher the initial pressure, the more vigorously will the powder ignite.

The same concepts apply to smokeless powders also.

6. COMBUSTION OF POWDER AT INCREASED PRESSURES

The rate of powder combustion depends very largely on the pressure. Although this fact was known for a long time, experiments determining the relations between the rate of combustion and the pressure were first conducted by Vel at the end of the 19th century.

It should be of interest here to show how he arrived at the determination of a characteristic for parallel layer combustion of powder on the basis of these experiments, and by also utilizing additional observations on [illegible] of the incompletely burned grains projected from the gun during firing.

[illegible]. After [illegible] observations on the incompletely burned grains which were projected from a gun when it was discharged,

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Kastan found that for densities of black powder $\delta \leq 1.64$ there are no residues; for $\delta \approx 1.72$ the residues had an irregular form; while at $\delta > 1.81$, the form of the residues was very similar to the original grains. (In fig. 9, residues are represented by the shaded areas.)

Vel prepared, from one and the same mixture of black powder, tablets and small cylinders of similar shape but of varying dimensions a_1 and a_2, and ignited them in a bomb, using the same charging density. Thus, the maximum pressure p_m was the same (for both). The times τ_1 and τ_2 for total combustion were determined by the aid of crusher pressure recordings.

In the first instance, at $\delta \leq 1.64$, the time of combustion of the powder was not governed by the dimensions; $\tau_1 \approx \tau_2$, regardless of the significant difference in dimensions.

In the second instance, at $\delta \approx 1.72$, the time increased with an increase of the thickness of the explosive blank; but not however proportionally to the dimensions. This corresponded to the irregular residues in the experiments by Kastan.

Fig. 9 – [illegible]

At $\delta \approx 1.80$, the result was a direct [illegible] combustion times to the linear dimensions [illegible]

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At an increased density of loading ($\delta \approx 1.70 - 1.72$), its particles fit closer one to another, the interstices become smaller, and the dispersal under pressure occurs later. Only a very dense mass ($\delta > 1.80$) does not disperse under pressure, and burns in concentric layers.

In this way, the character and rate of combustion of black powders at higher pressures depends on the density δ , and even to a greater degree on pressure. The rate of combustion increases with an increase in pressure.

The following characteristic of parallel layer powder combustion, which became known as Vel's characteristic, was determined on the basis of the experiments listed above.

If two powders, identical with respect to chemical composition and having a similar form of grain, but of different dimensions, are burned in a closed space with the same density of loading, and if their times of total combustion are related to each other as their [illegible] dimensions, or as their coefficients of similarity $\tau_2/\tau_1 = [illegible]/a_1$, then this is a characteristic of powder combustion by parallel layers.

For instance, if in the first instance the charge is composed of [illegible] with dimensions [illegible] x [illegible] mm, and in the second [illegible] of [illegible] x 10 x 10 [illegible] ratio of their times of [illegible] τ_2: τ_1, will [illegible] 1.

[illegible] the [illegible] of the [illegible] increase [illegible] in a [illegible] powders in a [illegible] so that [illegible] and its [illegible] the beginning and the end

[illegible], then all [illegible] points of the curves will [illegible] - they will [illegible].

[illegible] conducted in a [illegible] with [illegible] have shown that the latter satisfy [illegible], it can be [illegible] that they have [illegible].

[illegible] the [illegible] of [illegible] powders in open [illegible] black [illegible] in [illegible] pressure [illegible]. [illegible] by [illegible] of the [illegible] in a [illegible], [illegible] of [illegible] lasts only [illegible] of a second. [illegible] tablets [illegible] and [illegible] do not [illegible] during [illegible] (as in the case of [illegible] powder), [illegible].

[illegible] of powder [illegible] at times [illegible] guns in [illegible] and are [illegible] the air, [illegible] retain their [illegible], except for the [illegible] that their [illegible] with the combustion in [illegible], the strip [illegible] form of a [illegible]. [illegible] combustion, [illegible] in air by [illegible] a small [illegible] between the [illegible] combustion and [illegible], is not [illegible] higher [illegible] points to [illegible] that ignition [illegible] than in the [illegible], as a result of [illegible] igniter gases [illegible] a sufficiently [illegible].

If a strip is [illegible] in open air by throwing it into fire,

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then it will burst into flames instantly, practically in [illegible] and will burn out rapidly. During the combustion of [illegible] bombs and weapons, ignition is accomplished by auxiliary [illegible] made of pyroxylin or black powder.

These instantly produce a larger quantity of high [illegible] gases, which raise the pressure to 16-[illegible] atmospheres. [illegible] conditions, the powder begins to burn [illegible] over its entire surface, and the further [illegible] by concentric layers. That is why the [illegible] form.

Conducted observations served as a [illegible] Vel's law of combustion for smokeless [illegible] the mass of the powder is homogeneous [illegible] the grains of the charge are exactly [illegible] form, was accepted as a basis for [illegible] assumptions of this law are as follows [illegible]

1) The ignition of powder is [illegible]

2) The combustion of smokeless [illegible] layers, at a [illegible]

The [illegible] is charac[illegible] grain. [illegible] and [illegible] called [illegible]

[illegible] expresses only the superficial [illegible], and [illegible] the mass, and to [illegible] is a [illegible]tion of the [illegible] the entire charge [illegible] differing with [illegible] a [illegible] pressed [illegible] or [illegible] to the different [illegible]).

[illegible], the [illegible] of the charge [illegible] grains in the chamber (of the [illegible] for the [illegible].

[illegible] of the entire charge [illegible] for the individual grain of powder. [illegible] of a series of [illegible] the rate of [illegible] a basis [illegible] of the [illegible] by [illegible] aid it [illegible], the [illegible] of gas [illegible], and, by [illegible] the [illegible] of discharge.

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The subsequent studies of the twentieth century defined the geometrical principle of combustion more precisely, and introduced consideration of the influence of factors, inexplainable in the light of the basic assumptions.

The geometrical law was originated on the basis of an investigation of pyroxylin powders in a simpler form (strip, tablet, tube). Later, powders in a more complex form made their appearance. These were powders with a multitude of holes (American grain with three, seven and nineteen holes; in our country the grain of Kisniemskii with 36 holes), flegmatized powders with an uneven distribution of the flegmatizer over the depth of the powder mass, nitroglycerin powders, in which the nitroglycerin distributed itself unevenly over the powder in storage. Also introduced was the process of graphite covering the surfaces of certain powders, which retarded the ignition process.

The appearance of these new factors demanded a more precise definition of our ideas on the actual character of powder combustion, and required the introduction of corrections to the perfected scheme of combustion..

7. THE THEORY OF POWDER COMBUSTION

In 1908, the French research scientist Charbonnier presented in his study, entitled "Interior Ballistics" [3], a criticism of the basic assumptions of the geometrical principle of combustion, and expressed doubts as to their correctness.

By examining more closely strips of coarse powder projected in an incompletely burned state from small caliber guns during

[illegible] gun, one can find a large number of strips burning practically [illegible] parallel layers. However, some of the strips burn irregularly [illegible] identical, and the surface of [illegible] the appearance of [illegible]. The latter become larger and [illegible] this manner [illegible] the powder into fine tablets, which [illegible] the ground before the gun in large numbers. The cause for [illegible] irregularities of combustion, as well as for the crumbling of [illegible] strips, is partially included in the imperfection of powder [illegible]ture. However, an artillery specialist [illegible] firing guns and [illegible] to form a theory of their [illegible] operate on the basis of powders as they are in reality, [illegible] the basis of ideal powders perfect in [illegible] combustion. Even if we disregard the manufacturing defects [illegible] accept the principle of combustion by parallel layers, [illegible] that ideal principles evolved for one grain can be applied [illegible] modification to an entire bundle of [illegible] present in the [illegible] of a gun, or to a charge composed of several such bundles [illegible]ly not, for it is easy to show the basic [illegible] influence [illegible] from the [illegible] principle of combustion of one grain.

Ignition and Combustion. It was commonly accepted in interior [illegible] that the ignition of the entire charge is instantaneous [illegible], i.e., that the rate of transmission of fire is [illegible]ly high as compared with the rate of combustion. This [illegible]tion is [illegible], and is not [illegible] under any [illegible]tions in open air. It is prohibitive to assume that at [illegible] pressures, the rate of ignition will become infinitely higher in comparison to the rate of combustion.

It is by far more probable that the first bundle, located adjacent to the igniter, will ignite and burn somewhat ahead of the last one, which is located at the bottom of the projectile.

Propagation of ignition. Ignition propagates for the most part in the holes in and interstices between the powder grains. It will proceed more advantageously if the grains are distributed in the direction of chamber axis, than if they were placed irregularly.

The closeness of the grains to one another and to the wall of the gun increases the possibility of the occurrence of causes which result in the failure of the powder to burn by parallel layers.

These considerations were at one time of significance to the improvement of ideas on the actual combustion of powder.

However, Charbonne could not prove experimentally a number of the very probable assumptions on ignition which he put forth.

Considerably more complete investigations of the combustion of powder were conducted in the experiments of N.E. Serebriakov [4] (1924-1928), who developed the conical crusher of high sensitivity to small pressures, which permitted the registration of pressure increase beginning with 5 to 7 kg/cm^2.

The [illegible] conical crusher, [illegible] a complete [illegible] curve from [illegible] of [illegible] of the [illegible] and the [illegible] of [illegible] developed by [illegible], by evaluation [illegible] obtained in [illegible] to [illegible]

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to [illegible] on the basis of a series of experiments, the non-[illegible] of powder ignition and its dependence on the [illegible] the igniter gas pressure, as well as on the properties of [illegible] itself.

[illegible]ding a graphic representation of the intensity of gas formation, this methodology permitted the discovery of a series of [illegible] from the geometrical law of powder combustion, and [illegible] explaining them and showing that the [illegible] to be [illegible] more complex than it appears on the basis of the [illegible] of the geometrical law.

[illegible] discovered that the mass of powder [illegible] to layer, and that the exterior layers [illegible] at a higher rate [illegible] interior ones.

[illegible] shown by special experiments that the [illegible] depends not only on the form of the [illegible], but also [illegible] reciprocal situation of burning [illegible] they [illegible] situated to each other, the more [illegible] the [illegible] proceed.

[illegible]liarities of the combustion of flegmatized [illegible] with common powders, were shown on the [illegible] methodology. A method was also [illegible] of flegmatizer penetration into [illegible].

[illegible] character of the combustion [illegible] deep below, in which an increased [illegible] in the general intensity of gas [illegible] by experimental investigation. An [illegible] of these [illegible] resulted in the establishment of a [illegible] theory of

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irregular combustion of such powders, which took into consideration the influence of the varied intensity of combustion of individual elements of the charge.

The theory of irregular combustion shows that the difference in intensity of gas formation in the interior of powder holes, and on the exterior surface of the powder, depends on the density of loading, and varies during the combustion of powder in a gun in accordance with the motion of the projectile.

Since the discovered anomalies were not explainable by the assumptions of the geometrical law, but were caused by peculiarities of the physical-chemical properties of the powder or physical properties of powder gases, the sum total of data representing the actual law of gas formation were called the "physical law of combustion" [5]. For details see Section III.

Muraur [6] approached the process of combustion from the standpoint of physical chemistry, and introduced the following assumptions as a basis for a scheme of powder combustion:

1) Powder burns as a result of attaining its temperature of decomposition by virtue of the impact of previously formed gas molecules.

2) In the gaseous layer directly adjacent to the burning surface of the powder (contact layer), the reactions are still incomplete (presence of NO, which still fails to react with CO and H_2). The temperature of this contact layer is lower than the temperature of explosion obtainable by way of calculation. The rate of powder combustion depends on the temperature of this particular layer at a given pressure.

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3) The reactions end in layers of gas more distant from the surface of the powder, and the layers of gases become increasingly warmer.

4) Upon contact with the cool wall of the bomb, the gases transfer a portion of their heat and their temperature decreases.

The same author attempted to determine the influence of the radiant energy of powder gases on the heating up of powder at low densities of loading and pressures.

During recent years (from 1939 on), a number of studies of our science professors Ia. B. Zeldovich, A.F. Belinev and D.A. Frank-Kamenetski [8] were devoted to the problems relating to the mechanism of the combustion of powders and other explosive substances.

Beginning with the investigation of volatile liquid explosives, and having evolved a theory of their combustion, Professor Zeldovich applied that theory to the combustion of powders [9].

This so-called "thermal" theory considers the combustion of powder to be the result of the heating of its surface to the temperature of decomposition, with a consequent conversion of the powder into gases, and an increase of the temperature of these gases to the temperature of combustion.

In this connection, the rate of combustion depends on the temperature to which the powder is heated by the action of the hot gases surrounding it, while the depth of the heating and the increase of temperature depend on the rate of combustion, which is itself dependent on the pressure of the gases.

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A characteristic of this theory is the detailed chart of thermal energy distribution between the gases and the powder, and the extensive explanation of the significance of the initial temperature of decomposition, of the heating value and heat conductivity of the powder.

Even though this theory cannot be considered entirely complete, it still remains of interest, in view of the fact that it gives details on and perfects our ideas on the process of powder combustion.

8. THE MODERN THEORY OF POWDER COMBUSTION

(According to Ia. B. Zeldovich)

In accordance with contemporary views, the combustion of powder occurs as follows.

The nitrocellulose in the surface layer of the powder decomposes. The products of the decomposition come out on the surface (process of gasification) and react in the gaseous phase, increasing the temperature of the gases greatly. The temperature on the surface of the powder is now relatively low, and corresponds to the primary decomposition of nitrocellulose. The temperature distribution in the mass of the powder and in the gases forming during its combustion, is shown by the diagram of fig. 10.

T_0 - the temperature in the interior of the powder.

T_p - the temperature on the surface of the powder.

T_g - the temperature of combustion (temperature of the gases).

Chemical reactions occur in the shaded areas. In zone 1, gasification; in zone 2, reaction of gases liberated from the powder (reaction of combustion).

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The burning surface of the powder has a heated layer (x_g), whose thickness depends on the temperature conductivity of the powder and its rate of combustion. The decomposition reaction proceeds in a portion of this layer x_r. One of the fundamental tasks of the theory of powder combustion is to determine the relations existing between the rate of powder combustion and the kinetics of the chemical reaction.

The surface layer temperature value is important not only for determining the relations existing between the kinetics of gasification and the rate of combustion, but also for the investigation of problems connected with the ignition of the powder, unstable combustion etc.

In the tests of O.I. Leipunskii and V.I. Aristova, the following results were obtained in combustion of powder at atmospheric pressure:

For pyroxylin powder T_p = [illegible] ± 45°C = 528 ± 45°K

For nitroglycerin powders T_p = 330 ± 45°C = 603 ± 45°K

Fig. 10 - Distribution of Temperatures During Combustion of Powder (Zeldovich)

1) Powder; 2) gases.

The decomposition of pyroxylin does not occur in the entire surface layer, but only in that part of it where the temperature approaches T_p. The thickness of its x_r comprises only about 5% of the thickness of the heated-up layer x_g.

Below, certain characteristics of powders are listed in accordance with the data of O.I. Leipunskii (Table 7).

Table 7

Specimen of Powder	Calorific Value of Powder cal/g °C	Temperature Conductivity cm^2/sec.	Heat Conductivity cal/cm-sec. °C	Rate of Combustion at p = 1 kg/cm^2 mm/sec.
Pyroxylin	0.29	$1.2 \cdot 10^{-3}$	$5.5 \cdot 10^{-4}$	0.51
Nitroglycerin	0.34	$0.87 \cdot 10^{-3}$	$4.8 \cdot 10^{-4}$	0.45

CHAPTER 2 - CHARACTERISTIC EQUATION OF POWDER GASES

(Dependence of Powder Gases' Pressure on the Conditions of Loading)

During the combustion of powder, a large quantity of gases is formed. They have a high temperature and exert high pressure on the wall of the gun in which the combustion occurs. The general principles of physics and thermodynamics are applicable to these gases. Therefore the characteristics equation expressing the relations between pressure, temperature and specific volume of gases should also be valid for this case.

For "ideal" gases, whose molecules do not have a volume and fail to attract each other, or for sufficiently rarefied gases, the physical form of the equation for unit weight is expressed by the Klapeiron formula:

$$pv = RT,$$

if p is the pressure of gases;

v is the specific volume at $t = 0°C$ and a pressure of 1 atm;

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$T^{o} = 273 + t^{o}$ - absolute temperature of gases;

R is the gaseous constant.

Since in a combustion of powder in the manometric bomb, the gases have a very high density, then the following Van-der-Vaals equation for real gases should serve as the equation of their state of aggregation:

$$\left(p - \frac{a}{v^2}\right) (v - b) = RT,$$

where b is the characteristic of molecule volume, and a is the characteristic of cohesion of gas molecules.

The latter force (of cohesion) may be disregarded at higher temperatures, so that the equation will then be written in this simplified form:

$$p (v - b) = RT.$$

In this latter form, the equation is accepted in interior ballistics. This equation pertains to a unit of weight of gases.

If ω kg of powder is burned in the space W_0 and is converted entirely into gases whose temperature is equal to the temperature of combustion T_1, then the preceding equation will be written in this manner:

$$p\left(\frac{W_0}{\omega} - b\right) = RT$$

or

$$p (W_0 - \omega b) = \omega RT_1,$$

whence
$$p = \frac{\omega RT_1}{W_0 - b\omega} \qquad (1)$$

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The formula of Van-der-Vaals was evolved theoretically in the 1870's as a result of the assumptions of the kinetic theory of gases.

Investigations of powder combustion, and of gases forming during such process, were begun by Gay-Lussac (1824), and were continued by Bunsen and Shipkov (1850). These investigations were conducted at pressures around 1 atm. However, combustion in guns occurs at higher pressures. Therefore, studies began in the second half of the 19th century on the investigation of powder combustion at higher pressures in manometric bombs.

Detailed experimental investigations (initiated in 1868 and completed in 1880) were conducted in England by Nobel and Abel.

Some data of these experiments are quoted in the "Interior Ballistics" of A.F. Brink [10], and also in the study of I.P. Grave entitled "Pyrostatics" [11].

The experiments were conducted in manometric bombs, with a limitation of the maximum pressure by means of a cylindrical crusher, and included measurements of the volume of gases and an analysis of their composition. Black powders which were at that time used for military purposes, formed the object of investigation at loading densities $\Delta = 0.10$–0.90.

The experiments clarified the composition of the products of powder combustion, the quantity of gaseous products and their volume at 0°C and 760 mm, the quantity of non-gaseous products and the physical form in which they are found at the moment of explosion, the quantity of heat Q_w, the mean heat capacity of

decomposition products in a constant space, the temperature of decomposition products, the relations between the pressure and density of loading, the variations occurring in the products of decomposition with a change in the density of loading, the influence of the chemical composition of the powder on the resultant products of decomposition, heat and pressure, and also the effects of grain dimensions, their consistency and humidity content, etc.

Experiments on the clarification of the dependence of the maximum pressure on the density of loading, formed an object of special interest for interior ballistics.

1. THE FORMULA FOR THE MAXIMUM PRESSURE OF GASES

After conducting a large number of experiments, plotting the values for maximum pressure p_m and density of loading Δ in a diagram, drawing the curve p_m, Δ through the points obtained, and fitting an equation to this curve, Nobel and Abel established the following empirical relation between the density of loading Δ and the maximum pressure p_m:

$$p_m = \frac{f\Delta}{1 - \alpha\Delta}. \qquad (8)$$

In this formula, f and α are constant coefficients determined as a result of a number of experiments at various Δ.

Assuming $\Delta = 1/1 + \alpha$, we obtain:

$$p_m = \frac{f\dfrac{1}{1+\alpha}}{1 - \dfrac{\alpha}{1+\alpha}} = f.$$

At the first glance, we conclude that the value f has the dimensions of pressure. That type of dimension may be encountered in older courses of interior ballistics, under the theory of explosive

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substances. Actually, as it will be shown later, they express the work capacity of the powder gases.

The value f was termed the "energy" of the powder.

The value α represented the volume of liquid and solid residuals in the combustion products of black powder.

The proper physical meaning of the coefficients f and α is clarified by a comparison of formula (2) and a simplified formula (1).

In formula (2) we substitute in the place of Δ its meaning ω/W_0, and convert it by multiplication of the numerator and denominator by W_0. We obtain:

$$p_m = \frac{f\Delta}{1 - \alpha\Delta} = \frac{f\omega}{W_0 - \alpha\omega}.$$

Formula (1) has the following form for ω kg of gases:

$$p_m = \frac{\omega R T_1}{W_0 - b\omega}.$$

By comparing two formulas obtained experimentally and theoretically, we find that they are identical at $f = RT_1$ and $\alpha = b$. In formula (2), the value α represents the volume of gas molecules, as does value b in formula (1).

The physical meaning of the value f, the so-called energy of the powder, is explained by the expression

$$f = RT_1.$$

It is known from thermodynamics that the value R represents the work which the gas performs, if we heat it 1° at atmospheric pressure $p_a = 1.033$ kg/cm^2:

$$R = \frac{p_a w_1}{273}.$$

Since 1/273 is the coefficient of gas expansion when it is heated 1°, then $w_1/273$ is the expansion of the volume w_1 when the gas is heated 1°, while the product of p_a by $w_1/273$ is the work that is done by 1 kg of gases when heated up to T_1° at a constant pressure $p_a = 1.033$ kg/cm².

Consequently, $f = RT_1 = p_a w_1 T_1/273$ expresses the work which can be performed by 1 kg of powder gases, expanding during heating up to T_1° at a constant pressure $p_a = 1.033$ kg/cm².

Substituting values f in kg-dm/kg, α in dm³/kg, and $\Delta = \omega/W_0$, kg/dm³ in formula (2), we obtain the value of the pressure:

$$p_m = \frac{f\Delta}{1-\alpha\Delta} = \frac{\frac{kg\text{-}dm}{kg}\cdot\frac{kg}{dm^3}}{\text{abstract}\ \frac{dm^3}{kg}\cdot\frac{kg}{dm^3}} = \frac{kg}{dm^2};$$

the values f and α depend on the characteristics of the powder; f on w_1 and T_1, and $\alpha \approx 0.001 w_1$.

The Nobel formula, evolved on the basis of experiments with black powder, was further verified by other research scientists, for smokeless powders and for other explosives.

[illegible] formula was [illegible] empirically

for [illegible]

[illegible] express

the [illegible] divides

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dependence of p_m on Δ, represents a portion of a hyperbola, since upon elimination of the denominator it has the form $p_m - \alpha \Delta p_m - f\Delta = 0$ and the discriminant $a_{11}a_{22} - a_{12}^2 = -\alpha^2 < 0$.

If we assume that the formula is correct for points obtained further on in the experiment, then the curve p_m, Δ goes out into infinity, having approached an asymptote parallel to the axis $y(p_m)$ at $\Delta = 1/\alpha$.

Then

$$1 - \alpha\Delta = 0 \text{ and } p_m = \infty.$$

If we show the dependence of p_m on Δ in a graph, by plotting Δ on the axis of the abscissa and p_m on the axis of ordinates, then we obtain a curve p_m, Δ (fig. 11) passing through the origin of coordinates and having an asymptote in the form of a straight line running parallel to the axis of ordinates at a distance of $1/\alpha$ from the origin. Negative values are not discussed, in view of the fact that they do not have a physical meaning.

Since α is approximately equal to unity for pyroxylin powders, the critical value of Δ, at which p_m should be equal to infinity, is equal to one. For nitroglycerin powders, $\alpha \approx 0.8$ and, consequently, critical $\Delta = 1/0.8 = 1.25$.

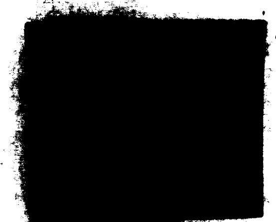

Fig. 11 - Dependence of p_m on Δ, in Accordance with Formula (2)

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In practice, densities of loading not higher than 0.25 are selected for combustion of powder in a bomb, because the resultant pressure in such a case is 2600 to 3000 atm. In extreme cases Δ up to 0.40 are selected, where p_m = 5500 atm. The gravimetric density of the most "[illegible]" rifle powder, in the form of small cylinders with one hole through them, amounts to 0.85-0.90, i.e., less than the required critical value. In this case, the calculated pressure would amount to about 50,000 kg/cm^2.

It is possible to obtain this type of density of loading, and even higher, in practice, by pressing the powder in the form of one compact cylinder fitting to the shape of the vessel in which the combustion occurs. In this instance, the density of loading $\Delta = \delta$ = the specific weight of the powder. At a density of loading higher than Δ, the process will change into an explosion even prior to the end of the powder combustion.

The dependence p_m, Δ can be represented in the form of a straight line, which can be used advantageously to determine two basic ballistic characteristics: the energy of the powder f and the covolume α.

By converting the equation (2), we obtain:

$$\frac{p_m}{\Delta} = f + \alpha p_m. \qquad (3)$$

If we accept p_m for x and p_m/Δ for y, then in the new system of coordinates formula (3) will be expressed by the lineal equation

$$y = f + \alpha p_m$$

where f represents the [illegible] intercept [illegible] of ordinates by the straight line; [illegible] of this straight line or the tangent of the angle γ, formed by this line with the axis x.

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Conducting a series of experiments with several Δ, and having obtained corresponding magnitudes of p_m, we find the ratio p_m/Δ, and we plot the points corresponding to each density of loading on axes p_m/Δ, and p_m of the graph (fig. 12). These points should be located on one straight line. Prolonging this straight line 1-1 to its intersection with the axis of ordinates, we find the energy f. The tangent of the angle formed by this straight line with the axis Δ gives the value of covolume α. Vel conducted experiments with a whole series of explosive substances and powders, and has constructed typical straight lines for them.

Fig. 12 - Dependence of p_m/Δ on p_m, in accordance with formula (3)

Fig. 13 - Characteristic Straight Lines of Three Basic Powders

If we take mean values of f and α for the common powders, then we [illegible] characteristic straight lines on the graph p_m/Δ, p_m [illegible]

[illegible]; α = 1.

[illegible],000; α = 0.8.

[illegible]5.

[illegible] for a number of other

2. DETERMINATION OF THE ENERGY OF POWDER AND OF THE COVOLUME OF POWDER GASES

The magnitudes f and α can be determined analytically and graphically.

We have a linear equation with two constant coefficients f and α:

$$\frac{p_m}{\Delta} = f + \alpha p_m. \qquad (3)$$

In order to determine f and α by the aid of this equation, it is necessary to conduct experiments with two densities of loading. Then, the values p_m and p_m/Δ will be known.

If in an experiment, at a density of loading Δ_1, the resultant pressure was p_1, while at a density of loading Δ_2, the pressure was p_2, we have a system of two equations:

$$\frac{p_2}{\Delta_2} = f + \alpha p_2; \qquad (a)$$

$$\frac{p_1}{\Delta_1} = f + \alpha p_1. \qquad (b)$$

Subtracting the terms of one equation from the other, we obtain:

$$\frac{p_2}{\Delta_2} - \frac{p_1}{\Delta_1} = \alpha (p_2 - p_1),$$

thus

$$\alpha = \frac{\dfrac{p_2}{\Delta_2} - \dfrac{p_1}{\Delta_1}}{p_2 - p_1} \qquad (4)$$

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and

$$f = \frac{p_1}{\Delta_1} \cdot \frac{p_2}{\Delta_2} \frac{\Delta_2 - \Delta_1}{p_2 - p_1}. \qquad (5)$$

In place of this equation, it is simpler to determine the value of f by substituting the obtained numerical value for α in equations (a) and (b). Obtaining an identical magnitude of f from either of the two equations serves as a verification of the correctness of calculations of f and α:

$$f = \frac{p_1}{\Delta_1} - \alpha p_1 = \frac{p_2}{\Delta_2} - \alpha p_2. \qquad (6)$$

The diagram on fig. 14 gives a graphical illustration of the application of derived equations (4) and (5) for the determination of f and α:

$$oa = p_1;\ ob = p_2;$$

$$ae = \frac{p_1}{\Delta_1};\quad bg = \frac{p_2}{\Delta_2};$$

$$gf = \frac{p_2}{\Delta_2} - \frac{p_1}{\Delta_1};\ ef = p_2 - p_1;$$

$$\alpha = \frac{gf}{ef} = \frac{\frac{p_2}{\Delta_2} - \frac{p_1}{\Delta_1}}{p_2 - p_1};$$

$$ec = \alpha \cdot p_1;\ gd = \alpha \cdot p_2;$$

$$f = oh = ae - ec = \frac{p_1}{\Delta_1} - \alpha p_1;$$

$$f = oh = bg - dg = \frac{p_2}{\Delta_2} - \alpha p_2.$$

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Thus, formula (4) for α and formula (6) for f both have a simple graphic interpretation.

In this manner, in order to determine the energy of powder f and the covolume α, it is necessary to conduct powder combustion experiments in a bomb at two densities of loading, and then to determine f and α either by use of formulas (4) and (6) or graphically. In this connection, it is not permissible to select densities of loading very close to each other, because the results would be less reliable and a larger error is possible. The best conditions exist when $\Delta_2 - \Delta_1 \sim 0.10$. For instance, for determining f and α for pyroxylin powders, it is advisable to conduct experiments at $\Delta = 0.15$ and 0.25, and for stronger nitroglycerin powders at $\Delta = 0.12$, and 0.20 or 0.22.

At densities of loading below 0.10, it is also possible to obtain unreliable results, because the linear dependence fails at pressures $p_m < 1000$ atm, and the points p_m/Δ, p_m are situated below the straight line: the lower they are, the smaller p_m is. This occurrence is explained by greater losses in heat emission through the walls of the bomb, because the combustion of powder takes place slowly at low loading densities. For the same reason, the resultant pressures are lower than in the case where loss of heat through the walls did not occur.

It will be shown below how losses through heat transfer are calculated in a determination of f and α.

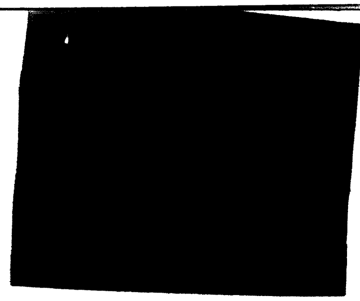

Fig. 14 - Graphic Determination of the Powder Energy and of the Covolume

In addition to determination of the values f and α on the basis of experimental data, the Nobel and Abel formula is also applicable to the following cases:

1) Knowing f and α, a calculation of p_m is undertaken by the aid of the value Δ.

$$p_m = \frac{f\Delta}{1 - \alpha\Delta} = \frac{f}{\frac{1}{\Delta} - \alpha} .$$

2) Knowing f and α, we undertake to calculate, by the aid of p_m, the Δ at which a given pressure would result.

Solving the equation for Δ , we obtain:

$$\Delta = \frac{p_m}{f + \alpha p_m} = \frac{1}{\frac{f}{p_m} + \alpha} .$$

The formula cited is used for solutions of a number of practical problems. For instance, at a given f and α, it can be determined what Δ should be in order to obtain a given magnitude of pressure (assume $\sim$ 3000), so that we may know the densities of loading for which powder may be burned in a bomb with an effective pressure of 3000

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atm. Or, for instance, to calculate the pressure produced by an igniter of a given weight in a chamber of a given volume, containing a charge of a given weight.

3. NUMERICAL EXAMPLES

In order to better explain the method of determining f and α, the following examples are given.

In solving examples, it is necessary to express all magnitudes by consistent units. The most used system of units: kilogram - decimeter - second.

Example 1. To determine f and α on the basis of the following data:

$$\Delta_1 = 0.15;\ p_1 = 1{,}470\ \text{kg/cm}^2 = 147{,}000\ \text{kg/dm}^2;$$

$$\Delta_2 = 0.25;\ p_2 = 2{,}750\ \text{kg/cm}^2 = 275{,}000\ \text{kg/dm}^2$$

We find the ratios $\frac{p_1}{\Delta_1}$ and $\frac{p_2}{\Delta_2}$.

$$\frac{p_2}{\Delta_2} = \frac{275{,}000}{0.25} = 1{,}100{,}000\ \text{kg-dm/kg}$$

$$\frac{p_1}{\Delta_1} = \frac{147{,}000}{0.15} = 980{,}000\ \text{kg-dm/kg}$$

$$\frac{p_2}{\Delta_2} - \frac{p_1}{\Delta_1} = 120{,}000;\ p_2 - p_1 = 128{,}000.$$

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We determine α:

$$\alpha = \frac{\frac{p_2}{\Delta_2} - \frac{p_1}{\Delta_1}}{p_2 - p_1} = \frac{120,000}{128,000} = 0.938.$$

f may be determined from the basic equation, by substituting in it the expression found for α.

$$f = \frac{p_1}{\Delta_1} - \alpha p_1.$$

$$\frac{p_1}{\Delta_1} = 980,000; \; \alpha p_1 = 0.938 \cdot 147,000 = 137,900.$$

$$f = 980,000 - 137,900 = 842,100 \text{ kg-dm/kg}$$

However, if we determine it in accordance with the general equation, then we obtain:

$$f = \frac{p_1}{\Delta_1} \cdot \frac{p_2}{\Delta_2} \frac{\Delta_2 - \Delta_1}{p_2 - p_1} = 980,000 \cdot 1,100,000 \frac{0.10}{128,000} = 842,100 \text{ kg-dm/kg}$$

Example 2. Given is $f = 850,000$ kg-dm/kg; $\alpha = 0.96$ dm^3/kg. To determine the Δ, at which the resultant $p_m = 3,200$ kg/dm^2 = 320,000 kg/dm^2. From the basic equation, we find the expression for Δ:

$$\Delta = \frac{p_m}{f + \alpha p_m}$$

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We substitute:

$$\Delta = \frac{320,000}{850,000 + 0.95 \cdot 320,000} = \frac{320,000}{850,000 + 304,000} = \frac{320,000}{1,154,000} = 0.2772$$

Exercise. A bomb withstands $p_m = 4000$ kg/cm^2. Determine the maximum permissible density of loading for each type of powder, pyroxylin, nitroglycerin and black, using data in the table for fig. 13.

4. THE EFFECT OF ERROR, IN DETERMINING PRESSURES p_1 AND p_2, ON ERRORS IN DETERMINING f AND α

Assume that errors δp_1 and δp_2 were made in determining the values for pressures p_1 and p_2. We will find the corresponding errors δf_1, and δf_2, also $\delta \alpha_1$ and $\delta \alpha_2$.

a. Errors in Determining f

For the purpose of investigation, we will take the equation

$$f = \frac{p_1}{\Delta_1}\,\frac{p_2}{\Delta_2}\,\frac{\Delta_2 - \Delta_1}{p_2 - p_1} \qquad \text{(a)}$$

$$\alpha = \frac{\dfrac{p_2}{\Delta_2} - \dfrac{p_1}{\Delta_1}}{p_2 - p_1}\,. \qquad \text{(b)}$$

We differentiate the expression (a) with respect to p_1, regarding all other values as constants:

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$$\delta f_1 = \frac{p_2}{\Delta_2}\,\frac{\Delta_2 - \Delta_1}{\Delta_1}\,\frac{p_2}{(p_2 - p_1)^2}\,\delta p_1; \qquad (c)$$

Dividing (c) by (a), we obtain

$$\frac{\delta f_1}{f} = \frac{p_2}{p_1}\,\frac{\delta p_1}{(p_2 - p_1)} = \frac{p_2}{(p_2 - p_1)}\,\frac{\delta p_1}{p_1}. \qquad (7)$$

Similarly:

$$\delta f_2 = \frac{p_1}{\Delta_1}\,\frac{\Delta_2 - \Delta_1}{\Delta_2}\,\frac{-p_1}{(p_2 - p_1)^2}\,\delta p_2; \qquad (d)$$

$$\frac{\delta f_2}{f} = -\frac{p_1}{p_2}\,\frac{\delta p_2}{(p_2 - p_1)} = -\frac{p_1}{(p_2 - p_1)}\,\frac{\delta p_2}{p_2}. \qquad (8)$$

An analysis of equations (7) and (8) shows that one and the same absolute error $\delta p_1 = \delta p_2$ exerts a varied influence on corresponding errors in the determination of the powder energy f.

The error $+\delta p_1$ at a lower density of loading Δ_1, increases the energy f; while the error $+\delta p_2$ at a higher density of loading decreases the powder energy f. At the same time, the effect of the magnitude δp_1 at a low Δ_1 is greater than the effect of δp_2 at a higher Δ_2, because in the first case at $\frac{\delta p}{p_2 - p_1}$ the multiplier is of the magnitude $\frac{p_2}{p_1} > 1$, while in the second case it is $\frac{p_1}{p_2} < 1$. Moreover,

both errors of determination increase with a reduction of the difference between the loading densities Δ_1 and Δ_2, which entails a reduction of the denominator $p_2 - p_1$ and an increase of $\delta f/f$. Thus, in order to reduce the error in determining f, it is necessary to increase the difference $p_2 - p_1$, selecting whenever possible a greater difference between Δ_2 and Δ_1. Still, a large error in the values for pressure p_1 results at too low Δ (<0.10), by virtue of losses in heat transfer. Therefore in practice $\Delta_1 = 0.15$ and $\Delta_2 = 0.25$ are selected for determination of the energy of pyroxylin powders, while $\Delta_1 = 0.12$ and $\Delta_2 = 0.20$–0.22 are taken for nitroglycerin powders.

b. Errors in Determining α

By differentiating the expression (b), we obtain:

$$\delta\alpha_1 = \frac{\frac{1}{\Delta_1} - \frac{1}{\Delta_2}}{(p_2 - p_1)^2}\, p_2 \delta p_1 = -\frac{f}{p_2 - p_1}\,\frac{\delta p_1}{p_1}; \qquad (9)$$

$$\delta\alpha_2 = \frac{\frac{1}{\Delta_1} - \frac{1}{\Delta_2}}{(p_2 - p_1)^2}\, p_1 \delta p_2 = \frac{f}{p_2 - p_1}\,\frac{\delta p_2}{p_2}. \qquad (10)$$

Contrary to δf_1 and δf_2, the error $\delta\alpha_1 < 0$ and $\delta\alpha_2 > 0$. At given magnitudes of Δ_1, Δ_2, p_1 and p_2, and at $\delta p_1 = \delta p_2$, the error ($\delta\alpha_2 > \delta\alpha_1$) since $p_2 > p_1$.

It is evident from a comparison of expressions (7) and (9), as well as (8) and (10), that one and the same error δp_1, or δp_2, affects f and α in opposite ways.

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Dividing (7) by (9) and considering the equality (a), we obtain:

$$\frac{\delta f_1}{f} = -\frac{p_2 - p_1}{\Delta_2 - \Delta_1}\,\frac{\Delta_2}{p_2}\,\frac{\Delta_1}{p_1}\,p_2\,\delta\alpha_1 = -\frac{p_2}{f}\,\delta\alpha_1 = -\frac{\delta\alpha_1}{\frac{1}{\Delta_2} - \alpha} \qquad (11)$$

and

$$\frac{\delta f_2}{f} = -\frac{\delta\alpha_2}{\frac{1}{\Delta_1} - \alpha}. \qquad (12)$$

It follows from a comparison of equations (11) and (12) that the error $\delta\alpha_2$, of the magnitude α, at a higher density of loading Δ_2 produces a smaller error in the magnitude f than does the error $\delta\alpha_1$ at a lower density of loading.

5. PRESSURE DURING THE INTERMEDIATE MOMENT. GENERAL FORMULA OF PYROSTATICS

The Nobel equations applies to the instant of attaining maximum pressure, when all the powder is burnt. For the intermediate moment, when all of the powder is not as yet burnt, but only a portion ψ of it is converted into gases, we use the physical state equation for $\omega\psi$ kg of gases:

$$p_\psi W_\psi = RT_1\omega\psi = f\omega\psi,$$

where the index ψ indicates that the given magnitude corresponds to the intermediate moment in which the portion of the charge ψ is burned and converted into gases.

W_ψ, the free volume of the bomb at the given moment, is equal to the volume W_0, of the bomb after the deduction of the volume of the still unburnt powder $\omega(1-\psi)/\delta$ and of the covolume of gases of the burnt powder $\alpha\omega\psi$.

$$W_\psi = W_0 - \frac{\omega}{\delta}(1-\psi) - \alpha\omega\psi.$$

In this way, the intermediate pressure for the moment in which

the portion of the charge ψ will burn out, will be found on the basis of the formula

$$p_\psi = \frac{f\omega\psi}{W_\psi} = \frac{f\omega\psi}{W_0 - \frac{\omega}{\delta}(1-\psi) - \alpha\omega\psi} . \qquad (13)$$

Combining the terms with ψ in the denominator, we write the general pyrostatics equation or the physical state equation for the intermediate moment in the second form:

$$p_\psi = \frac{f\omega\psi}{W_0 - \frac{\omega}{\delta} - \omega\left(\alpha - \frac{1}{\delta}\right)\psi} .$$

Dividing the denominator and the numerator by W_0, and replacing ω/W_0 by Δ, we obtain equation (4) in the following form:

$$p_\psi = \frac{f\Delta\psi}{1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi} = \frac{f\psi}{\frac{1}{\Delta} - \frac{1}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\psi} . \qquad (14)$$

Inserting given expressions of ψ, we can calculate the corresponding expressions of p_ψ on the basis of this equation.

If we assume $\psi = 1$ in equation (14), then it converts to a general equation:

$$p_1 = \frac{f\Delta}{1 - \alpha\Delta} .$$

The general pyrostatics equation, characterizing the magnitude

of the pressure in a combustion of powder, shows that, according to the degree of powder combustion completed, the free space W_ψ (in the denominator) increases to the value $\omega\psi/\delta$ as a result of liberation of the space from the combustion of powder, and decreases as a result of the addition of the molecular volume of formed gas (covolume) $\alpha\omega\psi$ (fig. 15).

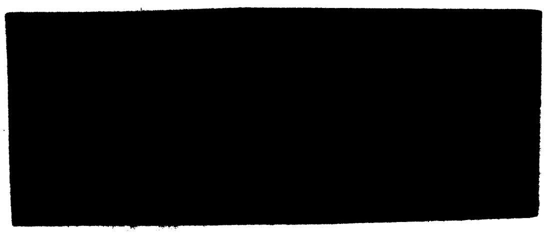

Fig. 15 – Scheme of Variations in the Free Volume of the Bomb During a Combustion of Powder

1) beginning of combustion; 2) intermediate moment; 3) end of combustion.

Investigation shows that in the final count, the free volume decreases with the progress of combustion. Consequently, the pressure p_ψ does not increase proportionately to the burnt fraction of ψ, but faster.

Actually, at the beginning of the combustion, at $\psi = 0$ and a density of loading Δ, the free space in the chamber (or in the bomb) $W_\Delta = W_0 - \omega/\delta$. At the end of the combustion, at $\psi = 1$, the free space $W_1 = W_0 - \alpha\omega$

Since for pyroxylin and nitroglycerin powders

$$\alpha > \frac{1}{\delta}\left(\alpha = 1.0 - 0.8, \quad \delta = 1.6 \text{ and } \frac{1}{\delta} = 0.625\right),$$

then

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$$\alpha\omega > \frac{\omega}{\delta} \text{ and } W_0 - \alpha\omega < W_0 - \frac{\omega}{\delta};$$

$$[illegible] > W_\psi > W_1,$$

i.e., the free [illegible] of the chamber, or of the bomb, is smaller at the end of the [illegible] than at the beginning of it.

6. [illegible] OF ψ ON p

The general [illegible] of pyrostatics is of great importance [illegible] interior ballistics. Namely, establishing relations between the burnt fraction of [illegible] charge ψ and the pressure at that instant, it also permits solving the inverse problem: to determine, on the basis of the magnitude of pressure at a given moment, what portion of the charge ψ [illegible] completely burnt up to that moment. The [illegible] data are of importance to the gas formation characteristic of the powder.

Actually, by selecting pressure magnitudes over given intervals of time, we can calculate the corresponding magnitudes of ψ on the basis of the curve which we obtained in the test with the [illegible] of pressure as a function of time. Consequently, we are in a position to judge the variations of the burnt fraction of the charge relative to time, and the rate of gas formation.

Solving equation (1d) for ψ, we obtain

$$\psi = \frac{\frac{1}{\Delta} - [illegible]}{\frac{f}{p_\psi} + \alpha - \frac{1}{\delta}} = \frac{p_\psi\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)}{f + p_\psi\left(\alpha - \frac{1}{\delta}\right)} \quad (1[illegible])$$

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Replacing f by $p_m\,(1 - \alpha\Delta)/\Delta$, and subtracting and adding $1/\Delta$ in the denominator, we can modify equation (15) to this form:

$$\psi = \frac{1}{1 + \dfrac{1 - \alpha\Delta}{1 - \dfrac{\Delta}{\delta}}\,\dfrac{p_m - p_\psi}{p_\psi}}, \qquad (16)$$

where p_m is the maximum gas pressure in the given experiment.

In this equation, the ratio $(1 - \alpha\Delta)/(1 - \Delta/\delta)$ is a value constant for a given experiment and characterizing the conditions of charging (we will designate it by ∂). p_m is also constant, with only p_ψ varying.

The magnitude $\partial = (1 - \alpha\Delta)/(1 - \Delta/\delta)$ represents the ratio of the free space of the bomb at the end of the combustion $W_1 = W_0\,(1 - \alpha\Delta)$ to its free space at the beginning of the combustion $W_\Delta = W_0\,(1 - \Delta/\delta)$. This value is always less than unity; at $\Delta = 0.25$, $\alpha = 1$, and $\delta = 1.6$, $\partial = 0.89$. At smaller Δ, the magnitude ∂ approaches unity.

7. CONSIDERATION OF THE INFLUENCE OF THE IGNITER

The general pyrostatics equation determines the pressure developed in a constant space by gases formed during the combustion of powder. In this connection, atmospheric pressure is disregarded, because of its smallness as compared with the pressure of powder gases.

In Nobel's experiments with black powders, black powder was also used for the igniter, and the weight of the igniter was included in the over-all weight of the charge for the purpose of calculating the density of loading.

Usually, during experiments in a bomb or discharges from guns,

smokeless powder is ignited by means of a different type igniter (black powder, pyroxylin). Burning itself out first, the igniter produces a certain pressure, and ignites the powder through its heated gases and glowing particles. The powder begins to burn at a certain initial pressure p_B developed by the gases of the igniter.

We adopt the designations: weight of igniter charge $= \omega_B$, igniter force $= f_B$, covolume $= \alpha_B$, pressure of igniter gases prior to the beginning of powder combustion $= p_B$.

Calculation of the pressure p_B is made on the basis of Abel's equation, taking into consideration the space ω/δ occupied by the powder itself prior to its combustion:

$$p_B = \frac{f_B\omega_B}{W_0 - \frac{\omega}{\delta} - \alpha_B\omega_B} \approx \frac{f_B\omega_B}{W_0 - \frac{\omega}{\delta}} = \frac{f_B\Delta_B}{1 - \frac{\Delta}{\delta}},$$

where $\Delta_B = \omega_B/W_0$. The magnitude for the covolume of igniter gases $\alpha_B\omega_B$ may be disregarded, in view of its smallness in comparison with the free space of the bomb $W_0 - \omega/\delta$.

For the intermediate moment in a combustion of the powder itself, the magnitude of pressure, with an allowance for the gases of igniter charge, is expressed in accordance with Dalton's law for the pressure of a gas mixture:

$$p_\psi = \frac{f_B\omega_B + f\omega\psi}{W_0 - \frac{\omega}{\delta}\left(1 - \psi\right) - \alpha\omega\psi - \alpha_B\omega_B}.$$

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Disregarding once more the magnitude $\alpha_B\omega_B$, as compared with the value for the free space $W_\psi = W_0 - \frac{\omega}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\omega\psi$, we obtain:

$$p'_\psi = \frac{f_B\omega_B + f\omega\psi}{W_\psi} = \frac{f_B\omega_B}{W_\psi} + p_\psi,$$

where p_ψ is the pressure of powder gases without allowance for the effect of the igniter, and is expressed by the general pyrostatics equation

$$p_\psi = \frac{f\omega\psi}{W_\psi}.$$

Since W_ψ diminishes, then $f_B\omega_B/W_\psi$ increases.

At the end of the combustion, the total pressure p'_m, with an allowance for the effects of the igniter, is expressed by the equation:

$$p'_m = \frac{f_B\omega_B + f\omega}{W_0 - \alpha\omega - \alpha_B\omega_B} \approx \frac{f_B\omega_B}{W_0 - \alpha\omega} + \frac{f\omega}{W_0 - \alpha\omega} = p'_B + p_m,$$

where

$$p'_B = \frac{f_B\omega_B}{W_0 - \alpha\omega} = \frac{f_B\Delta_B}{1 - \alpha\Delta} \quad \text{and} \quad p_m = \frac{f\omega}{W_0 - \alpha\omega} = \frac{f\Delta}{1 - \alpha\Delta}$$

([illegible]'s equation without consideration of the igniter).

Since $1 - \alpha\Delta < 1 - \Delta/\delta$ then $p'_B > p^*_B$.

Under the existing conditions of functioning in a manometric bomb ($\Delta < 0.25$; $\alpha \approx 1$; $\delta \approx 1.6$), [illegible] for smokeless powders is located in the range from 0.[illegible] to 1. That is, the difference between

p^*_B and p'_B does not exceed 10%. Since, generally speaking, the pressure of the igniter is small (from 20 to 50, and not more than 100 kg/cm^2), then it may be assumed with a sufficient degree of accuracy that $p'_B = p_B$ and, consequently, ψ_B may be regarded as a constant value.

Therefore, from here on we shall consider the following equality as applicable:

$$p_B = p^*_B = \frac{f_B \omega_B}{W_0 - \frac{\omega}{\delta}}.$$

In view of this assumption, which will greatly simplify all further calculations, the Nobel equation and the general [illegible] equation will be written in the following form:

$$p'_\Delta = p_B + \frac{f\Delta}{1 - \alpha\Delta} = p_B + \frac{f\omega}{W_0 - \alpha\omega},$$

where

$$p_B = \frac{f_B \omega_B}{W_0 - \frac{\omega}{\delta}} = \frac{f_B \Delta_B}{1 - \frac{\Delta}{\delta}};$$

$$p'_\psi = p_B + \frac{f\Delta\psi}{1 - \frac{\Delta}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\Delta\psi} = p_B + \frac{f\omega\psi}{W_0 - \frac{\omega}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\omega\psi}.$$

The equations show that the over-all pressure of gases, registered in a manometric bomb, is obtained as a sum of the

pressures: the pressure of igniter gases p_B, and the pressure of powder gases p_ψ; or p_m, determinable on the basis of the initial generalized pyrostatics equation, or Nobel's equation without consideration of the influence of the igniter.

Determining ψ from the last equation, we obtain the magnitude ψ taking into consideration the influence of the igniter:

$$\psi = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p' - p_B} + \alpha - \frac{1}{\delta}} = \frac{1}{1 + \frac{1 - \alpha\Delta}{1 - \frac{\Delta}{\delta}} \frac{p'_m - p'}{p' - p'_B}}$$

The included pressures p'_m and p' constitute the actual pressure registered by the crusher, or by any other instrument.

Since the equation renders calculations of a series of expressions for ψ very awkward in plotting an experimental pressure curve registered by the instrument, we have compiled tables for the purpose of expediting the work involved. From the magnitude of the ratio $(p' - p_B)/(p'_m - p_B)$, it is simple and easy to find, by the aid of these tables, the corresponding expressions of ψ as a function of time t, and to make a ballistic analysis of the powder.

The magnitude of the denominator $p'_m - p_B$ is constant for a given experiment; p' is taken from the measurement of the pressure curve; also from it, we mentally calculate the pressure p_B of the igniter gases, constant for all points of measurement. Then we determine, by the aid of a slide rule,

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$$\beta = \frac{p' - p_B}{p'_H - p_B} .$$

We make the conversion listed above.

$$\psi = \frac{p' - p_B}{p' - p_B + \partial (p'_H - p')} = \frac{p' - p_B}{p' - p_B + \partial [p'_H - p_B - (p' - p_B)]} =$$

$$= \frac{\dfrac{p' - p_B}{p'_H - p_B}}{\dfrac{p' - p_B}{p'_H - p_B}(1 - \partial) + \partial} .$$

At this time, the same ratio $\beta = (p' - p_B)/(p'_H - p_B)$ is introduced into the denominator and the numerator; and consequently ψ appears as a function of the constant magnitude ∂ and the variable β:

$$\psi = \frac{\beta}{\partial + (1 - \partial)\beta} .$$

The magnitude $\beta = (p' - p_B)/(p'_H - p_B)$ varies within the range from 0 to 1. The magnitude $\partial = (1 - \alpha\Delta)/(1 - \Delta/\delta)$ which depends on three constants α, δ and Δ (when α approaches 1, δ approaches 1.0), usually varies from 0.86 to 1, depending on the fluctuations of Δ, within the limits of 0.25-0.

The tables are arranged for each expression of ∂, from 0.86 to 0.97 by variations of 0.01, at 0.01 variations of the ratio $(p' - p_B)/(p'_H - p_B)$ within the range from 0 to 1.

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The arrangement of the table is analogous to tables of four-place logarithms.

The tables are in the supplement, which also includes instructions for their use.

Knowing the principle of ψ variation relatively to time, it is also possible to find the experimental law of variation of the magnitude $\frac{d\psi}{dt}$, i.e., <u>the rate of gas formation</u>. This value is one of the more important characteristics. Knowledge of it permits regulating the efflux of gases during a combustion of powder, and checking the law of gas pressure variation.

8. ON REDUCED LENGTHS

Among other things, the pressure in the bore of the gun appears as a function of the volume of initial air space, corresponding to the location of the projectile in the bore at a given instant. At the beginning of the powder combustion, this volume W is equal to the volume of the chamber W_0. Subsequently, it increases with the motion of the projectile to a volume equal to the volume of a cylinder having the cross section of the gun bore (including grooves) as its base, and the length of the travel l of the projectile as its height:

$$W = W_0 + sl.$$

Since, in practice, pressure is commonly expressed as a function of the travel accomplished by the projectile, rather than of the volume, it is more convenient to replace all volumes by corresponding lengths. However, in view of the fact that cross sections of the chamber are not identical at various points, and are larger than the cross section of the bore, the volumes of the chamber and of the bore

are not proportional to actual lengths. Therefore, to facilitate a more convenient operation with additional equations obtained in pyrodynamics, "reduced lengths" are introduced to replace chamber volumes in equations by cylinder volumes of equal magnitude and having the cross section area of the gun bore as their basis. The length of such cylinders is called "the reduced length of the chamber." It is designated by ℓ_0, and determined by the expression:

$$\ell_0 = \frac{W_0}{s}.$$

Since the actual cross section of the chamber will be larger than s, the reduced length ℓ_0 will be greater than the actual length of chamber ℓ_{km}. (*)

The volume of the initial air space will now appear as:

$$W = s\ell_0 + s\ell = s(\ell_0 + \ell).$$

After deduction of the volume of the still incompletely burnt powder and the covolume of the burnt portion of the charge, the free volume of the initial air space will be expressed in this way:

$$W_0 + s\ell - \frac{\omega}{\delta}(1 - \psi) - \alpha\omega\psi = W_\psi + s\ell$$

It can be represented in the form of a sum

$$s\ell_\psi + s\ell = s(\ell_\psi + \ell),$$

(*) The introduction of the reduced length is a purely mathematical operation. It fails to consider the influence of the form of the chamber, and of its cross section, on the gas formation law, and consequently on the magnitude of gas pressure.

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CHAPTER III - CALCULATING THE HEAT LOST TO THE WALLS DURING BURNING OF POWDER IN A CLOSED CHAMBER

When powder is burned in a closed chamber (in a bomb), a portion of the heat energy is lost on heating the walls of the bomb. As a result, the pressure p_m developed by the gases is somewhat lower than the theoretical pressure; the latter would be obtained if all the thermal energy emitted during the combustion of the given powder were utilized to increase the pressure of the gases.

This loss of heat depends on a number of loading factors.

Experiments conducted by professor S.P. Vukolov in the Naval Technical Research Laboratory back in 1895 and 1896 had shown that at $\Delta = 0.20$ the pressures p_m developed in a standard bomb and in a bomb whose interior surface was lined with a thin layer of nonconductive mica were different. The resultant pressures were: 2033 kg/cm^2 in the bomb without the mica layer and 2202 kg/cm^2 in the bomb containing the mica, the difference - 169 kg/cm^2 - comprises about 8%.

It was stated previously (in the theory of powder combustion), in the discussion of problems relating to powder ignition in a gun, that the contact of the powder grains with the cool surface of the chamber slows down the ignition process.

The following simple experiment in the open air will show this to be true. If a strip of powder is clamped vertically in a lockmith's metal vise and ignited from the top, the process of burning will be arrested upon reaching the vise. The strip will be extinguished because a considerable portion of the heat is taken up by the cold metal.

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It is known that a nonsimultaneous ignition distorts the initial shape of the grain and causes deviations from the ideal law of combustion. Consequently, the transfer of heat to the walls should have an effect not only on the magnitude of pressure, but also on the character of its development.

By disregarding losses in a bomb due to heat transfer we commit an error in the determination p_m and, consequently, an error in the magnitude of energy f and covolume α determined on the basis of experiments with a manometric bomb.

In view of this heat transfer, Nobel's formula will hold true for pressures above 1000 kg/cm^2. At lower pressures, corresponding to charging densities of $\Delta < 0.10$, the points in the diagram $\frac{p_m}{\Delta}$ versus p_m will fall below the straight line relationship expressed by the known equation $\frac{p_m}{\Delta} = f + \alpha p_m$.

Systematic experiments conducted by assistant A.I. Kokhanov in 1933, at charging densities of from 0.015 to 0.20, have shown that a hyperbolic curve abc, approaching asymptotically the straight line de (fig. 16), is obtained in the system of coordinates $\frac{p_m}{\Delta}$, p_m instead of a straight line. The smaller the value of Δ, the greater is the deviation from the theoretical relationship.

The results obtained in determining the powder energy f and covolume α may therefore differ, depending on the charging densities at which the tests were conducted. The higher the value of Δ, the greater will be the magnitude of f and the smaller the covolume α (points b and c). And, conversely, small values of Δ should produce a small energy and a large covolume (points a and b).

Our experiments have also disclosed the following.

If a powder of the same composition but of different thickness is burned in a bomb at the same value of Δ, the maximum pressure p_m will be the lower the thicker the powder. Therefore, the energy produced by thick powders, determined without taking the heat losses into consideration, will also be the smaller, the thicker the powder. This can be explained by the fact that a thick powder burns longer and, consequently, a larger portion of the heat is transmitted to the walls of the bomb.

If an identical powder is burned at the same Δ in bombs of different sizes, the value of p_m will be higher in the larger bomb, because the surface per unit weight of powder charge is smaller in the larger bomb, and hence the heat losses will be smaller in it.

All of these facts confirm the presence of cooling through the walls of the bomb. The considerable number of tests conducted by various investigators made it possible to determine quantitatively the corrections to be introduced in the charging of powders of various thickness under different conditions of loading, in order to obtain pressures corrected for heat transfer. Although these corrections are not final, nevertheless their introduction served to explain the phenomena discussed above. Miuraur's correction may be considered to be the best founded among corrections of this type.

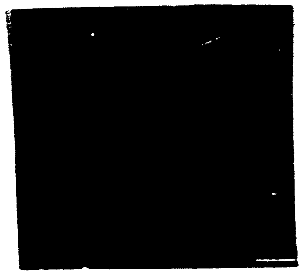

Fig. 16 - $\frac{p_m}{\Delta}$ as a Function of p_m, According to Kokhanov.

According to Miuraur the heat loss through transfer is proportional to the number of collisions between the powder gas molecules and the walls of the bomb, and this number is in turn proportional to the surface of the bomb S_6, pressure p and time t. For a fluctuating pressure, the loss is proportional to S_6 and $\int pdt$. Also, because $\int pdt$ does not depend on Δ, the loss of heat ΔQ through the surface of walls S_6 is constant for any powder charge ω or Δ. This was confirmed by Miuraur who conducted tests at different values of Δ. Inasmuch as the total quantity of the heat evolved is proportional to the weight of the charge ω, the relative loss $\frac{\Delta Q}{Q}$ is inversely proportional to ω or Δ. Consequently, $\frac{\Delta Q}{Q}$ is proportional to $\frac{S_6}{\omega} \int pdt$.

The magnitude of maximal pressure at a given value of Δ is governed by the volume of the bomb(*). Actually, an n% pressure

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(*) Consequently, magnitudes f and α will also depend on the volume of the bomb in which the combustion takes place, while the magnitude of $\int pdt$ should not change because of cooling through the walls of the bomb.

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drop entails a corresponding prolongation of the period of burning; the curve p,t becomes distorted, but the area $\int pdt$ remains unchanged.

All this was checked and verified by our tests with bombs of different sizes. The following important deduction can be reached from the above: the rate of burning $u_1 = \frac{e_1}{\int_0^1 pdt}$ can be determined with the same degree of accuracy in both large and small bombs, regardless of the heat lost through the walls.

Experiments for calculating heat losses. For quantitative determination of heat losses Miuraur conducted experiments at $\Delta = 0.20$ in a bomb measuring 150 cm^3 by volume. In one case the powder was burned under normal conditions, and in another a steel, trough-shaped insert with projections or ridges was inserted into the bomb in such a manner that its exterior surface did not come in contact with the surface of the bomb (fig. 17).

In the first instance, when the cooled surface S_1 was equal to the surface of the bomb $S_б$, the obtained pressure was p_1; in the second case, when the cooled surface S_2 was larger, i.e., when it consisted of the surface of the bomb $S_б$ and the surface of the insert $S_{ВКЛ}$ ($S_2 = S_б + S_{ВКЛ}$), the resultant pressure p_2 was lower. The pressu[re] difference $\Delta p = p_1 - p_2$ resulted in consequence of the surface area difference $S_2 - S_1 = S_{ВКЛ}$.

In order to determine the pressure in the absence of cooling, calculations were made of such an increment $\Delta p'$, which corresponded to a change in the surface area ΔS equal to the surface area of the bomb $S_б$:

$$\frac{\Delta p'}{\Delta p} = \frac{S_б}{S_{ВКЛ}} \quad \text{or} \quad \Delta p' = \Delta p \frac{S_б}{S_{ВКЛ}} .$$

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In this manner, pressure $p_1 + \Delta p'$ would correspond to the surface $S = 0$, i.e., it would correspond to the condition in which losses due to heat transfer were absent.

Knowing p_1 and $\Delta p'$, Miuraur determined the relative correction for heat transfer $\frac{\Delta p'}{p} = \frac{\Delta p_m}{p}$, which was equal to $\frac{\Delta T}{T_1}$ in a constant volume.

Such tests conducted with a large number of powders of varied thicknesses and properties established the significant dependence of pressure loss $\frac{\Delta p_m}{p_m}\%$ on the time of powder burning at $\Delta = 0.20$, the magnitude S_γ/ω in these tests being equal to 7.774 cm^2/g = 77.74 dm^2/kg.

The experiments were conducted with cylindrical crushers. Very strong igniters of gunpowder were used to determine the actual powder burning time t_k. The igniters developed a pressure of $p_n \approx 250$ kg/cm^2, which provided the crusher with small residual compression.

The obtained data was plotted, and the curve $\frac{\Delta p_m}{p_m}\%$, or $\frac{\Delta T}{T_1}\%$ as a function of the time of burning t_k, was termed "curve C" (fig. 18).

In order to determine the losses due to heat transfer under conditions other than those discussed, the powder must first be tested at $\Delta = 0.20$ and the time of burning t_k found, and the magnitude $C_M = \frac{\Delta p_m}{p_m}\%$ then determined from curve C. Losses under other conditions (in a different bomb and at a different density of loading), can be found by means of the following formula:

$$\frac{\Delta p_m}{p_m} = \frac{\Delta T}{T_1} = \frac{C_M\%}{7.774}\,\frac{S_\gamma}{\omega} = \frac{C_M\%}{7.774}\,\frac{S_\gamma}{W_0}\,\frac{1}{\Delta}. \qquad (17)$$

C_M depends on the thickness and nature of the powder, while S_γ/W_0 characterizes the relative surface of the bomb (exposed surface of

bomb) and can be calculated as the exposed surface of a uniform cylinder ($2e_1 = d$, length $= 2c$); Δ characterizes the conditions of loading:

$$\frac{S_\sigma}{W_0} = \frac{a}{e_1} = \frac{2 + \beta}{d : 2} = \frac{2 + \frac{d}{2c}}{d : 2} = \frac{4}{d} + \frac{1}{c}.$$

The relative surface of the bomb diminishes as the diameter d and length 2c are increased. When volume W_0 is increased 8 times, S_σ / W_0 diminishes approximately by one half.

Formula (17) shows that $\frac{\Delta p_m}{p_m}$ in a given bomb increases as Δ is reduced and hence the losses are particularly high at low values of Δ.

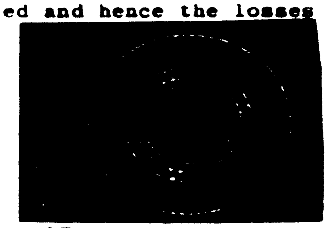

Fig. 17 - Test Arrangement for Calculating Losses due to Heat Transfer.

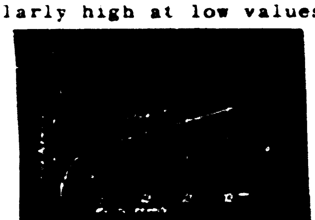

Fig. 18 - Curve C, Characterizing the Losses Due to Heat Transfer (according to Miuraur).

Ordinate: pressure loss in %; abscissa: milliseconds; 1) curve "C"

Large igniters are not necessary when working with conical crushers, because almost instantaneous ignition results at the low value of $p_n \approx 100\text{-}120$ kg/cm^2.

Therefore, Miuraur's data cannot be directly applied to our test results and to powder tests for determining heat transfer, without conducting special tests. However, using the theoretical formula as a basis, it is possible to compute the burning time at $\Delta = 0.20$ and $p_B = 250$ kg/cm^2 and to determine the order of the coefficient values C_M for powders of various thicknesses, in order to introduce the necessary correction for heat loss.

The table presented below contains the results of such calculations,

bomb) and can be calculated as the exposed surface of a uniform cylinder ($2e_1 = d$, length $= 2c$); Δ characterizes the conditions of loading:

$$\frac{S_\varepsilon}{W_0} = \frac{\alpha}{e_1} = \frac{2 + \beta}{d : 2} = \frac{2 + \frac{d}{2c}}{d : 2} = \frac{4}{d} + \frac{1}{c}.$$

The relative surface of the bomb diminishes as the diameter d and length 2c are increased. When volume W_0 is increased 8 times, S_ε/W_0 diminishes approximately by one half.

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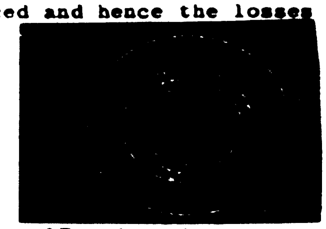

Fig. 17 - Test Arrangement for Calculating Losses due to Heat Transfer.

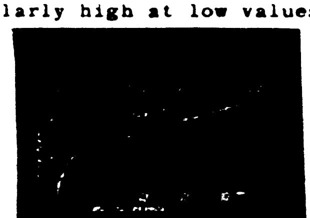

Fig. 18 - Curve C, Characterizing the Losses Due to Heat Transfer (according to Miuraur).

Ordinate: pressure loss in %; abscissa: milliseconds; 1) curve "C."

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Therefore, Miuraur's data cannot be directly applied to our test results and to powder tests for determining heat transfer, without conducting special tests. However, using the theoretical formula as a basis, it is possible to compute the burning time at $\Delta = 0.20$ and $p_B = 250$ kg/cm^2 and to determine the order of the coefficient values C_M for powders of various thicknesses, in order to introduce the necessary correction for heat loss.

The table presented below contains the results of such calculations,

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obtained from curve C.

t_k was determined from formula:

$$t_k = 2.303\tau \log \frac{p_m}{p_B},$$

where

$$\tau = \frac{e_1}{u_1} \frac{1 - \alpha\Delta}{f\Delta} = \frac{e_1}{u_1} \frac{1}{p_m}$$

(for a powder with a constant burning surface area) (Table 8).

Table 8 - Values of coefficient C_M for pyroxylin powders

Thickness of powder $2e_1$ in mm	0.30	0.30 flegm.	0.40	1.00	2.00	4.00
Rate of combustion u_1 $\frac{dm}{sec} : \frac{kg}{dm^2} \cdot 10^7$	90	70	80	75	72	62
Coefficient $\frac{1}{\tau}$	1220	950	813	305	146	69
Burning time in milliseconds, at $\Delta = 0.20$ and $p_0 = 250$ kg/cm^2	1.76	2.26	2.64	7.00	14.6	31.[illegible]
Heat transfer coefficient C_M in %	1.5	2.0	2.6	4.0	5.0	6.1

Corrections for heat loss during burning of powder in a bomb can be introduced by the aid of this table.

Let us introduce the correction for heat loss when determining f and α.

We shall assume that tests were conducted with a bomb having a volume $W_0 = 78.5$ cm^3 at two loading densities $\Delta_1 = 0.15$ and $\Delta_2 = 0.25$. The powder is 1 mm thick.

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$p_{m1} = 1435$ kg/cm^2, $p_{m2} = 2760$ kg/cm^2, $p_{m2} - p_{m1} = 1325$. We determine f and α without taking the heat-transfer losses into consideration:

$$\frac{p_{m1}}{\Delta_1} = 9570, \quad \frac{p_{m2}}{\Delta_2} = 11{,}020; \quad \frac{p_{m2}}{\Delta_2} - \frac{p_{m1}}{\Delta_1} = 1450.$$

$$\alpha = \frac{1450}{1325} = 1.096 \text{ dm}^3/\text{kg}$$

$$f = 957{,}000 - 1.096 \cdot 143{,}300 = 800{,}000 \text{ kg-dm/kg}.$$

We now introduce the correction for heat losses in pressures p_{m1} and p_{m2}, and determine the corrected values of energy f_0 and covolume α_0.

$$\frac{\Delta p_m}{p_m} = \frac{S_\delta}{W_0} \frac{1}{\Delta} \frac{C_M\%}{7.774}.$$

According to the table, $C_M = 4\%$.

For a bomb of volume $W_0 = 78.5$ cm^3

$$\frac{S_\delta}{W_0} = 1.30 \text{ cm}^2/\text{cm}^3.$$

$$\frac{S_\delta}{W_0} \frac{1}{7.774} = 0.1673; \quad 0.1673 \cdot 4 = 0.6692.$$

$$\Delta p_m = \frac{S_\delta}{W_0} \frac{C_M\%}{7.774} \frac{p_m}{\Delta} \begin{cases} \Delta p_{m1} = 0.6692 \cdot 9570 = 64 \text{ kg/cm}^2 \\ \Delta p_{m2} = 0.6692 \cdot 11{,}040 = 74 \text{ kg/cm}^2 \end{cases}$$

$$\begin{array}{l|l} p'_m = p_{m1} + \Delta p_{m1} = 1435 + 64 = 1499 \text{ kg/cm}^2 & \dfrac{p'_{m1}}{\Delta_1} = 10{,}000; \\ p'_{m2} = p_{m2} + \Delta p_{m2} = 2760 + 74 = 2834 \text{ kg/cm}^2 & \dfrac{p'_{m2}}{\Delta_2} = 11{,}340 \end{array}$$

$$p'_{m2} - p'_{m1} = 1335 \text{ kg/cm}^2;$$

$$\frac{p'_{m2}}{\Delta_2} - \frac{p'_{m1}}{\Delta_1} = 1340;$$

$$\alpha_0 = \frac{1340}{1335} = 1.004 \text{ dm}^3/\text{kg} \approx 1.00.$$

$$f_0 = 1{,}000{,}000 - 1.004 \cdot 1499 = 849{,}500 \text{ kg-dm/kg} \approx 850{,}000 \text{ kg-dm/}$$

As can be seen from the above results, the introduction of a correction for the heat lost has increased the energy of the powder from $f = 800{,}000$ to $850{,}000$ kg-dm/kg, i.e., by 6.25%, and reduced the magnitude of covolume from 1.096 to 1 dm^3/kg, i.e., by 9.6%.

The same powder burned in a bomb having a volume of 25.25 cm^3, an at the same loading density, will give $p_{m1} = 1405 \text{ kg/cm}^2$ and $p_{m2} = 272$ kg/cm^2. On the basis of this data, the resultant magnitudes of energy f and covolume α will be:

$$f = 774{,}000 \text{ kg-dm/kg} \qquad \text{and } \alpha = 1.16 \text{ dm}^3/\text{kg}.$$

For this bomb

$$\frac{S_6}{W_0} = 1.89 \text{ cm}^2/\text{cm}^3.$$

Corrections in pressure for heat transfer will be as follows:

$$\Delta p_{m1} = 95 \text{ kg/cm}^2 \quad \text{and} \quad \Delta p_{m2} = 110 \text{ kg/cm}^2;$$

$$f = 850{,}000 \text{ and } \alpha = 1.00.$$

In this way, magnitudes f and α for a bomb of small volume are obtained with a larger percentage of error: the energy will be

smaller and the covolume will be higher than their true values.

As shown by Kokhanov's experiments, these errors are extremely large for very low loading densities and thick powders.

The introduction of corrections for heat transfer losses by the above method provides practically identical values of f and α for all loading densities, and transforms the hyperbola $\frac{p_m}{\Delta}$, p_m obtained in Kokhanov's experiments into a straight line, as it should be [5].

The table presented below lists values of magnitudes $\frac{S_\delta}{W_0}$ and $\frac{S_\delta}{W_0} \frac{1}{7.774}$ for manometric bombs of the most typical dimensions (Table 9).

Table 9

W_0, cm^3	21.8	25.25	78.5	120	146.5	216	Krupp's 3320 bomb
d_0	2.2	3.0	4.4	4.4	3.0	4.4	8.0
$\frac{S_\delta}{W_0}$	2.17	1.89	1.30	1.16	1.48	1.05	0.53
$\frac{S_\delta}{W_0 \cdot 7.774}$	0.279	0.243	0.1673	0.1495	0.184	0.135	0.0682

Curve C_M is expressed as a function of the burning time t_k of powder at Δ = 0.20.

Inasmuch as the burning time of powder t_k is directly proportional to the total pressure impulse $\int_0^1 pdt$, the dependence of C as a function of I_k can be plotted independently of the loading density. Work of this type was conducted by M.I. Samarina at the Naval Artillery Research Institute in 1938, at which time experiments were repeated with bombs with and without steel inserts. The pressure was recorded

by means of conical crushers. The values of $\frac{\Delta p_m}{p_m}\% = C_A\%$ were plotted as a function of I_k (fig. 19). The form of the curve differs somewhat from curve $C_{M1}t_k$.

Fig. 19 - Curve C_A, Characterizing Losses due to Heat Transfer (according to the Naval Artillery Research Institute data).

Bomb tests for determining f and α involve heat losses in the form of heat transfer to walls of the bomb, and, therefore, pressures p_1 and p_2 contain an error in their determined values, δp_1 and δp_2 being inversely proportional to magnitudes $1 - \alpha\Delta_1$ and $1 - \alpha\Delta_2$, and $\delta p_2 > \delta p_1$; but $\frac{\delta p_1}{p_1} > \frac{\delta p_2}{p_2}$ because $\frac{\delta p}{p}\% = \frac{S_\delta}{w_0}\,\frac{1}{\Delta}\,\frac{C_M\%}{7.774}$, where C_M is the heat transfer coefficient.

Corrections for heat losses in the powder energy f and the magnitude of covolume can be determined after introducing corrections for heat losses in the values of pressures p_1 and p_2, by adding the equalities (7), (8), (9) and (10), taking into consideration equality (a) (see Chapter II):

$$\delta f = \delta f_1 + \delta f_2 = \frac{\Delta_2 - \Delta_1}{\Delta_2 \cdot \Delta_1}\,\frac{1}{(p_2 - p_1)^2}(p_2^2\delta p_1 - p_1^2\delta p_2) =$$

$$= \frac{\Delta_2 - \Delta_1}{p_2 - p_1}\,\frac{p_2}{\Delta_2}\,\frac{p_1}{\Delta_1}\,\frac{p_2^2\delta p_1 - p_1^2\delta p_2}{(p_2 - p_1)p_2 p_1} = f\,\frac{p_2\frac{\delta p_1}{p_1} - p_1\frac{\delta p_2}{p_2}}{p_2 - p_1}.$$

Finally,

$$\frac{\delta f}{f} = \frac{p_2 \frac{\delta p_1}{p_1} - p_1 \frac{\delta p_2}{p_2}}{p_2 - p_1}; \qquad (18)$$

$$\delta a = \delta a_1 + \delta a_2 = -\frac{\frac{1}{\Delta_1} - \frac{1}{\Delta_2}}{(p_2 - p_1)^2}(p_2 \delta p_1 - p_1 \delta p_2) =$$

$$= -f \frac{p_2 \delta p_1 - p_1 \delta p_2}{(p_2 - p_1) p_1 p_2} = -f \frac{\frac{\delta p_1}{p_1} - \frac{\delta p_2}{p_2}}{p_2 - p_1}. \qquad (19)$$

When the corrections for heat transfer are taken into account

$$\frac{\delta p}{p} \% = \frac{C_M \%}{7.774} \frac{S_\sigma}{W_0} \frac{1}{\Delta}.$$

For a given powder, C_M = const, and for a given bomb, $\frac{S_\sigma}{W_0}$ = cons[t]; only Δ changes.

Designating

$$\frac{C_M \%}{7.774} \frac{S_\sigma}{W_0} = D,$$

we get

$$\frac{\delta p_1}{p_1} = \frac{D}{\Delta_1}; \quad \frac{\delta p_2}{p_2} = \frac{D}{\Delta_2}.$$

Substituting these expressions in formulas (18) and (19), we obtain:

$$\frac{\delta f}{f} \% = \frac{D\left(\frac{p_2}{\Delta_1} - \frac{p_1}{\Delta_2}\right)}{p_2 - p_1}; \qquad (20)$$

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$$\delta\alpha = -fD\frac{\frac{1}{\Delta_1} - \frac{1}{\Delta_2}}{p_2 - p_1} = -\frac{f^2 D}{p_1 \cdot p_2}. \tag{21}$$

Let us apply the example presented above.

__Example.__ 1 mm thick powder is burned in a bomb having a volume of 78.5 cm^3; the powder coefficient is $C_M = 4\%$; and the value of $\frac{S_\delta}{W_0}$ for this bomb = 1.30 cm^2/cm^3.

$$\frac{S_\delta}{W_0}\,\frac{C_M\%}{7.774} = 0.1673 \cdot 4 = 0.6692\% = 0.006692 = D.$$

$\Delta_1 = 0.15;\ p_1 = 1433;\ \frac{p_1}{\Delta_1} = 9570$

$\Delta_2 = 0.25;\ p_2 = 2756;\ \frac{p_2}{\Delta_2} = 11{,}020$

Without correction for heat transfer losses
$f = 800{,}000$ kg-dm/kg;
$\alpha = 1.096$ dm^3/kg.

We shall now calculate the corrections for f and α taking the heat transfer into account:

$$\frac{\delta f\%}{f} = 0.6692\,\frac{\frac{2756}{0.15} - \frac{1433}{0.25}}{2756 - 1433} = 0.6692\,\frac{18{,}350 - 5730}{1323} = 6.4\%;$$

$$f_0 = f\left(1 + \frac{\delta f}{f}\right) = 800{,}000 \cdot 1.064 = 852{,}000 \text{ kg-dm/kg};$$

$$\delta\alpha = -\frac{f^2 \cdot D}{p_1 p_2} = -\frac{800{,}000^2 \cdot 0.006692}{143{,}300 \cdot 275{,}600} = -0.1085;$$

$$\alpha_0 = \alpha + \delta\alpha = 1.096 - 0.1085 = 0.9875 \approx 0.99.$$

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CHAPTER IV - THE LAW OF GAS FORMATION

1. DEFINITION

The study of the law of gas formation under the simplest conditi in a constant volume, permits the application of the obtained relatio ships to the determination of pressure in the bore of a weapon when t gun is fired, i.e., under conditions of variable volume.

The general pyrostatics formula

$$p_\psi = \frac{f\omega\psi}{W_0 - \frac{\omega}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\omega\psi} = \frac{f\omega\psi}{W_\psi}$$

shows that the magnitude of gas pressure at given conditions of loading (W_0, ω, f, α, δ) is governed by the amount of the burned portion of the charge ψ, where $\omega\psi$ is the gravimetric inflow of powder gases at a given instant, and $f\omega\psi$ is the inflow of the energy contained in this quantity of gas.

Keeping in mind that W_ψ varies slightly under conditions prevail in a manometric bomb, it may be said that the pressure is almost proportional to the burned portion of the charge ψ at the given powder energy f and the given loading density.

Correspondingly, the nature of pressure increase in time $\frac{dp}{dt}$ und the same conditions of loading is also determined in the main by the change of magnitude $\frac{d\psi}{dt}$ with time.

<u>The law of gas formation is a term defining the change of the magnitude of ψ with time and of its derivative $\frac{d\psi}{dt}$, known as the "rat of gas formation" or "volumetric rate of burning."</u>

An analysis of this magnitude $\frac{d\psi}{dt}$ makes it possible to determine the means by which the gas inflow during burning of powder can be regulated.

2. RATE OF GAS FORMATION

We shall derive the formula for the rate of gas formation, based on the burning of powder in parallel layers.

Let us assume that the burning of a powder grain of arbitrary shape proceeds in concentric layers at a constant rate in all directions. At some instant the grain, whose initial volume is Λ_1 and whose initial surface is S_1, has a volume Λ and a surface S (fig. 20). We shall also assume that a layer of thickness de is burned during the time interval dt. Then, the volume burned during the time interval dt will be expressed by formula:

$$d\Lambda_{cr} = Sde,$$

whence

$$\Lambda_{cr} = \int_0^e Sde;$$

$$\psi = \frac{\Lambda_{cr}}{\Lambda_1} = \frac{\int_0^e Sde}{\Lambda_1}.$$

Differentiating both sides of this equality with respect to t, we get:

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$$\Lambda_{cr} = \int_0^e Sde;$$

$$\psi = \frac{\Lambda_{cr}}{\Lambda_1} = \frac{\int_0^e Sde}{\Lambda_1}.$$

Differentiating both sides of this equality with respect to t, we get:

$$\frac{d\psi}{dt} = \frac{d\frac{\Lambda_{cr}}{\Lambda_1}}{dt} = \frac{S}{\Lambda_1}\frac{de}{dt} = \frac{S}{\Lambda_1}u.$$

Multiplying and dividing the right side by S_1, we get:

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u.$$

The first two multiples in the right side of the expression depend on the geometry of the powder grain:

$\frac{S_1}{\Lambda_1}$ - the initial exposed area of the powder grain or the specific surface per unit grain volume at the start of burning; it will be shown later that this area depends on the form and the dimensions of the grain;

$\frac{S}{S_1}$ - the relative surface of a powder grain; it varies during burning and depends only on the form of the grain and on the relative thickness of the burnt powder layer, but not o[n] its absolute dimensions.

As will be shown later, the third multiple, the linear rate of burning $u = \frac{de}{dt}$, depends on the type of powder, the pressure under which the powder burns, and on its temperature.

In order to determine the rate of burning in an actual test bomb under variable pressure, it is necessary to know the thickness variation of the burning layer per unit of time, and to this end it is necessary to establish the purely geometrical relation between the thickness of the burned layer and the volume of the powder gases in their various forms.

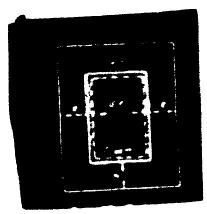

Fig. 20 – Diagram of Powder Burning in Parallel Layers.

If the pressure curve plotted as a function of time, or a table of the values of p versus t is available from the bomb test, the values of ψ, t can be calculated by means of the general pyrostatics formula (or from special tables), following which the geometrical law of burning can be applied to establish the relationship between the thickness of the powder, the value of ψ and $\frac{S}{S_1}$, and to derive the dependenc of the initial exposure $\frac{S_1}{\Lambda_1}$ on the form and dimensions of the grains. The numerical differentiation of ψ and e with respect to time t will permit obtaining from the experiment the rate of gas formation and the linear rate of burning, as well as their variation, during the burning of the powder.

The establishment of all these relationships will facilitate the analysis of those factors which can be used to control the quantity and the intensity of gas formation during the burning of powder and therefore, will permit the control of the phenomena of a gun discharge.

3. THE EFFECT PRODUCED BY THE GEOMETRY OF A POWDER GRAIN ON GAS FORMATION

The inflow of gases per unit of time for a powder of a given type (f, α, δ, u_1) can be regulated by the loading density or by the dimensions and the shape of the powder grains.

The effect of the geometry of the given grains on the rate of gas formation depending on the thickness of the powder burned at the given instant, can be determined by means of the basic conditions of the geometrical law of combustion.

The geometrical law of combustion permits determining the relationship between the relative thickness of the powder burned at the given instant $z = \frac{e}{e_1}$, the burnt portion of the grain $\psi = \frac{\Lambda_{cr}}{\Lambda_1}$ (Λ_{cr} being the volume of the burnt portion of the powder), and the relative surface of the powder $\sigma = S/S_1$ at the same instant. The dependence of the product $\Sigma = \frac{S_1}{\Lambda_1}\frac{S}{S_1}$ on the shape and the dimensions of the powder (grain) can be established at the same time, which product enters the formula for determining the rate of gas formation and has a great effect on the law governing the gas pressure developm[ent] during a discharge.

A. Relation Between the Burnt Portion of Powder ψ and the Relative Thickness of Powder z, Consumed at the Same Instant (Inflow of Gases)

Investigations show that the dependence of ψ on z for all forms [of] powder is expressed by a formula of the same type(*):

(*) In deriving this formula for one grain, as well as for an entire charge composed of many grains, it is assumed that all grains of the charge have strictly identical dimensions and are regular in form, th[e] latter being bounded by parallel planes, intersecting at right angle[s] or by surfaces of revolution whose axes coincide or are parallel to each other.

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$$\psi = \chi z \, (1 + \lambda z + \mu z^2), \tag{22}$$

where χ, λ, μ are shape characteristics - constant values depending on the shape of the grain; they possess a particular numerical value for each grain shape inherent to the given grain form.

The thickness of the burnt layer e varies during burning from 0 to e_1; the relative thickness z varies from 0 to 1; and the relative volume ψ fluctuates between 0 and 1.

We shall now derive the dependence of ψ on z for strip powder (a parallelepiped with three different dimensions) (fig. 21). We shall introduce the designations: $2e_1$ for the thickness of the strip, 2b for the width of the strip, and 2c for its length:

$$\frac{2e_1}{2b} = \alpha; \quad \frac{2e_1}{2c} = \beta.$$

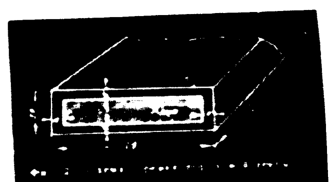

Fig. 21 - Burning Diagram of a Powder Strip.

Magnitudes α and β characterize the span of the strip in thickness and length. Furthermore, inasmuch as all the dimensions become reduced in all directions by a magnitude $2e_1$ during the full burning of the powder, $\alpha = \frac{2e_1}{2b}$ represents the relative reduction of the strip in width and $\beta = \frac{2e_1}{2c}$ represents the relative reduction of its length during the full burning period of the powder.

Inasmuch as

$$2e_1 < 2b < 2c,$$

$$1 > \alpha > \beta > 0.$$

Let us assume that at the given instant a layer of powder of thickness e will be burned on all sides. The volume burned can be determined more easily as the difference between the initial volume Λ_1 and the remaining volume $\Lambda_{\text{ост}}$.

We will have (see fig. 21):

$$\psi = \frac{\Lambda_{\text{сг}}}{\Lambda_1} = \frac{\Lambda_1 - \Lambda_{\text{ост}}}{\Lambda_1} = 1 - \frac{\Lambda_{\text{ост}}}{\Lambda_1};$$

$$\Lambda_1 = 2e_1 \cdot 2b \cdot 2c;$$

$$\Lambda_{\text{ост}} = (2e_1 - 2e)\ (2b - 2e)\ (2c - 2e);$$

$$\frac{\Lambda_{\text{ост}}}{\Lambda_1}\ \frac{e_1 - e}{e_1}\ \frac{b - e}{b}\ \frac{c - e}{c} = \left(1 - \frac{e}{e_1}\right)\left(1 - \frac{e}{b}\right)\left(1 - \frac{e}{c}\right);$$

but

$$\frac{e}{e_1} = z,\quad \frac{e}{b} = \frac{e_1}{b}\,\frac{e}{e_1} = \alpha z,\quad \frac{e}{c} = \frac{e_1}{c}\,\frac{e}{e_1} = \beta z;$$

then

$$\frac{\Lambda_{\text{ост}}}{\Lambda_1} = (1 - z)\ (1 - \alpha z)\ (1 - \beta z).$$

Upon removing the parentheses we obtain:

$$\frac{\Lambda_{ocr}}{\Lambda_1} = 1 - (1 + \alpha + \beta) z + (\alpha + \beta + \alpha\beta) z^2 - \alpha\beta z^2.$$

Substituting this expression in the formula for ψ, we find:

$$\psi = (1 + \alpha + \beta) z - (\alpha + \beta + \alpha\beta) z^2 + \alpha\beta z^3$$

or, reducing it to the general form of equation (22), we get:

$$\psi = (1 + \alpha + \beta) z \left[1 - \frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta} z + \frac{\alpha\beta}{1 + \alpha + \beta} z^2 \right].$$

Introducing the designations

$$1 + \alpha + \beta = \varkappa; \quad - \frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta} = \lambda; \quad \frac{\alpha\beta}{1 + \alpha + \beta} = \mu, \qquad (23)$$

we obtain a general type formula:

$$\psi = \varkappa z (1 + \lambda z + \mu z^2).$$

At the end of burning at $z = 1$ $\psi = 1$, and formula (23) assumes an equality in the form

$$1 = \varkappa (1 + \lambda + \mu), \qquad (24)$$

which must be satisfied by the numerical characteristics $\varkappa$, λ and μ.

This equality serves to verify the values of the characteristics calculated by means of formula (23).

The characteristics $\varkappa$, λ and μ depend on the ratio of the dimensions: the shorter and the narrower the strip, the greater is α and β; the longer the strip, the closer is $\varkappa$ to unity and λ and μ to zero.

B. The Law Governing the Change of the Powder Surface When the Powder is Burned.

Formula (22) is a general formula for all powder shapes; the difference will be only in the numerical values of the characteristics $\varkappa$, λ and μ. Using this formula as a basis, we shall derive a formula for depicting the relative surface $\sigma = \frac{S}{S_1}$ and the initial exposure $\frac{S_1}{\Lambda_1}$, characterizing the effect of the shape and dimensions of the powder on the rate of gas formation.

Differentiating ψ with respect to z, we find:

$$\frac{d\psi}{dz} = \varkappa(1 + 2\lambda z + 3\mu z^2). \tag{25}$$

Inasmuch as

$$\frac{d\psi}{dz} = \frac{d\psi}{dt}\,\frac{dt}{de}\,\frac{de}{dz},$$

and

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\cdot\frac{S}{S_1}\,u, \quad \frac{dt}{de} = \frac{1}{u}, \quad \frac{de}{dz} = \frac{de}{d\frac{e}{e_1}} = e_1,$$

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then

$$\frac{d\psi}{dz} = \frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} u \frac{e_1}{u} = \frac{S_1}{\Lambda_1} e_1 \frac{S}{S_1}.$$

Substituting this expression in the left side of equation (25), we obtain

$$\frac{S_1}{\Lambda_1} e_1 \frac{S}{S_1} = \kappa (1 + 2\lambda z + 3\mu z^2). \qquad (26)$$

At the start of burning $z = 0$, $S = S_1$, $\frac{S}{S_1} = 1$, and equation (26) at the start of burning will be written as follows:

$$\frac{S_1}{\Lambda_1} e_1 = \kappa. \qquad (27)$$

Dividing each term of (26) by (27), we will find the desired relationship:

$$\sigma = \frac{S}{S_1} = 1 + 2\lambda z + 3\mu z^2. \qquad (28)$$

At the start of burning, at $z = 0$,

$$\sigma = 1;$$

at the end of burning, at $z = 1$,

$$\sigma_K = \frac{S_K}{S_1} = 1 + 2\lambda + 3\mu.$$

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While the powder burns and z varies from 0 to 1, the change of σ will mainly depend on the magnitude and sign of the characteristic λ, since μ is small compared to λ.

If the surface of the powder diminishes while burning (strip, cube, bar, $\lambda < 0$) the shape of such a powder is called regressive; if the surface area becomes greater during burning (powder with multiple perforations, $\lambda > 0$), the powder is called progressive.

The σ function depends on λ and μ, i.e., on the shape of the grain and its dimensional ratio, rather than on the absolute dimensions of the powder. The greater the value of λ, the greater will be the surface change of the powder in burning.

From equality (27) we obtain an expression for the initial exposure which is of great importance in controlling the rate of gas formation:

$$\frac{S_1}{\Lambda_1} = \frac{\varkappa}{e_1}; \tag{29}$$

this equality shows that the initial burning area of the powder depends on its shape (characteristic $\varkappa$), as well as on its dimensions (e_1). The smaller the value of e_1, the thinner is the powder, and the greater will be the quantity of gases which it will evolve per unit of time.

Substituting (28) in (25), we obtain formula

$$\frac{d\psi}{dz} = \varkappa \cdot \sigma,$$

which will be used in plotting the graph for ψ, z.

C. Determination of ψ, z for Other Grain Shapes.

a) Tubular grain with a single perforation (fig. 22).

Designating:

$2e_1$ - web thickness

D - outside diameter

d - inside diameter

2c - length of tube;

$$\frac{2e_1}{2c} = \frac{e_1}{c} = \beta \text{ (which is the same as for strip powder);}$$

$$\Lambda_1 = \frac{\pi}{4}(D^2 - d^2)\, 2c;$$

$$\Lambda_{ост} = \frac{\pi}{4}\left[(D - 2e)^2 - (d + 2e)^2\right](2c - 2e);$$

$$\frac{\Lambda_{ост}}{\Lambda_1} = \frac{\left[(D - 2e)^2 - (d + 2e)^2\right]}{D^2 - d^2}\,\frac{(c - e)}{c}.$$

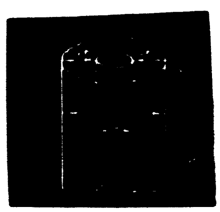

Fig. 22 - Burning of a Tubular Grain.

Removing the parentheses and bearing in mind that

$$D - d = 2 \cdot 2e_1,$$

and

$$\frac{e}{c} = \frac{e_1}{c}\,\frac{e}{e_1} = \beta \cdot z,$$

we get:

$$\frac{\Lambda_{oct}}{\Lambda_1} = \frac{\left[D^2 - d^2 - 2(D + d)2e\right]}{D^2 - d^2}\left(1 - \frac{e}{c}\right) = \frac{(D - d - 2 \cdot 2e)}{D - d}\left(1 - \frac{e}{c}\right) =$$

$$= (1 - z)\,(1 - \beta z) = 1 - (1 + \beta)z + \beta z^2;$$

$$\psi = 1 - \frac{\Lambda_{oct}}{\Lambda_1} = (1 + \beta)z - \beta z^2 = (1 + \beta)z\left(1 - \frac{\beta}{1 + \beta}z\right).$$

Assuming $1 + \beta = \varkappa$, $-\dfrac{\beta}{1 + \beta} = \lambda$ and $\mu = 0$, we get once again a general type formula

$$\psi = \varkappa z\,(1 + \lambda z),$$

where the terms μz^2 is missing, because $\mu = 0$.

It can be easily seen that a tube is equivalent to a strip whose dimension in width does not change in burning. This is equivalent to a strip of infinite width, where $\alpha = 0$.

In such a case, the characteristics of strip powder will be

2

$$\varkappa = 1 + 0 + \beta = 1 + \beta;$$

$$\lambda = -\frac{0 + \beta + 0}{1 + \beta} = -\frac{\beta}{1 + \beta};$$

$$\mu = 0,$$

i.e., the same as those obtained for tubular powder.

It is of interest to note that characteristics $\varkappa$ and λ for a tubular grain do not depend on the diameter, but, rather, on the web thickness $2e_1$ and the length of the tube $2c$.

b) Grain shapes which are derivatives of strip powder

It can be easily seen that the following grain shapes can be obtained from a strip as special cases of the latter:

1) square rod: $2b = 2c$; $\alpha = \beta$;

2) square slab: $2e_1 = 2b$; $\alpha = 1$; $\beta > 0$;

3) cube: $2e_1 = 2b = 2c$; $\alpha = \beta = 1$.

The characteristics of these shapes, as well as of strips and tubes, are given below in Table 10; also given in this table are the values of σ_K at the end of powder burning.

Table 10

Powder Shape	$\varkappa$	λ	μ	$\sigma_K = 1 + 2\lambda + 3\mu$
Tube (infinitely wide strip)	$1 + \beta$	$-\frac{\beta}{1 + \beta}$	0	$\frac{1 - \beta}{1 + \beta}$
Strip	$1 + \alpha + \beta$	$-\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}$	$\frac{\alpha\beta}{1 + \alpha + \beta}$	$\frac{(1 - \alpha)(1 - \beta)}{1 + \alpha + \beta}$
Square Rod	$1 + 2\beta$	$\frac{2\beta + \beta^2}{1 + 2\beta}$	$\frac{\beta^2}{1 + 2\beta}$	$\frac{(1 - \beta)^2}{1 + 2\beta}$
Square Slab	$2 + \beta$	$-\frac{1 + 2\beta}{2 + \beta}$	$\frac{\beta}{2 + \beta}$	0
Cube	3	-1	$\frac{1}{3}$	0

The data presented in the table shows that all the grain characteristics are increased in changing over from a tubular to a cube shape: $\varkappa$ increases from 1 to 3, $|\lambda|$ - from a small fraction to 1, μ - from 0 to 1/3.

Since $\varkappa$ characterizes the initial surface area $\frac{S_1}{\Lambda_1}$ for a given powder thickness $2e_1$, the increase of $\varkappa$ shows that in changing over from a strip to a cube, with the web thickness $2e_1$ remaining the same, the initial surface area is increased almost three times whereas the simultaneous increase of λ indicates a more drastic reduction of the surface area.

The diagram in fig. 23 clarifies the above: the heavy broken line divides the strip into square rods, the dotted line divides it into square slabs, while the dot-and-dash line divides it into cubes of the same thickness as that of the strip.

Increasing the tube length will ultimately result in a powder grain with a constant burning surface, for which

$$\varkappa = 1, \ \lambda = 0, \ \mu = 0;$$

consequently, the relations between ψ, z and σ, z will be expressed by the following formulas:

$$\psi = z; \quad \sigma = 1.$$

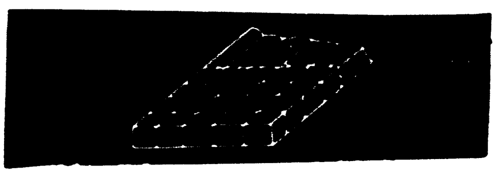

Fig. 23 - A Strip Divided into Der[illegible] [illegible]apes.

The same law governing the burning of powder will apply to tubular powder inhibited at its ends.

Examples of Calculating the Characteristics of Powder Shapes

1. Strip powder CП (SP); dimensions in millimeters: 1 by 18 by 300.

$$\alpha = \frac{2e_1}{2b} = \frac{1}{18} = 0.05555; \ \varkappa = 1 + \alpha + \beta = 1.0589;$$

$$\beta = \frac{2e_1}{2c} = \frac{1}{300} = 0.00333; \ \lambda = \frac{-(\alpha + \beta + \alpha\beta)}{\varkappa} = -\frac{0.05907}{1.0589} = -0.05575;$$

$$\alpha\beta = 0.00019; \ \mu = \frac{\alpha\beta}{\varkappa} = \frac{0.00019}{1.0589} = 0.00018;$$

$$\sigma_K = 1 + 2\lambda + 3\mu = 1 - 0.11150 + 0.00054 \approx 0.889;$$

$$\frac{S_1}{\Lambda_1} = \frac{\varkappa}{e_1} = \frac{1.0589}{0.50} = 2.1178 \text{ mm}^2/\text{mm}^2.$$

For the same powder reduced to 40 mm in length, the characteristics will change as follows:

$$\varkappa = 1.086;\ \lambda = -0.0758;\ \mu = 0.00129;\ \sigma_K = 0.852;\ \frac{S_1}{\Lambda_1} = 2.161.$$

2. A tubular grain having the same wall thickness and length as the strip powder:

$$2e_1 = 1;\ 2c = 300;$$

$$\alpha = 0,\ \beta = \frac{1}{300} = 0.00333;\ \alpha\beta = 0.$$

$$\varkappa = 1 + \beta = 1.00333;\ \lambda = -\frac{\beta_1}{1+\beta} = -\frac{0.00333}{1.0033} = -0.00332;$$

$$\sigma_K = \frac{S_K}{S_1} = 1 + 2\lambda = 1 - 0.00664 = 0.9934;$$

$$\frac{S_1}{\Lambda_1} = \frac{\varkappa}{e_1} = \frac{1.0033}{0.5} = 2.0066.$$

The same tube 40 mm in length:

$$\varkappa = 1.025;\ \lambda = -0.0244;$$

$$\frac{S_1}{\Lambda_1} = 2.050;\ \sigma_K = 0.9512.$$

Compared to a strip of the same length, a tube has a smaller effective surface area and a lower degree of regression. The surface area and regression in the burning area increase with decrease in length.

If we calculate the characteristics for shapes having the form of solids of revolution, we will find that their characteristics are expressed by precisely the same ratios as given in the table of characteristics for parallelepipeds; the only changes occur in the values of α and β, which are expressed in terms of the dimensions of the solids of revolution.

Table 11, below, contains shape characteristics for solids of revolution in the form of a solid rod, sphere and tube.

Table 11 - Shape Characteristics of Solids of Revolution

Basic dimensions			$\alpha = \frac{e_1}{b}$	$\beta = \frac{e_1}{c}$	$\alpha\beta$	$\varkappa = 1 + \alpha + \beta$	$\lambda = -\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}$	$\mu = \frac{\alpha\beta}{1 + \alpha + \beta}$
$2e_1$	$2b$	$2c$						
						Sphere		
$2R$	$2R$	$2R$	1	1	1	3	$-\frac{3}{3} = -1$	$\frac{1}{3}$
						Solid cylinder (rod)		
$2R$	$2R$	$2c$	1	$\frac{R}{L_0} = \beta$	β	$2 + \beta$	$-\frac{1 + 2\beta}{2 + \beta}$	$\frac{\beta}{2 + \beta}$
						Solid cylinder whose diameter equals its height		
$2R$	$2R$	$2R$	1	1	1	3	-1	$\frac{1}{3}$
						Cylindrical plate (pellet)		
$2e_1$	$2R$	$2R$	$\frac{e_1}{R} = \beta$	$\frac{e_1}{R} = \beta$	β^2	$1 + 2\beta$	$-\frac{2\beta + \beta^2}{1 + 2\beta}$	$\frac{\beta^2}{1 + 2\beta}$
						Tube		
$R-r$	∞	$2c$	0	$\frac{R - r}{[illegible]} = \beta$	0	$1 + \beta$	$-\frac{\beta}{1 + \beta}$	0

Table 11 (Cont'd.)

Ring-shaped pellet

$\frac{2e_1}{(2c_0)}$	∞	R-r	0	$\frac{2e_1}{R-r} - \beta$	0	$1 + \beta$	$-\frac{\beta}{1+\beta}$	0

A comparison of the data in Tables 10 and 11 will show that the characteristics $\varkappa$, λ, μ of some of the grain shapes listed in these tables are identical, and that therefore the law governing the change of volume ψ and surface area σ as a function of the relative burned thickness z is the same. Such shapes are termed <u>equivalent</u> shapes.

They are exemplified, for example, by a cube, a sphere, a cylinde whose height equals its diameter, or by square and round slabs, and square and round bars.

D. Graphical Illustration of the Relationships Between $\sigma - z$, $\psi - z$, $\sigma - \psi$

Knowing the general expressions for the inflow of gases

$$\psi = \varkappa z(1 + \lambda z + \mu z^2)$$

and for the law governing the change of relative surface

$$\sigma = 1 + 2\lambda z + 3\mu z^2,$$

we can by assigning specific values to z, calculate the corresponding values of σ and ψ and plot a graph for the investigated regressive grain shapes.

These diagrams are termed "progressive data sheets."

We shall consider the following shapes: 1 - tube; 2 - strip; 3 - square plate; 4 - a solid slab; 5 - cube or sphere.

a) σ, z diagram (fig. 24) (progressive data sheet).

Since λ is negative and μ is small in all the shapes under consideration, curves σ, z will always lie below the horizontal dotted line 1-1, which corresponds to a powder with a constant burning surface area.

At the start of burning, when $z = 0$, $\sigma = 1$ for all powder shapes. At the end of burning, when $z = 1$, the values of σ_K will vary for different grain shapes. The expressions for these values are given in Table 10. Inasmuch as for a tubular grain $\mu = 0$, the expression $\sigma = 1 + 2\lambda z$ will depict a straight line with an angular coefficient 2λ, where $\lambda < 0$; the straight line 1 is very close to the horizontal 1-1.

For strip powder, as well as all the other powder shapes, $\sigma = 1 + 2\lambda z + 3\mu z^2$ where $\lambda < 0$.

$$\frac{d\sigma}{dz} = 2\lambda + 6\mu z; \quad \frac{d^2\sigma}{dz^2} = 6\mu > 0.$$

Consequently, curve σ, z is convex with respect to the z-axis. At the start of burning

$$\left(\frac{d\sigma}{dz}\right)_0 = 2\lambda < 0.$$

Since the value of μ is small for a strip, the curvature of the line is very slight; the values of λ and μ increase as the shape

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becomes more regressive, the angle of inclination of the tangent at the origin of the coordinate axes increases, as does the curvature of the curve itself. In the case of a solid slab the curve has a pronounced downward bulge, and passes through zero ($\sigma_K = 0$) when $z = 1$ (curve 4).

In the case of a cube the downward slope and the convexity are maximum, when $\sigma_K = 0$, $z = 1$ (curve 5).

Thus the diagram shows that the cube is the most regressive of the five powder shapes considered here; the surface of the cube diminishes abruptly at the very start of burning and approaches zero at the end of burning. The least regressive powder shape is the tube, whose burning (surface) area remains practically constant at all times (reduction does not exceed 1%). Powder in the shape of a cube, at a given thickness of elements, gives off a maximum quantity of gases per unit of time at the start of burning; this quantity diminishes rapidly with burning. On the other hand, the quantity of gases given off by a tubular powder grain remains practically constant.

b) ψ, z Diagram (fig. 25):

$$\psi = \varkappa z(1 + \lambda z + \mu z^2).$$

Previously, we had the expression (9):

$$\frac{d\psi}{dz} = \varkappa(1 + 2\lambda z + 3\mu z^2) = \varkappa\sigma;$$

when $z = 0$ $\sigma = 1$ and $\left(\frac{d\psi}{dz}\right)_0 = \varkappa$.

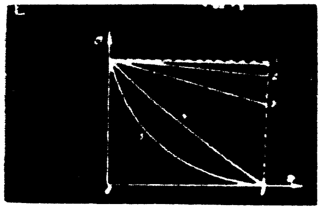

Fig. 24 - Change in area during regressive burning of powders of different shapes $\sigma = f(z)$.

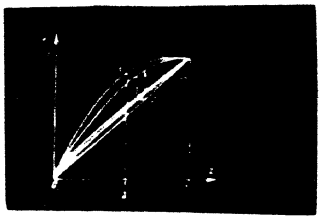

Fig. 25 - The effect of grain shape on gas inflow for regressive powders $\psi = F(z)$.

$\frac{d\psi}{dz}$ is the tangent of the slope angle of curve ψ, z with the abscissa, while the shape characteristic $\varkappa$ represents the slope angle of the curve at the origin of the coordinates.

When $z = 0$ $\psi = 0$, $\sigma = 1$; when $z = 1$ $\psi = 1$.

All the curves are located within a square whose sides equal unity.

All the curves originate at the origin of the coordinates and pass through point $\psi = 1$, $z = 1$; the tangent of the angle of inclination at the origin is $\left(\frac{d\psi}{dz}\right) = \varkappa$. Further variation of the angle of inclination is characterized by the value of σ. Inasmuch as σ diminishes in all the powder shapes considered here, the angle of inclination of all the curves diminishes also, and hence all the curves are convex upwards. In the case of a tubular grain $\varkappa$ approaches unity and the change of σ is small; curve 1 practically merges with a diagonal drawn from the origin of the coordinate system. For a cube, $\varkappa = 3$ is maximum; curve 5 has the greatest angle of inclination at the origin and the greatest variation of this angle corresponds to the variation of σ. In the case of a cube and a slab $\sigma_K = 0$,

and hence the curves are tangent to the horizontal line 1-1 when $z = 1$.

The arrangement of the remaining curves 2, 3, and 4 is obvious and does not require any explanation.

The diagram shows that for a given value of z the portion of the burnt grain ψ will be the larger, the more regressive is the powder and the greater is $\varkappa$. For example, in the case of tubular grain, at the instant the first half of the thickness ($z = 0.5$) is burned, the burnt portion of volume ψ will be equal to 0.5005, and in the case of a cube, when $z = 0.5$, $\psi = \frac{7}{8} = 0.875$.

Consequently, a more regressive powder gives off a larger quantity of gases during the first half of the burning process, and a smaller such quantity, during the second half.

c) The σ, ψ diagram has the greatest practical value, because it can be more easily compared with experimental data when evaluating the pressure curves obtained in the burning of powder in a manometric bomb. ψ is determined from the value of p by the aid of the general formula of pyrostatics, while σ goes into $\frac{d\psi}{dt}$ obtained by the numerical differentiation of the dependence of ψ on t. When this data is available, a comparison can be made of the theoretical and the experimental results.

Inasmuch as the equation for ψ, z is a third power equation, and σ, z is a second power equation, z is usually not eliminated when determining the dependence of σ, ψ; rather, by assigning definite values to z, the corresponding values of ψ and σ are computed, and the results are then plotted on the σ, ψ diagram.

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How will the σ, z diagram change after it is transformed into a σ, ψ diagram (fig. 26)?

Fig. 26 - Effect of the Shape of Regressive Powder Grains on the Change of the Powder Area During Burning - $\sigma = \varphi(\psi)$.

Similarly to the σ, z diagram, $\sigma = 1$ when $\psi = 0$. At the end of burning when $\psi = 1$, σ_K will have the same values as when $z = 1$. Consequently, the position of the initial and final points will not change.

Since all curves of ψ, z are situated above the diagonal dividing the square ψ, z in half, the magnitude $\psi > z$ will correspond to some value of z, and this magnitude will be the greater, the more regressive is curve ψ, z.

Therefore, when ψ is replaced by z, all the points on curves σ, z (see fig. 24), while remaining at the same height, will shift to the right and the amount of displacement will be the greater, the more regressive is curve ψ, z or σ, z (see fig. 26).

d) A binomial formula for the relationship ψ, z.

The examples given here for calculating the characteristics $\varkappa$, λ, μ for strip type powder show that μ is very small for strip and plate type grains, and that the term μz^2 does not appreciably affect the law governing the variation of ψ and σ. Therefore, in order to

simplify the expression subsequently entering the rather complex formulas of pyrodynamics in the solution of the basic problem, a binomial formula is used to express ψ for strip type powders without impairing the accuracy, by neglecting the term μz^2 in parenthesis. The influence of the neglected term μz^2 is compensated for by changing the remaining characteristics $\varkappa$ and λ, based on the following considerations.

Having a complete trinomial formula

$$\psi = \varkappa z(1 + \lambda z + \mu z^2)$$

with known characteristics, we shall replace it by a binomial formula with new characteristics $\varkappa_1$ and λ_1:

$$\psi = \varkappa_1 z(1 + \lambda_1 z).$$

In both equations $\psi = 0$ when $z = 0$. We shall establish the following conditions in order to determine coefficients $\varkappa_1$ and λ_1: 1) when $z = 1$ (end of burning), the binomial formula must give us $\psi = 1$, and 2) when $z = 0.5$, the value of ψ determined by means of the binomial formula must have the same value as ψ found by means of the trinomial formula at the same value of $z = 0.5$.

We thus obtain a system of two equations with two unknown coefficients $\varkappa_1$ and λ_1:

when $z = 1$

$$\varkappa(1 + \lambda + \mu) = 1 = \varkappa_1(1 + \lambda_1);$$

when $z = 0.5$

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$$\frac{\varkappa}{2}\left(1+\frac{\lambda}{2}+\frac{\mu}{4}\right)=\frac{\varkappa_1}{2}\left(1+\frac{\lambda_1}{2}\right).$$

Solving this system, we get [12]:

$$\varkappa_1=\varkappa-\frac{\varkappa\mu}{2}=\varkappa\left(1-\frac{\mu}{2}\right).$$

We can obtain from the first equation of the system

$$\lambda_1=\frac{1}{\varkappa_1}-1;$$

$$\lambda_1=\frac{\lambda+\frac{3}{2}\mu}{1-\frac{\mu}{2}}.$$

Since both λ_1 and $\lambda < 0$, for the absolute values thereof

$$|\lambda_1|=\frac{|\lambda|-\frac{3}{2}\mu}{1-\frac{\mu}{2}}.$$

When the values of the characteristics $\varkappa_1$ and λ_1 are determined in this manner, the second degree curve ψ, z and the third degree curve ψ, z will coincide at the starting point z = 0, then at z = 0.5, and finally at the end point z = 1.

Thus, in the case of strip-type powder, the curves practically coincide also at the intermediate points, when the values of $\varkappa_1$ and λ_1 are chosen as above.

Thus, the binomial formula can be used for strip and plate type powders also in the future

$$\psi = \varkappa_1 z(1 + \lambda_1 z).$$

Similarly, we shall have for the surface ratio

$$\sigma = 1 + 2\lambda_1 z.$$

Eliminating z from this system of equations, we get the dependence of σ on ψ in the following form:

$$\sigma = \sqrt{1 + 4\frac{\lambda_1}{\varkappa_1}\psi}.$$

For a given value of ψ, this relationship permits a direct calculation of the corresponding value of σ.

Hereafter, we shall drop the indexes of the characteristics $\varkappa$ and λ.

4. PROGRESSIVE-BURNING POWDERS

A. General Data

In all the regressive types of powder considered here, excepting tubular powders, the surface area always diminishes when the powder is burned, because burning proceeds inside the grain in concentric layers. A tubular grain is the exception in this respect: the surface of the perforation is displaced in burning from the axis of the tube outwards, thus increasing its area, and hence partly compensates

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for the reduction of the exterior surface area. The tubular type of powder would represent a powder of constant burning area bordering between progressive and regressive forms of powder, if the tube were not to burn from its ends and remain unchanged in length.

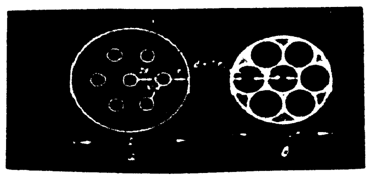

Fig. 27 - Grain with Seven Perforations:

a - before burning;
b - at the instant of decomposition.

At the start of burning the total surface of the tube of length 2c will be expressed by the following formula, if the end-areas and reduction in length are disregarded:

$$S_1 = 2\pi R 2c + 2\pi r \cdot 2c = 2\pi(R + r) \cdot 2c.$$

When the thickness burnt on the inside and the outside of the tube is e, the surface at that instant and at the same tube length will be

$$S = 2\pi(R - e)2c + 2\pi(r + e)2c = 2\pi(R + r)2c;$$

consequently, the surface area $S = S_1 = \text{const}$, because the reduction of the exterior surface is compensated for by the equivalent enlargement of the interior surface.

Under such conditions the constancy of the surface area does not depend on the diameter of the perforation. This property of the perforation surface to increase in burning indicates a means for obtaining a grain of the progressive type. This will require the use of grains with several perforations, whereby the increased surface area of the latter will compensate for the reduction of the outside area.

The grain with 7 perforations is based on this principle; the centrally disposed perforation along the axis of the grain compensates the decrease in the outside area, and the six radially disposed perforations in the vertices of a regular hexagon serve to increase the area during burning. The outside surface is located at a distance of $2e_1$ from the perforations. In such an arrangement (fig. 27a) the web thickness, i.e., the distance between the centrally located perforation and the outer perforations, as well as between the latter and the outer surface, will be the same, so that all the webs will burn simultaneously.

Burning from the perforation centers proceeds along concentric cylindrical surfaces, forming circles in section; when the latter converge and the thickness e_1 is burned in all directions, the grain disintegrates into 12 rods of irregular cross section (slivers): 6 inner, small rods and 6 outer larger rods (fig. 27b). These products of decomposition burn with a sharp reduction of the burning surface, similarly to a solid slab, and provide even greater regression because of the presence of sharp, rapidly burning projecting angles. Thus all progressive powder grains with several perforations, whose initial burning is accompanied by increased surface area disintegrate

at some instant into regressively burning products of decomposition. This is the undesirable characteristic of powders of the progressive type.

A grain with 7 perforations usually has a standard dimension ratio: the web thickness $2e_1$ between the perforations themselves as well as between the latter and the outer wall must be the same and equal to twice the diameter of the perforation (or the perforation diameter d must be equal to half of the web thickness e_1):

$$2e_1 = 2d;$$

accordingly, the outside grain diameter is

$$D = 4 \cdot 2e_1 + 3d = 11d = 11e_1.$$

The length of the grain 2c is not great, it is usually equal to (2-2.5) D or (20-25)d.

Fig. 28 - Products of Decomposition of a Grain With 7 Perforations.

At this dimension ratio, the surface increase at the time of decomposition amounts to about 37% ($S_s/S_1 = 1.37$, where S_s is the surface being burned at the instant decomposition occurs). At the same time, if the perforations are spaced correctly, about 85% of the

grain will be burned ($\psi_s \approx 0.85$). Consequently, about 15% of the grain is burned regressively with a sharp reduction of the surface area. If the spacing of the perforations is irregular, decomposition does not take place at the same instant: webs of the least thickness are burned first, followed by gradual burning of the thicker webs; a portion of the grain undergoes progressive burning and the remaining portion suffers a sharp reduction of its area. The maximum value of S_s is smaller than in a normal grain, and this corresponds also to a lower value of ψ_s.

If we inscribe a circle in the outer prism of decomposition (fig. 28), its radius $\rho = 0.1772(d + 2e_1)$ [12]; in the case of standard dimensions $\rho = 0.1772 \cdot 3e_1 = 0.5316e_1 \approx 0.532e_1$.

The radius of the circle inscribed in the inner prisms of decomposition $\rho' = 0.0774(d + 2e_1) = 0.2322e_1 \approx 0.232e_1$. Therefore, at the end of burning of the grain as a whole, when the burning surfaces merge at the center of the outside prisms of decomposition, the thickness burned will be $e_K = e_1 + \rho = e_1 + 0.532e_1 = 1.532e_1$.

Thus the burning of progressive powders is subdivided into two sharply differing phases: 1) prior to decomposition z varies from 0 to 1, ψ varies from 0 to $\psi_s < 1$; burning proceeds with a gradually increasing area; 2) after decomposition z changes from 1 to $z_K = \frac{e_1 + \rho}{e_1} = 1 + \rho/e_1$, ψ changes from ψ_s to 1; burning is progressive.

Some authors (V.E. Slukhotsky) consider it more expedient to consider $z_K = 1$ when $\psi = 1$; then, at the instant of decomposition, $z_s = \frac{e_1}{e_1 + \rho} < 1$ ($z_s = 0.653$ for standard dimensions).

Disintegration and regressive burning of the powder in the second phase are the defects of powders of the progressive type. In order to decrease the products of decomposition, a great many grain shapes were suggested by various authorities, of which the most interesting ones are the shapes suggested by Walsh and Kisnemsky.

Walsh's grain is an improved grain with 7 perforations, whose outer wall is not a continuous (single) cylindrical surface of diameter $D = 11d$, but, rather, consists of six cylindrical surfaces circumscribed about the axis of each perforation of radius $r = \frac{d}{2} + 2e_1$ (in the case of standard dimensions $r = 2.5d = 2.5e_1$). The cross section of such a grain is as shown in fig. 29a, and at the instant of decomposition its section takes on the form shown in fig. 29b.

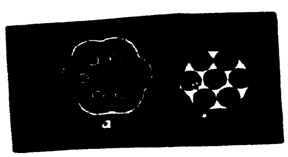

Fig. 29 - Walsh's Grain with 7 Perforations.

a) Before burning; b) at the instant of decomposition.

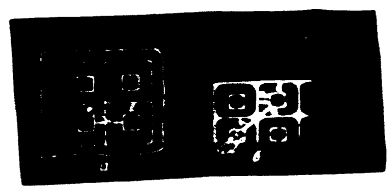

Fig. 30 - Burning of Kisnemsky's Grain.

A grain of this type is often called a "shaped grain."

The dimensions of the products of decomposition in such a grain are considerably smaller than in an ordinary grain with 7 perforations.

Calculations show that $\psi_s \approx 0.95$, i.e., that 95% of the grain undergoes progressive burning, and only 5% is regressive. Also, for the same dimensions d, $2e_1$ and 2c as in a standard grain, the surface area increase is the same, or 37%, so that $\sigma_s \approx 1.37$.

The Kisnemsky grain was designed for the purpose of eliminating the products of decomposition on the one hand, and for obtaining a grain of "greater progressivity" on the other. It is in the form of a square slab with square perforations disposed in two mutually perpendicular directions (fig. 30a). According to Kisnemsky, the square perforation should burn in parallel layers and retain its square form, so that burning would terminate without decomposition. Furthermore, a large number of narrow perforations should provide a highly progressive grain (according to calculations $\sigma_K \approx 2$). This is the powder recommended by Kisnemsky for obtaining extra-long-range firing; the KOCAPTOП (Kosartop) (Special Artillery Research Committee), headed by V.M. Trofimov, was the body engaged in the investigation of this type of powder at the time. All the theoretical work on the performance of this powder was carried out by V.A. Pashkevich.

It was found that Kisnemsky's estimates were not justified either with regard to burning without decomposition, or with regard to high progressivity. Tests had shown that the square-shaped perforation does not retain its shape, because burning from the corners of the square proceeds at the same rate as along a normal to the side of the square (there is no reason to believe that the powder would burn more rapidly along the diagonal than in other directions); it proceeds from each corner in concentric circles, whose radius equals the thickness

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of the burnt layer in the direction of normals to the sides of the square. As a result, the cross section is that of a square with rounded corners, and at the instant of decomposition the shape of the grain appears as in fig. 30b.

The products of decomposition amounting to 10% remain between the perforations. Hence ψ_s is not 1 but is about 0.90 (90% of the grain burns progressively). $\rho'' = (\sqrt{2} - 1)e_1$, because $e_1 + \rho$ is the diagonal of a square with side e_1.

For reasons which will be discussed later, the powder suggested by Kisnemsky did not show the high progressive quality anticipated by him.

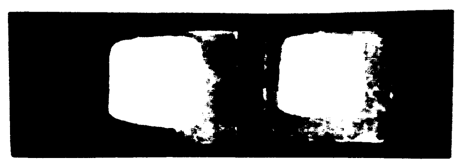

Fig. 31 - A Grain of Kisnemsky's Powder Before and After Disintegration (Obtained in Firing).

Figure 31, on the left, is a Kisnemsky grain with 35 perforations before burning; on the right is the same grain incompletely burned and ejected from the gun after firing. The photograph clearly shows the products of the starting disintegration having the form of the "ace of diamonds" in cross section; the perforations are almost round.

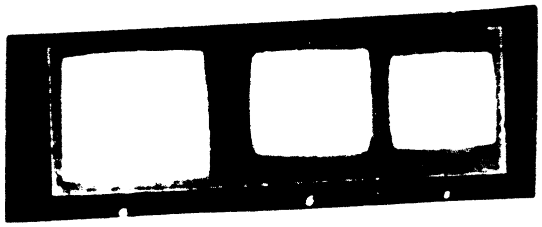

Fig. 32 - Transformation of a Square Perforation into a Round One.

Figure 32 is a large grain with a single perforation: a) before burning; b and c) intermediate states. Inasmuch as the side of the square perforation is small in comparison with the web thickness, the perforation is transformed into a circle.

B. Determination of the Characteristics of Progressive Powders

a) First phase, first method.

The law governing burning and the change of the surface of progressive type powders is expressed by the same general formulas:

$$\psi = \varkappa z(1 + \lambda z + \mu z^2),$$

$$\sigma = 1 + 2\lambda z + 3\mu z^2,$$

as for regressive types; the difference is only in the numerical values of the characteristics $\varkappa$, λ and μ and in their signs ($\lambda > 0$, $\mu < 0$). The derivation of the characteristics equations is obtained similarly to regressive powders, and only the obtained expressions themselves are somewhat more complex than for regressive powders.

We shall introduce the following designations: $2c$ - grain length, $2e_1$ - web thickness; d - diameter of perforation; D - grain diameter; n - number of perforations.

The burned portion of the charge is:

$$\psi = 1 - \frac{\Lambda_{ocт}}{\Lambda_1}; \quad \frac{2e_1}{2c} = \beta;$$

$$\frac{D + nd}{2c} = \Pi_1; \quad \frac{D^2 - nd^2}{(2c)^2} = Q_1.$$

It can be easily seen that the magnitude $\Pi_1 = D + nd/2c$ is the ratio of the perimeter of the grain's cross section to a circumference whose diameter equals the length of the tube $2c$. The magnitude $Q_1 = \frac{D^2 - nd^2}{(2c)^2}$ is the ratio between the base area and the area of a circle whose diameter is $2c$.

The initial grain volume is

$$\Lambda_1 = \frac{\pi}{4}(D^2 - nd^2)2c.$$

The unburned portion of the grain by volume at a given instant is

$$\Lambda_{ocт} = \frac{\pi}{4}\left[(D - 2e)^2 - n(d + 2e)^2\right](2c - 2e);$$

$$\frac{\Lambda_{ocт}}{\Lambda_1} = \frac{\left[D^2 - nd^2 - 2(D + nd)\,2e - (n - 1)(2e)^2\right]}{D^2 - nd^2} \cdot \frac{(2c - 2e)}{2c}.$$

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Bearing in mind that

$$\frac{2e}{2c} = \frac{2e_1}{2c}\,\frac{2e}{2e_1} = \beta z$$

and dividing each term in the parenthesis by the denominator, we get:

$$\frac{\Lambda_{OCT}}{\Lambda_1} = \left[1 - \frac{2(D + nd)}{D^2 - nd^2}\,2e - \frac{(n - 1)}{D^2 - nd^2}(2e)^2\right](1 - \beta z),$$

or, substituting $\Pi_1 \cdot 2c$ for $D + nd$ and $Q_1\,(2c)^2$ for $D^2 - nd^2$, we have:

$$\frac{\Lambda_{OCT}}{\Lambda_1} = \left[1 - \frac{2\,\Pi_1 \cdot 2c \cdot 2e}{Q_1(2c)^2} - \frac{n - 1}{Q_1(2c)^2}(2e)^2\right](1 - \beta z) =$$

$$= \left(1 - \frac{2\Pi_1}{Q_1}\beta z - \frac{n - 1}{Q_1}\beta^2 z^2\right)(1 - \beta z) =$$

$$= 1 - \left(1 + \frac{2\Pi_1}{Q_1}\right)\beta z - \left(\frac{n - 1 - 2\Pi_1}{Q_1}\right)\beta^2 z^2 + \frac{n - 1}{Q_1}\beta^3 z^3;$$

$$\psi = 1 - \frac{\Lambda_{OCT}}{\Lambda_1} = \left(1 + \frac{2\Pi_1}{Q_1}\right)\beta z + \frac{(n - 1) - 2\Pi_1}{Q_1}\beta^2 z^2 - \frac{n - 1}{Q_1}\beta^3 z^3 =$$

$$= \frac{Q_1 + 2\Pi_1}{Q_1}\beta z\left(1 + \frac{n - 1 - 2\Pi_1}{Q_1 + 2\Pi_1}\beta z - \frac{(n - 1)\beta^2}{Q_1 + 2\Pi_1} z^2\right).$$

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We thus get a general type formula:

$$\psi = \varkappa z(1 + \lambda z + \mu z^2),$$

where

$$\left.\begin{aligned} \varkappa &= \frac{Q_1 + 2\Pi_1}{Q_1}\beta; \\ \lambda &= \frac{n - 1 - 2\Pi_1}{Q_1 + 2\Pi_1}\beta; \\ \mu &= -\frac{(n-1)\beta^2}{Q_1 + 2\Pi_1}; \end{aligned}\right\}^{(*)} \quad \text{when } n = 7 \quad \left\{\begin{aligned} \varkappa &= \frac{Q_1 + 2\Pi_1}{Q_1}\beta; \\ \lambda &= \frac{2(3 - \Pi_1)}{Q_1 + 2\Pi_1}\beta; \\ \mu &= -\frac{6\beta^2}{Q_1 + 2\Pi_1}. \end{aligned}\right.$$

These formulas show that for powders with many perforations the characteristic $\mu < 0$ and hence the convexity of curve σ, z is directed upwards (when $n = 1$, $\mu = 0$). The sign of λ depends on the difference $(n - 1) - 2\Pi_1$, and in ordinary powders with 7 perforations of standard dimensions $\lambda > 0$. When the grain is shortened λ may become equal to zero: this will occur when $n - 1 - 2\Pi_1 = 0$ or $n - 1 - 2\frac{D + nd}{2c} = 0$, whence

---

(*) These formulas are suitable not only to progressive powders where $n > 1$, but also for regressive shapes - solids of revolution, for exampl for $n = 1$ (tube) and $n = 0$ (round slab without perforations).
Hence, they are general formulas: thus, for example, for a tube when $n = 1$, $\lambda < 0$, $\mu = 0$, when $n = 0$ (solid slab) $\lambda < 0$, $\mu > 0$, as was established in the previous formulas.

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$$\frac{D + nd}{2c} = \frac{n - 1}{2}.$$

Under such conditions, perforated powder is no longer progressive.

In particular, for a grain of standard cross section with 7 perforations, whose $D = 11d$, the condition $\lambda = 0$ is satisfied when

$$2c = 6d = 3 \cdot 2e_1,$$

and since $\mu < 0$, the area of such a short grain will diminish when burned.

A study of the λ coefficient shows that progressivity increases with the number of perforations, if the diameter of the perforations at the same web thickness decreases and the length of the slab increases.

The expression for the surface (area) change will have the following general form:

$$\sigma = 1 + 2\lambda z + 3\mu z^2.$$

In the case of rectangular shapes, such as the Kisnemsky grain, the magnitude Π_1 is the ratio between the perimeter of the cross section and the perimeter of a square with side 2c equal to the length of the slab, and Q_1 is the ratio between the cross-sectional area of the grain and the area of the same square with side 2c.

If we call the side of a square slab A_1, the side of the square perforation a_1, the length of the slab 2c and the number of perforations n^2 (n horizontal and vertical rows),

$$\Pi_1 = \frac{(A_1 + n^2 a_1)}{2c}, \quad Q_1 = \frac{A_1^2 - n^2 a_1^2}{(2c)^2}, \quad \beta = \frac{2e_1}{2c}.$$

These formulas hold true for Walsh's grain as well, but inasmuch as the cross section of this grain is more complex, the formulas for Π_1 and Q_1 are likewise more complex.

The outside wall of Walsh's grain can be considered as consisting of six arcs whose lengths equal 1/3 of a circumference of diameter $\delta_1 = d + 2 \cdot 2e_1$ and which are described from the centers of six perforations, and six arcs each measuring 1/6 of a circumference of diameter d_1 described from the outside vertices of equilateral triangles with side $a_1 = 2e_1 + d_1$, whose other two vertices lie in the center of the perforations (fig. 33).

In such a case the cross-sectional area S_T of the grain consists of the following elements:

1) 12 triangles with side a_1 less three sectors of a circle of diameter d_1 at their apexes,

2) six sectors each measuring 1/3 of a circle of diameter δ_1 less six sectors each equivalent to 1/3 of a circle of diameter d_1:

$$S_T = 12\left(\frac{\sqrt{3}}{4} a^2 - 3\,\frac{1}{6}\,\frac{\pi}{4} d_1^2\right) + 6\,\frac{1}{3}\,\frac{\pi}{4}(\delta_1^2 - d_1^2) =$$

$$= \frac{\pi}{4}\left(\frac{12\sqrt{3}}{\pi} a^2 - 8d_1^2 + 2\delta_1^2\right).$$

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The perimeter P of the grain will consist of:

1) six arcs each measuring 1/3 of a circumference of diameter δ_1;

2) seven circles of diameter d_1;

3) six arcs each measuring 1/6 of a circumference of diameter d_1 (in the angles included in the outside web.

$$P = \pi\left(6\,\frac{1}{3}\,\delta_1 + 7d_1 + 6\,\frac{1}{6}\,d_1\right) = \pi(2\delta_1 + 8d_1) = 2\pi(\delta_1 + 4d_1).$$

Then:

$$\beta = \frac{2e_1}{2c_1};$$

$$\Pi_1 = \frac{P}{\pi 2c_1} = \frac{2(\delta_1 + 4d_1)}{2c_1};$$

$$Q_1 = \frac{S_T}{\frac{\pi}{4}(2c_1)^2} = \frac{\frac{12\sqrt{3}}{\pi}a_1^2 + 2\delta_1 - 8d_1^2}{(2c_1)^2}.$$

Fig. 33 - Diagram of Walsh's Grain.

b) A binomial formula for the second phase.

Since for a standard grain with 7 perforations the characteristic μ is small, the law governing burning $\psi = f(z)$ can also be expressed with sufficient accuracy by means of a binomial formula:

$$\psi = \varkappa_1 z(1 + \lambda_1 z).$$

The characteristics $\varkappa_1$ and λ_1 will be found under the condition that when $z = 1$, $\psi = \psi_s$, and when $z = 0.5$, the value of ψ according to a trinomial formula would be equal to the value of ψ according to the binomial one. We thus obtain a system of two equations as for regressive powders:

when $z = 1$

$$\varkappa(1 + \lambda + \mu) = \psi_s = \varkappa_1(1 + \lambda_1);$$

when $z = 0.5$

$$\frac{\varkappa}{2}\left(1 + \frac{\lambda}{2} + \frac{\mu}{4}\right) = \frac{\varkappa_1}{2}\left(1 + \frac{\lambda_1}{2}\right).$$

Solving it we get

$$\varkappa_1 = \varkappa\left(1 - \frac{\mu}{2}\right),$$

and then

$$\lambda_1 = \frac{\psi_s}{\varkappa_1} - 1$$

(instead of $\lambda_1 = \dfrac{1}{\varkappa_1} - 1$ for regressive powders).

Example. Compute the shape characteristics of progressive powders. Grain with 7 perforations.

$$2e_1 = 1;\ d = 0.5;\ D = 5.5;\ 2c = 12.5 \text{ mm}$$

$$\beta = \frac{2e_1}{2c} = \frac{1}{12.5} = 0.08;\quad \Pi_1 = \frac{D + 7d}{2c} = \frac{9}{12.5} = 0.720;$$

$$Q_1 = \frac{D^2 - 7d^2}{(2c)^2} = \frac{30.25 - 7 \cdot 0.25}{156.25} = \frac{28.50}{156.25} = 0.1824;$$

$$Q_1 + 2\Pi_1 = 0.1824 + 2 \cdot 0.720 = 1.6224;$$

$$\varkappa = \frac{Q_1 + 2\Pi_1}{Q_1}\beta = \frac{1.6224}{0.1824} \cdot 0.08 = 0.712;$$

$$\lambda = \frac{(n - 1) - 2\Pi_1}{Q_1 + 2\Pi_1}\beta = \frac{6 - 1.44}{1.622} \cdot 0.08 = 0.225;$$

$$\mu = -\frac{(n - 1)\beta_1^2}{Q_1 + 2\Pi_1} = -\frac{6 \cdot 0.0064}{1.622} = -0.0237;$$

$$\psi_s = \varkappa(1 + \lambda + \mu) = 0.712(1 + 0.225 - 0.0237) = 0.712 \cdot 1.2013 = 0.855;$$

$$\sigma_s = 1 + 2\lambda + 3\mu = 1 + 0.45 - 0.0711 = 1.379.$$

In the case of the binomial formula $\psi = \varkappa_1 z(1 + \lambda_1 z)$:

$$\varkappa_1 = \varkappa \left(1 - \frac{\mu}{2}\right) = 0.712 \cdot \left[1 - \left(- \frac{0.0237}{2}\right)\right] = 0.712 \cdot 1.0118 = 0.720;$$

$$\lambda_1 = \frac{\psi_s}{\varkappa_1} - 1 = \frac{0.855}{0.720} - 1 = 0.1873;$$

$$\psi_s = \varkappa_1(1 + \lambda_1) = 0.720(1 + 0.1873) = 0.855;$$

$$\sigma_s = 1 + 2\lambda_1 = 1.375.$$

It can be seen that the difference between the values of σ_s corresponding to the instant of disintegration obtained by trinomial and binomial formulas would be very small (0.004), so that in practice the binomial formula can be used for powders with 7 perforations in the first phase of burning.

c) Second phase (after disintegration).

The products obtained after decomposition in the form of small prisms of triangular cross section (with curved sides) burn regressively with rapid surface reduction, similarly to a square or round slab.

A detailed investigation made by G.V. Oppokov [13], using the assumption that burning proceeds in strictly parallel layers, provides general expressions: one - prior to burning of the thinner inside prisms, and the other - until the end of burning of the outside prisms. Using the formulas suggested by him as a basis, Oppokov

prepared a table for the values of ψ as a function of z when the small and large prisms are burned and then when only the remainders of the large prisms are burned. An analysis of his data shows that the change in the surface area of the products of decomposition almost corresponds to the change in the surface of a cube, i.e., its burning is more regressive than in the case of a prismatic slab.

The binomial formula can be used for determining the relationship ψ, z in the second phase. Transferring the origin of the coordinates to the point of decomposition ($z_s = 1$, $\psi = \psi_s$) we will have

$$\psi - \psi_s = \varkappa_2(z - 1)\left[1 + \lambda_2(z - 1)\right]\ldots, \tag{30}$$

with z varying from 1 to z_K.

We shall determine $\varkappa_2$ and λ_2 by imposing the following requirements:

1) when $z = z_K$, ψ must equal unity according to formula (30);

2) at the end of burning when $z = z_K$ the surface σ must become equal to zero.

Differentiating equation (30) with respect to z, we get:

$$\frac{d\psi}{dz} = \frac{S}{\Lambda_1}e_1 = \varkappa_2\left[1 + 2\lambda_2(z - 1)\right].$$

The second requirement will be satisfied if:

$$1 + 2\lambda_2(z_K - 1) = 0.$$

The first requirement will be written thus:

$$1 - \psi_s = \varkappa_2(z_K - 1)\left[1 + \lambda_2(z_K - 1)\right].$$

Solving this equation we find that

$$\lambda_2 = \frac{-1}{2(z_K - 1)}; \quad \varkappa_2 = \frac{2(1 - \psi_s)}{z_K - 1}.$$

Calculations by means of these formulas show that for a standard grain with 7 perforations when $z_K = 1.532$ $\psi_s = 0.855$:

$$\lambda_2 = \frac{1}{2 \cdot 0.532} = -0.94; \quad \varkappa_2 = \frac{2 \cdot 0.145}{0.532} = 0.545;$$

for Walsh's grain when $z_K = 1.232$ and $\psi_s = 0.95$

$$\lambda_2 = \frac{-1}{2 \cdot 0.232} = -2.16; \quad \varkappa_2 = \frac{2 \cdot 0.05}{0.232} = 0.432.$$

C. Graphic Representation of Relations $\sigma - z$, $\psi - z$, $\sigma - \psi$ for Progressive Shapes

In order to construct diagrams $\psi - z$; $\frac{S}{S_1} - z$; $\frac{S}{S_1} - \psi$ for progressive shapes, we shall resort to the following general formulas:

$$\psi = \varkappa z(1 + \lambda z + \mu z^2) \quad \text{and} \quad \frac{S}{S_1} = 1 + 2\lambda z + 3\mu z^2,$$

which are applicable also to these shapes, but now when $z = 1$

$\psi = \psi_s < 1$; the end of burning occurs when $z > 1$, the coefficient $\mu < 0$ (not only for grains with 7 perforations, but also for the Kisnemsky grain.)

Accordingly, the diagrams will appear as shown in fig. 34 and 35:

I - grain with 7 perforations;

II - grain with shaped outer web;

III - Kisnemsky's grain.

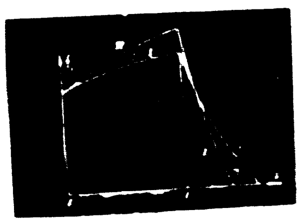

Fig. 34 - $\sigma = f(z)$ relationship for progressive grains.

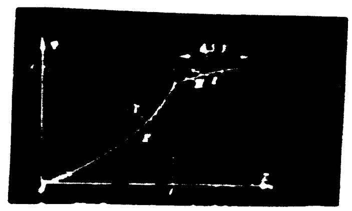

Fig. 35 - $\psi = f_1(z)$ relationship for progressive grains.

Inasmuch as $\mu < 0$, the convexity of the S/S_1, z curves is directed upwards. When constructing an S/S_1 diagram as a function of ψ, the corresponding points of the S/S_1,z diagram will be displaced to the left (the reverse of regressive grains), this displacement being the smaller the greater is the value of z; hence the S/S_1, ψ curves will likewise remain convex upwards as do the S/S_1, z curves (fig. 36).

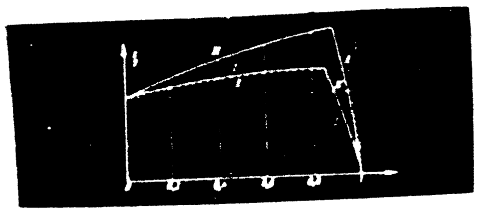

Fig. 36 - $s = f_2(\psi)$ [illegible] on for Progressive Powders.

a) Inhibited powders of high progressivity

Inhibited powders constitute one of the forms of highly progressive powders.

These powders first appeared in Russia soon after the appearance of Kisnemsky's powders and were developed jointly with the latter. This problem was studied by O.G. Filippov, an instructor at the Artiller[y] Academy, in 1920-1928.

Inhibited powder is obtained from ordinary tubular powder, whose outside surface is coated with a special nonburning substance.

When such a powder is ignited, only the inside surface of the grain burns, which surface increases in proportion to the diameters ratio $D_0 : d_0$. When the diameter of the perforation equals the thickness of the tube

$$\frac{S_K}{S_1} = \frac{D_0}{d_0} = 3.$$

The resulting progressiveness is greater than in a Kisnemsky grain with 36 perforations.

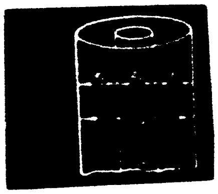

Fig. 37 - Inhibited Tubular Powder.

Notwithstanding the apparent simplicity of this idea and the theoretical possibility of obtaining high progressivity, the practical realization of same was found to be very difficult, because the inhibitor layer must not burn and must be capable of protecting the outer tube surface from burning. At the same time it must be sufficiently strong to withstand abrasion when shaken and be strong enough not to be torn off the surface by the action of the gases.

The undesirable property of inhibited powders is greater smoking when fired, due to the disintegration of the inhibitor at the instant the shell is ejected from the bore of the gun.

b) Characteristics of inhibited powders.

The outside surface and ends of the powder are inhibited and do not burn.

In contradistinction to an ordinary tube, the web thickness of an inhibited powder burns in one direction only and should therefore be denoted by e_1 rather than by $2e_1$, which is the usual designation.

Applying the general method, we get:

$$\Lambda_1 = \frac{\pi}{4}\left[(d_0 + 2e_1)^2 - d_0^2\right]2c = \frac{\pi}{4}\,2c\left[2d_0 2e_1 + 4e_1^2\right] = \pi 2c(d_0 e_1 + e_1^2);$$

$$\Lambda_{OCT} = \frac{\pi}{4}\left[(d_0 + 2e_1)^2 - (d_0 + 2e)^2\right]2c = \pi 2c\left[d_0 e_1 + e_1^2 - (d_0 e + e^2)\right];$$

$$\frac{\Lambda_{OCT}}{\Lambda_1} = 1 - \frac{d_0 e + e^2}{d_0 e_1 + e_1^2} = 1 - \frac{\frac{e}{e_1} + \frac{e_1}{d_0}\left(\frac{e}{e_1}\right)^2}{1 + \frac{e_1}{d_0}} = 1 - \frac{z + \frac{e_1}{d_0}z^2}{1 + \frac{e_1}{d_0}};$$

$$\psi = 1 - \frac{\Lambda_{OCT}}{\Lambda_1} = \frac{1}{1 + \frac{e_1}{d_0}}\, z\left(1 + \frac{e_1}{d_0}\, z\right);$$

Comparing it with the usual formula $\psi = \chi z(1 + \lambda z)$, we get:

$$\lambda = \frac{e_1}{d_0}, \quad \chi = \frac{1}{1 + \lambda} = \frac{1}{1 + \frac{e_1}{d_0}};$$

$$\sigma = \frac{S}{S_1} = 1 + 2\lambda z = 1 + \frac{2e_1}{d_0}z; \quad \sigma_K = 1 + \frac{2e_1}{d_0} = \frac{D_0}{d_0}.$$

When the tube thickness e_1 equals its diameter d_0,

$$\lambda = 1; \ \varkappa = \frac{1}{1 + \lambda} = 0.5; \ \sigma_K = 1 + 2 = 3.$$

By reducing the diameter of the perforation with the web thickness remaining the same, the geometric progressiveness can be considerably increased:

$$d_0 = \frac{e_1}{2}; \ \lambda = 2; \ \varkappa = \frac{1}{3}; \ \sigma_K = 1 + 2\lambda = 5.$$

CHAPTER V - BURNING RATE

The burning rate of powder mainly depends on its properties and temperature, and the pressure and temperature of the gases surrounding it.

Inasmuch as the temperature of the gases formed during burning of powder as yet does not lend itself to experimental determination, the burning rate is usually expressed as a function of its properties and gas pressure which is known from experiment at any given instant of time.

The functional dependence of the burning rate u on pressure of the form $u = f(p)$ is known as the "burning rate law," and this law is expressed by various empirical formulas as given by different authors.

Experimental determination of the burning rate of powder is possible on the basis of a test curve depicting pressure as a function of time; this involves the use of the fundamental relationship of the geometrical law of burning giving the relation between ψ and $z = e/e_1$.

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For determining the burning rate, use is usually made of strip, plate or tubular powder, of uniform thickness; its dimensions are carefully measured, the mean values of $2e_1$, $2b$, $2c$ or $2e_1$, D_0, d_0, $2c$ are determined, the characteristics of $\varkappa$ and λ are calculated (using the binomial formula), and a graph is constructed for $\psi = f(z)$.

The powder is then burned in a manometric bomb using a strong igniter, to insure simultaneous ignition along the entire powder surface so as not to impair the initial dimensions used for calculating the $\varkappa$ and λ characteristics.

The analysis of the bomb test data is conducted in the following manner.

Having obtained from the bomb test a curve of pressure p as a function of time t, and upon determining for this powder on the basis of the derived relationships the law governing the variation of ψ with z or e and constructing the corresponding ψ, z or ψ, e diagram, we can determine the rate of burning u at the given pressure. Indeed, knowing the values of p, we can determine by means of the general pyrostatics formula or from tables the values of ψ, for which we take the values of e from the ψ, e diagram. The difference between the neighboring values of e will give the increment Δe for the time interval Δt, known from measurement of the p, t curve; the $\Delta e/\Delta t$ ratio gives the burning rate u at p which is an average value for the given section. Thus, having at our disposal calculated data in the form of a table, we can determine the variation in the burning rate and of the given powder due to change of pressure p (Table 12).

Table 12 - Determining the Burning Rate u = f(p)

Auxiliary table or diagram compiled on the basis of the geometric law of burning			Table Derived from Test Data						
			1	2	3	4	5	6	7
z	ψ	e	t	p	p_{cp}	ψ	According to Auxiliary Diagram	Δe	$\frac{\Delta e}{\Delta t} = u$
0	0	0	Q	p_B	$\frac{1}{2}(p_B + p')$	0	0	$\Delta e' = e' - 0$	u'
z'	ψ'	e'	t'	p'	$\frac{1}{2}(p' + p'')$	ψ'	e'	$\Delta e'' = e'' - e'$	u''
z''	ψ''	e'	t''	p''		ψ''	e''		
.	.	.	.	.			.		
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.	.	.	.	.		.	.		
.	.	.	.	.		.	.		
1	1	e_1	t_K	p_m		1	e_1		

Plotting the obtained values of u, p_{cp} ($p_{cp} = p_{mean}$ - Translator) on a graph, we can find the change of u with change of pressure p, and thus determine the burning rate law.

The formulas most often used for determining the burning rate are the following.

a) Vieille's formula (exponential equation)

$$u = Ap^{\nu},$$

where A and ν depend on the nature of the powder, and, in particular, ν may be equal to unity.

The smaller the value of ν, the less sensitive is the powder to pressure changes.

For smokeless powders Vieille used $\nu = 2/3$, and for ordinary black powders $\nu = 1/2$. Our tests with slowly burning black powders for a time fuze gave the value of $\nu = 1/5$.

G.A. Zabudsky used $\nu = 0.93$ for pyroxyline powders. Some authors use $\nu = 1$ for cordites and $\nu = 1.07$ for ballistite.

b) Binomial formula

$$u = a + bp.$$

It was first used by Prof. S.P. Vukolov (1891-1897) in the Naval Technical Laboratory, and then by Wolf (1903) and Prof. I.P. Gravé (1904). Muiraur employed this formula considerably later (1930-1935).

In his thesis (1904) I.P. Gravé [14] compared formula $u = Ap^{\nu}$ with formula $u = ap + b$ by analyzing a large number of bomb tests conducted by himself and others and arrived at the following conclusion: "Both formulas can be considered equally valid for expressing the law governing the change of burning rate under varying pressure, because the mean errors obtained with the use of these formulas are

generally the same, and both formulas give practically identical results."

This deduction which appears strange at first glance, namely, that a parabola (semicubical) and a straight line not passing through the origin give identically accurate results, can be explained as follows.

The first tests in bombs since the year 1880 were conducted with the use of cylindrical crushers which do not permit recording pressures below 300-400 kg/cm^2 (fig. 38). The test data (points) are usually scattered to a certain extent and do not lie on a definite line. As a result, some of the investigators drew a parabola $u = Ap^{\nu}$ through these points, and others drew a straight line $u = ap + b$ which does not pass through the origin of the coordinates (curve 2).

The conclusion arrived at by Prof. Gravé confirms the fact that both lines pass sufficiently close to the points plotted on the basis of tests at pressures exceeding 400 kg/cm^2.

The relationship $u = f(p)$ could not be obtained experimentally at low pressures (< 400 kg/cm^2) at that time, and the question of the true relationship continued to remain open.

c) Formula $u = Ap$.

Charbonier (1908) first accepted the burning rate law in its general form $u = Ap^{\nu}$, and then, on the basis of his own analysis of test curves p, t obtained in bomb tests, arrived at the conclusion that for French strip-type B powders ν can be taken equal to $\nu = 1$.

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This law was accepted also by our Prof. N.F. Drozdov in his thesis [15] in the year 1910.

In 1913, Schmitz burned tubular powders in a large Krupp bomb, using an elastic bar with an optical method of recording pressures instead of a crusher. He succeeded in obtaining a full curve of the pressure increase in the bomb from the start to the end of powder burning. In order to evaluate the accuracy of either burning rate law, he introduced a new criterion, and proved by means of an actual test the justness of the burning rate law in the form $u = Ap$.

Fig. 38 - Dependence of Burning Rate on Pressure.

The criterion determining the justness of this law confirmed by experiment is presented below.

We shall assume that the following law holds true:

$$u = Ap;$$

since the burning rate is a ratio between the increment of the burned thickness de and the corresponding time element dt, i.e.,

$$u = \frac{de}{dt},$$

then

$$de/dt = Ap;$$

$$de = Apdt.$$

Integrating, we get:

$$\int_0^e de = e = A \int_0^t pdt \text{ or } \int_0^t pdt = \frac{e}{A}.$$

Burning terminates when thickness e_1 is burned. The full time of burning will be t_K, and we will have:

$$e_1 = A \int_0^{t_K} pdt,$$

whence

$$\int_0^{t_K} pdt = \frac{e_1}{A} = \text{const (for the given powder)}.$$

Magnitudes e_1 and A characterize the dimensions and nature of the powder and do not depend on the conditions of loading

It follows: that if the burning rate law $u = Ap$ is correct, the pressure impulse of the powder gases depends only on the burned thickness of the powder, on the burning rate coefficient, and on the characteristic properties of the powder, and does not depend on the loading density.

The full pressure impulse during the full time of burning equals half of the thickness e_1 of the burning layer divided by the burning rate coefficient A, and does not depend on the loading density.

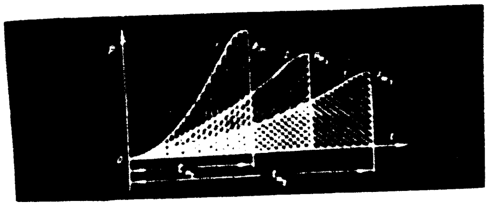

Fig. 39 - p, t Curves for Different Loading Densities.

When conducting tests with tubular powder of one kind in a Krupp bomb, with Δ varying from 0.12 to 0.26, and upon measuring the area under the pressure curves p = f(t), Schmitz had found that the areas $\int_0^{t_K}$ pdt found by experiment are actually equal to one another, which finding confirms the validity of the u = Ap law.

Figure 39 shows the form and arrangement of p, t curves.

The greater the loading density, the smaller is the burning time and the higher is the gas pressure p, t curve.

Were the u = de/dt = ap + b law valid, then, after transformation, we would have:

$$de = apdt + bdt$$

and

$$\int_0^{t_K} pdt = e_1/a - \frac{b}{a}t_K.$$

Inasmuch as t_K decreases when Δ increases, then, according to the u = ap + b law, the full gas pressure impulse should become greater with increase of loading density.

When applying the u = Ap^{ν} law, where $\nu < 1$, an analogous

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deduction is obtained.

Schmitz's tests have shown that $\int_0^{t_K} pdt$ does not depend on the loading density, which served to prove the validity of the $u = Ap$ law.

These tests had attracted the attention of many investigators and provided data for the verification of theoretical deduction and investigations. The latter include the work of Muiraur on the study of the burning rates of colloidal powders (1927-1928) and of calculating the amount of heat transferred to the walls during burning of powder (1924-1925).

M.E. Serebriakov's tests with pyroxyline and nitroglycerine powders confirmed the validity of the $u = Ap$ law. These tests will be discussed in greater detail in section III dealing with the physical law of burning.

Thus it may be assumed that the value of u as the rate of penetration of the burning reaction inside the grain is directly proportional to pressure, i.e., it is expressed by the formula:

$$u = Ap.$$

Here A can be expressed as the ratio between the burning rate at $p = 1$ (we shall designate it by u_1) and the magnitude of this pressure $p = 1$:

$$A = u_1/1$$

(the subscript "1" indicates that this burning rate refers to pressure $p = 1$).

This formula can be rewritten thus:

$$u = u_1 \cdot \frac{p}{1} = u_1 p;$$

when $p = 1$ $u = u_1$.

The dimensionality of the value of u_1 can be seen from the equality

$$u_1 = \frac{u}{p} \frac{dm}{sec} : \frac{kg}{dm^2},$$

i.e., this represents the rate referred to unit pressure.

Similarly to powder energy f and covolume α, the magnitude u_1 constitutes a fundamental ballistic characteristic of powder and, similarly to f and α, depends on the physical and chemical properties of the powder.

The value of u_1 for pyroxyline powders varies from 0.0000060 to 0.0000090 dm/sec : kg/dm^2. The thicker the powder, the greater is the content of volatiles and the slower is the burning of the powder. The higher the nitrogen content in pyroxyline, the more rapid is the burning. In nitroglycerine powders u_1 depends in the main on the nitroglycerine content itself, and the greater its content the more rapid is the burning. The admixture of dinitro-derivatives in powders in a nonvolatile solvent usually reduces the burning rate.

Varying of the content of volatiles by +1% lowers the burning rate of pyroxyline powders by 10-12%.

The following empirical formula for determining the burning rate of pyroxyline powders, introduced for the first time by

N.F. Drozdov [16] in our country, appears in American literature:

$$u_1 = 10^{-4} \frac{0.0025\varepsilon}{48\,(180 - t^o) + 2561h + 908.5h'},$$

where $\varepsilon = 69{,}400\,(N - 6.37)$ – powder energy in kg-m/kg;

t^o – temperature of powder;

h – content of volatile substances in %, removed by six hours of drying (moisture);

h' – content of residual solvent in % – not removed after six hours of drying;

N – nitrogen content in %.

This formula clearly shows the effect of various individual factors on the burning rate, but does not give sufficient satisfactory results as regards our own domestic powders.

The following formula is better adopted to our pyroxyline powders and is more convenient for performing the necessary calculations:

$$u_1 = \frac{0.175(N - 6.37)}{0.04(220 - t^o) + 3h + h'} \frac{mm}{sec} : \frac{kg}{dm^2} =$$

$$= \frac{0.175 \cdot 10^{-4}(N - 6.37)}{0.04(220 - t^o) + 3h + h'} \frac{dm}{sec} : \frac{kg}{dm^2},$$

where 220 is the ignition temperature of the powder;

$$\varepsilon = 700{,}000(N - 6.37\ \text{kg-dm/kg}).$$

Letan assumed on the basis of the kinetic theory of gases that the burning of powder is a process in which the powder molecules are split by the impact of gas molecules, and offers the following formula for determining u_1:

$$u_1 = \frac{g}{\delta\sqrt{\pi c_p}}\left(1 + \frac{c_1^2}{c_p^2}\right)e^{\frac{c_1^2}{c_p^2}},$$

where g - acceleration of gravity;

δ - physical density of powder;

c_p - probable velocity of molecules of gases formed in burning of powder;

c_1 - velocity of active gas molecules, whose kinetic energy is sufficient to split off at least one molecule when the surface of the powder undergoes an impact.

c_p and c_1 depend on the nature of the powder and of the gases formed during its combustion.

Schmitz had conducted his tests at loading densities Δ of from 0.12 to 0.26. Later tests had shown that at very low loading densities ($\Delta \geq 0.015$) the integral $\int_0^{t_K} pdt$ is a linearly decreasing function of time:

$$\int_0^{t_K} pdt = S_0 - \pi t.$$

As was shown above, such a relationship is obtained under the burning rate law u = ap + b.

In the tests conducted by M.E. Serebriakov [5] and A.I. Kokhanov it was shown that the integral $\int_0^{t_K} pdt$ changes with increase of the time of burning only in the case of powders of considerable thickness ($2e_1 > 0.5$); in the case of very thin powders, the full impulse, even at low loading densities, does not depend on the loading density. This shows that the speed of the process governing the heating of the entire powder mass is of importance, and that the increase of the powder temperature increases the burning rate u and reduces the value of the integral

$$\int_0^{t_K} pdt = e_1/u_1.$$

In order to determine the effect of heating on the burning rate of powder under a given constant pressure, tests were conducted with powder strips burned at different temperatures in open air. It was found that the time of burning of a strip of a given length varies from 14.1 seconds at $t = 15^\circ C$ to 9.4 seconds at $t = 50^\circ C$, and hence the rate of burning increases 1 1/2 times. The integral $\int pdt = p_a \cdot t_K$ was reduced in the same proportion, where p_a is atmospheric pressure.

The effect of heating on the burning rate of powder can be confirmed by the following tests.

If several charges of the same density are burned in succession in a bomb without cooling, the latter becomes quite hot. The powder inside the bomb becomes heated also, because the time between charging and the end of burning is considerable (several minutes). The value of the integral becomes smaller with each successive test, which

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condition points at an increasing rate of burning u_1 for the same powder thickness.

These tests permit the conclusion that the reduction of the integral I_K with decrease of Δ for thick powders when $\Delta < 0.10$ is the result of heating of the powder mass under the condition of relatively slow burning, whereby the degree of heating and hence the increase in the value of u_1 is the greater, the smaller the value of Δ, i.e., the slower is the burning of the powder at low pressures.

Inasmuch as the integral of I_K decreases with the decrease of Δ, the above will be theoretically valid if the burning rate laws $u = Ap^\nu$ and $u = ap + b$ are adhered to. Tests conducted by M.E. Serebriakov (1932) with powders with hard solvents showed that at pressures $p_m > 1000$ kg/cm^2 the linear law $u = Ap$ can be applied to determine the burning rate, and that at pressures $p_m < 1000$ kg/cm^2 the law expressed by the formula $u = A_1p^{0.82}$ will apply.

In analyzing later tests conducted by Prof. Yu. A. Pobedonostsev in bombs with nozzles at very low pressures (5 to 250 atm), Prof. Ya.M. Shapiro arrived at the relationship $u = 0.37\ p^{0.7}$ which, seemingly, is contradictory to the relationship $u = Ap$.

Actually, as was shown above, the decrease of the integral $\int pdt$ at small values of Δ can be explained also when applying the $u = u_1p$ law by the increase of the burning rate u_1 due to heating of the powder. This explanation is founded on the theory of Prof. Ya.B. Zeldovich mentioned earlier.

In any case a more accurate evaluation of either expression for the burning rate law requires further investigations (see Section III).

Inasmuch as powders in gun barrels burn under high pressures and under high loading densities, the following burning rate law may be considered valid for such powders:

$$u = u_1 p.$$

Going back to the formula for expressing the rate of gas formation, we can now write it as follows:

$$\frac{d\psi}{dt} = \frac{S_1}{\Delta_1} \frac{S}{S_1} u_1 p \qquad (31)$$

or

$$\frac{d\psi}{dt} = \frac{\varkappa}{e_1} u_1 \frac{S}{S_1} p = \frac{\varkappa}{I_K} \sigma p, \qquad (32)$$

where $\frac{S_1}{\Lambda_1}$ and $\frac{S}{S_1}$ depend on the geometry of the powder;

u_1 is the burning rate of powder when $p = 1$; it characterizes the nature of the powder and the degree to which it is heated;

p is the pressure at which the powder is burned; it characterizes the influence of the surrounding medium on the powder and depends on Δ, f, α, δ, ψ.

CHAPTER VI - PRESSURE VARIATION AS A FUNCTION OF TIME

We have derived above the following formulas: a) a formula for determining the rate of gas formation

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 p = \frac{\varkappa}{e_1} u_1 \sigma p = \frac{\varkappa}{I_K} \sigma p \qquad (33)$$

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and b) the general pyrostatics formula which takes the igniter into account

$$p = p_B + \frac{f\Delta\psi}{1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi} = p_B + \frac{f\Delta\psi}{\Lambda_\psi}, \qquad (34)$$

where $\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi = \frac{W_\psi}{W_0}$ is the relative free space in the bomb in which the powder is burned.

It is necessary to determine the dependence of the pressure change on time when the powder is burned in a constant volume, i.e., to give an analytical expression for the relationship between $p = f(t)$, dp/dt and the full time of burning t_K.

Differentiating equation (34) with respect to t, we get after simple transformations:

$$\frac{dp}{dt} = \frac{f\Delta\left[\Lambda_\psi + \Delta\left(\alpha - \frac{1}{\delta}\right)\psi\right]}{\Lambda_\psi^2}\frac{d\psi}{dt} = \frac{f\Delta}{(1 - \alpha\Delta)} \cdot \frac{\left(1 - \frac{\Delta}{\delta}\right)(1 - \alpha\Delta)}{\Lambda_\psi^2}\frac{d\psi}{dt}.$$

$1 - \frac{\Delta}{\delta}$ is the value of Λ_ψ at the start of burning;

$1 - \alpha\Delta$ the same at the end of burning,

$$1 - \frac{\Delta}{\delta} > \Lambda_\psi > 1 - \alpha\Delta.$$

It may be assumed with sufficient accuracy for practical purposes that when $\psi_{cp} = 1/2$

$$K_{\psi_{cp}} = \frac{\left(1 - \frac{\Delta}{\delta}\right)(1 - \alpha\Delta)}{\Lambda^2_{\psi_{cp}}} \approx 1.$$

Then, making use of the relation (33), we get:

$$\frac{dp}{dt} = \frac{f\Delta}{1 - \alpha\Delta}\frac{d\psi}{dt} = \frac{f\Delta}{1 - \alpha\Delta}\frac{u_1}{e_1}\varkappa\sigma p. \qquad (35)$$

In order to simplify further derivations, we shall consider a powder whose surface area changes little when burned, so that it may be assumed that $\sigma = \sigma_{cp} = $ const.

To such powders belong the tube and the strip, for which the binomial relationship $\psi = \varkappa z(1 + \lambda z)$ is valid, whereby for the end of burning ($z = 1, \psi = 1$)

$$1 = \varkappa(1 + \lambda);$$

$$\sigma_{cp} = \frac{1 + 1 + 2\lambda}{2} = 1 + \lambda.$$

Therefore

$$\varkappa\sigma_{cp} = \varkappa(1 + \lambda) = 1,$$

and for tubular or (to a lesser degree) strip powder

$$\frac{dp}{dt} = \frac{f\Delta}{1 - \alpha\Delta}\frac{u_1}{e_1}p = \frac{p_m - p_B}{I_K}p. \qquad (36)$$

We shall introduce the designation

$$\tau = \frac{e_1}{u_1} \frac{1 - \alpha\Delta}{f\Delta} = \frac{I_K}{p_m - p_B} \; [\text{sec}]; \tag{37}$$

then

$$\frac{dp}{dt} = \frac{p}{\tau}. \tag{38}$$

Upon separating the variables:

$$\frac{dp}{p} = \frac{dt}{\tau}.$$

Integrating, we get

$$\int_{p_B}^{p} \frac{dp}{p} = \frac{1}{\tau} \int_{0}^{t} dt \quad \text{or} \quad \ln\frac{p}{p_B} = \frac{t}{\tau},$$

whence on the one hand

$$t = 2.303\tau \log \frac{p}{p_B} \tag{39}$$

and for the end of burning

$$t_K = 2.303\,\tau \log \frac{p_m}{p_B}; \tag{40}$$

on the other hand

$$\frac{p}{p_B} = e^{\frac{t}{\tau}} \quad \text{or } p = p_B e^{\frac{t}{\tau}}, \tag{41}$$

and for the end of burning

$$p_m = p_B e^{\frac{t_K}{\tau}}. \tag{42}$$

Thus, all the required relationships are derived:

$$p = p_B e^{\frac{t}{\tau}},$$

$$t_K = 2.303\,\tau \log \frac{p_m}{p_B},$$

$$\frac{dp}{dt} = \frac{p_m - p_B}{I_K}\,p = \frac{f\Delta}{1 - \alpha\Delta}\,\frac{u_1}{e_1}\,p;$$

$$\tau = \frac{I_K}{p_m - p_B} = \frac{e_1}{u_1}\,\frac{1 - \alpha\Delta}{f\Delta} \text{ [sec]}.$$

The magnitude $\tau = \dfrac{e_1}{u_1(p_m - p_B)} = \dfrac{e_1}{u_m}$ constitutes the burning time of the powder if it burned at constant pressure $p_m - p_B$ throughout.

The full time of burning at a given loading density is proportional to τ and $\log p_m/p_B$, in other words, it is directly proportional to the

thickness of the powder and inversely proportional to the energy of the powder f and rate of burning u_1, and decreases with the increase of Δ and p_m (since the value of p_m in the denominator of τ is more effective than in the numerator under the logarithm); it also decreases with the increase of the igniter pressure p_B.

For tubular powder the relation $p = f(t)$ serves as a characteristic curve, whose slope angle ($dp/dt = \tan\varphi$) must continuously increase in proportion to the gas pressure during the entire combustion process, the rate of increase being the higher, the greater are the values of f, Δ, u_1 and the smaller the powder thickness e_1. At the end of burning the slope angle must be maximum (see figs. 39-42).

We shall derive the relation for the powder gas pressure impulse on the basis of formula (41)

$$\int_0^t p\,dt = p_B \int_0^t e^{\frac{t}{\tau}}\,dt = p_B\tau \int_0^t e^{\frac{t}{\tau}}\,d\frac{t}{\tau} =$$

$$= p_B\tau\left(e^{\frac{t}{\tau}} - 1\right) = \tau(p - p_B) = \frac{e_1}{u_1}\,\frac{p - p_B}{p_m - p_B}. \tag{43}$$

For the end of burning $p = p_m$ and

$$\int_0^{t_K} p\,dt = \frac{e_1}{u_1} = I_K. \tag{44}$$

The derived formulas confirm the general laws of powder burning

and show that the pressure, the time for complete combustion and the rate of pressure increase depend on the ballistic characteristics and density of loading. Thus, it is precisely the ballistic characteristics f, α, u_1, the shape and dimensions of the powder ($\varkappa$, λ, e_1) and the loading density Δ that can be used to control the magnitude and rate of pressure increase of the gases evolved during the burning of powder in a constant volume and to regulate the phenomenon of powder burning and gas formation.

<u>Example.</u> Determine the maximum pressure and time of burning of tubular powder ($2e_1$ = 1.00 mm) when $\Delta = 0.25$;

$$f = 900{,}000 \text{ kg-dm/kg}; \quad \alpha = 1.00 \text{ dm}^3/\text{kg};$$

$u_1 = 0.0000075$ dm/sec : kg/dm^2, igniter pressure $p_B = 50$ kg/cm^2;

$$p_m = p_B + \frac{f\Delta}{1 - \alpha\Delta} = 5000 + \frac{900000 \cdot 0.25}{1 - 1.0 \cdot 0.25} = 305{,}000 \text{ kg/dm}^2 = 3050 \text{ kg/cm}^2$$

$$I_K = \frac{e_1}{u_1} = \frac{0.005}{0.0000075} = 667 \text{ kg/dm}^2 \cdot \text{sec},$$

$$\tau = \frac{I_K}{p_m - p_B} = \frac{667}{300000} = 0.002223 \text{ sec};$$

$$t_K = 2.303\,\tau \log \frac{p_m}{p_B} = 2.303 \cdot 0.002223 \log \frac{3050}{50} = 2.303 \cdot 0.002223 \cdot$$

$$\cdot 1.7853 = 0.00913 \text{ sec}.$$

Curves $p = p_B e^{\frac{t}{\tau}}$ for the given powder will have the following form:

a) at different Δ and the same p_B (fig. 40)

$$\int_0^{t_K'} p\,dt = \int_0^{t_K''} p\,dt = \int_0^{t_K'''} p\,dt$$

b) at the same Δ and different p_B (fig. 41)

$$p_m' - p_m'' = p_B' - p_B''; \quad p_m' - p_m''' = p_B' - p_B'''$$

$$\int_0^{t_K'} p\,dt = \int_0^{t_K''} p\,dt = \int_0^{t_K'''} p\,dt.$$

Fig. 40 - p,t Curves at Different Values of Δ and the Same Value of p_B.

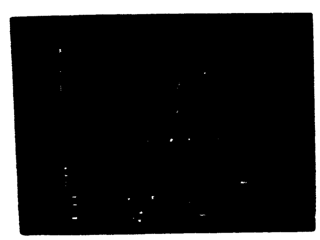

Fig. 41 - p, t Curves at Different Values of p_B and the Same Value of Δ.

For powders of the same kind but varying thickness $e_1' < e_1'' < e_1'''$ at the same loading density and the same igniter pressure p_B, curves p, t will have the form shown in fig. 42, where

$$t'_K : t''_K : t'''_K = e'_1 : e''_1 : e'''_1 .$$

When burning powder in small bombs of limited elongation (ratio between length of bomb and its diameter not exceeding 2-3), the curves of pressure increase are gradual in character.

If, however, the powder is burned in a long bomb (1m long by 22mm in dia.) with the powder concentrated at one of its ends, the pressure increase recorded by a meter takes the shape of a wave. In the case of thin powders this condition obtains at relatively low loading densities, of the order of 0.05-0.075, and in the case of thick powders, when Δ is of the order of 0.20-0.25. As the loading density is increased, the growth of the pressure recorder by the crusher increases also: thus, at $\Delta = 0.20$, for powders ~ 0.3 mm thick, the maximum pressure varied from 2200-2300 to 7500 kg/cm^2; in the case of thick powders at the same value of Δ the pressure was found to be ~ 4000 kg/cm^2.

Fig. 42 - p, t Curves at Various Values of e_1 and a Constant Value of Δ.

If the crusher cones are placed at the ends of a long bomb and

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the powder is ignited from the side at one of the ends, the curves of the pressure growth at both ends will appear wave-like in form if the charge is not distributed uniformly, whereby the maximum of one curve will correspond to the minimum of the other at the same instant of time. These tests show that a wave-like process of pressure distribution occurs in a bomb, where the pressure waves are reflected from one end of the closed pipe to the other.

All of the tests described above were conducted by Vieille in a special long bomb, using cylindrical crushers for recording the pressure growth at both ends. We had verified Vieille's deductions on a similar test set-up using conical crushers.

When the charge concentrated at one end of the bomb is ignited, a localized pressure increase occurs due to gases becoming separated or detached from the burning surface. This pressure increase is the greater, the larger the surface area of the charge, i.e., the thinner the powder. Due to the action of the localized pressure, the gases begin to move to the opposite end of the bomb in the form of a stream whose rate of flow increases rapidly, thus tending to form a vacuum at the point of the start of burning. Upon reaching the opposite end, the gas stream will stop abruptly at its forward end and its kinetic energy will be expended in compressing this part of the bomb until the pressure developed in it exceeds the pressure at the place occupied by the charge. The movement of the stream will then be reversed and the burning of the charge will proceed more intensely under the pressure of the gas stream, creating once again a localized pressure increase. This phenomenon will then be repeated.

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At the end of burning of the charge the phenomenon takes on the character of a damped oscillating gas motion inside the bomb.

Similar conditions may occur in the bore of a gun barrel. Therefore, when the loading density is low and the chamber is not completely filled with powder, it is recommended to uniformly distribute the charge along the entire length of the chamber.

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SECTION III - BALLISTIC ANALYSIS OF POWDERS ON THE BASIS OF THE PHYSICAL LAW OF COMBUSTION

CHAPTER I - METHOD FOR THE BALLISTIC ANALYSIS OF POWDERS

1. AN ATTEMPT TO CORRELATE THE THEORETICAL LAW WITH BOMB TESTS.

Formula $dp/dt = \tau p$ derived in the preceding chapter for tubular powders shows that dp/dt increases in proportion with p, and inasmuch as p itself continues to increase until the end of burning, the slope angle of the dp/dt curve must theoretically continue to increase also.

Nevertheless, in all the p, t curves obtained in burning powders in a manometric bomb, the maximum slope angle is obtained not at the maximum pressure at the end of the curve but, rather, at some $p_1 < p_m$. The point of inflexion i corresponds to pressure p_1, following which the p, t curve becomes concave instead of convex, and often approaches the end of burning when $dp/dt \approx 0$ (for strip and tubular powders).

The validity of the geometric law of burning was questioned for the first time by Charbonier, who attempted to investigate real powders and all their defects peculiar to manufacturing processes.

Using for his observations an imperfect cylindrical crusher and analyzing the shape of the pressure curves obtained in a manometric bomb, Charbonier introduced a special "shape function" to account for the actual burning of the powder, which was supposed to represent an analytical expression linking the relative surface area S/S_1 with the burned portion of the charge ψ.

The exponent of this function was determined not by the shape of the grain but, rather, on the basis of the bomb test.

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A. Derivation of a General Formula for the Shape Function

Let us find the relation between the value of the surface area ratio S/S_1 at a given instant and the burned portion of the charge Ψ for powders of the simplest shapes: for a sphere burning in parallel layers towards the center, for a solid cylinder, and for an infinitely wide strip.

a) Sphere. The initial volume of the sphere is $\Lambda_1 = 4/3\pi R^3$ (fig. 43). The volume of the burned portion $\Lambda_{cr} = \Lambda_1 - \Lambda_{ocr} = \frac{4}{3}\pi(R^3 - r^3)$. The burned portion of the grain

$$\Psi = \Lambda/\Lambda_1 = 1 - \frac{\Lambda_{ocr}}{\Lambda_1} = 1 - (\frac{r}{R})^3. \quad (45)(*)$$

The initial surface area $S_1 = 4\pi R^2$. The area at the given instant is $S = 4\pi r^2$.

Therefore,

$$\frac{S}{S_1} = \left(\frac{r}{R}\right)^2. \quad (46)$$

Eliminating $\frac{r}{R}$ from (45) and (46), we get:

$$\frac{S}{S_1} = (1 - \Psi)^{\frac{2}{3}}. \quad (47)$$

(*) Subscript cr = abbreviation of the word burned, subscript ocr = abbreviation of the word remainder - translator.

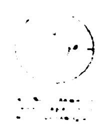

Fig. 43 - Burning Diagram for a Sphere (a Right Circular Cylinder).

b) Solid cylinder. Let the height of the cylinder h be great in comparison with its diameter and let us assume that the effect produced by the decrease in length on the volume change may be disregarded.

Then, referring to fig. 43:

$$A_1 = \pi R^2 h; \quad A_c = \pi (R^2 - r^2)\, h;$$

$$S = 2\pi r h; \quad S_1 = 2\pi R h;$$

$$\psi = 1 - \left(\frac{r}{R}\right)^2; \quad \frac{S}{S_1} = \frac{r}{R}.$$

Cancelling r/R from the expression for ψ and S/S_1, we get

$$S/S_1 = (1 - \psi)^{1/2}.$$

c) Infinitely wide strip (the effect produced by the changes along the edges may be disregarded). The surface S remains constant, i.e., $S/S_1 = 1$, and we can write:

$$S/S_1 = (1 - \psi)^0 = 1.$$

Thus in the case of typical regressive powder shapes the value of the area ratio S/S_1 is expressed as a function of the same kind

$$S/S_1 = (1-\psi)^{\beta},$$

where the exponent $\beta = 2/3$ for a sphere, $\beta = 1/2$ for a cylinder, $\beta = 0$ for an infinite strip (powder with a constant burning area).

Actually of course the burning of powder deviates from this ideal law, and Charbonier had determined the β exponent from an actual bomb test, by setting on the p, t curve the maximum pressure p_m and pressure p_i at the point of inflexion:

$$\beta = \frac{p_m - p_i}{p_i}. \tag{48}$$

The more uniform will be the burning of the powder, the higher will be the point of inflexion, the smaller the numerator, and the closer will the denominator and β exponent approach zero, and the more will the burning approach the condition of burning with a constant surface:

$$p_i = \frac{p_m}{1 + \beta}.$$

For a sphere $p_i = 3/5\ p_m$;

For a slab $p_i = 2/3\ p_m$; for $\beta = 0$ $p_i = p_m$.

For French cannon strip-type powders "B" it was determined by actual tests that $\beta = 0.2$ and for rifle plate-type powder BF - $\beta = 0.5$. This shows that in actual practice plate powders burn similarly to a

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theoretical solid cylinder of the more regressive type. The curve $\sigma = (1-\Psi)\beta$ when $\beta = 0.2$ and 0.5 is shown in fig. 45 (curves 1 and 2).

The σ, Ψ curves for the same powder shapes burned according to the geometric law are shown in fig. 45 in the form of a dotted line (1' - strip, 2' - plate). Curves 1 and 2 are arranged below curves 1' and 2', respectively.

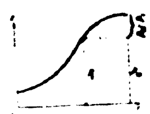

Fig. 44 - p, t Curve With a Point of Inflexion.

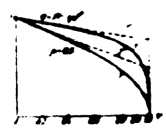

Fig. 45 - Shape Function $\sigma = f(\psi)$.

According to Charbonier: 1) surface y in all powders tends toward zero at the end of burning (because when $\Psi = 1$, $\sigma = 0$), whereby this sharp surface reduction starts the sooner, the more regressive is the powder; 2) the actual burning of the powder is more regressive than it should be according to the geometric law; 3) the possible reason thereof is the heterogeneity of the mass and the nonsimultaneous ignition of all the elements of the charge.

At the same time his investigations made it possible to establish a connection between the theoretical formula and the experimental data, using for this purpose the p, t curve for pressure increase, obtained by burning powder in a manometric bomb.

Thus Charbonier had introduced an evaluation of the progressivity of burning on the basis of bomb tests rather than on the basis of the powder shape, and had concluded correctly that the progressivity of the shape does not fully determine the progressivity of burning - a process depending not only on the geometry of the grain, but also on the physical and chemical properties and conditions of loading and ignition.

At the same time the "shape function" which is a step forward as regards the evaluation of the nature of burning, still does not sufficiently reflect the latter, inasmuch as of the entire pressure test curve p, t only two points were utilized for the determination of β: point p_m at the end of burning and point p_1 which is also close to the end of burning. The basic part of the curve was not utilized; this is partly explained by the fact that the curve was recorded by means of cylindrical crushers, and hence its form at the start of burning was unknown.

Nevertheless, maximum pressure is usually obtained in a gun after about half of the charge is burned, and hence the nature of burning from the start to the instant when the first half of the charge is burned must influence both the position of the maximum pressure in a gun as well as its magnitude. Actually, of course, the position of the inflexion point on the pressure curve in a bomb may not be closely associated with the first half of the burning process.

Therefore, the defect of the Charbonier method lies in the fact that the pressure curve remains unused on the whole.

In 1923-1924 M.E. Serebriakov obtained by means of a conical crusher full curves of the pressure increase of powder gases obtained by burning powder in a manometric bomb, and developed a new method for the analysis of powder burning utilizing the entire pressure curve for the purpose.

This analysis is conducted on the basis of the test characteristic of the burning progressivity of powder [4]. This characteristic is obtained by sectional analysis of the entire pressure curve from the start to the end of burning; it shows the change in the intensity of gas formation during the entire burning process.

Using this method, a series of new hitherto unknown peculiarities were disclosed of the actual process of powder burning and its deviation from the geometric law; also proven by means of direct tests were some of the formulations originally assumed by Charbonier.

The principles of this method follow.

2. TEST CHARACTERISTIC "Γ" OF THE PROGRESSIVE BURNING OF POWDER

The Use of Function Γ for the Analysis of the Burning of Powder.

In choosing a test characteristic for the progressive burning of powder, the expression used must be such as would be determinable on the basis of geometric data for the ideal case, assuming that the powder mass is fully homogeneous.

At the same time the numerical value of this characteristic must be found exclusively from such bomb test data whose values at any given instant are considered reliable within pre-established limits.

When powder is burned in a bomb, we get a pressure curve as a function of time, and it may be assumed that the pressure at every given instant is known to be correct within the limits of accuracy of the recording device itself.

If it is assumed that the powder energy f and its density δ are constant throughout the entire mass and that no cooling occurs through the walls of the bomb, i.e., if we make the usual assumption peculiar to ballistics, then, on the basis of the general pyrostatics formula, the pressure p at a given loading density is fully determined by the amount of the burned portion of charge ψ regardless of the powder shape and its rate of burning.

Indeed, the dependence of p on ψ is expressed by the formula

$$p - p_B = \frac{f\Delta\psi}{1 - \frac{\Delta}{\delta} - \Delta\psi\left(\alpha - \frac{1}{\delta}\right)},$$

into which time does not enter, and the pressure is determined by the burned portion of charge ψ when the other factors remain constant.

If, however, in addition to pressure its increase with relation to time must be known also, the magnitude of dp/dt will be determined by the velocity of gas formation $d\psi/dt$ and by its variation with time.

The values of this magnitude are determined directly from test, because in measuring the curve the values of p are known at definite time intervals t, as are the values of ψ corresponding to these

values of p. If the intervals taken are sufficiently small, the values of the increment $\Delta\psi$ can be found as the difference between two neighboring intervals, following which $\Delta\psi/\Delta t$ can be found as well (limit - $d\psi/dt$).

The value of $d\psi/dt$ which, in the case of the geometric law of burning, is expressed by the formula $\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 p$, depends on pressure. In order to compare various burning periods with respect to the rate of gas formation (increasing and decreasing rates) at constant pressure, as is usually done in the case of the geometric law, the obtained values $d\psi/dt$ must be reduced to constant pressure, i.e., a comparison must be made of the values $d\psi/dt : p = \frac{1}{p} \frac{d\psi}{dt}$.

If the value of $\frac{1}{p} \frac{d\psi}{dt}$ increases as burning progresses, the powder will burn progressively, if it decreases - the burning is regressive.

Henceforth we shall designate this magnitude by Γ (gamma):

$$\Gamma = \frac{1}{p} \frac{d\psi}{dt}.$$

It represents the specific rate of gas formation reduced to p = 1, which we shall hereafter call the intensity of gas formation.

Its variation during the burning process is characterized by the powder from the point of view of progressive burning, rather than by the shape of the grain alone. The dimensionality of Γ, which is equal to $\frac{1}{\frac{kg}{dm^2} \cdot sec}$, is the inverse of the dimensionality of pressure impulse.

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In the ideal case at constant pressure the value of Γ varies in proportion to the powder surface, as it does in the case of the geometric law of burning, and this value is therefore a characteristic of progressivity.

Actually, if ignition were to occur instantaneously along the entire area of the charge and the pressure during the entire process were to remain constant and equal to p_0, then a chemically homogeneous powder composition would burn according to the geometric law in parallel layers. In such a case the intensity of gas formation would vary in proportion to the change in area.

Indeed,

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1 p_0 .$$

The magnitude of Γ will be represented in the following form:

$$\Gamma = \frac{1}{p_0}\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1 .$$

When the composition of the powder is homogeneous, u_1 is constant and S_1/Λ_1 is a constant; hence the variation of Γ will be proportional to the S/S_1 ratio, i.e., the characteristic of the progressivity of burning will coincide with the powder grain characteristic.

Thus, as the test characteristic of the progressivity of burning, we can take the value $\Gamma = \frac{1}{p}\frac{d\psi}{dt}$ – the intensity of gas formation.

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The value of Γ is found by the sectional analysis of the pressure test curve in a constant volume.

The function Γ enables us to evaluate the progressivity of the actual burning of powders of any shape and size, of both a homogeneous and heterogeneous mass.

The rate of gas formation in burning powder in a bomb can be evaluated by its actual law of burning even if the shape and dimensions of the powder are not known. The nature of the burned powder is determined by the values

$$\psi \text{ and } \frac{1}{p}\frac{d\psi}{dt}.$$

The law of burning expressed by the function $\Gamma = \frac{1}{p}\frac{d\psi}{dt}$ and obtained by analyzing the pressure test curve p, t, wherein are reflected the peculiarities of the properties of actual powder and the deviations of its burning from that of an ideal powder, is called the experimental or physical law of burning.

Along with the Γ, ψ and Γ, t curves, the curve showing the pressure impulse variation $\int_0^t pdt$ as a function of ψ also serves as a characteristic of actual powder burning.

These integral curves and their values will be discussed in detail later in the text.

The procedure for analyzing the p, t curve for determining $\int pdt$, ψ and Γ, ψ is illustrated in Table 13.

When computing Δ, the mean value must be taken between the initial $\Delta_0 = \frac{\omega}{W_0}$ and Δ_K at the end of burning:

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$$\Delta_K = \frac{\omega}{W_0 + s\varepsilon},$$

where ε - compression produced by the crusher;

s - cross section of the piston.

The covolume effect of the igniter a_B can be disregarded:

$$\Delta_{cp} = \frac{\Delta_0 + \Delta_K}{2} = \frac{\omega}{W_0 + s\frac{\varepsilon}{2}}.$$

The Γ, ψ and Γ, t diagrams offer a visual interpretation of the change in the intensity of gas formation and enable one to analyze the processes occurring during ignition of the charge and during actual burning of the powder with all its peculiarities.

The diagrams in fig. 46, 47, 48 and 49 contain experimental p, t curves as a function of time for tubular powders (fig. 46), strip powders (fig. 47), powders with 7 perforations (fig. 48) and Kisnemsky's powder with 36 perforations (fig. 49), and also curves showing the variation of Γ as a function of t, where the corresponding Γ and p points lie on the same vertical.

For a constant value of u_1, the change of $\Gamma = \frac{S_1}{\Lambda_1} u_1 \frac{S}{S_1}$ must proceed in proportion to the change of the geometric surface ratio S/S_1, i.e., in the case of strip and tubular powders $\frac{S}{S_1}$ must be maximum at the start and undergo a very small decrease during burning; at the end, in the case of the geometric law of burning, $S_K/S_1 \approx 0.90$.

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Nevertheless, the Γ, t curves in fig. 46 and 47 are very peculiar in character: they start at some small value (pressure) and then proceed to ascend; after reaching the maximum at $t \approx 0.0045$ (which corresponds to a pressure of 150-170 kg/cm^2) the curve begins to descend, and after $t = 0.0115$ for strip powder and 0.0135 for tubular powder the Γ curve drops abruptly to zero. On the p, t curves this condition corresponds to the inflexion point p_1.

Table 13

1	2	3	4	5	6	7	8	9
t	p(*)	Δp	p_{cp}	$p_{cp}\Delta t = \Delta I$	$I = \Sigma p_{cp} \cdot \Delta t \approx \int p dt$	$\beta = \frac{p - p_B}{p_m - p_B}$	ψ(***)	$\Delta\psi$
0	p_B	$\Delta p'$	p'_{cp}	$\Delta I'$	0	0	0	$\Delta\psi'$
t'	p'	$\Delta p''$	p''_{cp}	$\Delta I''$	$I' = \Delta I'$	β'	ψ'	$\Delta\psi''$
t''	p''	$\Delta p'''$	p'''_{cp}	$\Delta I'''$	$I'' = I' + \Delta I''$	β''	ψ''	$\Delta\psi'''$
t'''	p'''	$\Delta p'''$	.	.	$I''' = I'' + \Delta I'''$	β'''	ψ'''	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
t_K	p_m				$I_K = \sum_0^K (\Delta I)$	1	1	

Remarks.

(*) When analyzing the pressure increase curve for the purpose of calculating the pow the portion representing the burning of the igniter is discarded and the analysis is star

(**) When plotting the curve on the diagram, the values of $\int pdt$ from column "6" are p of ψ, because both $\int pdt$ and ψ relate to the same pressure. As regards the value of Γ, i change in the rate of gas formation on the curve section representing the variation of ψ hence its values are plotted as a function of the mean ψ characterizing the given sectio

(***) When computing ψ by means of the tables, the entrant number is the parameter in steps of 0.01 from 0.86 to 0.97.

Table 13

5	6	7	8	9	10	11
$= \Delta I$	$I = \Sigma p_{cp} \cdot \Delta t \approx \int p dt$	$\beta = \frac{p - p_B}{p_m - p_B}$	ψ (***)	$\Delta\psi$	$\Gamma = \frac{\Delta\psi(**)}{\Delta I} = \frac{\Delta\psi}{p_{cp}\Delta t}$	ψ_{cp}
	0	0	0	$\Delta\psi'$	$\Gamma' = \frac{\Delta\psi'}{\Delta I'}$	ψ'_{cp}
	$I' = \Delta I'$	β'	ψ'	$\Delta\psi''$	$\Gamma'' = \frac{\Delta\psi''}{\Delta I''}$	ψ''_{cp}
	$I'' = I' + \Delta I''$	β''	ψ''	$\Delta\psi'''$	$\Gamma''' = \frac{\Delta\psi'''}{\Delta I'''}$	ψ'''_{cp}
	$I''' = I'' + \Delta I'''$	β'''	ψ'''	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	$I_K = \sum_0^K (\Delta I)$	1	1			

pressure increase curve for the purpose of calculating the powder characteristics, the burning of the igniter is discarded and the analysis is started at pressure p_B.

curve on the diagram, the values of $\int p dt$ from column "6" are plotted as a function and ψ relate to the same pressure. As regards the value of Γ, it characterizes the formation on the curve section representing the variation of ψ between ψ_i and ψ_{i+1}; ed as a function of the mean ψ characterizing the given section of the $\Delta\psi$ variations.

of the tables, the entrant number is the parameter $\partial = \frac{1 - \alpha\Delta}{1 - \Delta/\delta}$, varying ...

Γ, t curves with many perforations deviate from the theoretical to an even greater degree (fig. 48 and 49).

In fig. 48 the maximum Γ obtains at $t = 0.0095$ ($p \approx 240$), the rise of the Γ curve being rather gradual and smooth up to the point of maximum. Upon passing the maximum, the curve drops slowly throughout the entire burning process up to $t = 0.019$; this is followed by a sharper drop, which corresponds to the decomposition of the grain and the afterburning of the regressive products of decomposition.

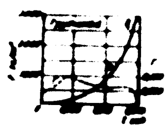

Fig. 46 – Γ, t Characteristic for Tubular Powder.

1) p kg/cm^2; 2) tubular; 3) t (sec).

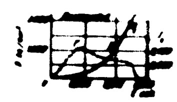

Fig. 47 – Γ, t Characteristic for Strip Powder

1) p kg/cm^2; 2) strip; 3) t(sec)

On fig. 49, Γ rises slowly at first, and at $t = 0.0075$ ($p = 120$) it proceeds to ascend very sharply; the ordinate increases almost two-fold and has a maximum at $t = 0.009$ ($p = 180$); after reaching the maximum the curve undergoes a continuous drop becoming more pronounced at $t = 0.015$, which corresponds to the instant the grain undergoes decomposition.

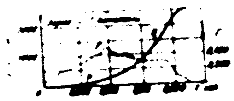

Fig. 48 – Γ, t Characteristic for a Grain with 7 Perforations

1) p kg/cm^2; 2) grain with 7 perforations; 3) t (sec).

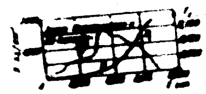

Fig. 49 – Γ, t Characteristic for Kishemsky's Grain with 36 Perforations.

1) p kg/cm^2, 2) Kishemsky's grain with 36 perforations; 3) t (sec).

Thus during the process of burning from $p \approx 200$ kg/cm^2 to the end, perforated grains burn with a seemingly decreasing surface area, whereas theoretically the area should continue to increase until decomposition occurs.

Γ, ψ curves. In order to obtain a more detailed comparison of the test data with theoretical data, the Γ curves are plotted as a function of the burned portion of the charge ψ, because if plotted as a function of time, when the pressure continuously increases and the process of burning is accelerated, the Γ curve becomes distorted in the direction of the abscissa (x – axis): the initial sections (of the curve) at low pressures are stretched, and those at higher pressures – at the end of burning – compressed.

The obtained curves of Γ plotted as a function of ψ are presented in the diagram of fig. 50-53, which diagrams also show theoretical curves of $\Gamma_T = \frac{S_1}{\Lambda_1} u_1 \frac{S}{S_1}$ when S/S_1 varies according to the geometric law of burning.

The average value of u_1 for pyroxylin powders is taken to be 0.075 mm/sec.

An analysis of experimental Γ, ψ curves for powders of simple shapes (fig. 50 and 51) will show that such curves consist of four distinct sections.

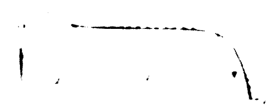

Fig. 50 - Γ, ψ Characteristic Curve for Tubular Powder.

Section I of the curve starts not at the maximum, as it should be in the case of instantaneous ignition, but, rather, increases from a small value to the maximum; the Γ maximum is obtained at ψ = 0.05 - - 0.08 and considerably exceeds the theoretical maximum.

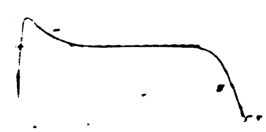

Fig. 51 - Γ, ψ Characteristic Curve for Strip Powder.

Section II - Γ drops from maximum to a point from which the curve follows a theoretical path - a section depicting accelerated burning; this section is confined between the limits of ψ = 0.05 - 0.08 and $\psi \approx 0.30$.

Section III depicts normal burning which coincides with the geometric law; ψ varies from 0.30 to 0.85 – 0.90.

Section IV from $\psi \approx 0.85-0.90$ to the end of burning. Here the experimental curve deviates from the theoretical downward and drops to zero at $\psi = 1$.

The Γ, ψ curves for powders with narrow perforations (figs. 52 and 53) have even a larger number of deviations from the theoretical. Furthermore, the reduction of the ordinates of Γ corresponding to powder decomposition commences at $\psi_{s\ on} = 0.70-0.75$ and has no sharp angle point. Decomposition proceeds gradually because in practice the web thicknesses in a grain are not uniform, and a partial decomposition commences after the smallest thickness is burned. Increasingly thicker elements are gradually burned away and progressive burning occurs simultaneously with regressive burning of the products of decomposition.

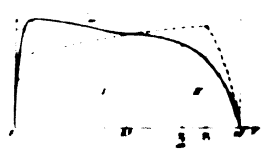

Fig. 52 - Γ, ψ Characteristic Curve for a Grain with 7 Perforations.

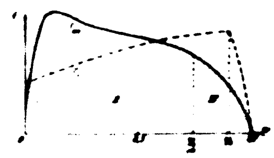

Fig. 53 - Γ, ψ Characteristic Curve for a Kisnemsky Grain with 36 Perforations.

As a result, the transition from burning with an increasing surface to the afterburning of the products of decomposition is gradual rather than abrupt in character.

CHAPTER 2 - BALLISTIC ANALYSIS OF THE ACTUAL BURNING OF POWDER

1. TESTS FOR INVESTIGATING THE IGNITION OF POWDER

A. Effect of the Size and Nature of the Igniter

a) Theoretical Data.

We had derived above (fig. 50) a formula for determining the full time of burning when the powder is ignited instantaneously:

$$t_K = 2.303\, \tau \log p_m / p_B$$

and shown that for a given loading density the time of burning decreases with the increase of the igniter pressure.

Calculations show that for $\Delta = 0.20$ at $p_m - p_B = 2000$ kg/cm^2 and at $p_B = 20;\ 40;\ 60$ and 120 kg/cm^2, t_K varies within the following limits (Table 14).

Table 14

p_B	20	40	60	120
t_K, sec	0.0140	0.0119	0.0107	0.0087
$\frac{t_K}{t_K\ 120}$	1.61	1.37	1.23	1.00

In this case the Γ, t and Γ, ψ curves (for strip powders) must start at the maximum and then descend slightly as the surface area of powder 6 decreases.

Hence, in the case of instantaneous ignition at $\Delta \approx 0.20$, if t_K 120 is taken as the unit time at p = 120 kg/cm^2, the time of burning t_K 20 will be increased by 61% if the igniter pressure is decreased to 20 kg/cm^2.

Actual tests show however that the difference in the periods of burning at such igniter pressures is considerably greater.

b) Test Data.

Strip powder "CП" about 1 mm thick (1 x 18 x 40) was burned in a manometric bomb at $\Delta = 0.20$ using batches of igniter material developing a pressure of p_B = 20, 40, 60 and 120 kg/cm^2.

The igniter used was dry powdered pyroxylin. Pressure was recorded by means of conical crushers. The test data are presented below in Table 15.

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Table 15

p_B	20	40	60	120
t_K, sec	0.0280	0.0160	0.0133	0.0090
$\frac{t_K}{t_{K\ 120}}$	3.10	1.60	1.48	1.00
$t_{K\ Teop}$	0.0140	0.0119	0.0107	0.0087

Note: $t_{K\ Teop}$ = t_K theoretical

A comparison of this data with the figures in the preceding table shows that the difference in the burning periods is considerably greater than the theoretical difference. Particularly great is the divergence between the test time t_K and the theoretical one when the igniter used was weak: $p_B = 20$ kg/cm^2 $\left(\frac{t_{K.\ on}}{t_{K\ Teop}} = 2\right)$; (*) this divergence becomes smaller as p_B increases and practically disappears at $p_B = 120$ kg/cm^2 (ratio $t_{K.\ on} : t_{K\ Teop} \approx 1$).

Analogous relationships were obtained with other samples (igniter materials).

Inasmuch as the p, t curves showed no sharp changes along their ascent, Γ, t and Γ, ψ curves obtained from the analysis of corresponding p, t curves from the start to the end of burning were utilized for the analysis of the processes of ignition.

(*) Subscript "on" stands for the word "test," subscript "Teop" stands for the word theoretical - translator.

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In order to determine experimentally, by the aid of function Γ, the process of ignition - whether instantaneous or gradual, it was necessary to determine the presence of the following conditions:

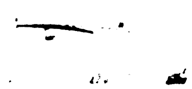

Fig. 54 - Theoretical Γ, t Curves at Different Values of p_B.

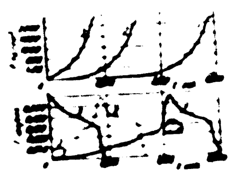

Fig. 55 - Experimental $p_1 t$ and $\Gamma_1 t$ Curves at Different Igniter Pressures.

a) Γ km/sec; b) p kg/cm^2; c) t (sec).

1. If the ignition is instantaneous, the curve of Γ variation for strip and tubular powders burning with a decreasing surface must start at the maximum.

2. If the ignition is instantaneous, the shape of the Γ curve must not depend on the size of the igniter, because after the entire surface is ignited its change must follow one and the same law.

The theoretical Γ, t, curves must have the form shown in fig. 54.

The above diagram (fig. 55) illustrates the experimental p, t curves obtained from the instant the powder became ignited; the Γ, t curves corresponding to them are plotted to the same scale of time t under the corresponding p, t curves. With the smallest igniter ω_B, developing a pressure of about 20 kg/cm^2, the Γ, t curve starts a very short distance above the origin and has a very long and gradually ascending section gradually changing to a steep slope, following which Γ maximum is reached (at p = about 225 kg/cm^2); this is followed by a rather sharp descent and, finally, by a very sharp drop at the end of the process. The growth of the ordinates of the curve at the start of burning corresponds to an increase of the burning area of the powder when its ignition proceeds gradually (curve 1).

When the igniter is increased by weight ($2\omega_B$ - curve 2) the starting ordinate of Γ increases, i.e., a larger area becomes enveloped at the same time, the length of the slowly ascending section of the curve becomes smaller. Otherwise the Γ, t curves practically remain unchanged; they seem to tend to shift to the left towards the origin of the coordinates, whereby the maximum value of Γ is the same as in the first case, at pressure p = 225-250 kg/cm^2. When the igniter is maximum $6\omega_B$ (p_B = 127 kg/cm^2 - curve 3), the curve Γ starts almost at the maximum point.

These results indicate that ignition at pressures of 20 to 60 kg/cm^2 does not proceed instantaneously, and is the slower, the lower the igniter pressure - and only at $p_B \approx 125$ kg/cm^2 is the ignition almost instantaneous.

Inasmuch as in guns the pressure developed by the igniter is between 10 and 40 kg/cm^2, the ignition will not be instantaneous. This is confirmed by the hangfire phenomenon.

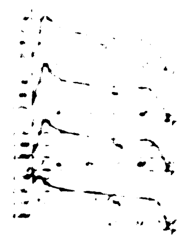

Fig. 56 - Experimental Γ, ψ Curves at Different Igniter Pressures.

If we plot on a diagram the same curves of Γ as a function of ψ to facilitate their comparison with curves of the variation of S/S_1 as a function of ψ, we shall obtain a diagram as illustrated in fig. 56.

When plotted as a function of ψ, the general appearance of the Γ, ψ curves changes, they resemble more curves S/S_1, ψ, but with certain deviations. As the value of ω_B increases, the maximum of Γ shifts from $\psi \approx 0.10$ towards the origin of the coordinates, and the descent of the curve becomes increasingly sharper whereby its end shifts from $\psi_1 = 0.85$ to $\psi = 1$.

This indicates that during the interval it takes for the entire surface of a strip powder charge to become ignited, about 5-10% of the entire charge will be burned.

The larger the igniter, the more rapidly will the flame envelope the surface of the powder and the later will the start of decomposition and afterburning of the strip occur.

In the tests mentioned, the point of inflexion, whose position on the pressure curve serves to determine the exponent β as a "function of the shape," varied as follows when the igniter changed from ω_B to $6\omega_B$: $\psi_1 = 0.85;\ 0.875;\ 0.90;\ 0.92$.

Therefore, the smaller the igniter, the longer will it take for the entire surface of the powder to become ignited, and the more heterogeneous will be the strip in thickness; this results in a curve in which the point of inflexion occurs at an earlier stage.

If the exponent β as a "function of shape" is calculated by the formula

$$\beta = \frac{p_m - p_1}{p_1} = \frac{p_m}{p_1} - 1 \approx \frac{1}{\psi_1} - 1,$$

then, as p_B varies from 20 to 120 kg/cm^2, we will obtain, respectively:

$$\beta = 0.18;\ 0.14;\ 0.11;\ 0.09.$$

The above deductions fully confirm in actual tests that the noninstantaneous ignition must affect the subsequent burning of the powder in a specific manner, increasing the value of β and making the burning process more regressive (transition from curve 4 to curve 1 in fig. 56). A comparison of the Γ, ψ curves in fig. 56 with

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the "shape function" graph for strip powders at $\beta = 0.2$ (fig. 45), shows an almost full coincidence of both the middle and terminal sections of these curves. However, the initial portions of the curves representing the first third of the process considerably differ from the theoretical form. They include: 1) ascending sections not present in the $\sigma = (1 - \psi)^{\beta}$ curve; 2) "ballooning" or a rather sharp increase of the ordinate compared with the Charbonier curve within the limits of $\psi \approx 0.10$ and $\psi \approx 0.30$.

"Ballooning," i.e., the sharp increase of the ordinate, represents an abnormal increase in the rate of gas formation at the start of burning, which ascent gradually levels off and coincides with the theoretical curve in the second third of the process; this phenomenon was not known to exist earlier.

Thus, it is proven by the aid of Γ, t and Γ, ψ curves, that the ascending first section of the curve from Γ to Γ_{max} at the start of burning represents a process of gradual ignition and the increase of the burning area of the powder due to noninstantaneous ignition. Ignition may be considered practically instantaneous only at $p_B = 120\text{-}150$ kg/cm^2.

If batches of pyroxyline and granulated black powder (of the rifle type) are prepared in such a manner as to produce the same pressure p_B, the ignition process will be more vigorous in the case of black powder, so that the latter will not succeed in getting fully burned by the time the basic charge of pyroxyline powder begins to burn (sic).

This is in full accord with the nature of the igniters. Whereas the products of decomposition of pyroxylin constitute a high temperature gas mixture, in the case of black powder these products also contain incandescent hard particles. The impacts of many such incandescent hard particles help to ignite the surface of the powder more rapidly than do the impacts of gas molecules.

2. THE NATURE OF "BALLOONING."

"Ballooning" is a term designating a condition where the test curve of progressivity Γ exceeds the theoretical curve (Section 11). This phenomenon is observed in powders with a volatile solvent, and is peculiar to a greater extent to thick powders than to thin ones, and to nitroglycerine powders than to pyroxylin ones. Powders with a solid solution produce practically no ballooning, - their burning approaches the geometric law more closely.

The cause of ballooning can be discovered by choosing powders of the same shape and size but of different properties.

In such a case the exposed grain area and the change in the surface area will be the same in both powders, and the difference in the values of Γ_{on} and $\Gamma_{\tau eop}$ (i.e., Γ_{test} and $\Gamma_{theor.}$ - respectively) can be obtained only because of the difference in the burning rate u_1, because

$$\Gamma = \frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} u_1 .$$

The shape of the Γ, ψ curves obtained by burning two samples of tubular powder of the same size is shown in fig. 57; curve 1

corresponds to a powder with a volatile solvent, and curve 2 to a powder with a solid solvent (trotyl + pyroxylin).

The first one produces considerable ballooning, and in the second there is practically no ballooning at all. The difference between the ordinates of these two curves is explained by the difference in their rate of burning because of the heterogeneous mass of the first sample produced by wetting the powder in water for the purpose of removing the excess of volatiles. When wetted, the powder becomes more porous on the outside and this tends to increase the rate of burning of the outer layers, as the burning layer is shifted inwardly, the rate of burning slows down. The inner portion of the powder layer is usually not affected by the wetting operation and therefore burns at a normal rate.

A somewhat higher rate of burning u_1 of the outer layers of powder with a solid solvent, homogeneous throughout its mass, can be partly explained by the more penetrating and intensive heating of the boundary layers of the powder at low pressures, while burning process proceeds with a relatively small absolute speed, and by reduced heating - as the burning process proceeds with a higher absolute rate of speed when the pressure increases to above 500 kg/cm^2. The layers of powder directly in contact with the burning surface will, in this case, become less heated and to a smaller depth, as a result of which the value of u_1, and with it the value of Γ, will become decreased.

This explanation suggested by the author in 1937 [5] is now substantiated in the theory of powder burning developed by Prof. Ya. B.

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Zeldovich, although the cause of ballooning has not been fully established as yet.

In any case, ballooning is the accelerated burning of the outer powder layers, occurring at relatively low pressures and, mainly, in the case of powders with volatile solvents.

The thicker the powder and the smaller its mean burning rate, the higher will be its relative degree of ballooning.

Ballooning becomes nil when the pressure is increased because of accelerated burning.

An analysis of the experimental Γ, ψ curves made it possible to establish the following empirical relationship between the burning rate u_1 and the depth of the layer:

$$u_1 = u_1' e^{-a\sqrt{z}},$$

where u_1' - the burning rate of the outer layer (u_1' for almost all pyroxylin powders is of the same order - 0.0000120 to 0.0000125 dm/sec : kg/dm^2);

z - relative thickness of the burned layer, equal to ψ for tubular powder and approaches ψ for strip powder;

a - coefficient characterizing the drop in the burning rate and determined from the Γ, ψ curve.

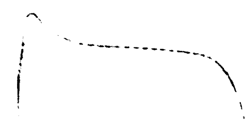

Fig. 57 - Intensity of Gas Formation in Powders of Different Properties.

This formula is valid for values of z from 0 to $z_c \approx 0.3$, and the magnitude of a can be found by the following formula:

$$a = \frac{\ln \frac{u_1'}{u_1}}{\sqrt{z_c}} = \frac{2.303}{\sqrt{z_c}} \log \frac{u_1'}{u_1},$$

where u_1 is the constant burning rate of internal layers after $z = z_c$.

The curves in fig. 58 show the variation of rate u_1 from one layer to another in the case of "СП" powder 1 mm thick and "Б$_{14}$" powder 6 mm thick; for the first powder $u_1' = 0.0_4 120$, $u_1 = 0.0_5 75$ and $a = 0.858$; for the second powder $u_1' = 0.0_4 125$, $u_1 = 0.0_5 60$ and $a = 1.34$.

The "Б$_{14}$" powder has a considerably higher content of volatiles, and hence its average burning rate $u_1 = 0.0_5 60$; the content of volatiles in "СП" powder is smaller and $u_1 = 0.0_5 75$. However, inas-

much as thick powders are subjected to longer periods of wetting, the burning rate of their outer layers is even higher than in the "CП" powder (0.0_4125 and 0.0_4120).

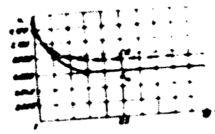

Fig. 58 – Change of u_1 During Burning.

The above formula shows that the burning rate of powders with a volatile solvent is not constant, as was assumed previously, but variable, being higher in the outer layers than in the inner ones. As a result, the effective burning of powder is more regressive than that assumed on the basis of a changing burning area S only, while considering the rate u_1 constant.

If the deviation of burning from the geometric law is due to the difference in the thickness of the elements of the charge and to the heterogeneity of the powder mass, would it be possible to obtain a Γ, ψ curve without ballooning? Is it possible to realize the geometric law of burning in actual practice? Well, the above is possible by observing certain conditions.

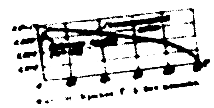

Fig. 59 - Γ, ψ Curve without Ballooning.

1) Test curve; 2) geometric law.

A solid cylindrical powder rod without a solvent (the mass is homogeneous) 7.5 mm in diameter by 42 mm long with rounded (spherical) ends, was fastened along the axis of a 21.5 cm^3 bomb by means of a frame made of thin copper wire. This arrangement facilitated ignition, and all the burning surfaces were subjected to identical conditions as regards the freedom of gas separation.

The weight of the igniter was such as would develop a pressure p_B = 160 kg/cm^2, and insure instantaneous ignition.

Figure 59 contains an experimental Γ, ψ curve and its corresponding curve of the geometric law of burning at u_1 = 0.069 mm/sec.

According to this diagram, no ballooning is observed on the test curve; the latter almost fully coincides with the theoretical curve upon reaching a maximum.

This shows that a powder which is entirely homogeneous in all its layers and is subjected to identical conditions as regards the freedom of gas separation from its surface elements, burns according to the geometric law.

A powder with a volatile solvent, whose burning rate varies from layer to layer deviates from the geometric law.

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Furthermore, it is impossible to distribute a charge in such a manner where all the surfaces would be placed under identical conditions as regards freedom of gas separation, and to obtain a condition where the deviation from the geometric law would always be the greater, the greater is the difference in the conditions of burning at different portions of the charge.

3. THE POINT OF INFLEXION ON THE PRESSURE CURVE.

When deriving the theoretical relationship between pressure increase and time, it was shown that in the case of tubular and strip powders in which the burning area varies little, the rate of pressure increase must grow continuously and have a maximum value at the end of burning. Indeed, in the expression

$$\frac{dp}{dt} = \frac{f\Delta}{1 - \alpha\Delta} \frac{u_1}{e_1} \times \frac{S}{S_1} p \qquad (49)$$

the value of $S/S_1 \approx$ const, and p grows continuously.

At the same time, the many tests conducted by various investigators show that when tubular and strip powders are burned in a bomb, an inflexion point invariably occurs on the p, t curves, following which dp/dt decreases and approaches zero, and the curve takes on a "beak-like" shape.

Charbonier had determined the exponent β from test from the position of the inflexion point in his suggested expression for the "shape functions."

Some of the authors were of the opinion that the hook in the curve is the result of the crusher's "setting" after the end of burning by the inertia of the small piston, and does not depend on the law of powder burning.

The analysis of formula (49) indicates that in order to obtain the deflection point (dp/dt = const), it is necessary to maintain the condition $S \cdot p$ = const, and inasmuch as the gas pressure in the bomb undergoes a continuous increase, the burning powder surface must decreas at the point of inflexion (S = const/p).

If, however, the reduction of the surface area proceeds faster than the pressure increase, then Sp and dp/dt will decrease in value, and the convex side of the p, t curve will be directed upwards.

Such curves are observed before the end of burning in the case of powders with 7 perforations, whose surface area rapidly decreases after decomposition.

It can be shown by test that the position of the point of inflexio when burning strip powders, depends on the degree of homogeneity of the thickness of the plates making up the charge. By carefully selecting the proper strip thickness and arranging them in such a manner that the igniter gases would immediately envelope their entire surface, and using a strong igniter in order to obtain a simultaneous and instantaneous ignition, a p, t curve can be obtained with practically no inflexion at all, or, at any rate, a curve without a "beak."

Contrariwise, if the charge is intentionally made up of powder strips of a given grade but of varying thickness, the point of inflexion can be made to appear considerably earlier than usual [5]

Moreover, by making up a charge of strips of different grades of powder, we had succeeded in converting the p, t curve into a rectilinear curve along most of its length, i.e., create what would appear a whole series of inflexion points.

The table below contains some of the data obtained by M.E. Serebriakov in his tests, in which he attempted to determine the reasons for the appearance of an inflexion point [Δ = 0.20; powder-Japanese strip, "СП"; igniter - dry pyroxylin].

In test No. 1 the charge was made up of strips, considerably varying in thickness; a weak igniter was used.

In test No. 2 the charge was made up of strips of uniform thickness using the same igniter.

In test No. 3 the charge was the same as in test No. 2; a strong igniter was used.

The following was determined in all of these tests: p_1 and ψ_1 at the point of inflexion, powder burning time t_k and exponent β.

Table 16

$W_0 = 78.5\ cm^3$

Test No.	$2e_1$, mm	p_B, kg/cm^2	$\Delta = \frac{\omega + \omega_B}{W_0}$	p_m kg/cm^2	p_i kg/cm^2	$\left(\frac{dp}{dt}\right)_{max}$ T/cm^2/sec	ψ_i	t_K sec	β
1	0.92-1.07	20	0.201	2115	1717	428	0.82	0.0355	0.230
2	1.00-1.01	20	0.201	2150	1950	480	0.92	0.0348	0.103
3	0.98-1.00	125	0.211	2310	2190	540	0.95	0.0084	0.055

Curves p, t - Γ, t and Γ, ψ are shown in fig. 60-61.

The obtained results offer a very clear graphic description of these tests.

In test No. 1 the pressure curve past the point of inflexion has a fairly long "beak," and the transition from the point of inflexion to the end of the curve is smooth ($\Delta t = 0.0030$ sec). No smooth transition after p_i occurs in test No. 2; there is a sharp break in the curve and the "beak" is considerably shorter ($\Delta t = 0.0013$ sec).

In test No. 3 ignition occurs instantaneously, the uniformity of the thicknesses is retained, the point of inflexion shifts to the very end of burning, $(dp/dt)_{max}$ increases in value, and the "beak" is totally absent on the p, t curve; the curve terminates at an angle approaching the maximum ($\Delta t < 0.0005$ sec). No "aftercompression" of the crusher was noted in this test.

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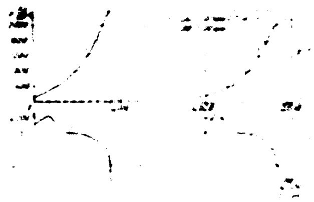

Fig. 60 - p, t and Γ, t Curves at Different Values of p_B and $2e_1$.

a) p, kg/cm^2; b) t (sec); 1) ... 1.07 mm "Cn" strip; 3) ... 1.00 mm "Cn" strip.

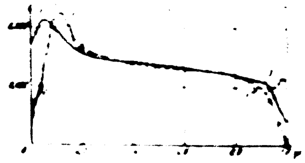

Fig. 61 - Γ, ψ Curves at Different Values of p_B and $2e_1$.

Fig. 62 - Γ, ψ Curves at End of Burning of Strips of Different Thicknesses.

Thus, the appearance of an inflexion point is linked with the sharp surface area decrease during burning of powder.

Its position, in the case of strip and tubular powders, depends on the uniformity of the thickness of the strips or tubes at the end of burning.

By choosing the proper conditions of ignition p_B and thickness of the strips, the position of the inflexion point on the pressure curve can be varied widely.

This point always occurs in the case of powders of nonuniform thickness or perforated powders, in which the burning surface area after decomposition undergoes a sharp decrease and tends towards zero.

4. REASONS LEADING TO A LOWERED INTENSITY OF GAS FORMATION IN THE LAST STAGE OF BURNING

In the case of regressive powders, the experimental Γ, ψ curves in their mid-section between $\psi = 0.3$ and $\psi = 0.8$-0.9 almost coincide with the theoretical curves; beyond $\psi = 0.8$-0.9 the ordinate of the curve begins to drop rapidly and tends towards zero at the end of burning when $\psi = 1$.

The reason for this drop lies in the nonuniformity of the plates making up the charge. The thinner the plate, the sooner will it burn. At the end of burning the surface area of such a plate undergoes a specific amount of reduction and this causes a decrease of the function $\Gamma = \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1$. The greater the nonuniformity of the charge in thickness, the earlier will the condition of lowered intensity of

gas formation occur. Furthermore, even an individual strip is usually nonuniform in thickness along its entire width, so that after it is pressed during drying the contraction of the strip is not uniform and the latter acquires a slightly lenticular cross section: it is thicker in the middle than at the ends.

A similar and even greater variation can occur in the web thickness of tubular powder as a result of a nonconcentric perforation. The resulting noninstantaneous ignition causes further impairment of the initial plate dimensions and this advances the time at which the lowered intensity of gas formation at the end of burning occurs. This was shown in Γ, ψ curves obtained in tests using igniters of different pressures, and also in fig. 61.

The results of calculating the change in the value of $\frac{S}{\Lambda_1} u_1$, for a charge consisting of plates of strip powder "СП" varying in thickness from 0.92 to 1.17 mm and tested in a manometric bomb, are presented below.

In order to simplify such calculations, the plates were split up into 6 groups according to thickness; the relative number of plates of the two middle groups taken was greater than of the remaining groups, namely, 0.20 instead of 0.15%.

Theoretically, for a powder of uniform thickness, the ratio (exposure) S/Λ_1 must vary during burning from 2.07 to 1.76 mm^2/mm^3, and function Γ - from 0.160 to 0.136.

$$\frac{1}{\frac{kg}{cm^2}\,sec} \text{ at } u_1 = 0.0775 \frac{mm}{sec} : \frac{kg}{cm^2} .$$

The following table gives the variation of S/Λ_1 and Γ as a function of ψ during the burning of the individual plate groups [4].

Table 17

Group No.	1	2	3	4	5	6	Remarks
Plate thickness $2e_1$ in mm	0.92	0.97	1.02	1.07	1.12	1.17	$2e_{1\,cp} = 1.05$ mm
Relative number of such plates in a charge	0.15	0.15	0.20	0.20	0.15	0.15	
ψ_1	0.894	0.931	0.961	0.982	0.994	1.000	
$\frac{S_1}{\Lambda_1}$	1.526	1.250	0.887	0.530	0.263	0	$\Gamma_{0\,cp} = 0.160$
$\Gamma_1 = \frac{S_1}{\Lambda_1} u_1$	0.118	0.097	0.069	0.041	0.020	0	$\Gamma_{1\,cp} = 0.136$

On the diagram in fig. 62, curve 1 corresponds to the theoretical variation of Γ for powders of average thickness; the descending portion 2 of the $\Gamma_{\text{теор}}$ curve is obtained on the basis of the above table; this diagram also includes curve 3 of the test curve Γ obtained on the basis of a bomb test. A comparison of the theoretical and test Γ curves shows that their behavior both at the mid-section and at the end of burning is similar. The fact that curve 3 begins to drop ahead of curve 2 is explained by the use of a weak igniter in the test, and by the fact that the noninstantaneous ignition leads to an increased thickness variation.

Conclusions

The above investigations clarify the reasons for the deviation of powder grains of plain shape from the geometric law of burning (Sections I, II, and IV in fig. 50).

The rapidly increasing section of the Γ, ψ curve at the start of burning constitutes a process of gradual ignition; a practically instantaneous ignition and the start of the curve directly from the maximum point is secured by the use of a high pressure igniter p_B = 120-150 kg/cm^2.

"Ballooning" is the accelerated burning of powders at low pressures and in the presence of layers of nonuniform thickness, occurring as a result of technological processes (wetting) and excessive heating of the powder at low rates of burning.

The point of inflexion on the p, t curve and the corresponding rapid decrease of the intensity of gas formation shortly before the end of burning are due to the nonsimultaneous burning of the elements of the charge of varying thicknesses. The smaller the igniter, the greater will be the variation in the thickness of the powder, the earlier will the inflexion point occur, and the more rapid will be the drop in the intensity of gas formation. By choosing charge elements of the proper thickness and a very weak igniter, a condition can be obtained whereby the burning of the powder at the end would proceed without a sharp drop in the intensity of gas formation, and a p, t curve can be obtained without a point of inflexion.

5. BALLISTIC ANALYSIS

A. The Application of Γ, ψ Curves to the Analysis of Burning of Flegmatized Powders

The powder must be flegmatized in order to make it a progressive powder, i.e., a substance tending to slow down the burning of the outer layers must be added to the powder mass. The distribution of the flegmatizer in the powder must be irregular - its concentration must be maximum in the outer layers and gradually diminish towards the center of the grain. Therefore, the burning rate u_1 at $p = 1$ kg/cm^2, which depends on the nature of the powder, must vary from a minimum in the outer layers to a maximum inside the grain. Progressive burning is thus obtained by changing the composition of the powder mass rather than from the shape of the powder.

Under actual factory conditions flegmatization may be inadequate or excessive, whereby the flegmatizer penetrates the full powder thickness and makes it slow burning rather than progressive in character.

It is of importance therefore to determine the depth of the flegmatizer's penetration, its distribution in the powder, and its effect on the burning rate u_1.

Usually the penetration of the flegmatizer is determined by coloring the flegmatizer solution with fuchsin (magenta red), and after flegmatization and drying the grain is cut and examined under a microscope.

This method is not very accurate however, because the penetrating properties of fuchsin and the flegmatizer may be different. The microscope enables one to determine the depth of penetration of fuchsin; it does not, however, permit one to determine the degree of distribution of the substance in the powder.

However, bomb tests and their analysis by means of the Γ function make it quite easy to obtain an accurate evaluation of the distribution of the flegmatizer throughout the mass of the powder.

Indeed, if we were to test in a bomb at a given loading density ordinary powder and then a flegmatized powder, and plot curves of the change of Γ as a function of ψ, the difference between these curves would be an appreciable one; this can be seen in fig. 63, which shows the test results obtained with powders with 7 perforations before and after flegmatization.

Whereas the nonflegmatized powder (curve 1) has "ballooning" present on the Γ, ψ curve and then drops to $\psi = 0.50$, flegmatized powder (curve 2) produces no ballooning, the ordinates of the curve move upward, and burning is progressive from the start up to $\psi \approx 0.50$; thereafter the Γ, ψ curves almost coincide. Therefore, the effect of flegmatization is felt until half of the grain is burned, following which it is terminated. The above makes it possible to calculate the depth to which the flegmatizer has penetrated.

Inasmuch as in both cases the powder grains were of the same shape and dimensions, it may be assumed that the change of S/Λ_1 with respect to ψ is the same in both cases. Hence the ratio of the ordinates Γ_2/Γ_1 gives the ratio between the elementary burning rates at a given instant.

$$\frac{\Gamma_2}{\Gamma_1} = \frac{\left(\frac{S}{\Lambda_1} u_1\right)_2}{\left(\frac{S}{\Lambda_1} u_1\right)_1} = \frac{(u_1)_2}{(u_1)_1};$$

upon determining the value of this ratio for successive values of ψ, a curve can be constructed showing the relative variation of the elementary burning rate, which depends on the distribution of the flegmatizer in the powder mass.

An example of a curve of this type is shown in fig. 64.

Fig. 63 - Γ, ψ Curves for a Powder Before and After Flegmatization.

Fig. 64 - Change of Burning Rate u_1 in a Flegmatized Powder.

The distribution of the flegmatizer in the powder is not uniform and becomes smaller as the depth of penetration is increased: this is indicated by the minimum value of the burning rate at the start and by the fact that the rate increases according to the law up to $\psi = 0.50$. Thereafter, the relative burning rate becomes equal to unity, i.e., the burning rates of the powders become the same. This indicates that the flegmatizer did not penetrate beyond $\psi = 0.50$ and its corresponding thickness.

Thus flegmatization serves to reduce the intensity of gas formation during the first half of the process. This permits increasing the charge in firing without exceeding the maximum pressure and to obtain a higher shell discharge velocity. But inasmuch as flegmatization usually lowers the powder energy f, in addition to the burning rate u_1, a portion of the increased charge is utilized for the purpose of maintaining the shell velocity which would have been obtained with nonflegmatized powder.

Inasmuch as the burning rate of powders containing uniformly distributed quantities of the flegmatizer can be determined from bomb tests, the variation of the flegmatizer concentration between the powder layers can be determined by the change in the burning rate of powders with different percentage contents of the flegmatizer material.

If the duration of the flegmatization process is overly long, a powder can be obtained in which the distribution of the flegmatizer is practically uniform throughout its entire thickness, and hence the burning of the powder will be slow but not progressive.

B. The Peculiar Burning Characteristics of Nitroglycerine Powders

Analysis by means of the Γ function had shown that English tubular cordite, notwithstanding its regressive shape, gives a somewhat increased value of Γ, after $\psi \approx 0.3$, i.e., its burning is progressive.

It has been established by means of special tests, that a Γ, ψ curve constructed for freshly manufactured cordite does not differ in anyway from an ordinary Γ, ψ curve for tubular pyroxylin powder (curve 1-1-1 in fig. 65), after the excess of the solvent (acetone) is removed.

But if the "life" of such a powder is shortened by placing it in a thermostat at $t = 50^{o}C$, its energy will be lowered after a while and the Γ, ψ curve, following ballooning and a descent, will begin to ascend again from $\psi \approx 0.3$ to $\psi = 0.9$ (curve 2-2). Thereafter, its drop to zero will proceed more sharply, similarly to pyroxylin powders during the end burning of the thicker ele[illegible]nts.

Fig. 65 - Γ, ψ Curves for Nitroglycerine Powder.

The fact that the energy f has decreased indicates that a portion of the nitroglycerin had evaporated through the outer surface of the powder. The nitroglycerine remaining in the layers close to the surface had shifted onto the surface causing ballooning during burning. This in turn had caused a redistribution of the nitroglycerin in the neighboring layers. Inasmuch as the rate of burning depends on the nitroglycerin content in the given layer, the rate should increase in the presence of such a nonuniform distribution as the

burning process penetrates the grain in depth. And since $\Gamma_T = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1$, Γ increases because of the increase of u_1, and the burning of the powder becomes progressive. Therefore the burning characteristic of cordite can change depending upon its "age" and the conditions for its storing, and this change can be easily disclosed by using the test function Γ as the analyzer.

CHAPTER 3 - BURNING PROPERTIES OF POWDERS WITH NARROW PERFORATIONS

It was shown above that test curves Γ, ψ of the progressivity of burning of powders with many narrow perforations deviate from the geometric law considerably more than do the curves of powders with grains of simple shapes (strip, short tube). These deviations are the greater, the longer the perforations, and, hence, the longer the powder grain itself.

The least understood phenomenon from the standpoint of the geometric law of burning are the regressive portions of the Γ, ψ curves in the case of perforated powders, whose surface area, theoretically, must undergo a continuous increase until the grain is decomposed.

If we consider such a change of the value of Γ from the standpoint of surface variation in a grain during burning, we get the impression that the grain becomes decomposed into separate parts in the form of rods after it is ignited, whose surface changes regressively. Such a decomposition of a grain during the initial stage of burning can occur for the reason that the gases formed inside

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narrow and long perforations do not succeed in fully escaping from the perforations, thus creating a higher pressure which serves to accelerate burning. This pressure becomes so high that the walls of the powder collapse and the grain disintegrates, following which the burning of the powder becomes regressive.

Such an explanation for the regressive form of the Γ curve for a powder with 36 perforations seemed natural at first glance.

Actually, however, an examination of the unburned powder rods obtained after firing has shown that no disintegration of the powder occurred at first. This was evidenced by the presence of many grains with strongly burned but otherwise intact perforations corresponding to $\psi \approx 0.60$, or by grains bearing signs of partial decomposition only (see right-hand photograph in fig. 31).

This condition indicates that the burning of powder with narrow perforations is more complex and depends on factors absent in the burning of powders of simple shapes and usually not taken into consideration.

It is therefore of importance to analyze the peculiarities involved in the burning of powders with narrow perforations and to develop a theoretical approach to the problem dealing with the deviation of such powders from the geometric law, as was disclosed in actual bomb tests.

1. THE EFFECT OF PROXIMATE CONTACT BETWEEN BURNING SURFACES

If two powder strips are burned separately in the open air, each strip will burn quietly. If, however, one strip is placed over the other while burning with the burning ends touching, burning at the points of contact will proceed more vigorously and gases will be forcefully ejected from the gap formed between the burning surfaces. This indicates that the gas pressure becomes increased.

An assumption has been made to the effect that if the surfaces of two grains were made to burn separately in one bomb and in close contact in another, the pressure between the contacting surfaces would be higher and the burning of the grains will proceed more vigorously.

A. Tests with a Rod and a Tube.

This assumption was confirmed in an actual test.

Identical powder grains were burned simultaneously in small bombs (21.5 cm^3) wherein the grains were arranged differently.

The charge consisted of a tube and rod of nitroglycerin powder. The rod diameter was such that it could be inserted into the tube with a certain small clearance between them.

In one test the rod was placed in the bomb alongside the tube, and the other rod was inserted into the tube.

Due to its higher burning rate, nitroglycerin powder was found to give a sharper distinction between these parallel tests.

The results were as follows (fig. 66 - p, t and Γ, t curves): in

the first case the p, t curve was smooth, and in the second (rod inside tube) a sharp pressure increase was obtained at $p \approx 100$ kg/cm^2, following which the p, t curve became smooth again while remaining above the first one.

In the first case, curve Γ, ψ (fig. [illegible]7) shows a small amount of ballooning at $\psi = 0.03$ and then slowly descends to $\psi = 0.70$, following which it drops rapidly because the web thickness of the tube is practically fully burned, and the final burning of the tube and the burning of the rod follow.

At the instant $\psi = 0.90$ the tube is fully burned (thickness of tube was 1.75 mm and diameter of rod 5 mm) and the rod undergoes the last stages of burning.

In the case of the rod inserted into the tube, the Γ, ψ curve balloons sharply at $\psi = 0.03$, its apex corresponding to $\psi = 0.04$. The maximum ordinate at this point is almost twice as great as that of the corresponding ordinate of Γ of the first test; this is followed by an almost vertical drop down to the first curve ($\psi = 0.06$), following which the curves are almost coincident.

In order to clarify these results, the test was repeated in open air. Upon igniting the rod simultaneously at both ends, it was ejected from the tube, apparently because of the pressure difference between the ends in the clearance present between the rod and tube.

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Fig. 66 - The Effect of Contacting Burning Surfaces on Γ, t.

1 - rod and tube side by side; 2 - rod inserted into tube.

Fig. 67 - The Effect of Contacting Burning Surfaces on Γ, ψ.

When tested in a bomb, the rate of gas formation rapidly increases because of the high burning rate of nitroglycerin powders, the close proximity of the burning surfaces and the small clearance, and ballooning occurs on the Γ curve. But due to the pressure difference which may occur at the opposite ends of the clearance, a string of relatively soft cordite would have been forced out of the tube. Thereafter, a close contact between the burning surfaces would have been obviated, and further burning would have continued under conditions almost analogous to the first test. This is indicated by the almost parallel path of the Γ, ψ curves.

The fact that the burning in this case proceeds more vigorously than in the first test is indicated by the condition that the burning of the tube and rod in the second test was concluded ahead of the first one, as can be seen from the Γ, t curves in the diagram.

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Actually, the drop of Γ corresponding to the end of the tube's burning occurs in the first case at $t = 0.0095$-0.0100 sec, and in the second case - at $t = 0.090$ sec. The full burning time in the first case is $\tau = 0.0140$, and 0.0130 sec in the second.

These tests are very valuable with regard to the theory of non-uniform powder burning, inasmuch as they show that the sharp increase in the rate of gas formation is not due to the increased surface area (which was the same in both tests), but, rather, to the increased rate of burning caused by the close contact of the burning surfaces and by higher pressure.

As soon as this contact is eliminated, the process proceeds normally. According to the geometric law of burning, all the powder surfaces must burn with the same rate, and a change in the mutual arrangement of the portions of the charge should not affect the law of gas formation.

B. Powders with Narrow Perforations

An identical phenomenon of accelerated gas formation should be observed in the case of powders with narrow and long perforations, wherein the burning surfaces are in close contact. The narrower the perforation, the closer is the contact between the perforation surfaces, and the more vigorous is the burning process. As the perforations are eroded, their surfaces spread apart and the intensity of burning decreases, and as a result the ordinates of the Γ, Ψ curve become gradually reduced.

The fact that the burning rate before decomposition, determined by the formula

$$u_1 = \frac{e_1}{\int_0^1 p\,dt}$$

is considerably greater than the rate after decomposition, at which time the rate is normal, indicates that the burning rate is actually greater in perforated grains than in strip powders. Thus, the burning rate of Kisnemsky's powder with narrow perforations before decomposition, corresponding to the point of inflexion on the pressure curve, is $u_1 \approx 0.100$ mm/sec, compared with strip powder of the same composition whose normal rate is 0.075 mm/sec.

2. THE EFFECT OF THE LENGTH OF PERFORATION ON THE PROGRESSIVE BURNING OF POWDER

According to the geometric law of burning, the narrower and longer the perforation, the more progressive is the shape of the grain.

However, the narrower and longer the perforations, the greater will be the difference between the conditions of burning inside the perforations and at the outer surface, the more difficult will it be for the gases to leave the perforations, and the greater will be the pressure developed in them; and as a result the deviation from the geometric law towards regressive burning will be the greater.

In order to confirm these deductions, we are presenting below the results of tests in a manometric bomb for determining the progressive burning of powders with a large number of narrow

perforations of uniform cross section, but of varying length, and also the results of firing.

Tests for determining the effect of the length of perforations were made with powder No. 8: 36 perforations, relative length in a normal slab $\frac{2c}{a_0} = 90$.

The charge consisted of normal slabs, of slabs of the same cross-sectional area reduced to 1/4 length ($\frac{2c}{a_0} = 22$), to 1/8 of the normal length ($\frac{2c}{a_0} = 11$), and slabs reduced to 1/10 of normal length ($\frac{2c}{a_0} = 9$). Perforation wall $a_0 = 0.42$ mm.

If for the sake of simplicity we assume that the powder burns to the very end without decomposing, then, as the length of the slabs is decreased, the geometric progressivity $\frac{S_K}{S_1}$ becomes smaller, and the exposed surface $\frac{S_1}{\Lambda_1}$ becomes greater, as can be seen in Table 18.

Table 18

	2c	$\frac{2c}{4}$	$\frac{2c}{8}$	$\frac{2e}{10}$
$\frac{2c}{a_0}$	90	22	11	9
$\frac{S_1}{\Lambda_1}$	1.37	1.53	1.76	1.89
$\frac{S_K}{S_1}$	2.17	1.83	1.39	1.20

This data shows that a full-length slab (2c) possesses a very high geometric progressivity (2.17), whereas a slab reduced 8 times in length has the progressivity of a grain with 7 perforations.

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Theoretical curves of the variation of S/S_1 corresponding to the geometric law of burning without decomposition are shown in the diagram of fig. 68a.

The geometric curves of progressivity ascend from the initial ordinate $S/S_1 = 1$ in the form of a diverging cluster. The ordinates have a maximum value at the end of burning; as the length of the slabs is increased, the slope of the curve, and hence the progressivity, increases.

The test characteristics $\Gamma = \frac{1}{p}\frac{d\psi}{dt}$ were calculated from these curves; then, in order to eliminate the influence of the varying exposure $\frac{S_1}{\Lambda_1}$ entering into the values of Γ, the latter were subdivided into corresponding values of S_1/Λ_1, i.e., reduced not to the initial volume, but, rather, to the initial surface area S_1. We shall designate this value of $\Gamma : \frac{S_1}{\Lambda_1} = \Gamma\frac{\Lambda_1}{S_1}$ by Γ_s; the obtained curves of Γ_s as a function of ψ are shown in fig. 68.

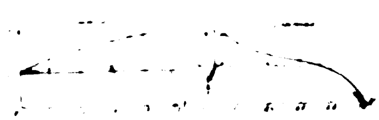

Fig. 68 - The Effect of the Length of Perforations on the Intensity of Gas Formation.

a) theoretical; b) test curves.

Were the geometric law applicable, $\Gamma = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1$; $\Gamma_s = \frac{S}{S_1}u_1$, i.e., th value of Γ_s at constant burning rate u_1 would vary in proportion

with S/S_1. Therefore the Γ_s, ψ curves should be of the same form and have the same relative arrangement as the curves of geometric progressivity $\frac{S}{S_1}$, ψ (fig. 68a).

An analysis of these tests shows the contrary to be true, as illustrated by the diagram in fig. 68b.

All the Γ_s curves have a sharp slope at first and a maximum at $\psi \approx 0.10$, following which their path is regressive. The highest curve is that for the slabs of the greatest length 2c, the shorter the slab, the lower is the Γ_s curve. The mutual disposition of the Γ_s curves is the same as of the S/S_1 curves. But whereas the S/S_1 curves proceed in the form of a diverging cluster and are the more progressive the greater the length of the slab, the experimental Γ_s curves show the reverse: they have a maximum divergence after a sharp ascent at $\psi = 0.10$, and then proceed in a converging cluster, whereby the greater the slab length, the more regressive is the curve.

For slabs $\frac{2c}{8}$ in length the curve has even a small horizontal section.

Thus the longer the slab, the more will the burning of perforated powders deviate from the geometric law, the steeper will be the slope (ascent) of the Γ_s curve at the start of burning (while the perforations are still narrow), and the more regressive will the curve be thereafter. Decomposition commences at $\psi \approx 0.70$ and the curves begin to drop to zero at $\psi = 1$.

The following conclusion can be derived from these tests: when powder is burned in a constant volume the change in the intensity of gas formation is the more regressive, the more progressive is the shape of the grain. The true or physical law of burning gives results which are opposite in character to those of the geometric law.

Further analysis will show that the Γ_s curve of the shortest slab 4 is more regressive than curve 3 representing a longer slab $\left(\frac{2c}{8}\right)$. Here the limited geometric progressivity begins to have its effect, and the conclusion is reached that for a powder grain of a given cross section there is such an optimum grain length at which the gas formation is most progressive (or least regressive)

$$\left(\frac{2c}{a_0} \approx 20\text{-}25\right) .$$

These bomb test results which are so paradoxical from the standpoint of the geometric law of burning were verified in actual firing tests (Table 19).

Table 19 - Results of Firing Using Powders of the Same Cross Section but of Different Lengths

Powder Specimen	$\frac{2c}{a_0}$	ω kg	p_m kg/cm^2	v_Δ m/sec	$\frac{S_1}{\Lambda_1}$	$\frac{S_s}{S_1} = \sigma_s$
Long ($2c = 10A_0$)	220	0.950	2285	613	1.39	1.89
Shortened ($2c = 3A_0$)	66	1.150	2290	655	1.44	1.81
Short ($2c = A_0$)	22	1.100	2285	648	1.59	1.54
Grade 9/7 (with 7 perforations)	24	1.200	2290	655	1.40	1.37

All the specimens gave the same value of p_m, though the longest specimen No. 1 having the most progressive shape produced this pressure with the smallest charge $\omega = 0.950$, which serves to explain why the velocity v_Δ was the lowest (613 m/sec).

The shortened specimen No. 2 having the least theoretically progressive shape permitted however, without elevating the pressure, to increase the charge to 1.150 kg and thus increase the muzzle velocity to 655 m/sec, i.e., it was actually found to be more progressive. Specimen No. 3, similarly to the bomb test, gave somewhat poorer results than specimen No. 2, producing the same pressure at a somewhat smaller charge (1.100 kg), and v_Δ was reduced to 648 m/sec.

These firing tests had shown that powders with narrow perforations give identical results in a bomb and when fired from a gun, and that powders with excessively long and narrow perforations are not profitable, notwithstanding their geometric progressivity. There is

a certain optimum length at which burning is most progressive; in guns at large values of Δ this length may differ from that in a bomb at small values of Δ.

Similarly, firing tests with tubular powders have confirmed the fact that longer tubes in an identical charge produce a higher pressure p_m and velocity v_Δ compared with shortened tubes.

These tubes show that powders with narrow perforations do not follow the geometric law and that their deviation from the latter is the greater, the longer the perforation.

A comparison of the results obtained in firing shortened Kisnemsky powders with 36 perforations (No. 2) and ordinary 9/7 powder (No. 4) indicate that notwithstanding the great difference between their geometric progressivity, the 9/7 powder produced the same results (at a considerably simpler manufacturing procedure).

3. THE FUNDAMENTAL THEORY OF NONUNIFORM BURNING OF PERFORATED POWDERS

As was mentioned above, a certain contradiction was disclosed between the actual and geometric law of burning even in powder grains of simple shapes.

Such contradicitions were particularly sharply defined in the case of powders with narrow and long perforations. Test curves of the progressivity of burning are entirely contradictory to the theoretical curves; these deviations increase with the length of the perforations. What is the explanation thereof?

The basic concept of the geometric law rests on the condition that the pressure is always uniform at all the elements of the burning surface of the charge, and that hence the burning rate $u = u_1 p$ is also uniform.

This condition would have been valid if the process of burning were to proceed very slowly to permit immediate equalization or balancing of the slightest pressure differences occurring at various points of the bomb, i.e., if the phenomenon were to proceed similarly to static processes.

Actually, of course, the burning process proceeds extremely rapidly to permit equalization of the pressure at different points of the charge, so that the burning rate is actually different at the various points of the burning surface. The difference in intensity must also be pronounced most sharply in the burning of powders with narrow perforations.

It can be easily proved that the rate of burning inside the perforations can be the same as the rate at the surface of the grain. In a chemically homogeneous powder composition the burning rates can be equal only at equal pressures. But if the pressure p" within the perforation equaled the outside pressure p', the gases formed inside the perforations could not escape, unless a pressure difference were present. Therefore, if the inside and outside pressures were equal, the bomb would become filled with the gases formed at the outside surface of the powder only, while the gases in the

perforations would remain where formed, which condition is entirely improbable. Thus a simple reasoning shows that the burning rate inside and outside the grain cannot be the same.

By separately applying the pyrostatics formulas to the surface of the perforation and the outside grain surface, it can be shown quantitatively that the pressure inside a narrow perforation cannot equal the pressure at the outside grain surface, and that the pressure increase inside the perforation must proceed considerably more rapidly.

We shall make a preliminary analysis of the values governing the pressure increase obtained in burning powders of simple shapes without perforations, assuming that the grains are uniformly distributed in the bomb, or that only one grain is being burned.

We have introduced the following expression for determining the pressure increase in a constant volume:

$$\frac{dp}{dt} = \frac{f\Delta\left(1 - \frac{\Delta}{\delta}\right)}{\Lambda_\psi^2} \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 p.$$

For the initial stage of the burning process $S \approx S_1$

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi \approx 1 - \frac{\Delta}{\delta};$$

then

$$\frac{dp}{dt} = \frac{f\Delta}{1 - \frac{\Delta}{\delta}} \frac{S_1}{\Lambda_1} u_1 p = \frac{f\omega}{W_0 - \frac{\omega}{\delta}} \frac{S_1}{\Lambda_1} u_1 p,$$

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but $\frac{\omega}{\Lambda_1} = \delta$ - powder density, and

$$\frac{dp}{dt} = f\delta u_1 \frac{S_1}{W_0 - \frac{\omega}{\delta}} p. \qquad (50)$$

Designating $W_0 - \frac{\omega}{\delta} = V$, we get

$$\frac{dp}{dt} = f\delta u_1 \frac{S_1}{V} p$$

and hence for a given type of powder (f, δ, u_1) and pressure p, the pressure increase dp/dt is determined by the ratio between the burning surface area of the powder and the volume in which the gas is separated from the powder surface.

We shall designate this ratio by ζ :

$$\zeta = S/V.$$

Burning inside the powder perforation can be viewed as consisting of two consecutive processes:

1) accumulation of gases separated from the surface of the perforation inside the perforation space;

2) outflow of the accumulated gases from the perforation if the inside pressure exceeds the pressure at the surface of the grain.

We shall apply formula (50) separately to burning at the inside surface of the perforations and to burning at the outside grain surface and compare the results.

Let us designate the initial volume of each perforation W_K and its surface area S_K; the total number of grains in the charge is N, and the number of perforations in each grain is n.

If we assume that the igniter pressure p_B is the same at the outer surface and inside each perforation, then:

$$\text{in the perforation } \frac{dp''}{dt} = f\delta u_1 p_B \zeta'',$$

where

$$\zeta'' = \frac{S_K}{W_K} = \frac{\pi d \cdot 2c}{\frac{\pi}{4} d^2 2c} = \frac{4}{d};$$

and at the outer grain surface S'

$$\frac{dp'}{dt} = f\delta u_1 p_B \zeta',$$

where ζ' is the ratio of the entire surface area S' to the volume within which the gases are separated from this surface, i.e.,

$$\zeta' = \frac{S'}{W_0 - \frac{\omega}{\delta} - NnW_K}.$$

Bearing in mind that $\frac{\omega}{\delta} = \Lambda_1$, where Λ_1 is the volume of the entire charge, and dividing the numerator and denominator ζ' by $\frac{\omega}{\delta} = \Lambda_1$, we get:

$$\zeta' = \frac{S'}{\Lambda_1} \frac{1}{\frac{W_0}{\Lambda_1} - 1 - \frac{NnW_K}{\Lambda_1}} = \frac{S_1}{\Lambda_1} \frac{S'}{S_1} \frac{1}{\frac{\delta}{\Delta} - 1 - \frac{NnW_K}{\Delta_1}} .$$

Calculations show that for a standard grain with 7 perforations at $\Delta = 0.20$ $\zeta'' \approx 70\zeta'$; therefore the ratio between the rate of pressure increase in the perforation and the rate at the outer surface will be considerably greater (of the order of several tens):

$$\frac{dp''}{dt} \gg \frac{dp'}{dt} \text{ and } p'' > p'.$$

Therefore, even if the pressure in a narrow perforation and at the outside surface are equal at a certain moment, the pressure inside the perforation will immediately begin to increase at a faster rate than at the outside surface, and hence the burning rate $u'' = u_1 p''$ will be higher.

All the reasonings presented above relate to the start of burning and would be entirely valid if no gases were to flow out of the perforations. Actually, inasmuch as the pressure will increase more rapidly inside the perforations, the gas will escape because of the resulting pressure difference, so that the free space within the perforation will be increased and the outside free space, wherein are also collected the gases separating from the outside surface, will be decreased. As a result, the pressure difference will become reduced when both the perforation and the outside surface are burning, and will gradually become equalized.

However, due to the extremely high speed of this phenomenon, the gases formed in the perforations are incapable of fully escaping from the narrow openings and become accumulated in the perforations, the pressure increases as a result and in turn increases the rate of burning, so that burning in the perforations at a given pressure within a bomb must invariably proceed at a higher rate than at the outside surfaces of the grain.

Thus the presence of narrow and long perforations in the powder will always cause nonuniform burning in the perforations and at the grain surface, and this nonuniformity results in the anomalous curves of progressivity Γ, ψ presented above (see diagrams in figs. 48, 49, 52 and 53).

If the loading density Δ is increased, the corresponding value of ζ'' for each perforation remains unchanged, the value of ζ' for the outside surface increases (because the entire ζ' fraction increases with the increase of Δ), and the ratio $\frac{\zeta''}{\zeta'}$ decreases and tends toward unity.

Tests have shown that the Γ, ψ curves at small values of Δ actually become more regressive.

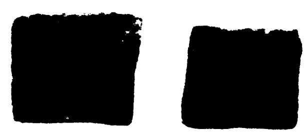

Fig. 69 - Burning of Kisnemsky's Powder with Very Narrow Perforations.

Fig. 70 - Burning of Kisnemsky's Powder with Very Narrow Perforations.

The presence of a considerably increased pressure in very narrow and long perforations can be seen in photographs of slabs of Kisnemsky's powders No. 9 and 10 (figs. 69 and 70) ejected from a gun before they were fully burned. The photographs on the left show slabs of Kisnemsky's powders before burning, the perforations are so narrow (0.1-0.2 mm) that they can be hardly seen because of their fused openings; the photographs on the right are of grains

ejected from a gun. Many of the perforations are eroded, which condition can be due only to the presence of a high pressure within the perforations; similar erosions were observed also on the side surfaces of the slabs. The perforations are almost circular in shape.

4. EFFECT PRODUCED BY NONUNIFORM BURNING OF PERFORATED POWDERS ON THE SHAPE OF THE Γ CURVE

We shall now show that nonuniform burning and excessive gas pressure in narrow perforations can serve to explain the steep ascent at the start and the continuous drop of the Γ, ψ curve obtained from the analysis of experimental p, t curves obtained with powders having narrow perforations.

We have the following designations:

S' – outside surface area of burning grain;

S'' – combined surface area of all the perforations in the grain

$S = S' + S''$ – total surface area of the grain;

Λ_1 – initial volume of grain;

u' – burning rate at the outside grain surface ($u' = u_1 p'$);

u'' – burning rate inside the perforations ($u'' = u_1 p''$).

In order to simplify the analysis of the phenomenon, let us assume that the burning rate inside the perforations is uniform and that the burning rate along the entire outside surface is likewise constant.

Let us see now how the test curve of progressivity $\Gamma = \frac{1}{p} \cdot \frac{d\psi}{dt}$ in the case of the geometric law will differ from the same curve based on actual burning proceeding with different rates at the outside surface and in the perforations.

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Let us assume that at a given instant the same portion of the charge ψ was burned in both cases, and that the pressure at the outside surface equaled p' and on the inside surface - p''; it may be assumed that the crusher used in the bomb test records a pressure p'.

Then, in the first case (geometric law of burning), the burning rate on all the surfaces will be the same:

$$u'' = u'; \; p'' = p'.$$

Using the general formula for the rate of gas formation as the basis, we can write:

$$\left(\frac{d\psi}{dt}\right)_I = \frac{S}{\Lambda_1} u = \frac{S' + S''}{\Lambda_1} u' = \frac{S' + S''}{\Lambda_1} u_1 p';$$

$$\Gamma_I = \frac{1}{p'} \left(\frac{d\psi}{dt}\right)_I = \frac{S' + S''}{\Lambda_1} u_1 .$$

In the second case (actual burning), the burning rate differs:

$$u'' > u'; \; p'' > p'.$$

The surface area of the perforations S'' burns with the rate u'', and the outside surface S' - with the rate u'.

We get:

$$\left(\frac{d\psi}{dt}\right)_{II} = \frac{S'u' + S''u''}{\Lambda_1} = \frac{\left(S' + S'' \frac{u''}{u'}\right)}{\Lambda_1} u' = \frac{\left(S' + S'' \frac{p''}{p'}\right)}{\Lambda_1} u_1 p';$$

$$\Gamma_{II} = \frac{1}{p'} \left(\frac{d\psi}{dt}\right)_{II} = \frac{\left(S' + S'' \frac{p''}{p'}\right)}{\Lambda_1} u_1 .$$

A comparison of the Γ_I and Γ_{II} expressions will show that they differ by their p''/p' factors alongside the surface S'' appearing in parenthesis.

Inasmuch as $\frac{p''}{p'} > 1$, $\Gamma_{II} > \Gamma_I$, as observed in comparing the theoretical and test curves of Γ.

Hence, when burning is not uniform, the behavior of the powder is such as if the surface area of its perforations were increasing with respect to p''/p'.

Actually this represents an increased rate of gas formation due to the increased burning rate in the perforations.

We thus arrive at the following conclusion: if the burning of the powder is not uniform, the pressure and burning rate in the perforations being higher than at the outside surface, then at a given ψ the intensity of gas formation Γ_{II} must always be greater than Γ_I, when calculated with the assumption that the burning of the powder is uniform.

The difference between the rates of gas formation based on actual and geometric laws of burning will depend upon the ratio

$$\frac{u''}{u'} = \frac{u_1 p''}{u_1 p'} = \frac{p''}{p'}.$$

As burning progresses, the diameter of the perforation increases, and the ratio $\frac{4}{d}$ and hence of $\frac{\zeta''}{\zeta'}$ becomes smaller; the $\frac{p''}{p'}$ ratio will decrease also and the Γ_{II} and Γ_I curves will converge. This is the very condition actually observed on the Γ, ψ curves.

Thus under conditions of actual burning of progressive powder with narrow perforations in a constant volume the intensity of gas formation will be considerably greater at the start of burning, and the Γ,ψ curve will be situated considerably higher than the theoretical curve.

This difference becomes smaller as burning progresses, and the intensity of gas formation will differ from the theoretical to a smaller degree. During the entire process of burning of progressive powder the value of Γ may generally decrease and hence the burning will be regressive.

The narrower the perforations and the greater their number, the higher will be the ratio between the perforation surface area S" and the total surface area S_1, the sharper will be the influence of nonuniform burning and the greater will be the deviation from the geometric law, so that progressive powder grains will actually burn regressively, which is the case observed in the burning of Kisnemsky's powders with 3o perforations.

5. BURNING OF POWDER WITH NARROW PERFORATIONS IN A GUN

Tests show that the Γ,ψ curves at small values of Δ are more regressive than at high values of Δ.

This behavior is most important in clarifying the actual burning of perforated powders in a gun, where the initial loading density is very high (of the order of 0.50-0.70), and decreases continuously as the shell moves through the bore.

It was shown in the theory of nonuniform burning of powder

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that as the loading density decreases, the difference increases between the values ζ' and ζ'' characterizing the rate of gas pressure increase at the outer surface and inside the powder perforations.

For powders with 7 perforations at $\Delta = 0.20$ $\zeta'' \approx 70\zeta'$, the Γ, ψ curve of the intensity of gas formation is also regressive.

The powder commences to burn in a gun at considerably higher loading densities ($\Delta_0 = 0.60$-0.70), at $\Delta_0 = 0.70$ $\zeta'' \approx 8\zeta'$ for the same type of powder with 7 perforations, i.e., the difference becomes equalized. Burning at this constant value of $\Delta_0 = 0.70$ would have occurred under more uniform conditions.

In addition, due to the movement of the shell through the bore, the space behind it increases continuously, and the loading density in this variable volume continues to decrease during the entire burning process ($\Delta = \frac{\omega}{W_0 + s\ell}$). This results in a continuous variation of the $\zeta'' : \zeta'$ ratio, whereby this ratio increases when Δ is reduced, which serves to increase the intensity of gas formation Γ and increases the progressive burning as the shell moves through the bore.

Thus in attempting to reach a conclusion regarding the burning of powder in a gun, we cannot consider its burning in a bomb at one constant loading density as being representative and assume that the obtained Γ, ψ curve characterizes the intensity of gas formation in a gun. Such a conclusion would be incorrect. The burning of powder must be considered in its relation to the conditions of loading and burning taken as a whole.

Therefore, in order to calculate the intensity of gas formation when burning powder in a gun, it is first necessary to determine the Γ, ψ curves at constant but different values of Δ by means of bomb tests, and to determine the effect of Δ on the characteristic change of intensity of gas formation. Then, taking into account the initial loading density Δ_0 in a gun, it is necessary to extrapolate the experimental Γ, ψ curves obtained in a bomb at smaller values of Δ for the given loading density Δ_0. Thereafter, bearing in mind that the loading density in a gun decreases continuously from Δ_0 to $\Delta_K = \dfrac{\omega}{W_0 + s l_K}$ (where l_K is the distance traversed by the shell at the end of burning of the powder), it is necessary to go over, as ψ increases, from the Γ, ψ curve at $\Delta = \Delta_0$ to the Γ, ψ curves corresponding to successively smaller constant densities of loading.

As a result, the Γ, ψ curve of the intensity of gas formation for a variable loading density will differ from each Γ, ψ curve obtained at constant values of Δ. As shown in fig. 71 by a solid curve, it can be even more progressive, the progressivity being the higher the greater the initial density of loading.

At low initial loading density Δ_0, the change in the regressive characteristic of the Γ, ψ curves obtained at constant Δ will be insignificant, and in such a case the burning may be regressive also at a variable value of Δ (fig. 72).

The effect of the value of Δ_0 on progressive burning will serve as an explanation of the following very interesting fact observed in the firing of guns with Kisnemsky's powders: in one gun a charge

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consisting of Kisnemsky's powder with 36 perforations at a high loading density gave better results than strip powder (p_{max} was lower at the same value of v_{Δ}), whereas another gun at a small value of Δ_0 loaded with the same powder gave poorer results (the same v_{Δ} velocity at a considerably higher pressure p_{max}).

This fact can be explained only by means of the theory of nonuniform burning.

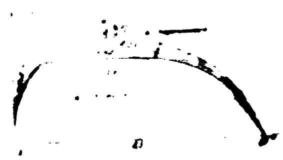

Fig. 71 - Intensity of Gas Formation in a Gun at a High Value of Δ_0.

1) Δ- variable; 2) Δ_0 - large.

The higher the loading density, the more uniform will be the conditions of burning inside the perforations and at the outside surface, and the more closely will actual burning approach the geometric law.

Fig. 72 - Intensity of Gas Formation in a Gun at a Small Value of Δ_0.

Fig. 72 - (Cont'd.)

1) Δ - variable; 2) Δ_0 - small.

Only by considering the process of burning in conjunction with all the factors influencing its characteristics, can we arrive at a correct conclusion, on the basis of the theory of nonuniform burning, regarding the true burning of a powder charge in the bore of a gun when the latter is fired. The theory of nonuniform burning had disclosed the fundamental laws governing the burning of powders with narrow perforations and had made it possible to explain the reasons for the unsatisfactory progressive burning of Kisnemsky's powders.

CHAPTER 4 - THE USE OF INTEGRAL CURVES

1. THE PRESSURE IMPULSE OF POWDER GASES AS THE BURNING CHARACTERISTIC OF POWDER

Using the pressure curve obtained from a bomb test, the corresponding values of ψ can be found from the values of the successively increasing values of p; the test characteristic of progressivity - the function $\Gamma = \frac{1}{p}\frac{d\Psi}{dt}$ with relation to t and ψ can then be calculated, as well as the successively increasing values of $\int_0^t pdt$.

The obtained data is used for constructing Γ, ψ and $\int_0^t pdt, \Psi$ graphs. We shall introduce the designation $I = \int_0^t pdt$.

The value of the Γ function as an analytic function of the powder burning process was discussed in detail earlier: its form depends on the dimensions and shape of the powder, on the characteristics and burning conditions of the powder, and takes into consideration the ignition characteristics and the heterogeneity of the powder composition - both chemical (flegmatization) and physical (porosity).

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The integral curve I, ψ is likewise a characteristic of the burning of powder, whose form changes depending upon the above-mentioned factors.

Actually, the tangent of the angle formed by a line tangent to curve I, ψ is the inverse of Γ (fig. 73):

$$\tan \beta = \frac{dI}{d\psi} = \frac{pdt}{d\psi} = \frac{1}{\Gamma}$$

or

$$\Gamma = \text{cotan}\ \beta.$$

Inasmuch as the value of Γ increases when the burning of the powder is progressive, angle β will become smaller and the concave side of the $\int_0^t pdt, \psi$ curve will be directed towards the ψ-axis; when burning is regressive, the convex side of the I, ψ curve will be directed towards the ψ-axis. Therefore, the I, ψ curve obtained from the bomb test pressure curve can also serve as a characteristic of progressivity under actual conditions of burning.

We are presenting below schematic diagrams of Γ, ψ and I, ψ curves obtained with a weak and strong igniter (figs. 74 and 75). The slower the full ignition, the longer will be the portion of the curve corresponding to gradual pressure increase, and the more rapidly will the area under the I curve increase at small variations of ψ. The inflexion of the I, ψ curve at point a corresponds to the apex of "ballooning" on the Γ, ψ curve (point a').

The nature of ignition considerably affects the form of the initial portions of the I, ψ curve (up to $\psi = 0.15$–0.20).

Therefore, in order to obtain results corresponding to actual firing conditions, the igniter pressure p_B for use in bomb tests must be the same as that used in firing of a gun with the same powder.

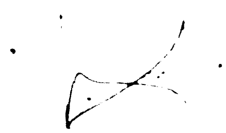

Fig. 73 - Relation Between I, ψ and Γ, ψ Curves.

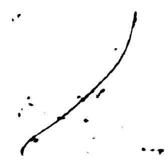

Fig. 74 - I, ψ and Γ, ψ Curves Obtained with a Weak Igniter.

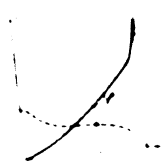

Fig. 75 - I, ψ and Γ, ψ Curves Obtained with a Strong Igniter.

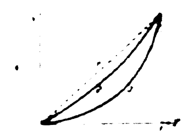

Fig. 76 - Relative Pressure Impulse I/I_K as a Function of ψ.

It is known from pyrostatics that $I_K = \frac{e_1}{u_1}$ and $I = \frac{e}{u_1} = \frac{e_1}{u_1}\frac{e}{e_1} = I_K z$; hence I is proportional to z, and the I, ψ curve is analogous to the z, ψ curve. The z, ψ curves were presented in the section of general pyrostatics; hence the theoretical I, ψ or $\frac{e_1}{u_1} z$ curves will have the form (when the coordinate axes change places) shown in fig. 76

Curve 1 in this figure corresponds to the change of pressure impulse of tubular powder, 2 - of strip powder, 3 - of a cube at the same value of $I_K = \frac{e_1}{u_1}$.

The thicker the powder at a given burning rate u_1, the higher will be the integral curve I, ψ on the diagram; the higher the burning rate at a given thickness, the lower will be the location of the I, ψ curve.

2. THE USE OF $\int_0^t p\,dt$ FOR DETERMINING THE BURNING RATE u_1.

The burning rate at $p = 1$, i.e., u_1, is determined by formula

$$u_1 = \frac{e_1}{\int_0^{t_K} p\,dt} = \frac{e_1}{\int_0^{\psi=1} p\,dt},$$

where e_1 is one half the thickness of the burning web, $\int_0^1 p\,dt = I_K$ is the complete integral determined from bomb tests.

Fig. 77 - The Application of $\int p\,dt$ for Determining u_1 for Regressive Powders.

This formula would apply if ignition were instantaneous and the thickness of the powder were uniform throughout and equal to an

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average value. Actually of course the powder thickness in a charge is not uniform. In addition to elements of an average thickness, there are present also thinner and thicker elements, and the complete integral $\int_0^1 pdt$ determined in bomb tests corresponds to the burning of an element of maximum thickness, which thickness is usually unknown. Actual powder measurements permit determining only the approximate average thickness of the webs of the grains composing the charge. Hence, in order to determine u_1, the value of $\int pdt$ used in the denominator must correspond to the mean powder thickness $e_{1\ cp}$ (e_1 average).

In order to determine $\int pdt = I_1$ corresponding to the average thickness $e_{1\ cp}$, a diagram must be constructed of the variation of I as a function of ψ. For strip and tubular powders the $\int pdt$, ψ curve usually undergoes a sharp deflection upwards after $\psi \approx 0.90$; this corresponds to the end burning process of the thicker elements. Hence, in order to determine $I_{1\ cp}$ corresponding to the burning of the mean thickness, the second half of the I, ψ curve must be extended or prolonged using an accurate French curve, when ψ changes within the limits of 0.5-0.9, until this extension along the basic direction of the curve intercepts the ordinate at $\psi = 1$ (fig. 77).

The ordinate thus obtained will be smaller than the full integral $\int_0^{\psi=1} pdt = I_K$, and will correspond to the average thickness $e_{1\ cp}$.

Then

$$u_1 = \frac{e_{1\ cp}}{I_{1\ cp}} .$$

Inasmuch as in the case of weak igniters a considerable variation will be obtained in the individual curves during the ignition process

(up to $\psi \approx 0.10$), then, in order to obtain a more reliable result as regards the basic region of burning conforming to the law, the following method is suggested for determining u_1.

Using the geometric law as the basis, with the shape of the powder and dimensions ratio known, the $\varkappa$, λ characteristics are computed for short tubes, strips or plates, or $\varkappa$, λ, μ for a slab or cube. Using formula

$$\psi = \varkappa z + \varkappa \lambda z^2$$

and assuming that $\psi = 0.10$ and 0.90, the corresponding values $z_{0.10}$ and $z_{0.90}$ are computed, and the thicknesses $e_{0.10} = e_1 z_{0.10}$ and $e_{0.90} = e_1 z_{0.9}$ and the burning rate in this region are determined by means of formula

$$u_1 = \frac{e_{0.9} - e_{0.1}}{\int_{0.10}^{0.90} p\,dt},$$

where $\int_{0.10}^{0.90} p\,dt$ is obtained directly from the I, ψ diagram (fig. 78).

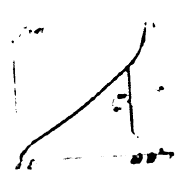

Fig. 78 - Determining u_1 at Weak Igniter Pressure.

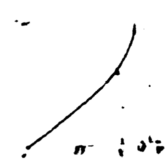

Fig. 79 - Determining u_1 for Powders of Progressive Shapes.

In the case of progressive powder grains, if the grain dimensions are known, it is necessary to calculate ψ_s at the instant of decomposition (using the assumption that the burning of the grain is progressive), obtain from the $\int pdt$, ψ diagram the value of $I_s = \int_0^{\psi_s} pdt$ (fig. 79), and, assuming that the mean web thickness already burned at the time is $e_{1\ cp}$, determine the value of u_1 by formula:

$$u_1 = \frac{e_{1\ cp}}{\int_0^{\psi_s} pdt} = \frac{e_{1\ cp}}{I_s} .$$

Inasmuch as in actual practice burning inside the perforations and at the outside surface (in powders with narrow perforations) proceeds at different rates, the determination of rate u_1 is conditional and depends on Δ, and can give only comparative results for a given loading density.

3. INTEGRAL CURVES AS CRITERIA FOR THE VERIFICATION OF THE BURNING RATE LAW

Using the burning rate law $u = u_1 p$ as a basis, it was shown above that the full pressure impulse $\int_0^{t_K} pdt = \frac{e_1}{u_1}$ does not depend on the loading density.

It can be easily shown that in the case of different burning rate laws $u = ap + b$ or $u = Ap^{\nu}$, where $\nu < 1$, the magnitude of $\int_0^{t_K} pdt$ must increase with the increase of the loading density.

Indeed, if $u = \frac{de}{dt} = ap + b$, then

$$de = apdt + bdt;$$

$$e_1 = \int_0^{t_K} pdt + bt_K;$$

$$\int_0^{t_K} pdt = \frac{e_1}{a} - \frac{b}{a} t_K;$$

but the full time of burning t_K decreases with increase of Δ (it depends on Δ). Hence, if the law is $u = ap + b$, $\int_0^{t_K} pdt$ will be the greater, the higher the loading density, i.e., it depends on Δ.

The same can be said for the law $u = Ap^{\nu}$:

$$u = \frac{de}{dt} = Ap^{\nu};$$

$$de = Ap^{\nu} dt \frac{p^{1-\nu}}{p^{1-\nu}} = \frac{A}{p^{1-\nu}} pdt.$$

When integrating, the average value of $p^{1-\nu}$ is taken out of the integral:

$$e = \frac{A}{(p^{1-\nu})_{cp}} \int_0^t pdt \text{ and } e_1 = \frac{A}{(p^{1-\nu})_{cp_K}} \int_0^{t_K} pdt,$$

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whence

$$\int_0^{t_K} p\,dt = \frac{e_1}{A}\,(p^{1-\nu})_{cp_K}.$$ (The subscript cp stands for "average.")

But as the loading density is increased, $(p^{1-\nu})_{cp_K}$ increases also, because the maximum and mean pressure become greater. Hence, for the law $u = Ap^{\nu}$, where $\nu < 1$, $\int_0^{t_K} p\,dt$ will be the greater, the higher the density of loading.

Thus the dependence or independence of the $\int_0^{t_K} p\,dt$ integral on the loading density constitutes the basic criterion in the validity of the given burning rate law.

If the full pressure impulse during the burning of powder does not depend on the loading density, this condition can prevail only in the case of the burning rate law $u = u_1 p$.

If, however, the impulse becomes greater as the loading density increases, this condition can apply only to the burning rate law $u = Ap^{\nu}$, where $\nu < 1$ or $u = ap + b$.

The above criterion can also be formulated as follows:

If upon increasing the loading density the integral curves $\int p\,dt$ as a function of ψ coincide with each other, the burning rate law $u = u_1 p$ (where p is of the first degree or $\nu = 1$) is valid.

If upon increasing the loading density the integral curves $\int p\,dt$ as a function of ψ proceed from the origin of the coordinates as a diverging cluster, the higher - the greater the value of Δ, then the law $u = Ap^{\nu}$, where $\nu < 1$, or the law $u = ap + b$ is valid.

4. THE USE OF INTEGRAL CURVES FOR VERIFYING THE BURNING RATE LAW

A. The Application of Diagrams to Powders of Simple Shapes.

In order to establish the burning rate law, we had conducted tests in 1924-25 with powders of simple shapes (strip, short tubes) in order to obtain the phenomenon in a purer form and eliminate the effect of narrow and long perforations [4].

We had chosen for our first tests strip powder "CП" ($2e_1 \approx 1$ mm), which has an exceptionally regular shape and is most uniform in thickness and in cross section. Plates were selected of the most uniform thicknesses and were tested in a bomb at a loading density of $\Delta = 0.159$, 0.209 and 0.259 using a strong igniter $p_B \approx 120$ kg/cm^2.

These plates were burned in simultaneous tests at a constant loading density of $\Delta = 0.209$ in order to obtain a picture of the scattering of the integral curves under identical test conditions. The results of both test series are presented in the diagrams of fig. 80a and b.

The I, ψ curves at different values of Δ lie just as closely to one another as do the curves at the same value of Δ; the difference between the values of $\int pdt$ at different Δ lies within the allowable test limits. No divergence of the integral curves (cluster) is observed.

Therefore, it may be considered proved that for strip powder of properly chosen thickness the value of $\int pdt$ for the given value of ψ does not depend on the loading density Δ, this condition being true only for the burning rate law $u = u_1 p$. Hence it may be considered

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proved that for pyroxylin strip powder the burning rate is proportional to pressure to the first power.

Thereafter, the following was established on the basis of many tests with powders of the most diversified compositions and with webs of different thicknesses.

1) Powders in the form of simple regressive shapes - strip, rod, short tube with a relatively wide perforation - burn in such a manner that at Δ varying from 0.12 to 0.25, $\int_0^{\psi} p\,dt$ does not depend on the loading density and the integral I, ψ curves proceed in the form of a coinciding cluster (figs. 81 and 82).

This coincidence is most complete for powders of uniform thickness and when using a strong igniter (p_B = 100-150 kg/cm^2), which insures almost instantaneous ignition in a bomb.

Tubular powders containing a solid solvent (trotyl + pyroxylin) produce Γ, ψ curves which are free of "ballooning" and integral I, ψ curves in the form of straight lines from $\psi = 0$ to $\psi \approx 0.90$-0.95 (see fig. 81).

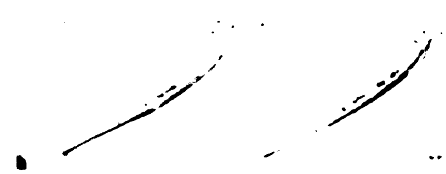

Fig. 80 - Integral Curves.

a) at different values of Δ ; b) at the same value of Δ ; 1) different values of Δ ; 2) same value of

Tubular powders with volatile solvents (pyroxylin and poorly volatile nitroglycerin) always produce "ballooning" on the $\bar{\ }, \psi$ curves, which gradually disappears in the first third of the burning process. However, the mutual disposition of the integral I, ψ curves does not change - they proceed in the form of a coinciding cluster and have a certain amount of curvature in the first third of the process (fig. 82).

Thus, also these tests with pyroxylin powders and powders with a solid solvent in the form of short tubes had shown that when they are burned in a bomb at loading densities of 0.15-0.25, the burning rate law $u = u_1 p$ is valid.

Similar data was obtained with large plates prepared by cutting up powder blocks with a solid solvent employed in certain types of rocket shells.

2) The same powders of simple shapes without narrow perforations burn at low loading densities ($\Delta \leq 0.10$) in a manner where the full pressure impulse $\int_0^{t_K} p\,dt = I_K$ decreases with the decrease of Δ; this behavior in the case of the $u = u_1 p$ law corresponds to the increase of the burning rate u_1 as Δ decreases.

Figures 83a and 83b show the nature of the variation of $\int_{\psi=0.05}^{\psi=0.75} p\,dt$ for "CП" powder ($2e_1 = 1$ mm) and the corresponding variation of the value of u_1 from 0.120 mm/sec : kg/cm^2 at $\Delta = 0.02$ to 0.077 mm/sec : kg/cm^2 at $\Delta = 0.12$. At $\Delta > 0.12$ I_K and u_1 retain their values.

The same condition, but with a more drastic change of I_K when Δ is decreased, is observed in A.I. Kokhanov's tests with powders 2.4 mm thick. I_K = const within Δ = 0.12-0.22, at Δ = 0.02 the value of I_K decreases almost 4-fold (fig. 84).

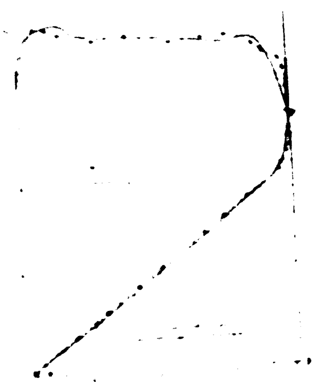

Fig. 81 - $\int pdt, \psi$ and Γ, ψ Curves for Tubular Powder with Solid Solvent.

a) Tubular powder with solid solvent.

Such a decrease of $\int pdt$ at low loading densities is observed only with powder thicknesses exceeding 0.7-1.0 mm.

3) In the case of thin pyroxylin powders in the form of small plates (hunting rifle powders "GLUKHAR," "VOLK" and others), the value of I_K remains practically unchanged when the loading density is varied from 0.157 to 0.02.

Therefore, in the case of pyroxylin powders of simple shapes, the variation of the $\int_0^{t_K}$ pdt integral with change of Δ, or its constancy, depends not on the nature of the powder mass, but, rather, on the burning characteristic of the powder.

Thick powders at high values of Δ and thin ones at both high and low values of Δ give a constant $\int_0^{t_K}$ pdt. In the case of thick powders at small values of Δ (< 0.10), $\int_0^{t_K}$ pdt decreases with decrease of Δ, which corresponds to an increase of the burning rate u_1 when the $u = u_1 p$ law applies.

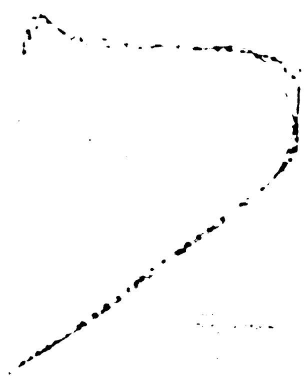

Fig. 82 – $\int$pdt,ψ and Γ,ψ Curves for Tubular Powder with Volatile Solvent.

[redacted]) Tubular powder with volatile solvent.

How can the above results be explained?

It is most reasonable and most simple to make the assumption [5] that in the case of slow burning thick powders at low pressures, the burning layers, notwithstanding their poor heat conductivity, will become heated under the influence of the surrounding gases and high temperature. Due to this increase of temperature at the outer powder layer, the burning reaction, similarly to any other chemical reaction, proceeds the faster, the lower the loading density and gas pressure; the lower the rate (u) of displacement of the burning layer towards the center of the grain, the deeper will be the penetration of heat through the layer and the higher will be the temperature of the latter.

Fig. 83a - The Dependence of $\int p\,dt$ on Δ.

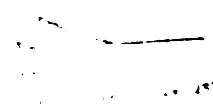

Fig. 83b - The Dependence of u_1 on Δ.

1) u m/sec.

This explanation is qualitatively confirmed by the modern "heat" theory of powder burning developed by Prof. Ya.B. Zeldovich.

A mathematical approach to this phenomenon will show that the divergence of the integral curves I,ψ at different values of Δ can be formally expressed by the burning rate law $u = Ap^{\nu}$, where $\nu < 1$.

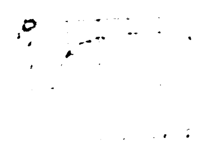

Fig. 84 - $\int p\,dt$ as a Function of Δ, According to Tests Conducted by Kokhanov.

Our tests with plates with a solid solvent have shown that for $\Delta > 0.10$ at a pressure of $p > 1000$ kg/cm^2 it may be assumed that for

$$u = u_1 p.$$

$\Delta < 0.10$ and pressure up to 800-1000 kg/cm^2 the nature of the variation of $\int p\,dt, \psi$ is such that the following law will apply

$$u = Ap^{\nu},$$

where $A = 0.240$ and $\nu = 0.83$.

Tests conducted by Prof. Yu.A. Pobedonostsev at very low pressures give a relation of the form $u = ap + b$:

$$u = 0.063p + 3,$$

where u is expressed in mm/sec and p in kg/cm^2.

Prof. Ya.M. Shapiro gives the relation $u = Ap^{\nu}$ for the same test data:

$$u = 0.37 \cdot p^{0.7}.$$

The values of u are practically the same when computed by means of these formulas for $p \doteq 25$-300 kg/cm^2.

Therefore, for simple shaped pyroxylin powders with a solid solvent, the following burning rate law holds true at $\Delta > 0.10$ and pressures above 800 kg/cm^2: $u = u_1 p$.

For these same powders at pressures < 800 kg/cm^2, the appropriate burning rate law is $u = Ap^{\nu}$, where $\nu < 1$.

When the pressure p varies from 300 to 800 kg/cm^2, the value of ν itself apparently changes also and approaches unity.

Thus the burning rate law is not the same for different conditions of burning; its form changes with change of pressure.

B. Applying the Diagrams to Powders with Narrow Perforations

In the case of progressive pyroxylin powders with many narrow perforations the integral curves I, ψ proceed in the form of a diverging cluster because of the nonuniform conditions of burning in the narrow perforations and at the outside surface, the disposition of the curves being the higher the greater the value of Δ. Starting with $\psi \approx 0.60$-0.65, the I, ψ curves become practically parallel (figs. 85 and 86).

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Fig. 85 – $\int p\,dt, \psi$ and Γ, ψ Curves for 7/7 Powder.

On the basis of the above criterion for the burning rate law, the divergence of the integral curves cluster leads to the following law:

$$u = Ap^{\nu},$$

where $\nu < 1$.

Tests involving a large number of powder specimens (over 100 specimens) consisting of powders with 7 perforations and Walsh's grades 7/7, 9/7, 12/7 and 15/7 give the value of $\nu = 0.83 \approx 5/6$ for our domestic pyroxylin powders with 7 perforations.

Once again we arrive at an apparent contradiction: for pyroxylin powders of plain shapes (plate and short tube), the burning rate law at $\Delta > 0.12$ is expressed by the formula

$$u = u_1 p.$$

For the same pyroxylin powders with narrow perforations at the same values of $\Delta > 0.12$ the burning rate law is expressed by the formula

$$u = Ap^{\nu},$$

because the integral I, ψ curves proceed in the form of a diverging cluster and are disposed on the diagram the higher, the higher is the loading density.

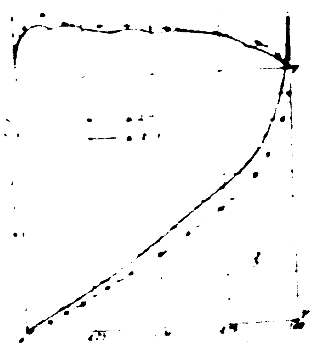

Fig. 86 - $\int pdt$, ψ and ¯, ψ Curves for 9/7 Powder.

This apparent contradiction can be easily explained on the

basis of the theory of nonuniform burning. It was shown above that location of the Γ, ψ curves is the lower, the higher the loading density, but Γ is the cotangent of the angle made by the I, ψ curves with the ψ - axis; hence, as Γ decreases with the increase of Δ, the slope angle of the integral curves increases and they continue to ascend higher and higher in the form of a diverging cluster.

Thus, in the case of the burning rate law $u = u_1 p$ corresponding to the nature of the powder and the conditions of loading ($\Delta > 0.12$), the integral curves nevertheless proceed as a diverging cluster because of the nonuniform burning of perforated powders at different values of Δ. Again, formally this divergence of the integral I, ψ curves with change of Δ can be expressed by the burning rate law

$$u = Ap^{\nu},$$

where $\nu < 1$.

The burning rate law for colloidal powders is expressed by the formula

$$u = u_1 p.$$

For powders with a solid solvent u_1 is a constant; for ordinary pyroxylin powders and also for nitroglycerin ones u_1 is a variable in the first third of the burning process; it depends on the nature of the powder and the conditions of burning.

Notwithstanding the fact that u_1 is a variable, the integral $\int^t pdt, \psi$ curves have the form of a converging cluster at different

Deviation from this law in the form of diverging $\int pdt, \psi$ curves for powders of simple shapes at small values of Δ is explained by the change of the burning rate u_1 due to the heating of powder layers when burning is gradual.

An analogous divergence of the integral curves for progressive powders at large values of Δ and the apparent deviation from the $u = u_1 p$ law is due to the nonuniform pressure distribution in the perforations and at the outside surface of the powder which depends on Δ.

This apparent deviation from the $u = u_1 p$ law can be expressed in a purely formal manner by means of formula $u = A_1 p^{\nu}$, but the law governing the displacement of the burning surface towards the center of the grain for each element of the powder surface remains the same: $u = u_1 p$, where p may be different at various elements of the surface, and u_1 may vary from layer to layer and will depend on the temperature of the layer near the burning surface. For powders with 7 perforations $\nu \approx 0.80$-0.83, and the A coefficient depends on the nature of the powder (contains nitrogen and volatiles).

Conclusions

The pyrostatic relations and the method of determining the various characteristics, as outlined above, make it possible to obtain a full analysis of the ballistic characteristics of powder and of the true law of combustion from tests in a manometric bomb.

The ballistic characteristics - energy f and covolume α - are determined from bomb tests at two or three loading densities by performing 3 to 5 tests at each value of Δ.

A correction for heat transfer is introduced into the test data (p_{m1}, p_{m2}, f, α) and the corrected values of f_0 and α_0 are determined.

The burning rate, u_1, at a pressure $p = 1$ is determined from the analysis of the integral curve $\int_0^t pdt, \psi$:

$$u_1 = \frac{e_1\, cp}{I_1} .$$

The true powder burning law is characterized by the test curve of the intensity of gas formation $\Gamma = \frac{1}{p}\frac{d\psi}{dt}$ as a function of ψ and t and by the curve of pressure impulse variation $\int_0^t pdt$ as a function of the burned portion of the charge ψ.

The burning rate law is determined from the convergence or divergence of the cluster of the integral I, ψ curves.

The Γ, ψ curve acts as the analyzer of the processes occurring during burning of powder and permits the evaluation of the various factors involved which could not be disclosed by any other methods (the process of gradual ignition, changes in the burning rate, effects of flegmatization, etc.).

In order to evaluate the burning of powder in the bore of a barrel when the gun is fired, tests must be conducted in a bomb at <u>different</u> but constant loading densities, and a determination made of the effect of loading density on the change in the progressivity of burning. A conclusion can be reached regarding the burning of powder in the gun's bore at a variable volume from the comparison and the analysis of the obtained data.

This entire method of investigation can be called the method of ballistic analysis of powders.

Inasmuch as this method permits determining the changes in the powder composition and in its dimensions under test conditions, it may be found very useful at powder manufacturing plants, particularly in the development of test specimens; when performing bomb tests and comparing the results with regular standard powder specimens it would permit establishing the deviation of the test specimen from standard samples and predicting its actual behavior in firing.

The following formula will serve as an example of the direct application to firing practice of the results of ballistic analysis obtained under laboratory conditions. It permits determining the relative weight of a test specimen from comparative bomb tests of two powder specimens - a standard and test specimen.

Designating the characteristics:

of the standard specimen - ω', f', I'_K; and

of the test specimen - ω'', f'', I''_K:

$$\frac{\omega''}{\omega'} = \frac{\frac{f'}{f''}\,\frac{I'_K}{I''_K}}{1 + \frac{\Delta'}{\delta}\left(\frac{f'}{f''}\,\frac{I''_K}{I'_K} - 1\right)},$$

where Δ' is the loading density of the standard specimen in a gun.

In order to avoid the errors usually obtained in taking the complete integral I_K until the end of burning, it is preferable to take the values of $\int_{0.05}^{0.75} p\,dt$ for I' and I''; by disregarding the

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initial section up to $\psi = 0.05$, the variation in ignition obtained with the use of relatively weak igniters ($p_B \sim 50$ kg/cm^2) will be eliminated.

The above formula permits determining without firing the approximate weight of a test powder specimen developing the same maximum gas pressure and muzzle velocity as a standard specimen.

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SECTION IV - THE PHYSICAL CONCEPTS OF PYRODYNAMICS

CHAPTER 1 - THE PHENOMENON OF A SHOT AND ITS BASIC RELATIONS

Pyrodynamics is the study of the phenomena occurring in the bore of a gun when the latter is fired, and a means for establishing the relation existing between the loading conditions and the various physical and chemical processes and mechanical phenomena occurring thereby.

The mutual relationship and interdependence of the various elements and factors involved are clearly manifested in the phenomenon of a shot. For example, the movement of a shell depends on gas pressure, whereas the pressure itself depends on both the burning of powder and the initial air space back of the shell, the latter in turn dependi[ng] on the speed of the shell.

The phenomenon of a shot may be considered to consist of the following periods.

1. PRELIMINARY PERIOD

The action of a negligible external impulse - such as the percussion of a firing pin or heating by an electric current - ig[nites] the composition of a percussion cap, and the resulting flame in turn ignites the igniter mixture in the primer cup (usually in the form of a tablet of black powder). The gases produced by the igniter and the incandescent particles of its combustion products enter the powder chamber through a special opening, and the resulting high temperature and pressure (p_B = 20-50 kg/cm^2) cause the ignition of the powder charge.

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When ignited, the powder burns at first in a constant volume until the gas pressure becomes sufficiently high to overcome the resistance of the rotating band and force it into the rifling grooves.

This period of a shot may be considered as being purely pyrostatic in character, because the powder burns in a constant volume (space).

Inasmuch as the rifling is provided with a forcing cone at its start, the rotating band enters the grooves gradually, and upon attaining the full depth of the thread its resistance undergoes a sudden drop and the shell proceeds through the bore with the band already fully notched.

The force Π_0 necessary for notching the band to the full depth of the grooves taken with relation to the cross-sectional unit area of the bore s, i.e., Π_0/s, is called the "pressure necessary to overcome the inertia of the projectile" and is designated as $p_0 = \Pi_0/s$ kg/cm^2.

Pressure p_0 may vary from 250 to 500 kg/cm^2 depending on the design of the rotating band and the rifling in the bore.

This period of a shot, when the powder gases commence to move the projectile and overcome the increasing resistance of the band until the latter is notched to the full depth of the grooves and traverses a specified distance, may be called the "forcing period" or the period of notching of the band. During this period the projectile traverses a distance equal to that measured from the initial position of the rear edge of the rotating band to the point at which the rifling grooves attain their full depth.

This period is considerably more complex than the pyrostatic period and is more difficult to analyze. Inasmuch as the initial chamber dimensions undergo a very small change during this displacement of the projectile, both periods are usually combined into a single preliminary period for the sake of simplicity, by assuming that the wedging of the band into the rifling occurs instantaneously and that the movement of the projectile commences as soon as the gas pressure equals the pressure p_0 (i.e., the pressure to overcome the inertia of the projectile).

This period is called the "preliminary period"; the pressure varies from 1 to p_B and then to p_0 and the change occurs during the period t_0.

In fig. 1 this period is depicted by the curve segment ab and the time t_0; in fig. 2, the element corresponding to it is the segment o-p_0 on the ordinate.

Methods are now available for the solution of the problem of interior ballistics which take into consideration the gradual breaking in of the rotating band into the rifling of the bore. These methods will be considered later.

2. FIRST PERIOD

The preliminary period is followed by the basic or first period of a shot, by the period of burning of powder and gas formation in a variable space, during which the powder gases impart a velocity to the projectile and thus perform the work at the expense of the energy confined in them and overcome a series of resistances.

This period, measured from the start of the projectile's movement until the end of powder burning in the bore, when the inflow of fresh gases stops, is the most complicated period: on the one hand the process of burning and the continuous inflow of gases increase the pressure inside the bore, whereas on the other hand the continuous acceleration of the projectile and the resulting increase of the initial "air space" tend to reduce this pressure.

At the start of the basic period, when the velocity of the projectile is still not very high, the quantity of gases increases at a greater rate than the volume of the initial air space, and the pressure increases until it reaches a maximum value p_m. However, the pressure increase and hence the increased acceleration of the projectile cause a rapid increase of the air space (volume) back of the projectile, so that notwithstanding the continued burning of the powder and the inflow of fresh gases, the pressure begins to drop until it attains a value p_K at the end of burning; at the same time the velocity of the projectile increases from zero to v_K. The powder gases perform most of their work during this basic period.

The maximum gas pressure is also developed during this period - this constitutes one of the fundamental ballistic characteristics of the powder and the gun in firing.

The maximum pressure serves as the basic data for establishing the wall thickness of the gun barrel and the projectile, whereas a knowledge of the associated maximum acceleration of the projectile is necessary for designing the inertia parts of time fuzes and firing devices.

3. SECOND PERIOD

The inflow of fresh gases stops at the end of burning of the powder, but inasmuch as the remaining gases still possess a very high reserve of energy, they continue to expand without an inflow of energy while the projectile completes its remaining path in the bore (up to the muzzle face), and thus continue to perform work and increase the velocity and kinetic energy of the projectile. This period constitutes a physical process in which a definite quantity of highly compressed and heated gases undergo expansion. Inasmuch as the velocity of the projectile is already high at the end of burning and continues to increase further, the projectile traverses the remaining portion of its path very rapidly. The ensuing heat losses through the walls of the gun barrel may be therefore disregarded and the entire period may be considered as "the period of adiabatic expansion of the gases." It is called the second period and terminates at the instant the base of the projectile passes the muzzle face of the gun barrel. The pressure drops from p_K to p_0, whereas the velocity of the projectile increases from v_K to $v_\text{Д}$ (see figs. 1 and 2).

Both periods occur during a very short period of time - varying from 0.001 to 0.060 second, depending on the length and caliber of the gun barrel.

If we introduce the designations:

s - cross-sectional area of bore;

p - gas pressure inside the bore at a given instant;

ℓ - distance traversed by projectile;

m - mass of projectile;

v - velocity of projectile,

then, according to the general theory of mechanics, to wit, that "the increment of work done by a force equals the increment of kinetic energy," we will have(*):

$$ps\,d\ell = d\left(\frac{mv^2}{2}\right).$$

Integrating, we get:

$$s\int_0^{\ell} p\,d\ell = \frac{mv^2}{2},$$

whence

$$v = \sqrt{\frac{2s}{m}\int_0^{\ell} p\,d\ell}.$$

The expression $\int_0^{\ell} p\,d\ell$ represents the area confined between the abscissa ℓ and the pressure curve; and inasmuch as it continuously increases, the velocity v will also undergo a continuous increase, whereby the nature of the increase of v will depend on the characteristic of the pressure curve. Inasmuch as the pressure, after reaching a maximum, undergoes a continuous drop, the area increase becomes smaller and smaller, and the velocity increment of the projectile gradually decreases at the end of its travel through the bore, i.e., the v, ℓ curve becomes flatter.

(*) ps - the product of pressure by the area equals the force applied to the entire area of the projectile's base.

According to the equation of the projectile's motion

$$ps = m\frac{dv}{dt},$$

the pressure curve drawn to a specific scale gives the curve of the projectile's accelerations:

$$\frac{dv}{dt} = \frac{s}{m} p,$$

where dv/dt is the tangent of the angle of inclination of the curve representing the velocity of the projectile as a function of time.

Inasmuch as the pressure continues to increase until it reaches its maximum, the velocity curve v, t proceeds with an increasing angle of inclination with its convex side directed downward. A point of inflexion is obtained at the point of maximum pressure, and thereafter, as the pressure decreases, the v, t curve continues with its convex side directed upwards:

$$v = \frac{s}{m}\int_{t_0}^{t} p dt.$$

4. THIRD PERIOD

After the projectile leaves the barrel, the gases flowing behind it with a high velocity continue to exert a pressure on the base of the projectile for a certain distance l_n, and thus continue to accelerate the projectile. This period of a shot is called the third period or the "after-effect period of the gases." The projectile acquires its maximum velocity v_{max} at the end of this third period.

following which its velocity begins to decrease under the action of air resistance.

In addition to this after-effect action on the projectile, the gases exert pressure also on the gun barrel; the latter action plays an important part in the design of the gun mount, and the fuzes. The duration of the after-effect of the gases on the gun mount is considerably longer than on the projectile.

In addition to the basic processes mentioned above, there is also a series of auxiliary processes affecting the phenomenon of a shot. Thus, for example, the movement of the projectile through the bore is accompanied by a non-uniform displacement of the gases in the initial air space and also by the recoil of the barrel. The projectile acquires a rotary or spinning motion in addition to the forward straight-line motion. Some of the gases escape through the clearance between the rotating band and the rifling of the bore, thus overtaking the projectile without first performing useful work; a portion of the heat energy is spent on heating of the barrel walls (losses due to heat transfer).

The following basic processes and relationships can therefore be established on the basis of the shot phenomenon discussed above.

1) The source of energy is derived from the expanding gases formed during the burning of the powder, and hence the laws of gas formation constitute the basic relationships expressing the process of burning of powder. The following laws apply to the science of pyrostatics:

a) gas formation governed by the burned thickness of the powd

$$\psi = \varkappa z(1 + \lambda z + \mu z^2),$$

or

$$\psi = \varkappa_1 z(1 + \lambda_1 z),$$

where

$$z = \frac{e}{e_1} \quad \text{and} \quad \psi = \frac{\Lambda c \Gamma}{\Lambda_1};$$

b) burning rate

$$u = u_1 p;$$

c) rate of gas formation

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 p = \frac{1}{I_K} \frac{S}{S_1} p.$$

The following relations are used in the case of the physical law of burning: $\psi = f(I)$ or $I = F(\psi)$, and also $d\psi/dt = \Gamma p$.

2) The gases formed during the burning of powder contain a large supply of heat energy; a portion of this energy is transformed into work when the gun is fired, which work is utilized mainly to impart kinetic energy to the projectile, the charge and the barrel and partly to overcome parasitic resistances. A portion of the heat is absorbed by the walls of the gun barrel. A major part of the

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energy is not used up however, and is ejected from the bore in the form of very hot gases after the projectile leaves the barrel.

Inasmuch as a shot is accompanied by transformation of energy, the first law of thermodynamics, i.e., the law of conservation of energy, gives the second basic relationship.

It is written thus:

$$Q = U + A\Sigma L,$$

where Q - quantity of heat supplied to the system from the exterior:

U - internal energy of powder gases;

ΣL - total amount of exterior work done by gases, including the work required to overcome parasitic resistances;

$\frac{1}{A} = E$ - mechanical equivalent, equal to 4270 kg · dm/cal.

This fundamental relationship is transformed in pyrodynamics into the so-called fundamental equation of pyrodynamics (see below).

3) The next fundamental relationship is the equation depicting the translation of the projectile.

It can be written two ways:

a) the first form of the equation of motion (Newton's law)

$$ps = m\frac{dv}{dt};$$

b) the second form of the equation of motion

$$sp = mv\frac{dv}{dt},$$

where s - cross-sectional area of the bore;

p - gas pressure;

$m = q/g$ - mass of projectile;

v - velocity of projectile;

ℓ - path traversed by projectile.

Other theorems and relationships of mechanics will be introduced later in the text in addition to the three fundamental relationships specified above.

4) Inasmuch as the charge-projectile-barrel system is brought into motion when a shot is fired by the action of the internal forces, i.e., by the pressure exerted by the powder gases, the following theorem of mechanics can be applied to it in the case of free recoil: "If a system is subjected to the action of internal forces, the displacement of its separate parts is such that the sum of the quantities of motion (the sum of moments) equals zero;"

$$mv + \mu U + MV = 0,$$

where M and V - mass and velocity of the recoiling parts;

μ and U - mass and velocity of charge.

This gives the relation between the velocity of the projectile and the velocity of the recoiling parts.

5) The equation of rotary motion of the projectile is obtained from the theorem: "The moment of a couple equals the moment of inertia multiplied by the angular acceleration":

$$rN = J\frac{d\Omega}{dt},$$

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where r - distance measured from the axis of the projectile to the center of the driving edge;

N - turning force;

J - moment of inertia of projectile relative to the axis of rotation;

Ω - angular velocity;

$\frac{d\Omega}{dt}$ - angular acceleration.

CHAPTER 2 - ENERGY EQUILIBRIUM WHEN A SHOT IS FIRED

When a shot is fired, a considerable portion of the energy developed by the powder gases is spent on performing work and is converted into kinetic energy of motion of the projectile. Furthermore, a part of the energy is expended on the performance of other work of lesser magnitude which must be taken into consideration, to obtain a full analysis of the equilibrium of energy when a shot is fired.

Say, a portion ψ of a charge ω is burned at the instant t, at which time a projectile whose weight is q has traversed a distance ℓ with a velocity v; the temperature of the burning powder is T_1. Inasmuch as the gases had performed work at the given instant and had cooled off, we shall designate their mean temperature by T, where $T < T_1$.

$Q\omega\psi$ cal of heat are evolved during the burning of $\omega\psi$ kg of powder, which quantity of heat is equivalent to work $\ni Q\omega\psi$, where $\ni$ = 4270 kg · dm/cal - the mechanical heat equivalent.

If we designate the mean heat capacity at constant volume for temperature T_1 by c_{w1}, $Q = \bar{c}_{w1}T_1$ and

$$E_1 = \ni\bar{c}_{w1}T_1\omega\psi \text{ kg} \cdot \text{dm}.$$

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This energy would be fully transformed into work, if the gas temperature were lowered to absolute zero.

Actually, this quantity of gas, having accomplished the work of moving the projectile and a series of other secondary items of work at the instant t, cools down only to a certain temperature $T < T_1$ and hence continues to retain a supply of unexpended energy equal to

$$E = \ni \bar{c}_w T \omega \psi \text{ kg} \cdot \text{dm},$$

where $\bar{c}_w$ is the mean heat capacity for temperature T.

Hence the energy expended at the instant t on the performance of external work will be expressed by the difference

$$E_1 - E = \ni \bar{c}_{w1} T_1 \omega \psi - \ni \bar{c}_w T \omega \psi.$$

Upon performing elementary transformation, we get:

$$E_1 - E = \ni \omega \psi \left[\left(A + b\frac{T_1}{2} \right) T_1 - \left(A + b\frac{T}{2} \right) T \right] = \ni \omega \psi \Big[A(T_1 - T) +$$

$$+ \frac{b}{2} (T_1^2 - T^2) \Big] = \ni \omega \psi (T_1 - T) \left(A + b\frac{T_1 + T}{2} \right) = \ni \Big|_{T_1}^{T} c_w (T_1 - T) \omega \psi,$$

where $\Big|_{T_1}^{T} c_w = A + b\frac{T_1 + T_2}{2}$ is the true heat capacity corresponding to the mean temperature in the interval between T_1 and T.

When a shot is fired, the gas temperature varies from T_1 to T_2, corresponding to the instant the projectile passes the face of the muzzle. The temperature interval is the one that is of practical value to interior ballistics.

Since the b coefficient is small, the change of $\Big|_{T_1}^{T} c_w$ is small and its average value can be considered to be constant for the entire process of the projectile's motion along the bore, i.e.,

$$\Big|_{T_1}^{T_k} c_w = A + b\frac{T_1 + T_k}{2};$$

we shall designate it by c_w' for short.

The graph in fig. 87 shows the variation of c_w with temperature.

$$Q_1 = \left(A + b\frac{T_1}{2}\right)T_1;$$

$$Q = \left(A + b\frac{T_1 + T}{2}\right)(T_1 - T);$$

the heat capacity c_w and the quantity of heat Q relate to a unit of gas by weight.

This graph shows that the heat quantity Q corresponding to a specific temperature difference is greater at high temperatures approaching T_1, than at low temperatures, because of the increased heat capacity.

According to the first law of thermodynamics, the energy balance when a shot is fired can be written as follows:

$$\text{Э} c_w'(T_1 - T)\omega\psi = \Sigma E_1,$$

where Э - mechanical heat equivalent (427 kg-m/kcal)

ΣE_1 - total amount of work done by the gases when a shot is fired, including the work necessary to overcome parasitic resistances.

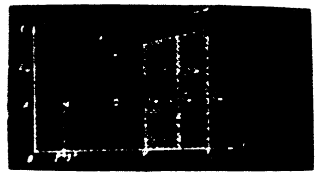

Fig. 87 - The Dependence of Heat Capacity of Gas on Temperature.

When a shot is fired, the energy confined in the gases is expended on the performance of the following forms of work:

1) Energy E_1 for the translation of the projectile, measured by the magnitude of the kinetic energy $mv^2/2$, is the basic form of energy expended during the movement of the projectile through the bore of the gun.

2) Energy E_2 is expended on the rotary motion of the projectile.

3) Energy E_3 is expended to overcome the frictional resistance between the rotating band of the projectile and the walls of the bore (bore + rifling grooves), and also for overcoming the friction between the walls of the projectile and the lands (of the rifling).

4) Energy E_4 is expended on the displacement of the gases of the charge itself and of the unburned portion of powder.

5) Energy E_5 is expended on the displacement of the recoiling parts and is measured by their kinetic energy $MV^2/2$.

6) Energy E_6 is used to force the rotating band of the projectile into the rifling grooves.

7) Energy E_7 is expended for heating the walls of the barrel, shell case and shell when the gun is fired - energy lost on heat transfer.

8) Energy E_8 is confined in the gases escaping through the clearances between the rotating band and the walls of the barrel.

9) Energy E_9 is expended on overcoming the air resistance and on the displacement of the air column present in the bore ahead of the projectile.

Of the above nine forms of expended energy, the first five must be accounted for directly, E_6 is accounted for directly or indirectly; E_7 is a form of heat energy which cannot be easily determined, and is accounted for indirectly for lack of a sufficiently satisfactory theory and test data to permit determining the heat lost to the walls of the barrel. The quantity of gas escaping through the clearances formed between the rotating band and the walls of the bore cannot be computed and has a random value; therefore, energy E_8 corresponding to it is not taken into consideration. This applies also to energy E_9 which is small in comparison with the other energy values.

The secondary work items will be discussed later. We shall note here (without offering proof) that E_2, E_3, E_4, E_5 are proportional to the main form of the work done by the powder gases, i.e., $E_1 = mv^2/2$. Hence, if each of these four forms of work is represented in the form

$$E_i = k_i\frac{mv^2}{2},$$

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where k_i - proportionality factors, determined by means of formulas which will be presented later, then the total amount of expended energy accounted for directly can be expressed in the form:

$$\sum_1^5 E_i = \sum_1^5 k_i \frac{mv^2}{2} = \frac{mv^2}{2}(1 + k_2 + k_3 + k_4 + k_5).$$

The sum of the coefficients in this expression is denoted by φ:

$$\varphi = 1 + k_2 + k_3 + k_4 + k_5.$$

The φ coefficient takes into account the secondary work items, and its numerical value for conventional type weapons varies between 1.05 and 1.20 depending on the loading conditions, and may exceed these values.

Thus, assuming that the expended values of energies E_6 and E_7 will be accounted for indirectly, the equation of energy balance during a shot will have the following form:

$$\vartheta \bar{c}_w T_1 \omega \psi - \vartheta \bar{c}_w T \omega \psi = \frac{\varphi m v^2}{2}.$$

This equation shows that the difference between two thermal conditions of the powder gases has become converted into a sum of external work items, where all the secondary work items are taken care of by the coefficient $\varphi > 1$. If this coefficient referred only to the mass m, rather than to the entire kinetic energy $mv^2/2$, we could assume that the work is performed by the gases for the purpose of imparting translation with the same velocity v to a heavier projectile of mass φm.

Thus, by introducing the coefficient φ, the actual motion of the projectile with the secondary work done by the gases taken into consideration, is replaced by a condition involving only the translation with the same velocity of a heavier projectile having a fictitious mass φm(*). The energy expended thereby remains the same. Coefficient φ is called the "fictitious mass coefficient."

The introduction of this fictitious magnitude helps to simplify without the introduction of an appreciable error calculations involving complex formulas.

It would be more correct to call φ the "secondary work coefficient," because this value depicts the relationship between the main and secondary work items (where the main work is taken to be equal to unity).

Derivation of the Fundamental Equation of Pyrodynamics

The energy balance equation depicts the relationship between the burned portion of the charge ψ, velocity of the projectile v, and the temperature of the gases formed at the given instant in the initial air space. Neither the length of travel ℓ of the projectile, nor the gas pressure p enters this equation. Nevertheless the basic problem of pyrodynamics is that of finding the relation between the distance ℓ traversed by the projectile, its velocity v and pressure

(*) The concept of a fictitious mass was introduced for the first time by Prof. N.A. Zabudsky, "DAVLENIYE POROKHOVYKH GAZOV V KANALE 3-DM PUSHKI" (Pressure Developed by Powder in the Bore of a 3-inch Cannon), 1894.

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p exerted by the gases on the projectile and the walls of the bore. Therefore the energy balance equation must be transformed in such a manner that it would depict the connection between the above-mentioned values p, v and l.

We know from thermodynamics that

$$c_p - c_w = AR = \frac{R}{\mathfrak{I}},$$

whence

$$\mathfrak{I} = \frac{R}{c_p - c_w}$$

$$\mathfrak{I} c_w = \frac{Rc_w}{c_p - c_w} = \frac{R}{\frac{c_p}{c_w} - 1} = \frac{R}{k - 1},$$

where R - gas constant;

$c_p/c_w = k$ - heat capacities ratio (adiabatic index).

We shall introduce for the sake of simplicity the denotation $k - 1 = \theta$; then:

$$\mathfrak{I} c_w = \frac{R}{\theta};$$

$$\theta = \frac{c_p - c_w}{c_w} = \frac{A_2 + bT - A_1 - bT}{A_1 + bT} = \frac{A_2 - A_1}{A_1 + bT}.$$

θ is a gradually decreasing function of temperature. When the gas temperature varies from T_1 to T_2, the mean value θ' will

correspond to the mean temperature $T_{cp} = \frac{T_1 + T_2}{2}$ and the corresponding value of c'_w:

$$\theta' = \frac{A_2 - A_1}{A_1 + b\frac{T_1 + T}{2}} = \frac{1}{A + BT_{cp}} .$$

Then

$$c'_w = \frac{R}{\theta'} .$$

Substituting this expression in the energy balance equation, we will get:

$$\frac{R}{\theta} T_1\omega\psi - \frac{R}{\theta} T\omega\psi = \frac{\varphi m v^2}{2} .$$

In order to exclude from this equation the variable T, we shall replace the expression $RT\omega\psi$, using as the basis the equation depicting the condition of the gas powders at the given instant, corresponding to the burned portion of the charge ψ:

$$pW = RT\omega\psi, \qquad (51)$$

where W is the free space in the initial air space (back of the projectile) at the given instant:

$$W = W_0 + s\ell - \alpha\omega\psi - \frac{\omega}{\delta}(1 - \psi) = W_\psi + s\ell;$$

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here W_ψ - free space in the powder chamber at the instant the portion of the charge ψ is burned in it;

s - cross-sectional area of the barrel;

$s\ell$ - added volume when the projectile had traversed the distance ℓ.

When using equation (51), it should be borne in mind that it applies to specific quantities of gas in a stationary condition, whereas we make the assumption that this equation is valid also for the conditions of a continuous gas formation and continuously changing gas pressure and the space occupied by them. Bearing in mind that

$$RT_1 = f,$$

we get:

$$\frac{f}{\theta'}\omega\psi - \frac{p(W_\psi + s\ell)}{\theta'} = \frac{\varphi m v^2}{2}. \qquad (52)$$

This is the fundamental equation of pyrodynamics depicting the relationship between ψ, p, v and ℓ. Actually, it is the equation of energy transformation by the equation of energy balance.

The left part of the equation depicts the change of internal energy $\omega\psi$ kg of the powder gases when their temperature is lowered from T_1^o to T^o, if the assumed mean values of heat capacity are c_w' and θ'. The right part of the equation represents the total external work done by the powder gases at the given instant due to the change of their thermal condition.

All the terms of the equation are expressed in units of work (kg · dm). This equation is called at times the "equivalence equation." The value of θ' is usually transposed to the right side, and the equation is solved for the second term:

$$p(W_\psi + s\ell) = f\omega\psi - \frac{\theta}{2}\varphi m v^2$$

or, replacing W_ψ by $s\ell_\psi$, we will have:

$$ps(\ell_\psi + \ell) = f\omega\psi - \frac{\theta}{2}\varphi m v^2. \tag{53}$$

This equation is also known as the Resál equation, first developed by the author in 1864. Hereafter we shall consider the value of θ in the balance equation and in the fundamental equation as a value corresponding to the mean value of $\Gamma_{cp} = \frac{T_1 + T_\Delta}{2}$, but without its prime index. The subject equation contains the following variables characterizing the elements of burning of powder and of the projectile's motion: the burned portion of the charge ψ, the gas pressure p, the length of projectile's travel ℓ, and its velocity v.

The length ℓ_ψ of the free space in the chamber at a given instant is a function of ψ. Actually, ψ is the independent variable, because the pressure imparting motion to the charge-projectile-barrel system is obtained only as a result of the burning of the powder and of gas formation. Nevertheless, all the variables are interconnected and affect one another. In order to establish the relation between four variables, additional equations must be had.

These additional equations are represented in one form or another by the aforementioned relations for the burning law and by the equatio[ns] of motion of the first and second forms.

If pressure p is determined from equation (52), we will have:

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$$p = \frac{f\omega\psi - \frac{\theta}{2}\varphi mc^2}{W_\psi + s\ell}.$$

In pyrostatics we had the following expression for depicting pressure at a given instant:

$$p = \frac{f\omega\psi}{W_\psi}.$$

Inasmuch as in the numerator of the first formula a value is substracted from $f\omega\psi$ proportional to the work $\frac{\varphi mv^2}{2}$, and in the denominator the volume of the bore corresponding to the distance ℓ traversed by the projectile is added to the free space of the chamber, it becomes entirely clear that at the "same loading conditions" the pressure in the barrel, while the projectile is in motion and while the work is performed by the gases, will be smaller than when the powder is burned at constant volume.

CHAPTER 3 - INVESTIGATION OF THE FUNDAMENTAL RELATIONS

1. THE BASIC ENERGY CHARACTERISTICS

The energy equilibrium equation is valid with regard to both the first and the second period when the charge is already fully burned, when $\psi = 1$ and when the gases expand adiabatically. In such a case only the two variables T and v enter into the equation.

$$\frac{f\omega}{\theta} - \frac{RT\omega}{\theta} = \frac{\varphi mv^2}{2}.$$

Inasmuch as $f = RT_1$,

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$$\frac{\varphi m v^2}{2} = \frac{f\omega}{\theta}\left(1 - \frac{T}{T_1}\right). \qquad (54)$$

The left side of the equation represents the total exterior work done by the powder gases when a shot is fired; it increases with the decrease of temperature T and would have attained a maximum value, were it possible to cool the powder gases at firing to T = 0, - a condition impossible in actual practice because it would correspond to an efficiency equal to unity.

Nevertheless, if we assume in equation (54) T = 0, we will get

$$\frac{\varphi m v_{\Pi p}^2}{2} = \frac{f\omega}{\theta}, \qquad (55)$$

i.e., the maximum amount of work performed by ω kg of powder gases if all the energy confined in them were utilized, i.e., if the gases were cooled to absolute zero.

$\frac{f\omega}{\theta}$ may be called the "full supply of energy" confined in ω kg of powder, and the velocity of the projectile $v_{\Pi p}$ ($= v_{limit}$) corresponding to the full utilization of energy - the limiting or maximum projectile velocity.

The full supply of energy of one kg of powder gas will be expressed by the formula

$$\Pi = \frac{f}{\theta}.$$

This value is called at times the powder "potential." Although

the above limiting expression for II has a theoretical meaning only, because in practice the gases cannot be cooled to absolute zero when a shot is fired, nevertheless it shows that the working capacity of powder gases can be increased either by increasing the force (energy) $f = \frac{p_a w_1}{273} T_1$, or by decreasing the value of $\theta = \frac{c_p}{c_w} - 1$.

The energy of the powder can be increased by increasing the specific volume of the powder gases w_1 (under normal conditions), or by elevating the burning temperature of the powder T_1.

As was shown above, θ depends on the composition and temperature of the gases: it decreases in value with increase of temperature and increases when the latter is decreased.

Hence a powder with a higher burning temperature will possess a greater supply of work not only because of energy f, but also because of the smaller value of θ.

Inasmuch as the gas temperature drops from T_1 to T_2 when a shot is fired (which corresponds to the projectile's passing through the muzzle face), the value of θ changes. However, this change is quite small and is usually considered to be a constant equal to the mean value of the given temperature interval. The value of θ can be found from the following formula:

$$\theta = \frac{c_p - c_w}{c_w} = \frac{A_1 - A_2}{A_2 + B_1 T} = \frac{1}{A + BT},$$

where

$$A = \frac{A_2}{A_1 - A_2} \quad \text{and } B = \frac{B_1}{A_1 - A_2}.$$

The average value of $\bar{\theta}$ can be found by means of the following formula:

$$\bar{\theta} = \frac{1}{(T - T_1)} \int_{T_1}^{T} \frac{dT}{A + BT} = \frac{A_1 - A_2}{T - T_1} \int_{T_1}^{T} \frac{dT}{A_2 + B_1 T} =$$

$$= \frac{2.303(A_1 - A_2) \log \dfrac{A_2 + B_1 T_1}{A_2 + B_1 T}}{B_1 (T_1 - T)}.$$

The variation of $\bar{\theta}$ for pyroxylin powder with temperature is given in table 20 [17].

Table 20

$\frac{T}{T_1}$	1	0.90	0.80	0.70	0.60	0.50	0.10
$T^{o}K$	2700	2430	2160	1890	1620	1350	270
$\bar{\theta}$	0.185	0.190	0.196	0.202	0.208	0.215	0.252

Inasmuch as $\frac{T_A}{T_1}$ is usually ≈ 0.70, θ approaches the value of 0.2. In most methods used for solving the fundamental problem of pyrodynamics the value of $\bar{\theta}$ is considered to be equal to 0.2. Theoretically it would have been correct to use different values of θ for the first and second periods: a smaller value for the first period while the gases have undergone little cooling, and a higher value for the second period at which time the gases had undergone a greater amount of cooling.

It should be noted that the values of the coefficients A_1, B, A_2 vary considerably with different authors, and this discrepancy

may affect the value of T_1 as well as the value of θ. Furthermore, the heat capacity values are expressed by more complex relationships than a linear one.

According to the latest data, these relationships deviate from a linear one in the low temperature range; in the range of 3000° to 1500-2000° the relationship C_w,t approaches a linear one even according to these data(*).

Solving equation (55) with respect to $v_{\Pi p}$, we shall find an expression for the so-called "maximum or limiting projectile velocity":

$$v_{\Pi p} = \sqrt{\frac{2}{\varphi}\frac{f}{\theta}\frac{w}{m}} = \sqrt{\frac{2g}{\varphi}\frac{f}{\theta}\frac{w}{q}}. \tag{56}$$

As was mentioned above, the maximum velocity of the projectile corresponds to the full utilization of the energy and an efficiency equal to unity. Although this value cannot be attained in practice, it enters as a factor into the formulas for the projectile velocity v of both the first and the second periods, and the true projectile velocity usually increases with increase of $v_{\Pi p}$. An analysis of formula (56) will show that the increase of $v_{\Pi p}$ depends on the supply of powder energy f/θ and on the relative weight of the charge w/q with respect to the weight of the projectile q: it decreases with the increase of φ. Although the concept of maximum velocity can be obtained by assuming that T = 0 in the energy equilibrium equation,

(*) A.M. Litvin, "TEKHNICHESKAYA TERMODINAMIKA" (Technical Thermodynamics), 1947.

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nevertheless, in order to apply the value of $v_{\Pi p}$ to actual practice, the value of θ must be taken as an average value in the temperature range of $T_1, \ldots, T_Д$ rather than in the $T_1, \ldots, 0$ range, because in computing the true projectile velocity which is proportional to $v_{\Pi p}$, the gas temperature in the first and second periods does not drop below $T_Д$. Thus the concept of maximum velocity is a conditional one, and it would be more appropriate to call it the "practical value of the maximum (or limiting) velocity."

Later on, when attempting to determine the dependence of φ on w/q, it will be shown that the maximum velocity $v_{\Pi p}$ tends towards a definite value, rather than towards infinity, even when the ratio of w/q is increased indefinitely.

The term "efficiency" signifies the ratio of the useful work done by the powder gases to the full supply of energy stored in a given powder charge.

The useful work performed by ω kg of gas is measured by the kinetic energy acquired by the projectile at the instant its base passes the muzzle face $\left(\frac{mv_Д^2}{2}\right.$, where $v_Д$ is the muzzle velocity of the projectile).

Denoting the efficiency by $r_Д$, we get:

$$r_Д = \frac{\frac{mv_Д^2}{2}}{\frac{f\omega}{\theta}} = \frac{\theta m v_Д^2}{2f\omega}.$$

In the case of ordinary weapons the value of $r_Д$ varies between 0.20 and 0.33.

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Certain authors incorporate into the efficiency expression the coefficient of the fictitious mass φ which takes into account the auxiliary work items. Thus

$$r'_{Д} = \frac{\varphi m v_{Д}^2/2}{f\omega/\theta} = \frac{v_{Д}^2}{v_{\Pi p}^2}.$$

Comparing it with formula (54), we will see that

$$r'_{Д} = \frac{T_1 - T}{T_1} = 1 - \frac{T}{T_1}.$$

But $\frac{T_1 - T}{T_1}$ is "the coefficient of the Carnot cycle performed by an ideal gas"(*), and would have represented the actual efficiency of the cycle in the absence of auxiliary or secondary work done by the gases and in the absence of parasitic resistances which the gases must overcome. Therefore, in order to correctly depict the efficiency of the powder in a weapon, φ should not be included in the efficiency expression.

The $r'_{Д}$ value is of great importance in the theory of ballistics developed in the USSR, because it takes into account the totality of the work done by the powder gases in the weapon.

In some textbooks the full amount of work is expressed by

(*) O.D. Khvolson, "KURS FIZIKI" (A Course in Physics), Vol. II, p. 451, 1919.

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the magnitude $II' = EQ$. But $EQ \neq f/\theta$ because Q is the quantity of heat determined experimentally in a calorimetric bomb when the gas of the burned powder is cooled from the burning temperature down to $t = 15^{\circ}C$ (or $288^{\circ}K$). The magnitude $\frac{f}{\theta}$ is the work the gases would be capable of doing if cooled from the burning temperature of the powder T_1 to 0° rather than to $288^{\circ}K$.

Hence the relation between EQ and $\frac{f}{\theta}$ will be expressed by the formula:

$$EQ = \frac{f}{\theta}\left(1 - \frac{288}{T_1}\right),$$

i.e., EQ is about 10% smaller than $\frac{f}{\theta}$, because $T_1 = 2700\text{-}2800^{\circ}K$.

This condition must be taken into account when determining the efficiency. If the value of the latter is given, it is of importance to know whether same is taken with respect to $\frac{f\omega}{\theta}$ (in which case it will be smaller) or with respect to FQ (in which case the efficiency will be greater).

"The coefficient expressing the utilization of a unit of charge η_{ω} by weight" is expressed by the muzzle energy of the projectile $\frac{mv_{\text{д}}^2}{2}$ per unit weight of charge ω:

$$\eta_{\omega} = \frac{mv_{\text{д}}^2}{2\omega} \quad kg \cdot dm/kg.$$

This value for specific gun systems approaches a constant, and depends mainly on the relative length of the gun, the powder thickness and the point at which it is burned (burning location).

For short-barreled, medium-caliber guns, $\eta_{\omega} = 1,200,000\text{-}1,400,000$ kg · dm/kg or 120-140 ton · m/kg; for small arms, $\eta_{\omega} = 100\text{-}110$ ton · m/kg.

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For fully charged howitzers η_ω = 150-160 ton · m/kg; η_ω decreased with the decrease of the charge. As the muzzle velocity of high-power artillery units is increased, the relative weight of the charge ω/q must increase also and with it the relative work necessary for displacing the charge itself (gases and powder); as a result, the relative useful work done in such guns becomes decreased and the value of η_ω drops to 90 ton · m/kg and lower.

The value of η_ω can be used for the approximate computation of the weight of charge ω necessary for imparting a given muzzle velocity $v_Д$ to a projectile of a given weight (mass) q:

$$\omega = \frac{mv^2_Д}{2} : \eta_\omega.$$

η_ω and $r_Д$ are linked by the simple relation:

$$r_Д = \eta_\omega : \frac{f}{\theta}.$$

In certain applications of interior ballistics of great importance is the ratio between the mean pressure p_{cp} at a given point of the projectile's travel and the maximum pressure p_m in the bore of the barrel ($\eta = \frac{p_{cp}}{p_m}$).

The average pressure during the period of time it takes the projectile to move from $\ell = 0$ to $\ell = \ell_Д$ is

$$\eta_Д = \frac{p_{cpД}}{p_m}.$$

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Fig. 88

Left: mean pressure when shot is fired.
Right: mean pressure as the characteristic of progressive burning of powder.

The mean pressure at a given point of the projectile's travel is that pressure which develops the same amount of work as a variable pressure, starting from p_0, passing through the maximum point and then decreasing in value.

Since the work done by the gases is depicted by the area $s\int_0^{\ell} p\,d\ell$ we can find p_{cp} from the condition

$$s p_{cp} \ell = s \int_0^{\ell} p\,d\ell.$$

In other words p_{cp} is the height of a rectangle whose area, when the base is ℓ, equals the area bounded by the pressure curve.

This is clarified by fig. 88,a; it also shows that as the length of bore travel ℓ is increased, p_{cp} decreases and has a minimum value when the projectile traverses the distance ℓ_g, i.e., at the instant the projectile leaves the gun barrel.

If after passing point p_m (fig. 88,b) one pressure curve 1 proceeds above the other curve 2, this condition indicates that the mean pressure and the ratio $\frac{p_{cp}}{p_m}$ for the first curve are likewise greater than for the second curve.

And inasmuch as curve 1 points at more progressive burning than curve 2, the coefficient $\eta_Д$ also serves to depict the progressivity of burning: the greater the value of $\eta_Д$, the more progressive is the burning of the powder in the bore of the barrel.

Inasmuch as the characteristic $\eta_Д$ of the progressivity of burning must be known under certain conditions of firing, and only p_m and $v_Д$ can be determined by test when s, $\ell_Д$, q and ω are known, whereas the change of pressure p with reference to ℓ is often unknown, p_{cp} and then $\eta_Д$ are computed on the basis of the following considerations.

It is known that the work done by gases along the path $\ell_Д$ equals

$$\frac{\varphi m v_K^2}{2} = s\int_0^{\ell_Д} p\,d\ell;$$

on the other hand,

$$s\int_0^{\ell_Д} p\,d\ell = s p_{cp Д}\ell_Д.$$

Therefore

$$s p_{cp.Д}\ell_Д = \frac{\varphi m v_Д^2}{2},$$

whence

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$$p_{cp.\text{д}} = \frac{\varphi m v_{\text{д}}^2}{2 s l_{\text{д}}}$$

and

$$\eta_{\text{д}} = \frac{p_{cp.\text{д}}}{p_m} = \frac{\varphi m v_{\text{д}}^2}{2 s l_{\text{д}} p_m} = \frac{\varphi m v_{\text{д}}^2}{2 W_{\text{д}} p_m} .$$

Inasmuch as the denominator in the expression for $\eta_{\text{д}}$ includes the swept volume of the bore $W_{\text{д}}$, $\eta_{\text{д}}$ is often called the "utilization coefficient of the swept volume of the bore."

Thus, in order to determine $\eta_{\text{д}}$ when a gun is fired, it is sufficient to know: the muzzle velocity of the projectile $v_{\text{д}}$, the maximum gas pressure p_m, the weight of the projectile, the cross-sectional area of the bore s, and the full length of travel of the projectile $l_{\text{д}}$ through the bore of the gun.

For cannons, the value of $\eta_{\text{д}}$ varies within the limits of 0.40 and 0.65.

The $\eta_{\text{д}}$ ratio can also be interpreted in a different manner. If we divide each term of the equation by $s l_{\text{д}} p_m$,

$$\eta_{\text{д}} = \frac{\varphi m v_{\text{д}}^2}{2 s l_{\text{д}} p_m} = \frac{s \int_0^{l_{\text{д}}} p\, dl}{s p_m l_{\text{д}}} = \frac{\int_0^{l_{\text{д}}} p\, dl}{p_m l_{\text{д}}} ,$$

where the right part represents the ratio of the area bounded by the true pressure curve and the x-axis along the path $l_{\text{д}}$ to the area of a rectangle of height p_m and base $l_{\text{д}}$. This ratio will thus show the

portion of the actual work done by the gases compared with the work done under ideal conditions if the pressure along the entire path $\ell_{\text{Д}}$ were equal to the maximum pressure p_m (fig. 89). For this reason $\eta_{\text{Д}}$ is often called the "coefficient of area closure on the indicator p, ℓ diagram."

The following characteristic can be introduced into the characteristic depicting the utilization of the entire barrel space, including the powder chamber space:

$$R_{\text{Д}} = \frac{\varphi m v_{\text{Д}}^2}{2s(\ell_0 + \ell_{\text{Д}})p_m} = \frac{\varphi m v_{\text{Д}}^2}{2W_{KH}p_m},$$

which may be called the "coefficient of ballistic utilization of the entire bore space."

It can be easily seen that

$$R_{\text{Д}} = \eta_{\text{Д}} \frac{W_{\text{Д}}}{W_{KH}} = \eta_{\text{Д}} \frac{\ell_{\text{Д}}}{\ell_0 + \ell_{\text{Д}}} < \eta_{\text{Д}}.$$

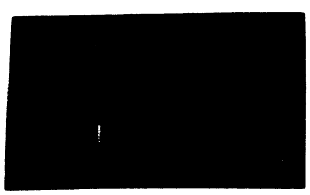

Fig. 89 - Utilization of the Swept Volume of the Bore.

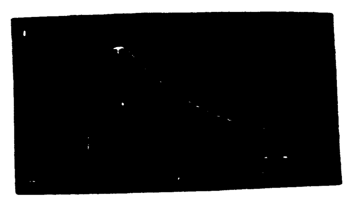

Fig. 90 - Utilization of the Entire Bore Space.

Graphically R_{Π} determines the ratio between the area bounded by the true pressure curve and the area of a rectangle of height p_m and base $\ell_0 + \ell_{\Pi}$ or $W_0 + W_{\Pi}$ (fig. 90).

2. THE DEPENDENCE OF PRESSURE CHANGE OF POWDER GASES IN THE GUN BARREL ON THE CONDITIONS OF LOADING

Using the pressure formula from the fundamental equation of pyrodynamics (53)

$$p = \frac{f\frac{\omega}{s}\psi - \frac{\theta}{2}\frac{\varphi m}{s}v^2}{l_\psi + l},$$

let us investigate the change of pressure with relation to time and the path traversed by the projectile. To do so, we shall find the derivatives

$$\frac{dp}{dt} \text{ and } \frac{dp}{dl} = \frac{dp\,dt}{dt\,dl} = \frac{1}{v}\frac{dp}{dt}.$$

Differentiating p with respect to t, we get

$$\frac{dp}{dt} = \frac{1}{(l_\psi + l)}\left[\frac{f\omega d\psi}{s\,dt} - \frac{\theta\varphi m}{s}v\frac{dv}{dt} - p\left(\frac{dl_\psi}{dt} + \frac{dl}{dt}\right)\right].$$

Bearing in mind that

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1 p = \frac{\chi}{I_K}\sigma p = \Gamma p;$$

$$\frac{\varphi m}{s}\frac{dv}{dt} = p;$$

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$$\frac{d\ell}{dt} = v;$$

$$\frac{d\ell_\psi}{dt} = \frac{d(\ell_\Delta - a\psi)}{dt} = -a\frac{d\psi}{dt} = -a\frac{\varkappa}{I_K}\sigma p = -a\Gamma p =$$

$$= -\frac{\omega}{s}\frac{\varkappa}{I_K}\frac{\sigma}{\delta_1}p;$$

$$\text{where } a = \frac{\omega}{s}\left(a - \frac{1}{\delta}\right) = \frac{\omega}{s}\frac{1}{\delta_1},$$

and substituting them in the dp/dt formula, we get

$$\frac{dp}{dt} = \frac{p}{(\ell_\psi + \ell)}\left[\frac{f\omega}{s}\frac{\varkappa}{I_K}\sigma - \theta v - \left(v - a\frac{\varkappa}{I_K}\sigma p\right)\right] =$$

$$= \frac{p}{(\ell_\psi + \ell)}\left[\frac{f\omega}{s}\frac{\varkappa}{e_1}\sigma u_1\left(1 + \frac{1}{\delta_1}\frac{p}{f}\right) - v(1+\theta)\right]. \tag{57}$$

This formula shows that the nature of pressure increase as a function of time depends on a large number of factors of varying influence.

At the start of motion when the rotating band is forced into the rifling grooves $p = p_0$, $\ell = 0$, $v = 0$, $\ell_\psi = \ell_{\psi_0}$, and formula (57) takes on the form:

$$\left(\frac{dp}{dt}\right)_0 = p_0\frac{f\omega}{s\ell_{\psi_0}}\frac{\varkappa\sigma_0}{I_K}\left(1 + \frac{1}{\delta_1}\frac{p_0}{f}\right) = p_0\frac{f\omega}{s\ell_{\psi_0}}\frac{\varkappa}{e_1}u_1\sigma_0\left(1 + \frac{1}{\delta_1}\frac{p_0}{f}\right) =$$

$$= p_0\frac{f\omega\Gamma_0}{s\ell_{\psi_0}}\left(1 + \frac{1}{\delta_1}\frac{p_0}{f}\right). \tag{58}$$

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Therefore the rate of pressure increase at the start of motion is proportional to the wedging pressure p_0, the powder energy f, the weight of the charge ω, the exposed area of the powder $\frac{S_1}{\Lambda_1} - \frac{\varkappa}{e_1}$, the rate of powder burning at $p = 1$, i.e., u_1, and inversely proportional to the free space of the powder chamber $s\ell_{\psi_0}$ at the instant wedging occurs. Inasmuch as the exposed area of the powder is inversely proportional to the thickness of the powder, $\left(\frac{dp}{dt}\right)_0$ is likewise inversely proportional to the powder thickness e_1. The first term in parentheses in formula (57) also depends on these magnitudes, which term expresses the intensity of energy developed by the powder gases.

If we replace $\frac{\varkappa}{e_1}u_1\sigma_0$ by $\frac{S_1}{\Lambda_1}u_1\sigma_0 = \Gamma_0$, we will obtain the additional condition where $\left(\frac{dp}{dt}\right)_0$ is proportional to Γ_0 for $\psi = \psi_0$; and since we had seen in our analysis of the physical law of burning that the outer layers of pyroxylin powders have an accelerated rate of burning (Γ develops ballooning), the pressure increase in this case will also be more intense than in the case of uniform burning u_1 assumed in the theoretical law.

Hence, all other conditions being equal, the pressure curve p, t in the case of the physical law of burning must proceed in the diagram above the corresponding p, t curve representing the theoretical law.

Formula (57) gives the tangent of the angle of inclination of the pressure curve as a function of time. At the start of motion the tangent of the angle has a specific limiting value depending on certain loading conditions (fig. 91,a); it becomes zero only when $p_0 = 0$. In this case the pressure curve as a function of time is tangent to the x-axis (fig. 91,b). This condition is not encountered in actual practice.

The nature of pressure increase as a function of path l is expressed by the following formula:

$$\frac{dp}{dl} = \frac{1}{v}\frac{dp}{dt} = \frac{p}{(l_\psi + l)}\left[\frac{f\omega}{s}\frac{\varkappa}{l_K}\frac{\sigma}{v}\left(1 + \frac{p}{f\delta_1}\right) - (1 + \theta)\right]. \qquad (59)$$

Fig. 91

Left: p, t curve at $p_0 > 0$; right: p, t curve at $p_0 = 0$.

At the start of motion $p = p_0$, $l = 0$, $v = 0$, $\sigma = \sigma_0$, $l_\psi = l_{\psi 0}$, and inasmuch as the first term in parenthesis is reduced to infinity,

$$\left(\frac{dp}{dl}\right)_0 = \infty .$$

Therefore, the ordinate is tangent to the p, l curve at the start of motion (fig. 92).

Fig. 92 - p, l curve; the p-axis is tangent to the p, l curve at the start of motion.

In order to obtain maximum pressure, $\frac{dp}{dt}$ or $\frac{dp}{d\ell}$ must be reduced to zero. Hence the condition at which p_m is obtained has the form:

$$f\,\frac{\omega}{s}\,\frac{\varkappa}{I_K}\,\sigma_m\left(1+\frac{1}{\delta_1}\,\frac{p_m}{f}\right)-v_m(1+\theta)=0,$$

where the m index indicates that the given magnitude corresponds to p_m or

$$f\,\frac{\omega}{s}\,\Gamma_m\left(1+\frac{1}{\delta_1}\,\frac{p_m}{f}\right)=(1+\theta)v_m,$$

where

$$\Gamma_m=\frac{\varkappa}{I_K}\,\sigma_m=\frac{S_1}{\Lambda_1}u_1\sigma_m.$$

If the requirements are such that the pressure must remain constant for a certain period of time after attaining a maximum value, a condition is obtained which must be satisfied by a change in surface area σ or by a change in the burning rate u_1:

$$f\omega\,\frac{\varkappa}{e_1}\,\sigma u_1\left(1+\frac{1}{\delta_1}\,\frac{p_m}{f}\right)=(1+\theta)vs.$$

This condition can be formulated as follows.

In order to maintain the maximum pressure constant for a certain portion of the projectile's path in the bore, it is necessary that the surface area σ of the powder or the burning rate u_1 change in proportion to the projectile velocity v, or that the energy imparted

by the powder gases at $p = l(f\omega\Gamma)$ be proportional to the rate of volume change in the bore when the projectile is in motion

$$\left(sv = \frac{sd\ell}{dt} = \frac{dW}{dt}\right).$$

The pressure decreases after passing point p_m, the expression in parenthesis becomes negative, $\frac{dp}{dt} < 0$.

For the end of burning at $\psi = 1$, we get

$$\left(\frac{dp}{dt}\right)_K = \frac{p_K}{\ell_1 + \ell_K}\left\{\frac{f\omega}{s}\frac{\varkappa}{I_K}\sigma_K\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_K}{f}\right] - (1 + \theta)v_K\right\}. \qquad (*)$$

Upon entering the second period the pressure equation takes on the form

$$p = \frac{f\omega}{s}\,\frac{1 - v^2/v^2_{np}}{\ell_1 + \ell}.$$

Differentiating, we get:

$$\frac{dp}{dt} = -(1 + \theta)\frac{v \cdot p}{\ell_1 + \ell}; \quad \frac{dp}{d\ell} = -(1 + \theta)\frac{p}{\ell_1 + \ell}$$

for the start of the second period

$$\left(\frac{dp}{dt}\right)_{(0)} = -(1 + \theta)\frac{v_K p_K}{\ell_1 + \ell_K}; \quad \left(\frac{dp}{d\ell}\right)_{(0)} = -(1 + \theta)\frac{p_K}{\ell_1 + \ell_K}.$$

Comparing this expression with (*), we note that at the instant of transition from the first period to the second, the derivatives dp/dt and $dp/d\ell$ undergo a drastic change (a jump), the pressure curve

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suffers a break, and the absolute value of the angle of inclination increases because of the disappearance of the first term in braces in expression(*).

Thereafter the angle of inclination of the p, t and p, ℓ curves becomes smaller, because p decreases and $\ell_1 + \ell$ increases; at the instant of the projectile's departure from the bore:

$$\left(\frac{dp}{dt}\right)_{\Pi} = -(1 + \theta)\frac{p_{\Pi} v_{\Pi}}{\ell_1 + \ell_{\Pi}}; \quad \left(\frac{dp}{d\ell}\right)_{\Pi} = -(1 + \theta)\frac{p_{\Pi}}{\ell_1 + \ell_{\Pi}}.$$

3. THE EFFECT OF DIMENSIONS AND SHAPE OF POWDER ON THE GAS PRESSURE AND PROJECTILE VELOCITY CURVES

An analysis of formulas (57) and (59) will show that the pressure increase with respect to time and as a function of the path traversed by the projectile in the bore mainly depends on the term in brackets $f\omega\Gamma = f\omega\frac{\varkappa}{e_1}u_1\sigma$ which, for a given powder energy f, depends on the product $\omega\Gamma$, Γ being the intensity of gas formation at p = 1.

By changing the shape and dimensions of the powder, the magnitude $\frac{\varkappa\sigma}{e_1} = \frac{S_1}{\Lambda_1}\sigma$, which we shall denote by Σ, can be varied at will within wide limits.

The dependence of the change of $\varkappa$ and σ on the change of the powder grain shape is known from pyrostatics. The change of the pressure curve p, ℓ and of the projectile velocity curve v, ℓ with respect to the following can be illustrated by means of an example:

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1) change of powder shape at the same powder thickness and the same charge ω, and

2) change of powder thickness at the same powder shape ($\varkappa$ = const, σ varies according to the same law) and at ω = const.

1. The effect of the grain shape when the thickness remains the same. We shall assume for the sake of simplicity that $e_1 = 1$, in which case $\Sigma = \varkappa\sigma$. At the start of burning at $z = 0$, $\sigma = 1$ and $\Sigma_0 = \varkappa$. The change of Σ corresponds to the change of σ. At the end of burning at $z = 1$

$$\Sigma_K = \varkappa\sigma_K = \varkappa(1 + 2\lambda + 3\mu).$$

By taking general formulas for five regressive powder shapes and using numerical data, we obtain table 21.

Table 21

	Shape of Powder	$\Sigma_0 = \varkappa$	σ_K	$\Sigma_K = \varkappa\sigma_K$	$\Sigma_0 = \varkappa$	σ_K	$\Sigma_K = \varkappa\sigma_K$
1	Tube	$1 + \beta$	$\frac{1 - \beta}{1 + \beta}$	$1 - \beta$	1.003	0.994	0.997
2	Strip	$1 + \alpha + \beta$	$\frac{(1 - \alpha)(1 - \beta)}{1 + \alpha + \beta}$	$(1 - \alpha)(1 - \beta)$	1.06	0.89	0.943
3	Plate	$1 + 2\beta$	$\frac{(1 - \beta)^2}{1 + 2\beta}$	$(1 - \beta)^2$	1.20	0.675	0.810
4	Slab	$2 + \beta$	0	0	~2.0	0	0
5	Cube	3	0	0	3	0	0

Upon plotting a graph of the change of Σ with respect to z, we will obtain the diagram shown in fig. 93.

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Fig. 93 - Change of Σ, z with Change of Powder Shape when ω and e_1 = const.

1) tube; 2) strip; 3) square plate; 4) rod; 5) cube.

The Σ, z diagram shows how the rate of gas formation changes when we cut up the same strip into square plates, then into strips and finally into cubes (see fig. 23).

The calculated action of these differently shaped powders (2, 4, 5) of the same thickness in the bore of a gun using the same charge ω by weight gives a diagram showing the changes of gas pressure as a function of the path traversed by the projectile in the bore (fig. 94).

These curves show that the strip generates a normal pressure p_m = 2380 kg/cm^2 and a muzzle velocity v_π = 590 m/sec, whereby the adiabatic curve of the second period attains the greatest height and the muzzle pressure is maximum.

Rod 4, whose exposed area is almost twice as great at the same powder thickness, generates almost double the pressure - p_m = 4600 kg/cm^2 and a considerably higher velocity v_π = 656 m/sec because of the large area of the p, ℓ curve depicting the work done by the gases. The adiabatic curve of the second period drops sharply at

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the end of burning and in descending intersects the curve of the first period of strip powder and lies below the adiabatic curve of the strip powder.

The point of maximum pressure moves towards the point of the start of motion, as does the point representing the end of burning. $\eta_K = \frac{\ell_K}{\ell_A} = 0.34$ for the strip, 0.18 for the rod, 0.128 for the cube. Inasmuch as the exposed area of the cube is three times as great, the cube generates a pressure of $p_m = 6200$ kg/cm^2 and a velocity $v_A = 681$ m/sec because of its still greater area $\int pd\ell$ than that of the rod (slab). The adiabatic curve of the second period lies still lower and gives the lowest muzzle pressure.

Thus, using the same powder thickness e_1 and the same charge by weight, we find that of the three powder shapes compared above the lowest pressure and the smallest velocity are produced by strip powder. The rod (slab) increases the pressure by almost 100%, whereas the velocity v_A increases only by 11%; the cube increases p_m almost 2.6 times and the velocity by 15.5%. In this case the regressive shape produces the maximum v_A, but at a pressure which is almost three times the normally allowable pressure. Hence, if the requirement, as is the case in actual practice, calls for the same pressure p_m with powders of different shapes, the area of the $\int pd\ell$ curves obtained with powders of greater regressivity will be smaller than the area obtained with strip powder, and the velocity v_A will be smaller. This represents the advantage of strip powder as the more regressive type.

For purposes of comparison, p, ℓ and v, ℓ curves are presented in the diagrams for the case (o), in which the powder is fully burned

in the chamber before the projectile starts moving. This is the case of "instantaneous powder burning" in which the dimensions of the powder are infinitely small. This would obtain in practice when using dry powder-like pyroxylin.

In such a case the pressure curve starts from the maximum point, which pressure (11,700 kg/cm^2) is calculated according to the Noble formula. Thereafter the p, ℓ curve is entirely adiabatic in character, and the curves corresponding to various powder shapes arrange themselves in a proper manner (according to the established law) with respect to same. The obtained velocity v_A was found to be equal to 690 m/sec - i.e., the greatest, but the pressure p_{m0} was almost 5 times as great as p_m when strippowder was used (p_{m2} = 2380 kg/cm^2).

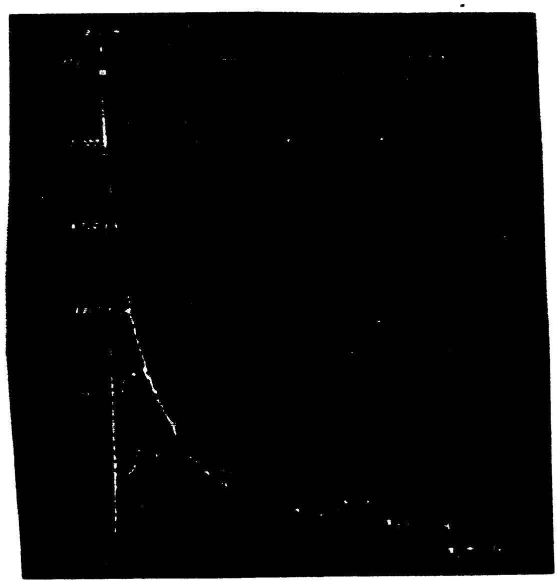

Fig. 94 - The Effect of the Grain Shape on the Pressure Curve Inside the Bore.

1) instantaneous burning (0); 2) cube (5); 3) slab (4); 4) strip (2); ordinate: g/cm^2.

2. The effect of web thickness on grains of the same shape. Grain shape - strip; thickness: $2e_1 = 1.5$, 2.0 and 2.5 mm.

$$\Sigma = \frac{\varkappa}{e_1}\sigma; \quad \Sigma_0 = \frac{\varkappa}{e_1} = \frac{1.06}{e_1};$$

$$\sigma_K = 0.889; \quad \Sigma_K = \frac{\varkappa}{e_1}\sigma_K = \frac{1.06 \cdot 0.889}{e_1} = \frac{0.943}{e_1}.$$

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Table 22 - Magnitude

Thickness of Strip $2e_1$	$\Sigma_0 = \frac{1.06}{e_1}$	$\Sigma_K = \frac{0.943}{e_1}$	p_m, kg/cm^2	v_A m/sec
1.5	1.414	1.256	3540	632
2.0	1.060	0.943	2040	575
2.5	0.848	0.744	1450	486

The (Σ, z) diagram in fig. 95 shows that thinner powder generates a greater quantity of gas in a unit of time than thicker powder. The gas supply during burning can be regulated by changing the thickness of the powder.

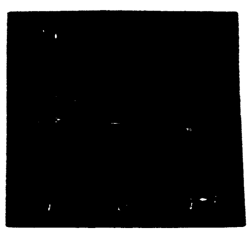

Fig. 95 - The Dependence of the Intensity of Gas Formation on the Powder Thickness.

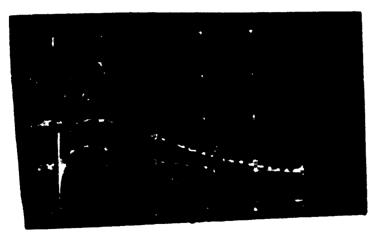

Fig. 96 - The Effect of Powder Thickness on p,ℓ Curves in a Gun.

The calculated results of the action produced by these same powders in a gun using the same charge can be seen in the diagram of fig. 96; it shows that thin powder 1 generates a p_m which is 68% greater than normal pressure, whereas thicker powder 3 generates a pressure p_m which is 31% lower than normal pressure 2.

The muzzle velocities v_A are 632, 575 and 488 m/sec, respectively, where in the last case the thick powder does not succeed in getting

fully burned inside the gun barrel, and hence the energy of a portion of the charge is not utilized at all.

The two examples cited above show the importance of the shape and dimensions of powder when firing a gun, and how the inflow of gases can be regulated and the desired rate of pressure change can be obtained by varying the thickness of the powder in combination with its shape.

If the requirement is such that p_m must not exceed a given value, the most regressive powder will be most suitable for the purpose, namely, tubular powders of the various shapes considered here, which approach the closest a powder with a constant burning area. It is possible however to obtain a progressive powder shape whose surface area increases with burning, and thus improve the efficiency of the weapon.

CHAPTER 4 - FORCES DEVELOPED IN A GUN WHILE THE PROJECTILE IS MOVING ACROSS THE RIFLING

1. THE RIFLED BORE. BASIC DESIGN DATA.

The bore of a gun barrel is rifled for the purpose of imparting a spinning motion to the projectile. The angle α formed by the grooves with the generatrix may be constant (rifling with a uniform twist) or variable, with the angle of twist increasing towards the muzzle (rifling with an increasing twist). The stability of the projectile's flight at a given velocity depends on the angle of twist α. The projectile's spinning motion is imparted by the pressure of the driving edges of the lands, which, in the case of a right-handed thread (clockwise rotation), is provided by the right-hand edge of the lands (fig. 97, a and b). These edges, upon encountering the rotating band of the

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projectile, develop a resistance N, which force is applied to the center of the protruding band and forces the projectile to rotate clockwise. A similar but a directly counteracting force N' is imparted by the projectile on the driving edge of the thread. Due to the elastic properties of the walls of the bore and the rotating band, radial forces Φ and frictional forces $\nu\Phi$ originate on the contacting surfaces (see fig. 98).

Fig. 97 - Rifling in a Gun Barrel

1) barrel; 2) projectile.

Fig. 98 - Forces Acting in the Rifling Grooves.

In addition to the twist angle α, the grooves are characterized by the land width a, the width b at the bottom of the groove, depth of grooves t_H and length (height) of driving band b_0. The force

N is distributed over the area b_0t_H : cos α, but inasmuch as at $\alpha = 8^o$, cos $\alpha = 0.99 \approx 1$, the area b_0t_H is used for computing the stress in the groove and the rotating band. In artillery pieces the value of t_H is usually taken as $t_H \approx (0.01-0.02)d$, where d is the caliber of the bore or the diameter between the lands; $b_0 \approx 0.15d$.

For small-caliber weapons $t_H = (0.02-0.04)d$.

When the projectile moves through the bore, the gases exert a pressure on the base of the projectile as well as the ridges of the rotating band formed when the latter is forced into the grooves. Therefore the cross-sectional area s of the bore is greater than $\pi d^2/4$ and is calculated approximately by the formula $s = (0.80-0.83)d^2$ or by means of the more exact formula

$$s = \frac{\pi}{4}\left(\frac{a}{a+b}d^2 + \frac{b}{a+b}d'^2\right) = \frac{\pi}{4}\left(\frac{ad^2 + bd'^2}{a+b}\right) =$$

$$= \frac{\pi}{4}d^2\left[\frac{a + b\left(\frac{d'}{d}\right)^2}{a+b}\right].$$

The latter is obtained if we subdivide the whole area into pairs of sectors of diameters d and d' resting respectively upon arcs a and b. Of the two sectors subtended by the given angle, the sector resting on the land of the rifling occupies the portion $\frac{a}{a+b}$, and the one resting on the bottom of the thread occupies the portion $\frac{b}{a+b}$.

If we equate this area s to the area of an equidimensional circ[le] the diameter of the latter d_1 will represent the true caliber of the bore. It may be assumed that the force N spinning the projectile ab[out]

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its axis has a lever-arm d_1. For artillery pieces $d' \approx 1.02$; $d_1 \approx 1.01d$;

$$d_1 = \sqrt{\frac{ad^2 + bd'^2}{a + b}}.$$

The required number of grooves n is usually determined from the formula

$$n = (3\text{-}3.5)d_{cm} \text{ or } n = 2d_{cm} + 8$$

rounded off to a multiple of four ($n = 4$ for small arms), in order to be able to cut the bore simultaneously with four cutters.

In addition to the angle of inclination α, the rifling is also characterized by the lead of the thread h, i.e., by the length of the generating line equivalent to a full turn of the thread (fig. 99):

$$h = \pi d \cot \alpha.$$

The $\frac{h}{d}$ ratio is called the rifling twist or the lead of the rifling in calibers:

$$\eta = \frac{h}{d} = \frac{\pi}{\tan \alpha}.$$

h/d is usually given in round numbers (20, 25, 30...60) and the angle of inclination α is determined from them:

$$\alpha = \arctan \frac{\pi d}{h}.$$

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Table 23

η	50	40	35	30	25	20
α	3°35'6"	4°30'	5°07'5"	5°58'7"	7°09'45"	8°56'

Rifling Equation. When the cylinder of the bore with an increasing rifling twist is developed on a plane, the rifling appears as a parabola (fig. 100), whose origin and angle of inclination $\alpha = 0$ lie below the actual thread on the extension of the thread curve.

The equation for this parabola is:

$$x^2 = ky \text{ or } y = \frac{x^2}{k},$$

$$\tan\alpha = \frac{dy}{dx} = \frac{2x}{k}; \quad \frac{d\tan\alpha}{dx} = \frac{2}{k} = \text{const},$$

i.e., the change of the angle of inclination versus the distance x remains constant.

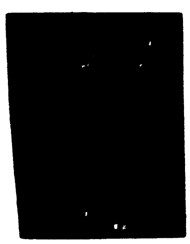

Fig. 99 - Uniform Twist Rifling Diagram

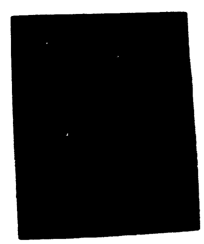

Fig. 100 - Increasing Twist Rifling Diagram

Since the angles of inclination α_1 and α_2 at the beginning and end of the rifled portion of the bore are known, the constant k can be determined. Indeed:

$$\tan \alpha_1 = \frac{2c}{k}; \qquad \tan \alpha_2 = \frac{2(c + L_{Hp})}{k}, \tag{60}$$

whence

$$k = \frac{2L_{Hp}}{\tan \alpha_2 - \tan \alpha_1}, \tag{61}$$

and hence

$$\frac{d \tan \alpha}{dx} = \frac{\tan \alpha_2 - \tan \alpha_1}{L_{Hp}} = \text{const}.$$

The last expression enters into the formula expressing the pressure exerted on the driving edge.

Bearing in mind that $x = c + \ell_n$, where ℓ_n is the path traversed by the rotating band in the bore, and finding from (60) that

$$c = \frac{k}{2} \tan \alpha_1 = \frac{\tan \alpha_1}{(\tan \alpha_2 - \tan \alpha_1)} L_{Hp},$$

we will obtain the relation $\tan \alpha = \frac{2}{k} x$ from the path ℓ_n in the form

$$\tan \alpha = \frac{2(c + \ell_n)}{k} = \tan \alpha_1 + (\tan \alpha_2 - \tan \alpha_1)\frac{\ell_n}{L_{Hp}}. \tag{62}$$

ℓ_n varies from zero to $\ell_n = \ell_A - a$, where a is the distance between the base of the projectile and the forward edge of the rotating band.

We are presenting below the characteristic of several guns with relation to the weight of the recoiling parts Q_0, the weight of the

projectile q, the weight of the charge ω and the type of rifling used(*) (Table 24).

2. THE RESISTANCE ENCOUNTERED BY THE ROTATING BAND WHEN FORCED INTO THE RIFLING GROOVES; PRESSURE TO OVERCOME THE INERTIA OF THE PROJECTILE.

When the projectile is properly seated, the forcing cone of the band must bear against the chamber cone and partly enter the tapered end of the rifling (fig. 101); this prevents the escape of the gases from the chamber. As the gas pressure increases, the band takes the grooves and fully enters the latter when the rear edge of the band a approaches the end of the groove taper (point b). The rotating band resistance is maximum at this point. The force Π_0 acting on the cross-sectional area of the bore s, i.e., $\frac{\Pi_0}{s} = p_0$ kg/cm^2, necessary to drive the band for its full length, is called the "pressure to overcome the inertia of the projectile."

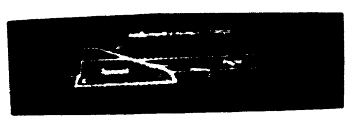

Fig. 101 - Rotating Band Entering the Rifling Grooves

1) connecting taper; 2) rotating band; 3) rifling.

(*) For rifling with an increasing twist, $\frac{h}{d}$ is given for the angle of inclination at the muzzle face of the bore.

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Upon entering the grooves to its full length, the copper band does not undergo further deformation, and the projectile continues to move with the ridges fully formed on its rotating band, following which the pressure undergoes a sudden drop.

Fig. 102 - Change of Resistance as the Rotating Band is Forced into the Grooves.

a) kg/cm^2; b) path of projectile.

The resistance of the walls and grooves of the bore against the rotating band as the projectile moves through the barrel is determined by forcing the projectile through the bore by means of a mechanical or hydraulic press. This method was used by M.F. Rozenberg at the former Obukhov Plant in 1898 and by A.G. Maturin at the former Putilov Plant in 1899. However, this slow, static cold broaching operation usually produces high resistances between the rotating band and the bore due to the presence of the N. μN, $\dot{\Phi}$ and $\nu\dot{\Phi}$ forces.

The diagram in fig. 102 shows the change in pressure p - II/s when forcing the band into the grooves and moving the projectile through a 76-mm gun 1902 issue. This diagram was obtained in tests conducted by (KOSARTOP) in 1925. It shows that the pressure developed in the 76-mm gun 1902 issue by the gradual forcing of the band into the grooves increases from 150 to 250 kg/cm^2, and abruptly drops to 70

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kg/cm^2 after the band is fully wedged in the grooves, following which it slowly decreases to 30 kg/cm^2 at the muzzle face of the gun.

Under actual firing conditions, the walls of the barrel near the rotating band undergo elastic deformation under gas pressure p, whereby the walls are displaced or stretched from position a to position a' (fig. 103); this deformation is transmitted for a certain distance forward and weakens the action of forces $\dot{\Phi}$ and $\nu\dot{\Phi}$.

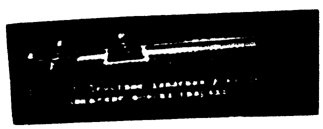

Fig. 103 - The Action of Pressure p in Displacing the Rotating Band of the Projectile

Table 24 - Gun Characteristics

Type of gun	Weight of recoiling parts	Weight of projectile	Weight of charge	Muzzle energy $E_1 = \frac{mv^2}{2}$ ton-m	$\frac{d}{h}$	Angle of twist	Depth of groove, mm	Width of land	of bo... of g...
	kg								
76-mm mountain gun 1909 issue	287	6.20	0.365	45.85	25	$7^{\circ}9'45''$	0.76	3.05	
76-mm cannon 1902/30 issue	570	6.20	1.080	146.10	25	$7^{\circ}9'45''$	0.76	2.10	
107-mm gun 1910/30 issue	1300	17.18	2.79	393.20	25	$7^{\circ}9'45''$	1.0	3.0	
152-mm gun 1910/34 issue	1650	43.56	7.56	952.40	20	$3^{\circ}54'25''$ $8^{\circ}55'37''$	1.5	3.0	
152-mm howitzer 1909/30 issue	1435	40	2.125	311.90	20	$3^{\circ}42'00''$ $8^{\circ}55'37''$	1.25	3.81	
122-mm howitzer 1910/30 issue	570	21.76	1.170	147.10	20	$3^{\circ}42'00''$ $8^{\circ}56'$	1.015	3.04	

Table 24 - Gun Characteristics

ight of rojectile	Weight of charge	Muzzle energy $E_1 = \frac{mv^2}{2}$ ton-m	$\frac{d}{h}$	Angle of twist	Depth of groove, mm	Width in mm of land	Width in mm of bottom of groove	Number of grooves	Length of rifled part L_{Hp} in mm
kg									
6.20	0.365	45.85	25	7°9'45"	0.76	3.05	6.91	24	1060
6.20	1.080	146.10	25	7°9'45"	0.76	2.10	5.38	32	2663
17.18	2.79	393.20	25	7°9'45"	1.0	3.0	7.47	32	3419
43.56	7.56	952.40	20	3°54'25" 8°55'37"	1.5	3.0	6.97	48	3591
40	2.125	311.90	20	3°42'00" 8°55'37"	1.25	3.81	9.47	36	1809
21.76	1.170	147.10	20	3°42'00" 8°56'	1.015	3.04	7.60	36	1260

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This was substantiated by the KOSARTOP tests. The pressure necessary for the translation of the projectile was determined by firing a shortened 76-mm cannon. By using reduced charges and by selecting such charges whereby a half of the number of projectiles fired would be ejected from the bore and the other remain in it, it was determined that a pressure of 150 kg/cm^2 was not sufficient to drive the band into the grooves and would only produce a hardly perceptible imprint of the grooves on the forward portion of the rotating band. At a pressure of 225 to 275 kg/cm^2 some of the projectiles remained in the bore and a number of them was ejected from the bore and dropped near the gun. Thus only a small additional pressure was sufficient to eject the projectile from the bore; the action of forces ϕ and $\nu\phi$ was negligible. In consequence, we shall disregard these forces in our future analysis.

Thus the pressure p_0 necessary to overcome the inertia of the projectile is equal in the given case to 250 kg/cm^2. This value may vary considerably depending on the rifling and band design. In compiling his tables, Prof. N.F. Drozdov used the value of p_0 = 300 kg/cm^2; certain authors use p_0 = 400 kg/cm^2 in their calculations.

Krantz uses different values for p_0 varying from 270 kg/cm^2 for a 76-mm cannon to 550 kg/cm^2 for rifles, in which the grooves are relatively deep and the entire bullet rather than just the rotating band is forced to make the rifling.

Special tests conducted by Asst. Prof. P.N. Shkvornikov indicate a pressure of p_0 = 300-400 kg/cm^2 for rifles.

This pressure p_0 will change if the diameter of the band d_0 or

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its profile is changed.

The value of p_0 usually specified for medium-caliber guns is

$$p_0 = 250\text{-}350 \ \mathrm{kg/cm^2}.$$

Gabeau offers the following expression for determining p_0:

$$p_0 = 4\nu U \frac{H}{d'} \cos \alpha \left[1 + \sin \alpha \frac{\sin \alpha + \nu \cos \alpha}{\cos \alpha - \nu \sin \alpha}\right],$$

where U - Elastic limit of the rotating copper band, attained by the band while being forced into the grooves;

$H(b_0)$ - diameter of rotating band (reduced);

d' - caliber - reduced;

α - angle of twist;

ν - coefficient of friction.

U is determined from the expression

$$U = 3000 \left(\frac{\Sigma}{100}\right)^{0.6} + 350,$$

where

$$\frac{\Sigma}{100} = 0.1 + 1.1 \frac{D_0 - d'}{d' - d_{CH}};$$

D_0 - maximum diameter of rotating band,

d_{CH} - diameter of projectile body at the rotating band.

If we disregard the resistance present after the band is fully driven into the grooves, the motion of the projectile may be assumed to start at the instant the pressure attains the value p_0 produced by the partial burning of the charge ψ_0.

In order to obtain pressure p_0 at constant volume, it is necessary that a portion of charge ψ_0 determined from the following general pyrostatics equation be burned:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}.$$

Thus the pressure to overcome the inertia of the projectile is indirectly accounted for by the magnitude ψ_0 of the portion of the charge burned by the time the projectile starts moving and by the corresponding relative part of the burned thickness z_0. The initial value $\left(\frac{dp}{dt}\right)_0$ also depends on the value of p_0. The higher the value of p_0, the greater $\left(\frac{dp}{dt}\right)_0$, the steeper the p, t curve at the start of the projectile's motion, and the higher the maximum pressure p_m.

3. FORCES DEVELOPED AT THE DRIVING EDGES WHEN THE PROJECTILE IS IN MOTION

The angle formed by the bore axis and the direction of the rifling grooves creates reaction forces between the driving edges of the rifling and the rotating band as the projectile moves through the bore. These forces N at each groove are directed perpendicularly to the surfaces of contact and create friction forces $\nu_1 N$ along the driving edge in a direction opposite to that of the projectile's motion.

These forces and their components (acting in the direction of the bore axis and a plane perpendicular to it) impart a spinning motion to the projectile and develop forces which counteract the translation of the projectile (force R).

In order to determine the reaction force N of the groove and the

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braking force R, let us imagine the bore surface developed in plane xy, with the x-axis parallel to the bore axis (fig. 104). Curve OO depicts a groove with an increasing twist. Point A corresponds to the center of the driving edge, where $\frac{ps}{n}$, N and νN represent the forces acting on each driving edge, and n is the number of grooves.

We shall disregard the radial force Φ and the force of friction $\nu\Phi$.

Let us resolve forces N and νN into their components along the x and y axes:

$$N'' = N \sin \alpha; \quad N' = N \cos \alpha.$$

$$\nu N' = \nu N \cos \alpha; \quad \nu N'' = \nu N \sin \alpha.$$

According to the law of mechanics, we will have the following equation of motion:

1) The sum of the projections of forces along the x-axis equals $m \frac{d^2 x}{dt^2}$,

2) The sum of the moments of rotating forces equals $I \frac{d^2 \varphi}{dt^2} = I \frac{d\Omega}{dt}$,

where I is the moment of inertia of the projectile about the longitudinal axis;

Ω is the angular velocity of the projectile.

The moment of inertia of a projectile of mass m is

$$I = \Sigma \Delta m_1 \cdot r_1^2 = \int r^2 \, dm,$$

where Δm - an element of the mass, located a distance r_1 from the axis of rotation, can be represented as:

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$$I = m\rho^2;$$

here ρ is the radius of gyration, which is determined as follows.

Fig. 104 - Forces Acting in the Rifling Grooves.

If the entire mass of the body rotated about the axis were concentrated in an infinitely thin cylindrical layer at a distance ρ from the axis, and the value of ρ is so chosen that the moment of inertia $m\rho^2$ is equal to the true moment of inertia I, then this distance from the axis of rotation would represent the radius of gyration.

Let us write the equation of rotary motion for the projectile:

$$nrN(\cos a - \nu \sin a) = I \frac{d\Omega}{dt}. \tag{63}$$

Inasmuch as

$$\Omega = \frac{v \tan \alpha}{r},$$

$$\frac{d\Omega}{dt} = \frac{1}{r}\left(\tan \alpha \frac{dv}{dt} + v \frac{\tan \alpha}{dx} \frac{dx}{dt}\right) = \frac{1}{r}\left(\tan \alpha \frac{dv}{dt} + v^2 \frac{d \tan a}{dx}\right);$$

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for rifling with an increasing twist

$$\frac{d \tan \alpha}{dx} = \frac{\tan \alpha_2 - \tan \alpha_1}{L_{n_p}} = k_\alpha = \text{const}$$

and for rifling with a uniform twist

$$k_\alpha = 0.$$

Upon substituting the expression for I and $\frac{d\Omega}{dt}$ in formula (63) and determining the value of N, we will get:

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 \frac{\left(\tan \alpha \, m \frac{dv}{dt} + k_\alpha m v^2\right)}{\cos \alpha - \nu \sin \alpha}. \qquad (64)$$

In order to determine the value of $m \frac{dv}{dt}$ in parenthesis, we shall write the equation of translation:

$$p_{CH} s - nN(\sin \alpha + \nu \cos \alpha) = m \frac{dv}{dt}, \qquad (65)$$

where p_{CH} is the gas pressure acting on the base of the projectile;

$$nN(\sin \alpha + \nu \cos \alpha) = R$$

is the resisting force due to the reaction of n driving edges.

$$p_{CH} s - R = p_{CH} s \left(1 - \frac{R}{p_{CH} s}\right) = m \frac{dv}{dt},$$

The value of $\frac{R}{p_{CH} s}$ is small compared with unity, and

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$$\frac{1}{1 - \frac{R}{p_{CH}s}} \approx 1 + \frac{R}{p_{CH}s} = \varphi_1,$$

where φ_1 is a value slightly exceeding unity; this value will be determined with greater accuracy later on.

Therefore,

$$m\frac{dv}{dt} = \frac{p_{CH}s}{\varphi_1}.$$

Substituting this expression in formula (64), we get:

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 \frac{\tan\alpha\, sp_{CH} + \varphi_1 k_\alpha m v^2}{\varphi_1(\cos\alpha - \nu\sin\alpha)}.$$

The expression in the denominator closely approaches unity:

$$\varphi_1(\cos\alpha - \nu\sin\alpha) \approx 1.$$

We thus get the final expression for the reaction N of the groove if the rifling has an increasing twist:

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 (\tan\alpha\, sp_{CH} + \varphi_1 k_\alpha m v^2). \tag{66}$$

For grooves having a uniform twist $k_\alpha = 0$ and the force is

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 \tan\alpha\, sp_{CH}. \tag{67}$$

The value of $\left(\frac{\rho}{r}\right)^2 = \lambda$ depends on the type of the projectile and varies between 0.48 for a bullet and 0.68 for a thin-walled high-

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explosive percussion shell.

For example:

	$\left(\frac{\rho}{r}\right)^2$
for a circular solid cylinder....................	0.50
for a solid bullet............................	$\sim$ 0.48
for armor-piercing thick-walled shells.........	$\sim$ 0.56
for thin-walled percussion shells..............	0.64-0.68.

In order to compute the stress in the band metal, the force N must be referred to the contact area between the driving edges of the rifling and the band, i.e., to $b_0 t_H$, where t_H is the depth of the rifling and b_0 is the width of the rotating band.

Formula (67) shows that for rifling with a constant twist the pressure exerted by the band of the projectile on the driving edge of the rifling and, inversely, the pressure exerted by the driving edge on the band while the projectile is in motion, varies in proportion to the pressure exerted by the gases on the base of the projectile. Therefore, curve 1 representing the change of force N as a function of ℓ is similar to the pressure curve (fig. 105), and the band of the projectile and the driving edge are subjected to a maximum stress at the instant the pressure and the velocity of the projectile are at a maximum.

Fig. 105 - Effect Produced by Rifling on the Pressure N in the Rifling Grooves.

Formula (66) indicates that by decreasing the initial groove angle α_1, the first term in parenthesis for the instant at which p_m is developed can be considerably decreased. Inasmuch as the velocity of the projectile at this instant is still small (as is the doubled kinetic energy of the projectile, mv^2), then at the instant of maximum developed pressure the value of N obtained according to formula (66) (in the case of an increasing twist) may be smaller than that obtained by formula (67) (for rifling with a uniform twist).

As the pressure continues to decrease, the tan α increases, as does the second term $k_\alpha mv^2$. Hence, in the case of an increasing twist, the pressure acting on the driving edge varies more uniformly than in the case of rifling with a uniform twist. The force N may be varied considerably by changing the angles α_1 and α_2.

The curves in fig. 105 show the change of force N as a function of the path of the projectile for rifling grooves with α = const and for two riflings with a variable α:

1) $\alpha = 10^\circ$ = const;
2) $\alpha_1 = 5^\circ$, $\alpha_2 = 10^\circ$;
3) $\alpha_1 = 2^\circ$, $\alpha_2 = 10^\circ$.

If the rifling equation is known, the dependence of tan α on the path of the projectile ℓ can be determined according to formula (62) as a means for calculating the change of N:

$$\tan \alpha = \tan \alpha_1 + (\tan \alpha_2 - \tan \alpha_1) \frac{\ell_n}{\ell_{\text{Hp}}} = \tan \alpha_1 + k_\alpha \ell_n;$$

and curves p, ℓ and v, ℓ can then be plotted and the values of α, p and v for the same values of ℓ substituted in formula (66). By substituting expression (62) in equation (66), we will get:

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$$N = \frac{\lambda}{n}(\tan \alpha_1 sp + k_\alpha \ell_n sp + k_\alpha m v^2) = \frac{\lambda}{n} \tan \alpha_1 \, sp + \frac{\lambda}{n} k_\alpha (\ell sp + m v^2).$$

The first term represents that pressure N_{α_1} which would obtain in a rifling with a uniform twist whose angle $\alpha = \alpha_1$. This change is similar to the change of the pressure curve p, ℓ, i.e., it first increases and then decreases. The second term depends on ℓ, p and v, whereby, ℓ, p and v increase until the pressure attains a maximum value, following which ℓ and v continue to increase and p decreases. This formula makes it possible to analyze the influence of each variable p, v and ℓ on the pressure exerted on the driving edge.

The resistance offered by the rifling against the translatory motion of the projectile is

$$R = nN(\sin \alpha + \nu \cos \alpha) = nN \cos \alpha(\tan \alpha + \nu). \qquad (68)$$

For rifling with a uniform twist (assuming $\cos \alpha \approx 1$):

$$R = \left(\frac{\rho}{r}\right)^2 (\tan^2 \alpha + \nu \tan \alpha) sp_{CH}, \qquad (69)$$

i.e., R is proportional to the gas pressure on the base of the projectile.

The magnitude φ_1 introduced above, which takes into account the braking effect of the rifling grooves on the motion of the projectile, is also constant when the rifling twist is uniform:

$$\varphi_1 = 1 + \frac{R}{sp_{CH}} = 1 + \left(\frac{\rho}{r}\right)^2 (\tan^2 \alpha + \nu \tan \alpha).$$

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The equation of translatory motion of the projectile can be written as follows:

$$sp_{CH} = \varphi_1 m \frac{dv}{dt}.$$

Since $\varphi_1 > 1$, the resistance offered by the rifling is the same as if the mass of the projectile were increased. In the expression $\varphi_1 = 1 + \lambda \tan^2 \alpha + \lambda \nu \tan \alpha$ the value of the coefficient $\lambda \tan^2 \alpha$ varies between 0.0025 $\left(\frac{1}{4}\%\right)$ for a small pitch with a rifling pitch $n = 45$ calibers and 0.025 (2.5%) for a very steep pitch ($h = 15$ calibers).

For a medium angle of twist $\alpha = 6\text{-}7^\circ$ $\lambda \tan^2 \alpha \approx 0.01$ (1%).

The value of the coefficient $\lambda \nu \tan \alpha$ depends on both the angle α and the coefficient of friction ν which is usually taken between 0.16 and 0.20; on the average $\lambda \nu \tan \alpha \approx 0.01$ (1%).

Investigations made during the past few years have shown that ν decreases as the velocity of the projectile increases; at $v \approx 200$ m/sec $\nu \approx 0.10$, and at $v = 1000$ m/sec $\nu \approx 0.05$.

φ_1 is usually taken to be equal to $\varphi_1 = 1.02$.

For rifling with an increasing twist

$$R = \frac{\lambda}{n}(\tan \alpha \; sp_{CH} + \varphi_1 k_\alpha m v^2)(\tan \alpha + \nu),$$

and the magnitude

$$\varphi_1 = 1 + \frac{R}{sp_{CH}}$$

will no longer be a constant value.

4. THE WORK DONE IN OVERCOMING THE RESISTANCE R OFFERED BY THE RIFLING GROOVES.

In order to overcome the resistance R, the powder gases must do a certain amount of work.

For rifling with a uniform twist:

$$R = nN \cos \alpha(\tan \alpha + \nu) = \lambda(\tan^2 \alpha + \nu \tan \alpha) s p_{CH}.$$

The work done in overcoming this resistance is

$$\int_0^l R d\ell = \lambda(\tan^2 \alpha + \nu \tan \alpha) s \int_0^l p_{CH} d\ell,$$

but

$$s \int_0^l p_{CH} d\ell = \frac{mv^2}{2}.$$

Therefore,

$$\int_0^R R d\ell = \lambda \tan^2 \alpha \frac{mv^2}{2} + \lambda \nu \tan \alpha \frac{mv^2}{2}.$$

It can be shown that the first term represents the work done in imparting a spinning motion E_2 to the projectile, and the second term represents the work done in overcoming friction E_3.

Both types of work are proportional to $\frac{mv^2}{2} = E_1$ (to the work done in imparting translation to the projectile) and can be written as:

$$E_2 = \lambda \tan^2 \alpha \frac{mv^2}{2} = k_2 \frac{mv^2}{2},$$

where $k_2 = \lambda \tan^2 \alpha$;

$$E_3 = \lambda \nu \tan \alpha \frac{mv^2}{2} = k_3 \frac{mv^2}{2},$$

where $k_3 = \lambda \nu \tan \alpha$.

Comparing it with the expression for φ_1, we find that

$$\varphi_1 = 1 + k_2 + k_3.$$

For rifling with an increasing twist:

$$R = nN(\sin \alpha + \nu \cos \alpha) = \lambda(\tan \alpha sp_{CH} + \varphi_1 k_1 mv^2)(\sin \alpha + \nu \cos \alpha) =$$

$$= \lambda \cos \alpha(\tan \alpha sp_{CH} + \varphi_1 k_1 mv^2)(\tan \alpha + \nu).$$

Substituting therein the expressions

$$\tan \alpha = \tan \alpha_1 + k_1 \ell$$

and

$$\tan^2 \alpha = \tan^2 \alpha_1 \left[1 + 2n_\alpha \frac{\ell}{L_{np}} + n_\alpha^2 \left(\frac{\ell}{L_{np}}\right)^2\right],$$

where

$$n_\alpha = \left(\frac{\tan \alpha_2}{\tan \alpha_1} - 1\right),$$

we will get an expression for R as a function of the projectile's path ℓ and its velocity v, and using the numerical value of $\int_0^1 R d\ell$ we can

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determine the work done in overcoming the resistance offered by a rifling with an increasing twist.

CHAPTER 5 - DERIVATION OF FORMULAS FOR DETERMINING THE SECONDARY TYPES OF WORK INVOLVED

1. WORK DONE IN SPINNING THE PROJECTILE

The work done in imparting a spinning motion to the projectile is expressed by the formula

$$E_2 = \frac{I\Omega^2}{2},$$

where I = moment of inertia of the projectile about the axis of rotation ($I = m\rho^2$).

Ω = angular speed of rotation.

It was shown above that all four items of work under consideration are proportional to the basic work $E_1 = \frac{mv^2}{2}$, and the expression for E_2 can be reduced to the form:

$$E_2 = k_2 \frac{mv^2}{2},$$

from which we can determine the value of k_2. We shall substitute the linear speed v for the angular speed of the projectile Ω:

$$\Omega = \frac{v \tan \alpha}{r};$$

$$E_2 = \frac{m\rho^2 v^2 \tan^2 \alpha}{2r^2} = \left(\frac{\rho}{r}\right)^2 \tan^2 \alpha \frac{mv^2}{2} = k_2 \frac{mv^2}{2},$$

where

$$k_2 = \left(\frac{\rho}{r}\right)^2 \tan^2 \alpha.$$

The coefficient k_2 indicates the portion of the total work done in spinning the projectile. This magnitude depends on the design or type of the projectile $\left(\frac{\rho}{r}\right)^2$ and on the rifling of the bore - the angle of twist α.

2. THE WORK DONE IN OVERCOMING THE FRICTION IN THE RIFLING GROOVES

The component of the force of friction on the driving edge resisting the projectile's motion is expressed by:

$$n \nu N \cos \alpha.$$

The work done in overcoming this resistance is

$$E_3 = \int_0^{\ell} n\nu N \cos \alpha \frac{d\ell}{\cos \alpha},$$

because the path traversed along the rifling is $\ell \cos \alpha$.

Substituting here the expression for N:

$$E_3 = \left(\frac{\rho}{r}\right)^2 \nu \tan \alpha \, s \int_0^{t} p_{CH} d\ell = \left(\frac{\rho}{r}\right)^2 \nu \tan \alpha \frac{mv^2}{2}.$$

we thus get

$$k_3 = \left(\frac{\rho}{r}\right)^2 \nu \tan \alpha,$$

i.e., the expression derived earlier.

The numerical values of k_2 and k_3 were likewise discussed earlier in the text.

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3. THE WORK DONE IN DISPLACING THE CHARGE

In displacing the projectile through the bore, the powder gases move together with it, whereby the unburned portion of the charge may move likewise under the action of non-uniform pressures developed in the bore. A portion of the developed energy is thus spent on the displacement of certain portions of the charge and on imparting kinetic energy to them, which must be taken into consideration.

Inasmuch as no accurate data is available on the distribution of the gas mass and the unburned portion of the charge in the initial air space, certain allowances must be resorted to when these factors are taken into account.

It is known that when a shot is fired wave motions may occur when the gases impinging on the base of the projectile rebound and encounter other gases flowing towards them, thus creating a localized pressure rise. Furthermore, in entering the narrower bore from the chamber, the gas stream becomes smaller in cross section, and this may also make it more difficult to express the law of motion in the form of analytic functions. As a result, it is necessary to resort to certain simplified expressions and allowances when determining the work done in the displacement of the gases.

This problem is presented as follows: an expression must be obtained for the kinetic energy of the portions of the charge moving with a variable speed, and an expression for linking this with the kinetic energy of the projectile.

In solving this problem, we shall make the following allowances:

1) The bore, including the powder chamber, has the same area,

equal to the cross-sectional area s.

2) At each position occupied by the projectile, the mass of the charge is distributed evenly throughout the entire space between the base of the projectile and the base of the bore.

3) The elements of the charge have a translatory motion only, and the velocities between its layers increase from zero at the base of the chamber to v at the base of the projectile, according to the linear law.

4) The velocities of the particles at a given cross section are the same, and no friction exists between the particles of the charge and the walls of the bore.

The diagram in fig. 106 clarifies the above.

We shall designate:

v - velocity of the projectile,

v_ω - velocity of a charge element in a given layer;

$\mu = \omega/g$ - mass of charge;

Λ - distance between chamber base and base of projectile.

At the instant the projectile had traversed a distance ℓ, Λ is constant; it varies with time, whereas we are considering the condition of the charge masses in the initial air space at various distances x from the base of the chamber at every instant.

We shall separate an elementary layer of cross section s and height dx and designate its mass by $d\mu$. The layer moves with a velocity v_ω; its elementary kinetic energy will be expressed thus:

$$dE_4 = \frac{d\mu v_\omega^2}{2}.$$

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In order to obtain an expression for the full kinetic energy, this expression must be integrated for ℓ between zero and Π. We will then find the kinetic energy of the charge, whose elements move in the initial air space according to a given law.

We have from the condition of uniform mass distribution:

$$\frac{d\mu}{\mu} = \frac{dx}{\Pi} \text{ or } d\mu = \frac{\mu}{\Pi}\,dx.$$

Fig. 106 - Distribution of Velocities of the Gas Layers Back of the Projectile

a) chamber base; b) projectile base.

From the condition that the velocity v_ω changes according to the linear law and from similar triangles:

$$v_\omega : v = x : \Pi,$$

whence

$$v_\omega = \frac{v}{\Pi}x.$$

Upon substituting, we get:

$$dE_4 = \frac{v^2}{\Pi^2}\,\frac{x^2}{2}\cdot\frac{\mu}{\Pi}\,dx = \frac{\mu v^2}{2\Pi^3}x^2dx.$$

Integrating between the limits of zero and Π, we get:

$$E_4 = \int_0^{\Pi} \frac{v_\omega^2 d\mu}{2} = \int_0^{\Pi} \frac{\mu v^2}{2\Pi^3} x^2 dx$$

or

$$E_4 = \frac{\mu v^2}{2\Pi^3} \int_0^{\Pi} x^2 dx = \frac{\mu v^2 \Pi^3}{2\Pi^3 \cdot 3} = \frac{1}{3} \frac{\mu v^2}{2} = \frac{1}{3} \frac{\mu}{m} \frac{m v^2}{2},$$

because

$$\frac{\mu}{m} = \frac{\omega}{q},$$

then

$$E_4 = \frac{1}{3} \frac{\omega}{q} \frac{m v^2}{2} = k_4 E_1 .$$

Hence, in consequence of the allowances made, the work done in moving the gases and the charge is proportional to the basic work E_1. The coefficient $k_4 = \frac{1}{3} \frac{\omega}{q}$ varies considerably depending on the relative weight of the charge $\frac{\omega}{q}$. In low-power guns and howitzers $\frac{\omega}{q} \approx 0.10 \div 0.15$, in high-power guns it is about $0.30 \div 0.40$. Consequently,

$$k_4 = \frac{1}{3} \frac{\omega}{q} = 0.03 \div 0.13 .$$

This coefficient was obtained under specific assumptions regarding

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the distribution of masses and velocities. In actuality this phenomenon is much more complex and permits certain allowances. For example, F.F. Lender, in assuming that the greater portion of the charge is concentrated nearer the chamber base and hence has a smaller velocity, obtained a coefficient $b_4 = \frac{1}{6}$, other authorities assume b equal to 1/4 or 3/11.

4. THE EFFECT OF WIDENING THE CHAMBER ON THE WORK DONE IN DISPLACING THE GASES

The motion of gases in the presence of a widened chamber represents a complex problem in gas dynamics which has not been solved to this day. We shall resort to a simpler and less accurate relation which takes into account the effect of a widened chamber on the k_4 coefficient.

The following allowances must be made in deriving this relation.

1) The gas mass is distributed uniformly in the initial air space, but only that part of the mass is in motion whose cross-sectional area s equals the cross-sectional area of the bore. The outer layers adjoining the chamber walls do not participate in this movement. As usual, the interval gas friction and the friction between the gases and the walls of the bore are disregarded.

Fig. 107 - Motion of Gases in the Presence of a Widened Chamber.

2) The velocity of the gas layers participating in the motion

varies linearly from zero at the base of the chamber to that of the velocity of the projectile at its base.

By using this purely mechanical representation which does not take into account the gas dynamics relating to the compression of the gas stream, we obtain the diagram shown in fig. 107.

The weight of the gases ω' participating in this motion relative to the over-all weight of the charge ω is expressed by the following formula:

$$\frac{\omega'}{\omega} = \frac{s(\ell_{KM} + \ell)}{W_0 + s\ell} = \frac{\ell_{KM} + \ell}{\ell_0 + \ell} = \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1},$$

where $\chi = \frac{\ell_0}{\ell_{KM}}$ is the coefficient of expansion (widening) of the chamber and $\Lambda = \frac{\ell}{\ell_0}$.

$$\omega' = \omega \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}.$$

The formula for k_4 derived above is applicable to this gas mass:

$$k_4' = \frac{1}{3}\frac{\omega'}{q} = \frac{1}{3}\frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}\frac{\omega}{q} = b\frac{\omega}{q}$$

when $\chi = 1$, $k_4 = \frac{1}{3}\frac{\omega}{q}$, $b = \frac{1}{3}$.

As Λ changes with the movement of the projectile, the coefficient b varies from $b = \frac{1}{3}\frac{1}{\chi}$ at the start of motion when $\Lambda = 0$ and tends towards $b = \frac{1}{3}\frac{\omega}{q}$ as Λ increases.

Because Λ varies, when integrating the equation of interior

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ballistics, the value of b must be taken as the average value between 0 and Λ_k:

$$b_{cp} = \frac{1}{3}\left(\frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}\right)_{cp} = \frac{1}{3}\,\frac{1}{\Lambda}\int_0^{\Lambda}\frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}\,d\Lambda = \frac{1}{3}\left[1 - \left(1 - \frac{1}{\chi}\right)\cdot 2.303\,\frac{\log\left(\Lambda + \frac{1}{\chi}\right)}{\Lambda}\right].$$

A table is compiled for the values of b_{cp} (i.e., $b_{average}$) at two limits Λ and χ, at $\chi = 1$ $b = \frac{1}{3}$ (Table 25).

Table 25

χ \ Λ	0.6	1.0	2.0	3.0	5.0	7.0	10.0
1.1	0.309	0.312	0.316	0.319	0.322	0.324	0.326
1.5	0.246	0.256	0.272	0.282	0.293	0.300	0.306
2.0	0.203	0.218	0.242	0.256	0.273	0.284	0.293
3.0	0.159	0.179	0.211	0.230	0.253	0.267	0.280
4.0	0.137	0.160	0.180	0.280	0.244	0.259	0.273

According to this table the coefficient b increases when Λ increases and χ decreases and tends towards $\frac{1}{3}$ as the limit.

5. WORK DONE IN DISPLACING THE RECOILING PARTS

If we designate the weight and mass of the recoiling parts by Q_0 and M, respectively, and the velocity by V, the work done in moving the recoiling parts will be expressed in the form:

$$E_5 = \frac{MV^2}{2} = \frac{Q_0 V^2}{2g}.$$

The mass M is known, whereas the velocity of recoil V can be found from the theorem of conservation of momentum of the system barrel-charge-projectile which is subjected to the action of internal forces.

However, it is first necessary to find an expression for the absolute velocity of the projectile v_a (with respect to the ground) and for the average velocity of the charge v_ω portions of which move behind the projectile and behind the barrel itself.

Fig. 108 - Diagram of the Distribution of Gas Velocities when the Barrel is Recoiled

1) chamber base, 2) projectile base.

In order to determine the average absolute velocity v'_ω of the charge, we shall assume, as before, that the mass of the charge is distributed uniformly in the initial air space at each given instant and that the velocity v'_ω changes linearly from V at the base of the chamber to v_a at the base of the projectile, whereby

$$v_a = v - V,$$

where v is the relative velocity of the projectile in the bore of the gun barrel (fig. 108).

If the velocity of the elements of the charge varies linearly, the

average velocity will be expressed as the half sum of the end velocities, i.e.,

$$v_{\omega\ cp} = \frac{-V + v_a}{2} = \frac{-V + v - V}{2} = \frac{v}{2} - V.$$

In constructing the equation of momentum and by assuming that the velocities in the direction of the projectile's motion are positive and those in the opposite direction are negative, we will get:

$$-MV + \mu v'_{\omega\ cp} + mv_a = 0,$$

or, replacing $v'_{\omega\ cp}$ and v_a by their values in terms of the relative velocity v of the projectile, we have:

$$-MV + \mu\left(\frac{v}{2} - V\right) + m(v - V) = 0,$$

whence

$$V = \frac{m + \frac{\mu}{2}}{M + m + \mu} v.$$

Substituting the value of V in the expression for E_5, we get:

$$E_5 = \frac{MV^2}{2} = \frac{M}{2} \frac{\left(m + \frac{1}{2}\mu\right)^2}{(M + m + \mu)^2} v^2 = \frac{Mm^2}{2M^2} \frac{\left(1 + \frac{1}{2}\frac{\mu}{m}\right)^2}{\left(1 + \frac{m}{M} + \frac{\mu}{M}\right)^2} v^2 =$$

$$= \frac{m}{M} \frac{\left(1 + \frac{1}{2}\frac{\mu}{m}\right)^2}{\left(1 + \frac{m}{M} + \frac{\mu}{M}\right)^2} \frac{mv^2}{2}.$$

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Replacing the masses by their weights, we get

$$F_5 = \frac{q}{Q_0} \frac{\left(1 + \frac{1}{2}\frac{\omega}{q}\right)^2}{\left(1 + \frac{q}{Q_0} + \frac{\omega}{Q}\right)^2} \cdot \frac{mv^2}{2}.$$

The factor at $\frac{mv^2}{2}$ is the coefficient k_5:

$$k_5 = \frac{q}{Q_0} \frac{\left(1 + \frac{1}{2}\frac{\omega}{q}\right)^2}{\left(1 + \frac{q}{Q_0} + \frac{\omega}{Q_0}\right)^2}.$$

The factor k_5 depends in the main on the ratio between the weight of the projectile q and the weight of the recoiling parts Q_0; the second factor incorporates only a small change in this ratio. An approximate expression can be used in actual practice. Inasmuch as $\frac{q}{Q_0} + \frac{\omega}{Q_0}$ as well as $\frac{1}{4}\left(\frac{\omega}{q}\right)^2$ are small and approximately equal to each other,

$$k_5 \approx \frac{q}{Q_0}\left(1 + \frac{\omega}{q}\right).$$

If we express $v_{\omega\,cp}$ in terms of the absolute velocity of the projectile v_a, we will get:

$$-MV + \mu\left(\frac{v_a - V}{2}\right) + mv_a = 0,$$

whence

$$V = \frac{m + \frac{1}{2}\mu}{M + \frac{1}{2}\mu} v_a = \frac{q\left(1 + \frac{1}{2}\frac{\omega}{q}\right)}{Q_0\left(1 + \frac{1}{2}\frac{\omega}{Q_0}\right)} v_a .$$

The coefficient k_5 characterizing the relative work of recoil is higher for howitzers than for cannon, because for the same caliber and projectile weignt the weight of the recoiling parts Q_0 is considerably smaller on howitzers than on cannon of the same caliber.

6. SUMMATION OF THE AUXILIARY WORK DONE

Upon investigating the auxiliary work items involved and determining a general expression for each in the form

$$E_i = k_i \frac{mv^2}{2} = k_i E_1,$$

an expression can be compiled for expressing the total work done in the energy equilibrium equation:

$$\frac{f\omega}{\theta}\psi - \frac{ps(l_\psi + \ell)}{\theta} = E_1 + E_2 + E_3 + E_4 + E_5 = \Sigma E_i,$$

whereby

$$\Sigma E_i = E_1(1 + k_2 + k_3 + k_4 + k_5).$$

The sum in parenthesis is usually denoted by φ:

$$\varphi = 1 + k_2 + k_3 + k_4 + k_5,$$

or

$$\varphi = 1 + \left(\frac{\rho}{r}\right)^2 \tan^2 \alpha + \left(\frac{\rho}{r}\right)^2 \nu_1 \tan \alpha + \frac{1}{3}\frac{\omega}{q} + \frac{q}{Q_0}\left(1 + \frac{\omega}{q}\right).$$

Thus the total exterior work done by the gases when a shot is fired

$$\Sigma E_i = \frac{\varphi m v^2}{2},$$

where φ is a coefficient representing the auxiliary or secondary work done.

Upon determining φ, when performing the necessary transformations for the solution of the fundamental problem of pyrodynamics, the sum of the work $E_1 \ldots E_5$ in the right side of the equation may be omitted and replaced by the coefficient $\frac{\varphi m v^2}{2}$. φ is the coefficient representing the secondary or auxiliary work items involved. It increases in the main with the increase of the relative weight of the charge ω/q.

In such cases where design data on the gun rifling are not available, φ is calculated by means of simplified formulas.

For example, Prof. V.E. Slukhotsky offers the following general expression for φ:

$$\varphi = K + \frac{1}{3}\frac{\omega}{q},$$

by introducing into K the sum of all the k_i's except k_4, which is separately expressed by $\frac{1}{3}\frac{\omega}{q}$. The value of K varies with the type of gun:

for howitzers........................K = 1.06
for medium-power cannon...............K = 1.04-1.05
for high-power cannon.................K = 1.03
for small arms........................K = 1.10

The difference in the value of the K coefficient for cannon and howitzers is mainly obtained because of the work done by the recoil (k_5 coefficient):

Sugot, offers the following expression for φ:

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$$\varphi = 1.05 \cdot \left(1 + \frac{1}{4}\frac{\omega}{q}\right).$$

CHAPTER 6 - SUPPLEMENTARY PROBLEMS

1. RELATION BETWEEN THE PRESSURES EXERTED ON THE BASE OF THE BORE AND THE BASE OF THE PROJECTILE

When the projectile moves under the action of the powder gases, the pressure in the initial air space is not uniform because the gases are in motion and move with varying rates of speed.

In order to determine the pressure exerted on the base of the shell and the base of the bore, let us consider two sections of the initial air space: at base of the bore and the base of the shell, and construct an equation of motion of the masses located to one side (to the right) of these sections.

Then the projectile of mass m under the action of force $p_{CH}s$ will be subjected to an acceleration j, and the equation of motion for the section at the base of the projectile, with the resisting forces taken into account, will be written as follows:

$$p_{CH}s = \varphi_1 mj.$$

The gas pressure $p_{\text{Д}H}$ at the base of the shell will impart an accelerated motion not only to the shell, but also to the entire gas mass to the right of this section, between the base of the bore and the base of the projectile. Therefore the equation of motion at this section will be written thus:

$$p_{\text{Д}H}s = \varphi_1 mj + \mu j_3,$$

where μ - mass of gases generated by the charge (including the still unburned portion of the powder);

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j_3 - mean acceleration of the charge moving with a varying velocity from one layer to another.

Dividing these equations term by term, we get:

$$p_{\Omega H} = p_{CH}\left(1 + \frac{\mu j_3}{\varphi_1 m j}\right) = p_{CH}\left(1 + \frac{j_3 \omega}{j \varphi_1 q}\right) .$$

The ratio between the accelerations developed by the charge and the projectile is governed by the hypothesis applied with regard to the distribution of the mass of the charge in the initial air space and by the law of the change of accelerations at the various sections of the space.

If the law governing the gas velocity change is linear, the ratio $j_3/j = \xi$ equals 0.5 and this magnitude, usually called the Piober coefficient, is the one used in most textbooks on the subject.

The problem dealing with the distribution of gas pressure in the initial air space is analyzed in great detail by Prof. I.P. Gravé [18] and, recently, by Asst. Prof. P.N. Shkvornikov [7] who gives a solution to this problem based on the assumptions commonly used in gas dynamics with relation to gas motion under condition of uniform density of the mixture of gas and unburned portion of powder at a given position of the projectile in the bore. All the relations are derived from the fundamental equations of motion of the powder gases and the burning charge which, in turn, are obtained from the general equations of gas dynamics for a one-dimensional unstable gas motion.

The fundamental formulas developed by P.N. Shkvornikov follow. Using the designations:

v - relative velocity of projectile;

v_a - absolute velocity of projectile;

V - velocity of recoiling parts;

q - weight of projectile;

Q_0 - weight of recoiling parts;

φ_1 and φ_2 - coefficients representing the resistances encountered by the projectile in moving through the gun bore and by the recoiling parts.

Introducing the designation

$$i = \frac{\varphi_1 q + \frac{1}{2}\omega}{\varphi_2 Q_0 + \frac{1}{2}\omega};$$

then

$$V = i v_a = \frac{i}{1 + i} v$$

and

$$v_a = \frac{1}{1 + i} v.$$

The relation between $p_{ДН}$ and p_{CH}(*) will be expressed by the formula:

(*) $p_{ДН}$ - pressure at base of bore; p_{CH} - pressure at base of projectile

$$p_{\text{ДH}} = p_{CH}\left[1 + 0.5\,\frac{\omega}{\varphi_1 q}\,(1 - i)\right].$$

The maximum pressure develops not at the base of the bore but, rather, at a distance $x_m = \frac{1}{1 + i}\,\Lambda$ at the section where the absolute velocity of the powder gases is $v_a = 0$.

In deriving the fundamental equation of pyrodynamics and assuming that $pw = RT\omega\psi$, the pressure p was understood to represent a certain mean pressure which is identical at all points in the initial air space; it is assumed thereby that the powder burns under precisely such a pressure. The relation between p, p_{CH} and $p_{\text{ДH}}$ is given in the following form:

$$p = p_{CH}\left[1 + \frac{1}{3}\,\frac{\omega}{\varphi_1 q}\left(1 - \frac{i}{2}\right)\right];$$

$$p = p_{\text{ДH}}\,\frac{1 + \frac{1}{3}\,\frac{\omega}{\varphi_1 q}\left(1 - \frac{i}{2}\right)}{1 + \frac{1}{2}\,\frac{\omega}{\varphi_1 q}(1 - i)}.$$

For a recoilless barrel $i = 0$ and the formulas are simplified:

$$p = p_{CH}\left(1 + \frac{1}{3}\,\frac{\omega}{\varphi_1 q}\right).$$

Multiplying both parts of this equation by φ_1, we get:

$$\varphi_1 p = p_{CH}\left(\varphi_1 + \frac{1}{3}\,\frac{\omega}{q}\right) = p_{CH}(1 + k_2 + k_3 + k_4).$$

Hence, if we disregard the work done in recoiling the barrel,

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$$\varphi_1 p \approx \varphi p_{CH} \quad \text{or} \quad \frac{p_{CH}}{\varphi_1} = \frac{p}{\varphi}.$$

In such a case the equation of the projectile's motion

$$s p_{CH} = \varphi_1 m \frac{dv}{dt}$$

will be equal to the equation of the projectile's motion

$$sp = \varphi m \frac{dv}{dt},$$

where p – mean gas pressure;

φ – a coefficient which takes into account all the auxiliary or secondary work done.

This constitutes a very important deduction, because it permits to consider p and φ to be identical values in both the fundamental equations of pyrodynamics

$$ps(l_\psi + l) = f\omega\psi - \frac{\theta\varphi m v^2}{2}$$

and in the equation of the projectile's motion

$$ps = \varphi m \frac{dv}{dt} \quad \text{or} \quad ps = \varphi m v \frac{dv}{dl}$$

whereby v is the relative velocity of the projectile (relative to the bore), which is computed in solving the problem of interior ballistics.

For cannon and howitzers $\varphi_1 = 1.02$, for small arms using ordinary bullets $\varphi_1 = 1.05$, for armor-piercing bullets $\varphi_1 = 1.07$.

The coefficient i depending on the distribution of the moving mass in the system barrel-charge-projectile, may be considered to be

equal to:

$i = 0.0015$ for rifles and machine guns.

$i = 0.0030$ for antitank guns

$i = 0.020$ for cannon

$i = 0.035$ for howitzers

This data was arrived at from the analysis of the results obtained by P.N. Shkvornikov for some of our (Russian) artillery systems and small arms.

2. HEAT LOST TO THE BARREL WALLS THROUGH HEAT TRANSFER

When considering the heat lost to the walls of the barrel when a gun is fired, we shall take into account the heat transfer resulting from the direct contact of the hot gases with the cold walls of the gun barrel; we shall not consider the heating of the walls due to mechanical reasons (energy of translation of projectile, friction by the rotating band, and deformation of barrel).

The data entering these calculations were presented in Part I of this book. The basic allowance used is the same as that assumed in computing the heat transferred to the walls of a manometric bomb, namely, that the heat loss is proportional to the number of impacts made by the molecules against the wall of the bomb, which in turn depends on Σ, p and t, i.e., on $\Sigma \int pdt$, where Σ is the surface area of the bore.

However, whereas the area $S_б$ remains constant in a bomb, Σ varies from the chamber area Σ_0 to area Σ_{KH} of the entire barrel surface. The chamber area is constantly subjected to the action of the gases and thus takes up a portion of the heat energy, whereas the area of the rifled portion of the bore participates in the cooling of

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the gases only while the projectile is in motion and thus gradually tends to increase the initial air space. The nearer the rifled portion to the muzzle, the shorter will be the period during which it will be subjected to the action of the gases, and the less heat will it take up when a shot is fired.

After the projectile's departure, the heat transfer to the walls continues along the entire area of the bore at a constantly decreasing pressure, but inasmuch as this heat transfer no longer affects the actual shot (reduced pressure and velocity), we shall not consider it for the time being.

In order to take into account the effect of heat transfer when a shot is fired, it is first necessary to determine by means of bomb tests the time t_K it takes for the powder to burn at $\Delta = 0.20$, and also the coefficient C_M% from the Muiraur C curve.

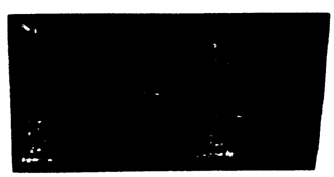

Fig. 109 - Heat-Transfer Diagram According to Miuraur

If the powder were burned in the chamber at constant initial charge density Δ_0, the correction for heat transfer would be determined by the following formula:

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$$\left(\frac{\Delta I_{\text{я}}}{T}\right)_0 = \frac{C_M}{7.774}\frac{\Sigma_0}{W_0}\frac{1}{\Delta_0},$$

where Σ_0 is the chamber surface area and $\frac{\Sigma_0}{W_0} = \frac{4}{D} + \frac{1}{l_{KM}}$;

l_{KM} is the length, and D is the mean diameter of the chamber.

If the powder had burned all the time throughout the entire bore space at a loading density of $\Delta_{KH} = \frac{\omega}{W_0 + s l_{Д}} = \frac{\omega}{W_{KH}}$, the loss would be expressed by the formula:

$$\left(\frac{\Delta I_k}{T}\right)_{Д} = \frac{C_M}{7.774}\frac{\Sigma_{KH}}{W_{KH}}\frac{1}{\Delta_{KH}}, \tag{70}$$

where Σ_{KH} is the surface area of the whole bore.

Actually, the loss suffered when a shot is fired is confined between these two values, and in order to account for same when the pressure and the cooling surface of the bore change simultaneously, these changes whose increments are proportional to the path traversed by the projectile must be known as a function of time.

With the p, t and Σ, t or l, t curves at his disposal, Muiraur suggested the following method for determining losses due to heat transfer.

The p, t (I) and $\frac{\Sigma}{\Sigma_{KH}}$, t(II) curves are plotted on the same diagram (fig. 109); point C' represents the end of burning of the powder ($p = p_K$).

The area of the pressure curve $Op_0p_mp_KC'O$ corresponds to $I_K = \int_0^{t_K} p\,dt$ obtained from bomb tests; the area $C'p_Kp_ДC'' = I_{II}$ corresponds

to the additional cooling which would obtain in the second period after the end of burning of the powder just before the projectile leaves the barrel.

If the surface area were constant, the right side of the equation would have to be multiplied by the area ratio in order to take the heat transfer into account:

$$\frac{Op_0 p_m p_Д C''O}{Op_0 p_m p_K C'O} = \frac{l_K + l_{II}}{l_K} > 1.$$

But since the area varies from the relative value $\frac{\Sigma_0}{\Sigma_{KH}}$ to $1 = \frac{\Sigma_{KH}}{\Sigma_{KH}}$ acc rding to the law expressed by curve II, a correction must be introduced into formula (70) by multiplying the ordinates of curve I by the ordinates of curve II; the obtained products are given in the form of curve III.

The ratio of the area $OABp_Д C''O = \int \frac{\Sigma}{\Sigma_{KH}} p\,dt$ to l_K will then show the part of the losses computed by means of formula (70) that must be taken in order to determine the loss obtained due to the simultaneous change of the area and the pressure. Miuraur had found that this ratio must be equal to 0.43÷0.46.

Thus the loss due to heat transfer and the relative drop in temperature due to this loss will be expressed by the following formula:

$$\frac{\Delta T}{T}\% = \frac{C_M}{7.774} \frac{\Sigma_{KH}}{\sqrt{\omega_{KH}}} \frac{1}{\Delta_Д} \frac{1}{l_K} \int_0^t \frac{\Sigma}{\Sigma_{KH}} p\,dt. \tag{71}$$

The losses computed in this manner for various weapons amount to

1% in the case of a 152-mm cannon and to about 15% for a rifle.

Therefore cooling through the walls when a shot is fired varies within very wide limits and amounts to from 1 to 15% of the total heat energy; thus whereas an error of 1% could be disregarded, no such allowance can be made in the case of a 15% error, and the latter error must therefore be accounted for.

The method used by Miuraur for determining losses incurred during a shot requires the availability of pressure and distance or path curves as a function of time, which data are usually obtained by means of numerical integration or from АНИИ (ANII) or ГАУ (GAU) tables.

The author had shown that the heat lost to the walls of the barrel can be calculated also in the absence of the p and ℓ curves as a function of time.

Bearing in mind that $W_{KH}\Delta_{KH} = W_0\Delta_0 = \omega$, $pdt = dI$, the above formula can be rewritten thus:

$$\left(\frac{\Delta T}{T}\right)\% = \frac{C_M}{7.774}\frac{\Sigma_{KH}}{\omega}\int_0^I \frac{\Sigma\, dI}{\Sigma_{KH} I_K} = \frac{C_M}{7.774}\frac{\Sigma_0}{\omega}\int_0^I \frac{\Sigma}{\Sigma_0}\frac{dI}{I_K}. \qquad (72)$$

But from the equation of motion

$$\varphi m dv = spdt = sdI,$$

whence

$$dI = \frac{\varphi m}{s}dv, \quad I_K = \frac{\varphi m}{s}v_K' = \frac{\varphi m}{s}(v_0' + v_K),$$

where v_0' is the velocity the projectile would have attained if it had started to move as if there were no rifling at the instant a

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portion of the charge ψ_0 is burned, corresponding to the true pressure:

$$v_0' = \frac{s}{\varphi m} I_0 = \frac{s}{\varphi m} \int_0^{\psi_0} p dt.$$

This value serves as a means for determining the heat lost in the chamber due to transfer before the projectile starts moving. The ratio $\frac{dI}{I_K}$ can be replaced by the ratio

$$\frac{dv}{v_K'} = \frac{dv}{v_0' + v_K}.$$

$\sum_{KH}$ represents the total surface area of the chamber $\sum_0$ and of the rifled portion of the bore $\pi d'' \ell_\pi$, where d'' is the reduced diameter of the circle having the same perimeter as the perimeter of the cross section of the bore including the rifling bores. If the number of grooves is n, and their depth is t_H,

$$\pi d'' = \pi (d + t_H) + 2nt_H = \pi d \left[1 + \frac{t_H}{d} \left(1 + \frac{2n}{\pi} \right) \right].$$

Inasmuch as in the barrels of artillery pieces, $t_H = 0.01\text{-}0.02d$,

$$d'' = d[1 + (0.01...0.02)(1 + 0.64n)]$$

when n = 28.

$$d'' = d[1 + (0.01...0.02)18.9] = d[1.19...1.38] = d[1 + \alpha_1].$$

The numerals in parenthesis show that the increase of the cooling surface due to the presence of rifling is very great and becomes greater with the increase of n and t_n/d.

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Inasmuch as the mean diameter of the powder chamber is

$$D = d\sqrt{\chi},$$

where $\chi = \dfrac{\ell_0}{\ell_{KM}}$ is the widening coefficient of the chamber greater than unity,

$$\Sigma_0 = \pi d\sqrt{\chi}\,\ell_{KM} + 2\,\frac{\pi}{4}d^{\prime\prime 2} = \pi d\,\ell_{KM}\left\{\sqrt{\chi} + \frac{1}{2}\,\frac{d}{\ell_{KM}}\,[1 + a_1]\right\}.$$

Assuming for the sake of simplicity that

$$\ell_{KM}\left\{\sqrt{\chi} + \frac{1}{2}\,\frac{d^{\prime\prime}}{\ell_{KM}}\,[1 + a_1]\right\} \approx \ell_0\,[1 + a_1],$$

where $\ell_0 = \dfrac{w_0}{s}$ is the reduced length of the chamber, we obtain expressions for the areas of the bore at the start, at the end and at an intermediate instant:

$$\Sigma_0 = \pi d(1 + a_1)\,\ell_0;$$

$$\Sigma_{KH} = \pi d(1 + a_1)(\ell_0 + \ell_{\text{д}});$$

$$\Sigma = \pi d(1 + a_1)(\ell_0 + \ell);$$

$$\frac{\Sigma}{\Sigma_0} = \frac{\ell_0 + \ell}{\ell_0} = 1 + \frac{\ell}{\ell_0};\quad \frac{\Sigma}{\Sigma_{KH}} = \frac{\ell_0 + \ell}{\ell_0 + \ell_{\text{д}}}.$$

Replacing $\dfrac{dI}{I_K}$ and $\dfrac{\Sigma}{\Sigma_0}$ in formula (72) by their corresponding expressions, we get a relationship for the heat transfer occurring while the projectile moves through the bore in the form:

$$\frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\int_0^I \frac{\Sigma}{\Sigma_0}\,\frac{dI}{I_K} = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\int_0^v \frac{(\ell_0+\ell)\,dv}{\ell_0(v_0'+v_K)}. \tag{73}$$

To this must be added the heat losses in the chamber during the preliminary period, determined by the analogous formula

$$\frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{I_0}{I_K} = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{\ell_0 v_0'}{\ell_0(v_0'+v_K)}. \tag{74}$$

Adding formulas (73) and (74), we get an expression for the gas temperature drop occurring while the projectile is in motion due to heat lost to the walls:

$$\left(\frac{\Delta T}{T}\right)\% = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{\ell_0(v_0'+v)+\int_0^{v_{Д}}\ell\,dv}{\ell_0(v_0'+v_K)} = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\,\eta. \tag{75}$$

Prior to the start of motion, $v = 0$, and expression (75) is transformed into (70). Prior to the end of travel in the bore, $v = v_{Д}$, so that:

$$\left(\frac{\Delta T}{T}\right)_{Д}\% = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{\ell_0(v_0'+v_{Д})+\int_0^{v_{Д}}\ell\,dv}{\ell_0(v_0'+v_K)} = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\,\eta_{Д}. \tag{76}$$

A curve depicting the velocities of the projectile as a function of the traversed path ℓ is adequate for the purpose of calculating this loss.

The ℓ, v diagram in fig. 110 constructed by the author indicates

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that η_R represents a ratio of the areas

$$\eta_R = \frac{\text{area aoefhda}}{\text{area abcd}};$$

the rectangle aghd characterizes the loss incurred in the chamber, the area of the curvilinear figure oefg represents the loss in the rifled portion of the barrel, and the rectangle aokd represents the loss in the preliminary period.

The factor η_R represents the ratio between the total heat lost to the walls of the entire bore, which takes into account the fact that the rifled portion enters into action gradually, as the projectile moves through it, and the heat lost in the chamber at the end of burning of the powder; the area of the chamber includes the area of the base of the projectile.

By analogy, the coefficient η characterizes the heat transfer at a given time.

When computing the cross-hatched area in fig. 110, v is the independent variable and ℓ is the dependent one. But the situation will not change if we were to turn the graph around in such a way as to obtain an ordinary curve of velocities v as a function of the path ℓ. In such a case we would have a v, ℓ graph (fig. 111) instead of an ℓ, v graph, wherein the cross-hatched area lies above the v, ℓ curve. The numerical values of η and η_R will remain the same, but due to the change of the coordinates the expression for the area in the numerator will be different:

$$\eta = \frac{\ell_0(v_0' + v) + v\ell - \int_0^{\ell} v\,d\ell}{\ell_0(v_0' + v_K)};$$

$$\eta_Д = \frac{\ell_0(v_0' + v_Д) + v_Д \ell_Д - \int_0^{\ell_Д} v\,d\ell}{\ell_0(v_0' + v_K)}.$$

The relation between v and ℓ can be found in the ANII or GAU tables by means of the ratio

$$\frac{\ell}{\ell_0} = \ldots$$

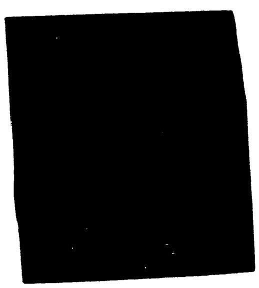

Fig. 110 - Diagram for Computing the Heat Transfer in Terms of ℓ, v Coordinates

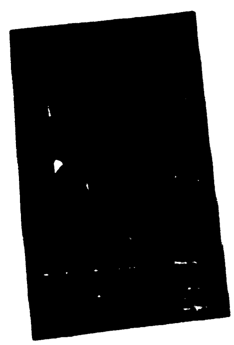

Fig. 111 - Diagram for Computing the Heat Transfer in Terms of v, ℓ Coordinates

Dividing the numerator and denominator of η by ℓ_0, disregarding the value of v_0' in the numerator, and bearing in mind that

$$v_0' + v_K = \frac{sI_K}{\varphi m} = v_1,$$

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we get

$$\eta = \frac{v(1+\Lambda) - \int_0^{\lambda} v\,d\lambda}{v_1} = (1+\Lambda)\frac{v}{v_1} - \int_0^{\lambda} \frac{v}{v_1}\,d\Lambda;$$

$$\frac{v}{v_1} = \frac{v_{table}\sqrt{\varphi\frac{\omega}{q}\varphi_{\text{п}}}}{sl_K} = v_{table}\sqrt{\frac{\omega\varphi^2{}_{\text{п}}{}^2}{\omega g s^2 l_K^2}} = v_{table}\sqrt{\frac{f\omega\varphi_{\text{п}}}{s^2 l_K^2 f g}} =$$

$$= \sqrt{\frac{1}{B}}\,\frac{v_{table\,\text{д}}}{\sqrt{fg}}.$$

On the basis of the constants assumed in the ANII tables 95,000 kg-m/kg; $K = 9.81$ m/sec^2; $\varphi = 1.05$; $\sqrt{fg} = 955$ and $\frac{v}{v_1} =$
$= \frac{v_{table}\ \text{m/sec}}{955\sqrt{B}}$.

Therefore

$$\eta = \frac{1}{955\sqrt{B}}\left[(1+\Lambda)v_{table} - \int_0^{\Lambda} v_{table}\,d\Lambda\right];$$

$$\eta_{\text{д}} = \frac{1}{955\sqrt{B}}\left[(1+\Lambda)v_{table} - \int_0^{\Lambda_{\text{д}}} v_{table}\,d\Lambda\right].$$

Incorporating these expressions in formulas (75) and (76), we will find the heat transfer losses incurred when a shot is fired from the ANII tables.

It should be noted that when calculating $\frac{\Sigma_0}{\omega} = \frac{\Sigma_0}{W_0 \Delta_0}$, the Σ_0 and W_0 must be expressed in cm^2 and cm^3.

Results of Calculations. Calculations for a 76-mm cannon of 1902 issue measuring 30 calibers in length show that for a normal charge $\left(\frac{\Delta T}{T}\right)\% \approx 1.8$; when the charge is reduced the loss increases somewhat and amounts to 2.2%. Upon increasing the length of the barrel to 50 calibers the loss incurred with a normal charge equals 2.5%, and is 3.1% for a reduced charge.

If expression (76) is presented in the form

$$\left(\frac{\Delta T}{T}\right)_Д \% = K_T \eta_Д,$$

where

$$K_T = \frac{C_M}{7.774} \frac{\Sigma_0}{W_0} \frac{1}{\Delta},$$

we shall have the following data (Table 26) for a 76-mm cannon of 1900 issue, using CП strip powder ($2e_1 = 1.03$ mm) (based on N.A. Zabudsky's tests) and $C_M = 4.6\%$, when varying the charge.

Table 26

ω	1.041	0.892	0.725	0.558
Δ	0.613	0.525	0.426	0.328
K_T	0.553	0.646	0.795	1.034
$\eta_К$	1.36	1.51	1.75	2.24
$\eta_Д$	3.01	2.83	2.54	2.11
$\left(\frac{\Delta T}{T}\right)_К \%$	0.75	0.98	1.39	2.32
$\left(\frac{\Delta T}{T}\right)_Д \%$	1.67	1.83	2.00	2.18
$\ell_К + \ell_0$	7.79	9.98	14.23	25.05

This table shows that when the charge is decreased, the heat transfer loss increases from 1.7 to 2.2%. When the barrel length is increased to 50 calibers, the percentage value increases correspondingly to 2.3 and 3.1. When the charge is decreased the drop in temperature at the end of burning becomes greater due to the increased cooling surface area, because the end of burning is transposed towards the muzzle (see line $\ell_K + \ell_0$).

Calculations for systems of different calibers have shown that when the caliber is increased the heat transfer losses decrease because of the reduced value of $\frac{\Sigma_0}{w_0}$, and notwithstanding the increase of CM which increases slowly in the case of thick powders, η_λ fluctuates between the limits of 2.2% and 3.7%. The heat loss in a rifle amounts to 10% and must be accounted for by reducing correspondingly the powder energy $f = RT_1$, i.e., as if the burning temperature of the powder were reduced. For guns varying from 37 to 76 mm in caliber, the energy f determined in bomb without corrections for heat transfer may be considered acceptable, because the heat losses obtained in such guns and in bombs are about the same (2-3%).

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SECTION V - PHENOMENA ASSOCIATED WITH GAS DISCHARGE

GENERAL COMMENTS

The fundamental equations of internal ballistics permit the solution of the problem dealing with the motion of the projectile up to the instant at which the base of the shell traverses the muzzle face of the barrel.

However, the action of the gases continues or persists even after the projectile's departure from the bore. Both the projectile and the barrel experience for a certain period of time the so-called "after-effect" of the powder gases in the form of continued gas pressure exerted on both the projectile and the barrel. While it is possible to approximately determine the theoretical relation between the pressures acting on the base of the bore and the base of the projectile while the latter is travelling through the bore, this relationship will be entirely different after the projectile has left the bore. That portion of the gas which, upon leaving the bore, continues to exert a pressure on the projectile for a certain period of time, will be subjected to conditions differing sharply from the conditions to which the gases still remaining in the barrel are subjected.

The gases remaining in the barrel continue in their motion along the axis of the bore from the base of the barrel to its muzzle face, and upon leaving the bore commence to disperse somewhat from their basic direction of motion. The resulting reaction imparts additional acceleration to the recoiling barrel and the maximum speed of recoil obtains after the projectile's departure from the barrel. Almost the entire mass of gases generated by the charge participates in the reaction on the barrel. After the projectile's departure, only

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the central portion of the gases which had retained its axial motion along the bore continues to react on the base of the barrel; the greater portion of the gas either temporarily overtakes the projectile or is dissipated laterally in the form of a gas cloud. Inasmuch as the density of the gas discharge rapidly drops as the gases expand, the gases lose speed very rapidly and fall behind the projectile. Nevertheless, the projectile's speed will continue to increase somewhat even while the gases are lagging behind it. During this period the fuze mechanisms are triggered and go into action. A study of the projectile's motion after its departure from the bore constitutes one of the problems of internal ballistics.

The period of gas after-action, or the third period, which is a direct continuation of the shot phenomenon accompanied by gas discharge from the gun barrel, constitutes a problem involving the derivation of special fundamental relations for its solution. This, in turn, requires a knowledge of the fundamental laws of gas discharge.

General relations of gas dynamics are also required for the solution of special problems arising from the complex structure of artillery weapons and with the appearance of new systems, in which the gases are discharged from ports of various types.

Such systems may include:

1) Automatic weapons in which the gases are discharged from the bore before the projectile's departure, or guns with muzzle attachments.

2) Weapons with separate powder chambers with gas discharge through a single or multiple nozzles (weapons operating on the principle of gas dynamics or hydrodynamics).

3) Recoilless guns - in which the gases are discharged through an opening in the breechblock.

4) Muzzle brakes, in which the gases are discharged through passageways laterally, and by exerting pressure on the walls of these passageways brake the recoil and reduce the recoiling speed.

5) Mine throwers, in which a portion of the gases overtakes the mine while it is moving through the bore; special mine throwers with a remotely controlled valve, in which a portion of the gases is discharged through the valve and does not participate in the action exerted on the mine; furthermore, mine throwers are provided with means for discharging the gas from the inner chamber, containing the tail cartridge and the main charge, into a chamber containing additional charge.

6) Special manometric bombs with nozzles used for studying the powder burning phenomenon by means of gas discharge through a nozzle.

7) Rocket chambers.

In all the systems mentioned above, the gas is discharged through ports of varying shapes and sizes under high pressure. In order to establish the proper relations, taking into account all of these phenomena and their peculiarities, use must be made of the general formulas relating to gas discharge. Therefore, all the derived relations constitute first approximations only and require further refinement on the basis of test data obtained during firing of weapons.

CHAPTER 1 - GENERAL INFORMATION ON GAS DYNAMICS

1. RATE OF GAS DISCHARGE

Using the designations:

U, V, W - gas velocities projected on the coordinate axes;

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X, Y, Z - projections of external volume forces on the same axes;

ρ' - density of a unit mass of gas;

p - pressure,

then the fundamental Euler's equation of hydrodynamics with respect to the x-axis will read as follows:

$$\frac{dU}{dt} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} = X - \frac{1}{\rho'}\frac{\partial p}{\partial x} \qquad (77)$$

and will be analogous in character with respect to the other axes.

We shall consider the gas discharge as a single-dimensional motion in the direction of the x-axis, due to pressure difference, in the absence of external forces (X = 0):

$$\frac{dU}{dt} = \frac{\partial U}{dt} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho'}\frac{\partial p}{\partial x}. \qquad (78)$$

The values of p and U for a stabilized motion do not depend on t and are functions of x only. In this case $\frac{\partial U}{\partial t} = 0$:

$$U\frac{dU}{dx} = -\frac{1}{\rho'}\frac{dp}{dx}$$

or

$$-\frac{dp}{\rho'} = UdU = d\,\frac{U^2}{2}.$$

Replacing the mass density ρ' by gravimetric density ρ and bearing in mind that $\rho = 1/w$, where w is the specific volume of gas, we get:

$$-w\,dp = d\,\frac{U^2}{2g}. \tag{79}$$

If we designate the pressure, specific volume and velocity in the vessel from which the gases are discharged by p_1, w_1, U_1, respectively, then, upon integrating expression (79), we will get

$$-\int_{p_1}^{p} w\,dp = \int_{p}^{p_1} w\,dp = \frac{U^2 - U_1^2}{2g}. \tag{80}$$

In order to integrate the left side of equation (80), the dependence of w on p for the process taking place in the gas must be known. We shall consider the polytropic process, of which the adiabatic process constitutes a particular case:

$$pw^k = p_1 w_1^k = \text{const},$$

whence

$$w = \frac{1}{\rho} = w_1\,\frac{p_1^{1/k}}{p^{1/k}}.$$

Substituting this expression in equation (80) and integrating, we get:

$$w_1 p_1^{1/k} \int_{p_1}^{p} \frac{dp}{p^{1/k}} = \frac{U^2 - U_1^2}{2g};$$

$$w_1 p_1^{1/k} \frac{k}{k-1}\left(p_1^{k-1/k} - p^{k-1/k}\right) = \frac{k}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right] =$$

$$= \frac{U^2 - U_1^2}{2g},$$

whence

$$U = \sqrt{U_1^2 + \frac{2gk}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]}. \tag{81}$$

This is Saint-Venant's formula.

If we assume that $U_1 = 0$ when the discharge is from a very large vessel, we shall get an expression for the velocity of the gas discharged into space with a pressure p from the vessel under pressure p_1.

$$U = \sqrt{\frac{2gk}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]}. \tag{82}$$

The maximum velocity will obtain when the discharge is into vacuum, when $p = 0$; we will have:

$$U_{max} = \sqrt{\frac{2gk}{k-1} p_1 w_1}.$$

however, we know from physics that $\sqrt{gkp_1w_1} = \sqrt{gkRT_1} = C_1$ is the speed of sound in gas, corresponding to the given condition p_1 and w_1 or T_1 of the gases present in the vessel from which the gases

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are discharged. Hence,

$$U_{max} = \sqrt{\frac{2}{k-1}}\, C_1 .$$

Fig. 112 - Dependence of Velocity of Gas Discharge on p/p_1.

Substituting this expression in equation (82), we get:

$$U = U_{max}\sqrt{1-\left(\frac{p}{p_1}\right)^{\frac{k-1}{k}}} = C_1\sqrt{\frac{2}{k-1}\left[1-\left(\frac{p}{p_1}\right)^{\frac{k-1}{k}}\right]} .$$

The dependence of the velocity of discharge on pressure p or the ratio p/p_1 is depicted in fig. 112. When $p = 0$, $U = U_{max}$. As the back pressure p increases, U decreases and has a point of inflexion ($U_{cr.}$, $x_{cr.}$); and when $p/p_1 = 1$ it becomes zero, i.e., the discharge ceases.

We shall discuss the values $x_{cr.}$ and $U_{cr.}$ later in the text.

2. THE GRAVIMETRIC CONSUMPTION OF GASES G_{sec} PER SECOND

If the stream of gases discharged under a high pressure has a velocity U, density $\rho = 1/w$ and a cross-sectional area s, the gas consumption per second will be:

$$G_{sec} = sU\rho = s\rho\sqrt{\frac{2gk}{k-1}p_1 w_1\left[1-\left(\frac{p}{p_1}\right)^{k-1/k}\right]}. \qquad (83)$$

Since for a polytropic process

$$\rho = \frac{1}{w} = \frac{1}{w_1}\left(\frac{p}{p_1}\right)^{1/k},$$

then, substituting this expression in the formula for determining the expenditure per second, we get:

$$G_{sec} = s\frac{1}{w_1}\left(\frac{p}{p_1}\right)^{1/k}\sqrt{\frac{2gk}{k-1}p_1 w_1\left[1-\left(\frac{p}{p_1}\right)^{k-1/k}\right]} =$$

$$= s\sqrt{\frac{2gk}{k-1}\frac{p_1}{w_1}}\sqrt{\left(\frac{p}{p_1}\right)^{2/k}-\left(\frac{p}{p_1}\right)^{k+1/k}} \qquad (84)$$

(Tseiner's formula).

Designating the p/p_1 ratio by x and the constant $\sqrt{\frac{2gk}{k-1}\frac{p_1}{w_1}}$ by a_1, we find:

$$G_{sec} = a_1 s\sqrt{x^{2/k} - x^{k+1/k}} = a_1 sf(x).$$

When the motion is steady, the consumption per second is constant; therefore,

$$sf(x) = s\sqrt{x^{2/k} - x^{k+1/k}} = \frac{G_{sec}}{a_1} = \text{const}$$

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and

$$s = \frac{\text{const}}{f(x)},$$

i.e., the cross section of the stream varies inversely with the change $f(x)$ depending on p/p_1.

Investigations show that $f(x)$ has a maximum when the value $\frac{p_{cr.}}{p_1} = x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1}$; therefore, the cross-sectional area s will be minimum at this value. The pressure ratio $p_{cr.}/p_1$ at which the cross section of the stream is minimum and the flow through a unit cross-sectional area is maximum is called the critical pressure ratio, and the cross section is called the critical cross section.

The value $x_{cr.}$ depends on the polytropic index, though to a small degree only. The following table gives the values of $x_{cr.}$ with relation to k (Table 27).

Table 27

k	1.41	1.30	1.25	1.20	1.10
$x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1}$	0.527	0.546	0.555	0.565	0.585

Substituting the value $x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1}$ in formula (82) for the discharge velocity, we get the following expression for the "critical" gas velocity:

$$U_{cr.} = \sqrt{\frac{2gk}{k+1}p_1 v_1} = \sqrt{\frac{2}{k+1}}c_1.$$

This value approaches the speed of sound in a gas located in a vessel, from which the discharge takes place, and whose equation of

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state is determined by the values p_1 and w_1.

Since from the adiabatic equation $p_1 w_1 = p_{cr.} w_{cr.} \frac{k+1}{2}$, the expression for critical velocity will take on the form:

$$U_{cr.} = \sqrt{gkp_{cr.} w_{cr.}} = c_{cr.},$$

i.e., the critical velocity at the minimal cross section at the point of critical pressure equals the velocity of sound, corresponding to the state of gas at this critical pressure. This velocity is shown in fig. 112 in the form of segment $U_{cr.}$ at $x_{cr.}$.

Having determined the critical pressure and velocity of the gases, we shall now find the consumption through the smallest cross section (which we shall designate by s_m). To do this, we substitute in the right side of formula (84) the value

$$\frac{p_{cr.}}{p_1} = x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1};$$

$$G_{sec} = s_m (x_{cr.})^{1/k} \sqrt{\frac{2gk}{k-1} \frac{p_1}{w_1} \left[1 - x_{cr.}^{k-1/k}\right]} =$$

$$= s_m \left(\frac{2}{k+1}\right)^{1/k-1} \sqrt{\frac{2gk}{k+1} \frac{p_1}{w_1}} = K_0 s_m \sqrt{\frac{p_1}{w_1}}.$$

Here the coefficient $K_0 = \sqrt{\frac{2gk}{k+1}} \left(\frac{2}{k+1}\right)^{1/k-1}$ is a constant which, depending on the exponent k, varies within small limits in accordance with Table 28 ($g = 98.1$ dm/sec^2).

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Table 28

k	1.25	1.20	1.15	1.10
$\sqrt{\frac{2gk}{k+1}}\left(\frac{2}{k+1}\right)^{1/k-1} = K_0$	6.518	6.424	6.325	6.224

For the vessel in which the powder is burned, the expression for G_{sec} can be presented differently, by multiplying and dividing the expression under the root by p_1. Replacing, approximately, $p_1 w_1 = RT_1$ by f, we get

$$G_{sec} = \frac{K_0}{\sqrt{f}} s_m p_1 = A s_m p_1, \qquad (85)$$

where $A = \frac{K_0}{\sqrt{f}}$ is a constant depending on the nature of the gases and their temperature, inasmuch as k is a function of the temperature;

s_m is the minimal cross section of the gas stream, which may be assumed to be an orifice with rounded edges or the minimum cross section in the Laval nozzle;

p_1 is the pressure at which the gases are discharged from the vessel.

Coefficient A, characterizing the consumption of gas at $s = 1$ and $p_1 = 1$ is measured in (sec^{-1}) and varies with f and k.

The coefficient A was first introduced by V.M. Trofimov who assumed A = 0.007 for pyroxylin powders and A = 0.006 for nitroglycerine powder.

Actually, when gas is discharged from a vessel, even in the case where powder is burned in the vessel, the temperature T inside the

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vessel will be lower than T_1, and the value $p_1 w_1 = RT$, where $T < T_1$. Hence, it would be more correct to state:

$$G_{sec} = \frac{K_0}{\sqrt{p_1 w_1}} s_m p_1 = \frac{K_0}{\sqrt{RT}} s_m p_1 = \frac{K_0}{\sqrt{f}\sqrt{\tau}} s_m p_1 = \frac{A}{\sqrt{\tau}} s_m p_1,$$

where $\tau = T/T_1$ (see below).

Table 29

f \ k	1.1	1.2	1.3
1,000,000	0.00622	0.00642	0.00661
900,000	0.00656	0.00677	0.00697
850,000	0.00675	0.00695	0.00717
800,000	0.00695	0.00718	0.00739

3. FULL GAS CONSUMPTION

The full gas consumption Y over a period t can be obtained from the expression

$$Y = \int_0^t G_{sec}\, dt.$$

We proved earlier that

$$G_{sec} = A s_m p_1,$$

where p_1 - pressure in the vessel from which gas is discharged;

s_m - cross-sectional area of opening or orifice through which the gas flows.

If we apply this formula to a bomb with a nozzle, in which

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pressure p is developed when the powder is burned, then

$$G_{sec} = As_m p,$$

where $p = f(t)$,

$$Y = As_m \int_0^t p\,dt = As_m I,$$

but $I = \frac{e}{u_1}$.

During the entire period that the powder is burned $\int_0^t p\,dt = I_K$, and hence the full consumption during the entire period of powder burning will be

$$Y_K = As_m I_K = As_m \frac{e_1}{u_1}.$$

When the cross section s_m of the nozzle, the nature of the powder gases and their temperature [$A = f(k, f)$], the thickness of the powder and its rate of burning are known, this formula enables us to compute in advance the consumption of powder by weight during the period the powder is burned in the chamber or in a bomb with a nozzle. This formula has been satisfactorily confirmed in bomb tests, in which the start of burning of cylindrical grains with very narrow perforations - 1 to 3 mm in diameter, at pressures p_m = 2000-2500 kg/cm^2 has been investigated.

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4. THE DEPENDENCE OF GAS PRESSURE ON THE CROSS SECTION OF THE STREAM(*)

If the gases are discharged through a tapered diverging nozzle, the pressure in the direction of flow will decrease, whereas the velocity of discharge will increase. The pressure magnitudes at various sections can be found from the equation of continuity, because $G_{cr.} = G_x$, where G_x is the flow through section s_x. The equation of continuity will be written in the form:

$$\frac{s_m U_{cr.}}{w_{cr.}} = \frac{s_x U_x}{w_x},$$

but

$$\frac{s_m U_{cr.}}{w_{cr.}} = s_m K_0 \sqrt{\frac{p_1}{w_1}} = s_m \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2gk}{k+1} \frac{p_1}{w_1}},$$

and according to formula (84)

$$\frac{s_x U_x}{w_x} = s_x \sqrt{\frac{2gk}{k-1} \frac{p_1}{w_1}} \sqrt{x^{\frac{2}{k}} \left(1 - x^{\frac{k-1}{k}}\right)}.$$

Equating the right sides of these equations and assuming that $k = \text{const}$ from one section to another, and reducing by $\sqrt{2gk \frac{p_1}{w_1}}$, we get:

(*) The derivation and numerical data are taken from the book by Prof. I.P. Grave, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics). Pyrodynamics, 3rd Edition, p. 217.

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$$s_x x^{1/k} \sqrt{\frac{1 - x^{k-1/k}}{k - 1}} = s_m \left(\frac{2}{k+1}\right)^{1/k-1} \sqrt{\frac{1}{k+1}},$$

whence

$$\frac{s_x}{s_m} = \left(\frac{2}{k+1}\right)^{1/k-1} \frac{\sqrt{\frac{k-1}{k+1}}}{x^{1/k}\sqrt{1 - x^{k-1/k}}}. \qquad (86)$$

This equality gives the dependence of the relative pressure x in the region back of the minimum cross section on the relative cross section of the nozzle s_x/s_m. Upon assigning values of x for various values of k, a table can be compiled for the values of s_x/s_m, according to which an inverse problem can be solved: i.e., the value of the ratio $x = p/p_1$ (Table 30) can be found from the ratio of the cross section of the flow at a given point to its minimal cross section.

Table 30 – Values of s_x/s_m for Various x and k

k \ x		1/2	1/3	1/4	1/5	1/6	1/10	1/15	1/20
1.1	1	1.018	1.180	1.373	1.569	1.762	2.500	3.364	4.180
1.2	1	1.010	1.143	1.309	1.477	2.640	2.260	2.967	3.625
1.25	1	1.007	1.128	1.282	1.438	1.590	2.162	2.802	3.405
1.3	1	1.005	1.115	1.258	1.404	1.545	2.075	2.670	3.214
1.4	1	1.002	1.093	1.218	1.346	1.470	1.931	2.440	2.900

__Example.__ Determine how much the cross section of the stream must be increased in order to obtain a 10-fold pressure decrease (at k = 1.25). At k = 1.25 and x = 1/10, we get $s_x/s_m = 2.162$.

5. EXPRESSION FOR DETERMINING THE REACTION PRESSURE DEVELOPED DURING GAS DISCHARGE THROUGH AN OPENING IN THE WALL OF THE VESSEL (TRACTIVE FORCE)

Let us assume (fig. 113) that a gas in a closed vessel is under pressure p. To each element of the surface s there is applied a force sp or $s(p - p_a)$, where p_a is atmospheric pressure. The velocity of the gas inside the vessel is $U_1 = 0$. When the opening of area s is opened, the gases will be discharged through it, and the vessel will be subjected to a reacting force composed of the following components:

Fig. 113 - Diagram of Gas Discharge and the Reaction Pressure

1) The force $R' = s(p - p_a) \approx sp$, acting in all directions before the orifice is opened, and reacting in a direction opposite to the flow of the gases when a portion of the wall of area s disappears.

2) The force R'', originating in consequence of the gas discharge through orifice s under the action of internal forces and determined on the basis of the mechanics theory concerned with momentum and force impulse.

The elementary mass dm discharged through area s during time dt acquires a velocity U and an increment of momentum dmU; it creates a force impulse R''dt in the reverse direction:

$$R''dt = dmU = \frac{\gamma}{g}\, sUdtU = \frac{G_{sec}}{g} dtU,$$

whence

$$R'' = \frac{G_{sec}}{g} U.$$

The full reaction pressure or the tractive force originating upon the discharge of gas through opening s will be expressed by the formula:

$$R = R' + R'' = \frac{G_{sec}}{g}\, U + sp.$$

It is assumed thereby that the gas velocity inside the vessel is $U_1 = 0$ and that hence no change in gas momentum takes place in a vessel of a sufficiently large capacity.

If we apply this formula to the discharge opening of a diverging tapered nozzle of cross section s_a, whereby $p = p_a$ and $U = U'_a$, then

$$R = \frac{G_{sec}}{g} U_a + s_a p_a.$$

Replacing G_{sec}, U_a, s_a and p_a by values relating to the minimum cross section s_m and internal pressure p_1, we get, on the basis of formulas (82) and (86)(*):

(*) I.P. Grave, "PIRODINAMIKA" (Pyrodynamics), Part III.

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$$R = \frac{G_{sec} \cdot U_a}{g} + p_a s_a =$$

$$= \frac{s_m}{g} \sqrt{gk} \left(\frac{2}{k+1} \right)^{k+1/k-1} \sqrt{\frac{p_1}{w_1}} \sqrt{\frac{2gk}{k-1} \sqrt{p_1 w_1}} \sqrt{1 - \left(\frac{p_a}{p_1} \right)^{k-1/k}} +$$

$$+ \frac{p_a}{p_1} p_1 \frac{s_a}{s_m} s_m = s_m \left[k \left(\frac{2}{k+1} \right)^{1/k-1} \sqrt{\frac{k+1}{k-1}} \, p_1 \sqrt{1 - x_a^{k-1/k}} + \frac{x_a p_1 \left(\frac{2}{k+1} \right)^{1/k-1} \sqrt{\frac{k-1}{k+1}}}{x_a^{1/k} \sqrt{1 - x_a^{k-1/k}}} \right] =$$

$$= k x_m \sqrt{\frac{k+1}{k-1}} \sqrt{1 - x_a^{k-1/k}} \left[1 + \frac{k-1}{2k} \, \frac{x_a^{k-1/k}}{1 - x_a^{k-1/k}} \right] p_1 s_m. \qquad (87)$$

Only the k, x_a, p_1 and s_m values enter this expression. It may be thus concluded that the value of R is proportional to pressure p_1 inside the vessel and the area of smallest cross section s_m; it depends on the exponent k and is determined by the degree of divergence of the nozzle which, in turn, depends on the ratio s_a/s_m. Formula (87) can be presented in an abbreviated form:

$$R = \zeta \, s_m p_1,$$

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assuming that

$$\zeta = k\left(\frac{2}{k+1}\right)^{k/k-1}\sqrt{\frac{k+1}{k-1}}\sqrt{1 - x_a^{k-1/k}}\left[1 + \frac{k-1}{2k}\,\frac{x_a^{k-1/k}}{1 - x_a^{k-1/k}}\right]. \tag{88}$$

The coefficient ζ for a given nozzle depends on k only; it depends on the nature of the powder only to the extent that it determines the value of k; it does not depend on the charging density, nor on the value of p.

Langevin calls this coefficient the propulsive action coefficient. In the absence of a nozzle, and if only an opening were present in the wall, then, at

$$x_a = x_m = \left(\frac{2}{k+1}\right)^{k/k-1}$$

we would obtain

$$\zeta_0 = k\left(\frac{2}{k+1}\right)^{k/k-1}\left(1 + \frac{k-1}{2k}\,\frac{\frac{2}{k+1}}{\frac{k-1}{k+1}}\right) =$$

$$= (k+1)\left(\frac{2}{k+1}\right)^{k/k-1} = (k+1)x_m. \tag{89}$$

If the nozzle were infinitely large and permitted infinite divergence, and if the outside pressure were disregarded (in other words, if the discharge were into vacuum), then:

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$$\zeta_{max} = k \sqrt{\frac{k+1}{k-1}} \left(\frac{2}{k+1}\right)^{k/k-1} = k \sqrt{\frac{k+1}{k-1}} x_m .$$

The following table gives the dependence of coefficient ζ on the ratio between the discharge opening diameter d_a and the diameter of minimum cross section (Table 31).

Table 31

$\frac{d_a}{d_m}$	1	2	3	4	5	6
ζ	1.24	1.62	1.72	1.80	1.86	1.89
$\frac{s_a}{s_m}$	1	4	9	16	25	36

It can be seen from the table that as the outfit diameter of the nozzle increases, the reaction pressure increases rapidly at first and then slower and slower, approaching asymptotically the value ζ_{max}. In rocket shells it is customary to take $\frac{d_a}{d_m} > 3$ in order not be make the shell unnecessarily heavy. The coefficient ζ changes very little with the change of k.

6. FUNDAMENTAL FORMULAS

Thus, as a result of applying the laws of gas dynamics, the following relations have been established.

Gas discharge velocity:

$$U = \sqrt{\frac{2gk}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]} ,$$

where p_1 and w_1 are the pressure and unit volume of gases in the vessel from which the discharge takes place.

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Gas consumption through cross section s_m per second:

$$G_{sec} = As_m p_1,$$

where A - a coefficient depending on the nature of the powder (f and $k = 1 + \Theta$);

$A \approx 0.007$ for pyroxylin powders;

$A \approx 0.0065$ for nitroglycerine powders.

The gas consumption in time t is

$$Y = \int_0^t G_{sec} dt = As_m \int_0^t p_1 dt.$$

If the gases are formed in the vessel in consequence of powder burning, then

$$\int_0^t pdt = \frac{e}{u_1}; \quad Y = As_m \frac{e}{u_1};$$

the full gas consumption during the powder burning period is

$$Y_K = As_m \frac{e_1}{u_1}.$$

The reaction force of the discharged gases is

$$R = \frac{G_{sec}}{g} U + sp_1 = \zeta s_m p_1,$$

where ζ is given in the above table and is practically independent of the coefficient $k = 1 + \Theta$. Thus the basic values G_{sec}, Y and R are very simple functions of ballistic elements and of the gas pressure

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characteristics p in the vessel and of the pressure impulse at a given instant:

$$\int_0^t pdt = I = \frac{e}{u_1}$$

and at the end of burning

$$I_K = \int_0^{t_K} pdt = \frac{e_1}{u_1}.$$

In some cases the movement of gas inside the vessel from which it is discharged cannot be disregarded, as, for example, in the case of gases discharged from the bore of a gun, wherein the gas velocity varies linearly from zero at the base of the chamber to U_Δ at the face of the muzzle. In such a case an additional term will be added to the two components of the reaction pressure (force) depicting the change of gas momentum in the bore of the barrel:

$$R = \frac{G_{sec}}{g} U + sp_1 + \frac{dI}{dt}.$$

When the gas velocity changes linearly:

$$\frac{dI}{dt} = \mu \frac{U}{2} = \frac{\omega}{g} \frac{v_{Ag}}{2}.$$

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CHAPTER 2 - THE APPLICATION OF BASIC FORMULAS OF GAS DISCHARGE

1. GAS DISCHARGE FROM A VESSEL OF SPECIFIC VOLUME

Say, a volume W_0 contains ω kg of gas. The initial state of the gas is characterized by the values p_1, T_1, $w_1 = \frac{W_0}{\omega}$. It is necessary to determine the law governing the pressure and temperature drop as a function of time. As in the case of the general theory of discharge, we shall consider the process an adiabatic one. Then

$$\left(\frac{p}{p_1}\right)^{1/k} = \frac{w_1}{w},$$

but

$$w_1 = \frac{W_0}{\omega}, \quad w = \frac{W_0}{\omega - \int_0^t G_{sec}\,dt}.$$

Therefore,

$$\left(\frac{p}{p_1}\right)^{1/k} = \frac{\omega - \int_0^t G_{sec}\,dt}{\omega} = 1 - \frac{\int_0^t G_{sec}\,dt}{\omega}, \tag{90}$$

whereby

$$G_{sec} = sK_0 \sqrt{\frac{p}{w}}.$$

Inasmuch as

$$\frac{p}{w} = p_1 \frac{p}{p_1} \frac{1}{w_1} \left(\frac{p}{p_1}\right)^{1/k} = \frac{p_1}{w_1} \left(\frac{p}{p_1}\right)^{k+1/k},$$

$$G_{sec} = sK_0\sqrt{\frac{p_1}{v_1}}\left(\frac{p}{p_1}\right)^{k+1/2k}.$$

Differentiating expression (90), we get:

$$\frac{1}{k}\,\frac{p^{1-k/k}}{p_1^{1/k}}\,dp = -\frac{G_{sec}}{\omega}\,dt = -\frac{sK_0}{\omega}\sqrt{\frac{p_1}{v_1}}\left(\frac{p}{p_1}\right)^{k+1/2k}dt.$$

Separating the variables and integrating:

$$\left(\frac{p}{p_1}\right)^{1-k/k}\left(\frac{p_1}{p}\right)^{k+1/2k}\frac{dp}{p_1} = -\frac{ksK_0}{\omega}\sqrt{\frac{p_1}{v_1}}\,dt = -\,bdt,$$

where

$$b = \frac{ksK_0}{\omega}\sqrt{\frac{p_1}{v_1}};\quad x^{1-3k/2k}\,dx = -\,bdt;\quad \int_1^x x^{1-3k/2k}\,dx = -bt$$

or

$$\frac{2k}{k-1}\left[1 - \frac{1}{x^{k-1/2k}}\right] = -\frac{ksK_0}{\omega}\sqrt{\frac{p_1}{v_1}}\,t = -bt.$$

This enables us to find the duration of discharge when the pressure drops from the initial value p_1 to the given value p.

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$$t = \frac{1}{B'}\left[\frac{1}{x^{k-1/2k}} - 1\right],$$

where

$$B' = \frac{k-1}{2} s_m \frac{K_0}{\omega}\sqrt{\frac{p_1}{\omega_1}} \text{ and } K_0 = \sqrt{\frac{2gk}{k+1}}\left(\frac{2}{k+1}\right)^{1/k-1}.$$

This relationship is valid until the ratio between the outside and inside pressures becomes equal to the critical value. When the discharge is into the atmosphere

$$x_{cr.} = \frac{p_a}{p_{cr.}},\ p_{cr.} = \frac{p_a}{x_{cr.}} \approx 1.8 \text{ kg cm}^2.$$

The full time of discharge is

$$t_n = \frac{1}{B'}\left[\frac{1}{x_{cr.}^{k-1/2k}} - 1\right]. \qquad (91)$$

These formulas show that the length of discharge up to a given pressure is inversely proportional to the cross section of the nozzle s_m. Solving the formula with respect to $p = p_1 x$, we get the relative pressure change as a function of time:

$$p = \frac{p_1}{(1 + B't)^{2k/k-1}}. \qquad (92)$$

If the values of t are given, p can be calculated and a p,t curve can be constructed. The larger the cross section of the opening and the greater the value of p_1, the more rapid will be the relative pressure drop within the vessel. Inasmuch as

$$w = w_1 \left(\frac{p_1}{p}\right)^{1/k} \quad \text{and} \quad \frac{T}{T_1} = \left(\frac{w_1}{w}\right)^{k-1},$$

$$w = w_1(1 + B't)^{\frac{2}{k-1}} \tag{93}$$

and

$$T = \frac{T_1}{(1 + B'T)^2}. \tag{94}$$

A comparison of formulas (94) and (92) shows that the gas temperature drop inside the vessel occurs much slower than the pressure drop.

2. GAS DISCHARGE FROM THE BORE OF A GUN AFTER THE PROJECTILE LEAVES THE GUN

Applying the relations obtained above to the gases discharged from the barrel bore after the projectile leaves the latter, we will have the following: prior to the start of discharge the barrel will contain ω kg of gas, so that the specific gas volume within the entire volume of the bore $W_0 + sl_{\text{Д}}$ will be

$$w_{\text{Д}} = \frac{W_0 + sl_{\text{Д}}}{\omega} = \frac{W_{KH}}{\omega} = \frac{\Lambda_{\text{Д}} + 1}{\Delta}.\ (*)$$

(*) Subscripts Д and KH stand for "muzzle" and "bore," respectively.

The pressure at the atart of discharge equals the muzzle pressure p_{Δ}, the gas temperature is T_{Δ}, the cross-sectional area of the flow equals the cross-sectional area of the bore s.

Designating by B' the constant parameter

$$B' = \frac{k-1}{2} K_0 \sqrt{\frac{p_{\Delta}}{w_{\Delta}}} \frac{s}{\omega} = \frac{k-1}{2} K_0 \frac{s}{\omega} \sqrt{\frac{p_{\Delta} \Delta}{\Lambda_{\Delta} + 1}}$$

for pressure, gas temperature and the time of gas discharge, we will obtain the following expressions:

$$p = \frac{p_{\Delta}}{(1 + B't)^{2k/k-1}};$$

$$T = \frac{T_{\Delta}}{(1 + B't)^{2}};$$

$$t = \frac{1}{B'} \left[\left(\frac{p_{\Delta}}{p} \right)^{k-1/2k} - 1 \right].$$

All of these relations are valid while $x > x_{cr.} = 0.565-0.545$, i.e., up to a pressure of $p_{cr.} = \frac{p_a}{x_{cr.}} \approx 1.8$ kg/cm^2. The total duration of the after-action (or after-effect) of gases on the gun mount is determined by the following formula:

$$t_{\pi} = \frac{1}{B'} \left[\left(\frac{p_{\Delta}}{1.8} \right)^{k-1/2k} - 1 \right].$$

Example. Given a 76-mm gun, $s = 0.4693$ dm^2, $\Lambda_{\Delta} = 9.0$, $\omega = 1.080$ kg,

$\Delta = 0.70$, $p_{Д} = 600$ kg/cm^2, $k = 1.2$, $K_0 = 6.424$.

$$B' = \frac{0.2}{2}\, 6.424\, \frac{0.4693}{1.080} \sqrt{\frac{60000 \cdot 0.70}{10}} = 0.2793 \cdot 64.8 = 18.10;$$

$$\frac{p_{Д}}{1.8} = \frac{600}{1.8} = 333.3; \quad \log\left(\frac{p_{Д}}{1.8}\right) = 2.523; \quad \frac{k-1}{2k} = \frac{1}{12};$$

$$\frac{1}{12} \log\left(\frac{p_{Д}}{1.8}\right) = 0.2103; \quad \left(\frac{p_{Д}}{1.8}\right)^{1/12} = 1.623; \quad t_{\Pi} = \frac{0.623}{18.10} =$$

$$= 0.03443 \text{ sec}.$$

The discharge time until the pressure is 20 atm

$$t_{20} = \frac{1}{18.10}\left[\left(\frac{600}{20}\right)^{1/12} - 1\right] = \frac{0.327}{18.10} = 0.01807 \text{ sec}.$$

i.e., the time is almost one-half the full period of discharge down to $p_1 = 1.8$.

If $T_{Д} = 0.70 \cdot T_1 = 0.70 \cdot 2800 = 1960°K$, then

$$T_{\Pi} = \frac{1960}{(1 + 18.1 t_{\Pi})^2} = \frac{1960}{(1 + 0.623)^2} = \frac{1960}{2.635} = 744°K = 471°C.$$

3. THE AFTER-ACTION OF GASES ON THE GUN MOUNT

The relations introduced in Section 2 enable us to investigate the after-action of gases on the recoiling parts after the projectile leaves the gun, and, in particular, to determine the highest velocity

of recoil, necessary for the design of the gun mount.

Fig. 114 - Velocity of Recoil During the Period of After-Action.

The reaction force R, arising as a result of gas discharging from the bore of the gun, imparts an added impulse to the recoiling parts and increases the velocity of recoil.

The action of the gases ceases at the end of their discharge, at which time the recoiling parts attain their maximum velocity V_{max}.

The curves in fig. 114 depict the gas pressure $p_{ДН}$ on the base of the bore and the velocity of the recoiling parts V. $V_Д$ corresponds to the instant the projectile leaves the bore, V_{max} corresponds to the period of after-action, $t_Д$ is the time of recoil prior to the projectile's departure from the bore, $t_П$ is the period of gas after-action.

If the recoil is free, the relation between the velocity of recoil V and the velocity of the projectile (absolute) v_a is expressed by the formula:

$$V = \frac{q + \frac{1}{2}\omega}{Q_0 + \frac{1}{2}\omega} v_a \approx \frac{q + \frac{1}{2}\omega}{Q_0} v_a,$$

because $\frac{1}{2}\omega$ is small compared with Q_0.

Prior to the instant the projectile leaves the bore

$$v_A = \frac{q + \frac{1}{2}\omega}{Q_0} v_{A.a},$$

where $v_{A.a}$ is the absolute velocity of the projectile at the instant it leaves the gun. The recoil velocity increment $\Delta V = V_{max} - V_A$ is obtained as a result of the action produced by the reaction force impulse developed during gas discharge:

$$\frac{Q_0}{g} V_{max} - \frac{Q_0}{g} V_A = \int_0^{t_n} R dt. \qquad (95)$$

When the recoil is subjected to a braking effect

$$\frac{Q_0}{g} V_{max} - \frac{Q_0}{g} V_A = \int_0^{t_n} R dt - \int_0^{t_n} F dt,$$

where F is the resultant of the forces braking the recoil.

The problem dealing with the force R under conditions of powder gas discharge from the barrel bore has been considered in considerable detail in a series of special texts.

We shall assume some of the simplest allowances, to wit: 1) the cross section s of the barrel bore is the critical one; 2) the velocity of the gas at the instant the projectile is ejected from the gun equals the velocity of the projectile $v_{A.a}$; 3) we take into account the change of momentum ω_{kg} of the gases when the mean rate of motion drops from $U_A = \frac{v_A}{2}$ at the start of discharge to zero

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$(U = 0)$ at the end of the period of after-action.

In such a case it may be assumed that

$$\int_0^{t_n} R\,dt = \zeta_0 s \int_0^{t_n} p\,dt + \frac{\omega}{g} \int_{\frac{v_{\text{д.а}}}{2}}^{U=0} dU = \zeta_0 s \int_0^{t_n} p\,dt - \frac{\omega}{g}\,\frac{v_{\text{д.а}}}{2};$$

$$\zeta_0 = (k+1)x_{cr.};\quad \zeta_0 = 1.22 - 1.24.$$

p is the mean gas pressure in the bore of the barrel.

The dependence of p on t is expressed by the formula:

$$p = \frac{p_{\text{д}}}{(1 + B't)^{2k/k-1}},\quad \text{where } B' = \frac{k-1}{2} K_0 \frac{s}{\omega} \sqrt{\frac{p_{\text{д}}\Delta}{\Lambda_{\text{д}} + 1}}.$$

Then

$$\int_0^{t_n} R\,dt = \zeta_0 s p_{\text{д}} \int_0^{t_n} \frac{dt}{(1 + B't)^{2k/k-1}} - \frac{\omega}{g}\,\frac{v_{\text{д.а}}}{2}.$$

Integrating, we get:

$$\int_0^{t_n} R\,dt = \frac{\zeta_0 s p_{\text{д}}}{B'} \int_0^{t_n} \frac{B'dt}{(1 + B't)^{2k/k-1}} - \frac{\omega}{g}\,\frac{v_{\text{д.а}}}{2} =$$

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$$= \frac{\zeta_0 s p_Д}{B'} \frac{k-1}{k+1} \left[1 - \frac{1}{(1 + B' t_n)^{k+1/k-1}} \right] - \frac{\omega}{g} \frac{v_{Д.a}}{2}.$$

Substituting the value of B', reducing, and bearing in mind that the expression in square brackets can be assumed to be equal to unity (0.995), we get

$$\int_0^{t_n} R dt = \frac{\zeta_0 2\omega p_Д}{K_0 (k+1)} \sqrt{\frac{\Lambda_Д + 1}{p_Д \Delta}} - \frac{\omega}{g} \frac{v_{Д.a}}{2}.$$

Introducing here the values

$$\zeta_0 = (k+1) \left(\frac{2}{k+1} \right)^{k/k-1} \quad \text{and } K_0 = \left(\frac{2}{k+1} \right)^{(1/2)(k+1/k-1)} \sqrt{gk},$$

and multiplying the numerator and denominator of the first term by $\sqrt{gk}$ and bearing in mind that

$$p_Д \sqrt{\frac{\Lambda_Д + 1}{p_Д \Delta}} = \sqrt{p_Д \frac{W_{KH}}{\omega}} = \sqrt{p_Д w_Д} \quad \text{and} \quad \sqrt{gkp_Д w_Д} = c_Д,$$

where $c_Д$ is the speed of sound in a gas under conditions corresponding to the start of discharge, we get:

$$\int_0^{t_n} R dt = \frac{2\omega (k+1) \left(\frac{2}{k+1} \right)^{k/k-1} c_Д}{kg \left(\frac{2}{k+1} \right)^{(1/2)(k+1/k-1)} (k+1)} - \frac{\omega}{g} \frac{v_{Д.a}}{2} =$$

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$$= \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{\omega}{g}c_A - \frac{\omega}{g}\frac{v_{A.a}}{2}.$$

Inserting the obtained expression in formula (95), we get:

$$\frac{Q_0}{g}v_{max} = \frac{Q_0}{g}v_A + \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{\omega}{g}c_A - \frac{\omega}{g}\frac{v_{A.a}}{2}.$$

Inasmuch as

$$v_A = \frac{q + 0.5\omega}{Q_0}v_{A.a},$$

$$v_{max} = \frac{q + 0.5\omega}{Q_0}v_{A.a} + \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{\omega}{Q_0}c_A - \frac{\omega}{Q_0}\frac{v_{A.a}}{2}.$$

Presenting v_{max} in the form

$$v_{max} = \frac{q + \beta\omega}{Q_0}v_A = \frac{q}{Q_0}\left(1 + \beta\frac{\omega}{q}\right)v_A,$$

we get:

$$v_{max} = \frac{q}{Q_0}\left\{1 + \frac{\omega}{q}\left[\frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{c_A}{v_A}\right]\right\}v_A,$$

where the coefficient $\beta = \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{c_A}{v_A}$ is called the coefficient of gas after-action on the gun mount and depends in the main on the value c_A/v_A. Since c_A varies within narrow limits, the predominating

effect on β is produced by the initial (muzzle) velocity of the projectile.

At k = 1.2 we get:

$$\beta = 1.59\ c_Д / v_Д,$$

where

$$c_Д = \sqrt{gkp_Д w_Д} = 10.85 \sqrt{\frac{p_Д(\Lambda_Д + 1)}{\Delta}};$$

at k = 1.25

$$\beta = 1.51 \frac{c_Д}{v_Д};\quad c_Д = 11.06 \sqrt{\frac{p_Д(\Lambda_Д + 1)}{\Delta}}.$$

These formulas tie in the coefficient β with the charging conditions and with the design data In addition to this theoretical formula, there are also empirical formulas for the β coefficient, for example:

$$\beta_1 = \frac{1400}{v_Д\ m/sec} + 0.15 \text{ or } \beta_2 = 1300/v_Д.$$

All of these formulas point at the predominating effect of the projectile velocity at the instant of its ejection from the gun.

<u>Example.</u> A 76-mm cannon, $p_Д$ = 600 kg/cm^2 = 60,000 kg/dm^2; $\Lambda_Д + 1 = 10.0$; $\Delta = 0.70$; $v_Д$ = 6800 dm/sec.

$$\beta = 1.59 \cdot 10.85 \sqrt{\frac{60000 \cdot 10}{0.70}}\ \frac{1}{6800} = 17.26\ \frac{926}{6800} = 2.35.$$

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Using the empirical formulas, we get:

$$\beta_1 = \frac{1400}{680} + 0.15 = 2.21; \quad \beta_2 = \frac{1300}{680} = 1.912.$$

It can be seen that the numbers obtained by means of the empirical formulas are smaller than those obtained by the theoretical one.

At a high speed $v_A = 1000$ m/sec, $\Lambda_A + 1 = 5.0$, $\Delta = 0.72$, $p_A = 120{,}000$ kg/dm^2 we get:

$$\beta = 17.26 \sqrt{\frac{120000 \cdot 5}{0.72} \frac{1}{10000}} = 17.26 \frac{913}{10000} = 1.575;$$

$$\beta_1 = \frac{1400}{1000} + 0.15 = 1.40 + 0.15 = 1.55; \quad \beta_2 = \frac{1300}{1000} = 1.30.$$

The value of β_1 closely approaches that of β. If in the first case $Q_0 = 80q$, then at $\omega/q = 0.16$

$$v_{max} = \frac{1}{80}(1 + 2.35 \cdot 0.16) \cdot 680 = \frac{1.376}{80} 680 = 11.7 \text{ m/sec};$$

in the second case $Q_0 = 150q$; $\omega/q = 0.50$ and

$$v_{max} = \frac{1000}{150}(1 + 1.575 \cdot 0.50) = 11.92 \text{ m/sec}.$$

4. THE AFTER-ACTION OF GASES ON A PROJECTILE

The action of gases on the projectile after it leaves the barrel is as follows. The velocity of the gases discharged in the wake of the projectile exceeds that of the projectile, and the gases

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surround and overtake the latter, so that the projectile actually moves for a certain period of time within a mobile medium. At the same time the gases continue to exert a pressure on the base of the projectile, thus increasing its velocity even after the projectile has left the bore.

Thus the maximum velocity is not the muzzle velocity, but the velocity at some point a short distance ahead of the gun muzzle face. Nevertheless all the known methods of computation in internal ballistics permit to conclude the computations at the muzzle face. The muzzle velocity v_0 obtained on the basis of test data is computed by reducing the velocity v_c recorded by the chronograph to that of the muzzle face, under the assumption that the velocity beyond the muzzle face is continuously decreased by the action of air resistance.

Fig. 115 - Projectile Velocity in the After-Action Period

I) frame-target; II) frame-target.

Figure 115 characterizes the relation between the velocities of the projectile at different points of its trajectory.

The projectile leaves the muzzle face with an absolute velocity $v_{a.a}$ which increases to v_{max} during the period of after-action,

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following which it decreases because of air resistance. Using a chronograph and two frame-targets, v_c (the average velocity between frame-targets I and II) is determined. Then, using formula

$$v_0 = v_c + \Delta v_c,$$

where

$$\Delta v_c = \frac{iqX_c}{d^2 \Delta D(v)},$$

we can compute the so-called "initial" velocity of the projectile, whereby the effect of the after-action of the gas powders is not taken into consideration, and the correction Δv_c is computed using the normal resistance law. In consequence we get the following relations:

$$v_0 > v_{max} > v_{A.a}; \quad v_A = v_{A.a} + v_A > v_{A.a}; \quad v_A \approx v_0.$$

Therefore, it may be assumed in practice that the relative muzzle velocity v_A, calculated in solving the problem of internal ballistics, is approximately equal to the "initial" velocity v_0 of the projectile determined by test by means of a chronograph.

The law governing the change of velocity and pressure in a stream of discharged gases, as well as the law governing the change of the stream's shape when the projectile is situated in the stream, lend themselves to experimental analysis only with great difficulty.

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Gas dynamics offers only an approximate relationship for determining the gas velocity in the absence of solids distorting the stream, which relationship does not take into account the external pressure.

In view of the absence of reliable theoretical relations for determining the velocity increment of the projectile, we are presenting here certain test data on gas after-action. Spark photography and ultra high-speed photography make it possible to study the phenomena occurring during the motion of the projectile after it has left the barrel and during the discharge of the gases from the bore.

We shall not attempt to enumerate here all the tests of this type and limit our discussion to the firing of small arms and howitzers. When a shot is fired, the air present in the bore is ejected causing a spherical impact wave at the face of the muzzle. Next there appears a small quantity of gas escaping through the clearances between the walls of the bore and the surface of the bullet or projectile, following which there appears the bullet or projectile itself.

Next in order is the discharge of powder gases causing a shock wave upon encountering the outside air, which is responsible for the report of the gun.

The powder gases surround the bullet or projectile and tend to move forward with a velocity considerably exceeding the velocity of the bullet.

Air resistance and friction cause the powder gases to rapidly lose their velocity. A certain distance away from the muzzle face (about 35 cm) the bullet begins to overtake the gases, and the ballistic or bow wave usually accompanying the flight of the projectile

originates at this instant. The photographs in figs. 116, 117, 118 and 119(*) taken by D.F. Chernyshev show that in addition to the ballistic wave around the bullet, a large number of similar waves accompanies the unburned flying particles of powder ejected from the bore.

When the projectile velocity exceeds the speed of sound, the ballistic wave gradually emerges from the spherical sound wave in the form of a cone (see figs. 116, 117, 118 and 119). This is accompanied by clearly defined masses of condensed gas accompanied by eddies and by the appearance of stationary waves when the powder gases are discharged. The latter phenomenon is explained by the following: As the pressure drops gradually, the gases in receding from the muzzle face cause local increases of pressure (pressure jumps), the pressure becoming maximum at the points where the gases become condensed. As the gases are discharged, the position of the first maximum changes - it is gradually displaced toward the muzzle face. The occurrence of such masses of condensed gas is mainly explained by the gradually increasing effect of air resistance.

As the pressure changes, the gas velocity increases at first, mainly at the center of the stream, where the gas is not affected by outside friction; however, at the points of condensation and increased pressure, the gas velocity again undergoes a considerable decrease.

(*) These photos are missing from the original text. Editor.

According to observations made by Kampe-de Ferier in firing a 37-mm cannon having a muzzle velocity of about 720 m/sec, approximately 0.0015 sec before the shell leaves the bore, poorly luminous gases begin to appear from the bore and disperse with a velocity of about 300 m/sec. Directly after the shell leaves the muzzle face, a lateral gas discharge occurs through an annular clearance between the walls of the bore and the base of the projectile with a velocity of about 2000 m/sec. Then, as soon as the base of the projectile recedes from the muzzle face, the expanded gases rushed forward with a velocity of the order of 1400 m/sec, and inasmuch as this velocity greatly exceeds the velocity of the projectile (720 m/sec), the entire gas mass catches up with and overtakes the projectile, and completely surrounds it.

The velocity of the forward layers of the gas mass begins to decrease thereby in the following order:

t, sec..........	0.001	0.002	0.003	0.004	0.005	0.007	0.009
v, m/sec.......	780	750	580	470	370	320	310

The velocity of the projectile continues to approach the value of 720 m/sec, and at t = 0.007 sec it emerges from the gas mass and is relieved of its influence, its distance from the muzzle face being 5 m at that instant.

The gas cloud explodes about 0.019-0.028 sec later, at which time the velocity of the forward layers of the gas increases from 120 to 180 m/sec.

Tests were conducted by Okosi in Japan in 1913 to determine the change in the velocity of a rifle bullet. A special chronograph was

used in these tests permitting the use of several targets simultaneously.

It was found that in 10 cases out of 14 (71%) the velocity was maximum; in the remaining cases (29%) a minimum was observed followed by a maximum, where the maximum velocity exceeded the muzzle velocity in all cases. Okosi concluded that for the Japanese rifle of 1898 issue the maximum velocity is obtained at a distance of about 1.5 m from the muzzle face, and that the increment constitutes only about 0.8% on the average. At a distance of 5 m, the velocity again drops to that of the muzzle velocity.

Tests were conducted by N.M. Platonov for the purpose of determining the period of gas after-action on the base of the projectiles in howitzers with relation to the distance traversed. Curves were obtained showing the change in projectile velocity and the pressure acting on the projectile's base (fig. 120a and b) during the period of after-action. The curves were obtained by means of slow-motion photography.

Figure 121 represents a curve of the pressure exerted on the base of the projectile for a reduced change, obtained from the analysis of the v, X curve.

A comparison of the v, X curves obtained with a full and reduced charge (fig. 120a) disclosed that the length of the period of after-action is approximately doubled in changing from a reduced to a full charge. Curve p_{CH}, x (fig. 121) shows that the pressure exerted by the powder gas on the base of the projectile during the period of after-action rapidly decreases as the distance traversed by the projectile increases.

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It should be noted here that in line with the positive results cited here, tests conducted by other investigators employing different methods have produced opposite results. It may be concluded therefore that the subject problem is still in the stage of experimental study and that most of the attention should be directed towards the development of new methods for the study of bullet motion during the period of after-action and for the establishment or determination of errors peculiar to the different methods used.

Fig. 120 - Change of Projectile Velocity During the Period of After-Action

a) v, m/sec; b) full charge; c) reduced charge.

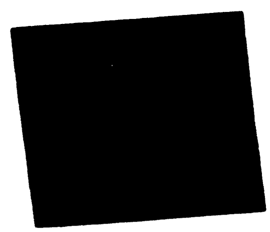

Fig. 121 - Pressure Drop During the Period of After-Action

a) kg/cm^2; b) reduced charge.

CHAPTER 3 - BURNING OF POWDER IN AN INCOMPLETELY CLOSED SPACE

1. PRESSURE EXERTED BY GASES WHEN DISCHARGED THROUGH A NOZZLE DURING BURNING OF THE POWDER

The process considered here applies in practice to the following:

1) When powder is burned in a special manometric bomb with nozzle, for the purpose of investigating the burning under conditions simulating powder burning in a gun - where the pressure rises and drops;

2) In a separate combustion chamber of a gas-actuated gun;

3) In the chamber of a rocket shell.

In all of the above cases the gas inflow due to the burning of powder is simultaneously accompanied by the discharge of a portion of the gas through a nozzle. Therefore, the pressure during the burning process may drop as well as increase. Under these conditions the process will vary, depending on the pressure maintained in the chamber: the lower the gas pressure in the chamber, the easier it is to keep it constant. We shall first consider the case of high pressures, for which the burning rate law $u = u_1 p$ is valid.

Let us compile a formula for determining pressure in constant volume W_0 at a given instant under the condition that a portion of the gas is discharged through an opening (nozzle).

We shall designate the quantity of gas discharged at a given instant (in kg) by Y and the ratio $\frac{Y}{\omega} = \eta$.

We had derived above the formula for gas discharge in time t:

$$Y = \int_0^t G_{sec}\, dt,$$

where $G_{sec} = Asp_1$; the pressure in the space from which the gas is discharged equals p_1 and the cross section of the nozzle is s; A is the discharge coefficient.

The pressure p_1 is not constant when burning occurs in a bomb with a nozzle; it changes continuously and hence the discharge process will not be a stable one. In order to evaluate the process, let us assume as a first approximation that the relation derived for a stabilized discharge process also applies to our case, wherein p

varies constantly with respect to time. Then, denoting by p the current gas pressure in a chamber with a nozzle and by s_m the cross section of the nozzle, we get:

$$G_{sec} = As_m p;$$

$$Y = As_m \int_0^t p\,dt = As_m I$$

and at the end of powder burning

$$Y_K = As_m I_K.$$

We arrive at the conclusion that the gas discharge during burning of the powder is proportional to the pressure increase impulse at the given instant, and at the end of burning - to the full impulse

$$I_K = \frac{e_1}{u_1}.$$

Inasmuch as the impulse I_K depends only on thickness $2e_1$ and the rate of burning u_1, the gas discharge does not depend on the shape of the powder and its burning progressivity. It may be assumed for sufficiently small cross sections s_m and $\tau = \frac{T}{T_1} \approx 1$.(*) In such a case the expression depicting the pressure at a given instant will be:

(*) The relations taking into account the lowering of gas temperature in the presence of large openings are analyzed in special texts.

$$p = \frac{f(\omega\psi - Y)}{W_0 - \alpha(\omega\psi - Y) - \frac{\omega}{\delta}(1 - \psi)} = \frac{f\omega(\psi - \eta)}{W_0 - \frac{\omega}{\delta}(1 - \psi) - \alpha\omega(\psi - \eta)} =$$

$$= \frac{f\Delta(\psi - \eta)}{1 - \frac{\Delta}{\delta}(1 - \psi) - \alpha\Delta(\psi - \eta)}. \qquad (96)$$

At the end of burning, we will have:

$$\psi_K = 1, \quad \eta_K = \frac{Y_K}{\omega} = As_m \frac{I_K}{\omega};$$

$$p_K = \frac{f\omega(1 - \eta_K)}{W_0 - \alpha\omega(1 - \eta_K)} = \frac{f\Delta(1 - \eta_K)}{1 - \alpha\Delta(1 - \eta_K)}. \qquad (97)$$

Using the designation $\Delta(1 - \eta_K) = \Delta_K$, the formula will be transformed into the usual Noble formula:

$$p_K = \frac{f\Delta_K}{1 - \alpha\Delta_K}, \qquad (98)$$

where Δ_K is that charging density at which the maximum pressure $p_m = p_K$ would obtain in a closed space.

The simple rule for calculating the powder charge or the density of the charge producing the required pressure p_K at the end of burning follows from the above. Using Noble's formula, the values are found of Δ_K or ω_K at which the pressure $p_m = p_K$ would obtain in a closed space:

$$\Delta_K = \frac{p_K}{f + \alpha p_K} \quad \text{or} \quad \omega_K = \frac{W_0 p_K}{f + \alpha p_K}.$$

Then, using formula

$$Y_K = As_m I_K = As_m \frac{e_1}{u_1}$$

the weight is determined of the gases discharged through a nozzle of cross section s_m during the period that the powder is burned with impulse I_K. The sum of $\omega_K + Y_K$ will give the full charge which, when burned in a bomb with nozzle s_m, will produce pressure p_K.

$$\omega_1 = \omega_K + Y_K; \quad \Delta_1 = \frac{\omega_1}{W_0}.$$

The value of $I_K = \int_0^1 pdt$ can be found beforehand from a test in a closed bomb, inasmuch as the magnitude of the pressure impulse for powders of simple shapes does not depend on Δ and should not depend on whether the pressure increases according to the law applicable to a closed bomb, or decreases more slowly, or even drops in consequence of the discharge of a portion of the gas through the nozzle. Indeed, if we designate the pressure in a closed bomb by P, and that in a bomb with a nozzle by p, and the times τ and t, respectively, then upon burning powder of the same thickness in a closed space, $de = u_1 Pd\tau$. When powder of the same thickness is burned in a chamber with an opening, $de = u_1 pdt$. Because as a result of partial gas discharge $p < P$, the time interval necessary for the burning of the same thickness de at the smaller pressure p will be correspondingly longer, and the total time will therefore be

$Pd\tau = pdt,$

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and hence

$$\int_0^1 P d\tau = \int_0^1 p dt = I_K.$$

Therefore, in order to determine the gas flow through the nozzle during the time the powder is burned, use can be made of the impulse $I_K = \int_0^1 P d\tau$ calculated from a test in a closed space (a manometric bomb).

Once the values of I_K and Y_K are known and the value of $I = \int_0^{\Psi} p dt$ is obtained from the pressure curve, the value of γ can be found for any given instant and the corresponding value of ψ then determined.

$$Y = \gamma\omega = Y_K \frac{I}{I_K} \text{ or } \gamma = \gamma_K \frac{I}{I_K}.$$

Solving formula (96) with respect to ψ, we get:

$$\psi = \frac{p\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) + \gamma(f + \alpha p)}{f + p\left(\alpha - \frac{1}{\delta}\right)} = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p} + \alpha - \frac{1}{\delta}} + \frac{f + \alpha p}{f + \left(\alpha - \frac{1}{\delta}\right)p}\gamma.$$

The first term of this equation represents the usual expression for the portion ψ of the charge at which, in a closed space at the same charging density Δ as in a chamber with a nozzle, the obtained

pressure is p; the second term takes into account the influence of the discharged gases.

The magnitude η_K is precisely the one characterizing the relative intensity of gas flow, or the relative gas discharge during burning of the powder; it is the greater, the greater s_m and I_K and the smaller the charge ω, but always $\eta_K < 1$.

2. PRESSURE CURVE OBTAINED IN A NOZZLED CHAMBER WHEN THE DISCHARGE OPENING IS SMALL

We shall consider, as in the case of general pyrostatics, the case of a powder with a constant burning area ($\varkappa = 1$, $\lambda = 0$, $\sigma = 1$). The instantaneous pressure is expressed by the formula:

$$p = \frac{f\omega(\psi - \eta)}{W_0 - \frac{\omega}{\delta}(1 - \psi) - \alpha\omega(\psi - \eta)} = \frac{f\omega(\psi - \eta)}{W_\psi + \alpha\omega\eta}.$$

The denominator in the right side shows that if a portion of the gas ($\omega\eta$) is discharged, the free space during burning will be greater and hence undergo a smaller change than W_ψ - the free space obtained during burning in a closed space. Hence, as in the case of general pyrostatics, the mean value of the free space can be used to determine the general character of the phenomenon. Assuming that $\psi_{av.} = \frac{1}{2}$ and $\eta_{av.} = \frac{\eta_K}{2} = \frac{As_m I_K}{2\omega}$, we get the following expression for the average value of the free space in the chamber:

$$W_{av.} = W_{\psi av.} + \alpha\omega\eta_{av.} = W_0 - \alpha'\omega + \frac{\alpha}{2}\omega\eta_K.$$

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The pressure formula will take on the form:

$$p = \frac{f\omega}{W_{av.}} (\psi - \eta).$$

Differentiating with respect to t and bearing in mind that for a powder with a constant burning area or for a strip $\varkappa\theta_{av.} = 1$ and

$$\frac{d\psi}{dt} = \frac{u_1}{e_1} p = \frac{p}{I_K}, \qquad \frac{d\eta}{dt} = \frac{As_m p}{\omega},$$

we will get:

$$\frac{dp}{dt} = \frac{f\omega}{W_{av.}} \left(\frac{1}{I_K} - \frac{A}{\omega} s_m \right) p = \frac{f\omega}{W_{av.} I_K} \left(1 - \frac{As_m I_K}{\omega} \right) p =$$

$$\frac{f\omega}{W_{av.} I_K} (1 - \eta_K).$$

Denoting the constant

$$\frac{f\omega}{W_{av.} I_K} (1 - \eta_K) = \frac{1}{\tau_1},$$

separating the variables and integrating the obtained equations we get:

$$\ln \frac{p}{p_B} = \frac{t}{\tau_1}$$

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or

$$p = p_B e^{t/\tau_1}.$$

Fig. 122 - Pressure Increase Curves when Burning Powder in a Bomb with a Nozzle.

We have obtained the same formula as when burning powder in a constant closed space, although the constant τ_1 corresponds to the burning of charge $\omega(1 - \eta_K) < \omega$ rather than charge ω, and hence the process of pressure increase is slower, i.e., the same as it would have been in a closed space at $\Delta_K = \frac{\omega(1 - \eta_K)}{W_0}$ which is smaller than the actual $\Delta = \frac{\omega}{W_0}$.

The full time of burning under these conditions is determined by the formula:

$$t_K = 2.303\ \tau_1 \log \frac{p_K}{p_B}.$$

It should be noted that also the pressure p_B of the igniter under conditions of partial gas discharge will not be equal to the rated pressure under conditions of a closed space; a correction must be made for the gas discharge.

The curves in fig. 122 show the pressure increase: 1 - when the powder is burned in a closed space; 2 - when the same charge is burned with a portion of the gas discharged through a nozzle.

Both curves are theoretical ones under the assumption that burning proceeds according to the geometric law. It has been shown however that the true characteristic of pressure increase differs from the theoretical by the fact that the pressure curve is bent at the end and that it approaches the horizontal tangentially rather than at an angle.

Therefore, when powder is burned in a chamber with a nozzle, curve 2 will likewise be distorted.

The problem dealing with the effect of the charging conditions and of the burning of powder on the law governing the pressure increase when a portion of the gas is discharged through a nozzle, can be solved graphically in its first approximation.

Indeed, the input of gases per second as a result of powder burning at high pressures is expressed by the well-known formula:

$$\omega \frac{d\psi}{dt} = \omega \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 p \quad \text{kg/sec} = \omega \Gamma p;$$

and the gas discharge (output) per second is expressed by the following formula:

$$G_{sec} = \frac{dY}{dt} = \omega \frac{d\eta}{dt} = As_m p.$$

If the input exceeds the output, the pressure in the chamber will rise; if the procedure is reversed, the pressure will drop.

If the input equals the output, the pressure must be maximum or remain constant. Hence the pressure change depends on the ratios $d\omega/dt$ and $d\gamma/dt$. A simple relationship is obtained between the charging conditions and the powder burning characteristics, permitting a direct answer to the problems dealing with the nature of the pressure curve and with the condition of obtaining the maximum pressure before the end of burning.

Everything depends on the ratios:

$$\omega\Gamma = \omega \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 \text{ and } As_m,$$

where As_m, characterizing the flow conditions, is a constant, and $\omega\Gamma$ is usually a variable and becomes constant only for powders with a constant burning area. Therefore, these values can be made equal and the pressure p_{max} can be obtained only at a certain instant. Thereafter the pressure will begin to drop or rise because S/S_1 usually varies in one direction only.

We thus get a simple graphic solution for the problem. If it is required to find out whether the end of burning will obtain after the maximum p_{max} is reached and the value of ψ_m to which the maximum pressure will correspond, the answers will be obtained by constructing a curve for the given powder depicting the progressivity of burning $\omega\Gamma$ as a function of ψ, and a straight line aa' must be then drawn parallel to the abscissa at a distance As_m from it (fig. 123). If

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the entire line aa' lies below line 1-1, the gas input during the entire burning process exceeds the gas discharge through the nozzle, the pressure curve rises continuously, and the maximum pressure coincides with the end of burning. The angle of inclination of the curve $\left(\frac{dp}{dt}\right)_K$ will be maximum at the end of burning (curve 1-1 in fig. 124). If line aa' starts below the $\omega\Gamma$ curve (see fig. 123) and then intersects it at point b and continues above it, it means that the pressure will first increase, pass the maximum at point $b(\psi_m)$ where the gas input equals the gas discharge, and will then drop, because the gas discharge As_m per second will exceed the gas input $\omega\Gamma$.

A pressure curve 2-2 (fig. 124) is thus obtained with a smooth inflection at point p_m and a descending portion showing a pressure drop between p_m and p_K.

In fig. 123 curve Γ_T (3-3) lies below the line aa'. This indicates that the discharge will always exceed the input, that the pressure will continuously decrease (curve 3-3 in fig. 124), and that the powder may burn slowly and may even tend to die out.

The Γ curves in fig. 123 represent strip powders of varying thicknesses: 1-1 for thin strip, 2-2 for strips of average thickness, and 3-3 for thick powder. Hence, with the same powder shape, by varying the thickness of the strip and leaving the charge and cross-sectional area of the nozzle unchanged, we can obtain all three forms of the pressure (increase) curve.

Contrariwise, for the same powder of given dimensions, by varying the nozzle opening s_m or the weight of the charge ω, the position of line aa' or $\omega\Gamma$ can be changed and with it the characteristic of

the curve depicting the pressure increase in a chamber with a nozzle. Therefore, a chamber with a nozzle, if provided with means for recording the rise and drop of pressure, makes it possible to test powders at considerably greater charging densities and under conditions approaching those of powder burned in a weapon; i.e., not only under conditions of pressure rise, but also under conditions of pressure drop.

All of the above conclusions and the possibility of obtaining maximum pressure before the end of burning are based on the analysis of theoretical curves of progressivity Γ calculated according to the geometric law. Actually, of course, the test curves Γ, ψ differ in character, i.e., they differ in regard to regressive powders by their beginning and end portions, whereas in regard to progressive powders they differ along their entire zero-to-unity interval. Typical diagrams for strip (or tubular) powder (fig. 125) and for powders with many perforations (fig. 126) are presented below.

Fig. 123 - Gas Input and Discharge Characteristic Curves.

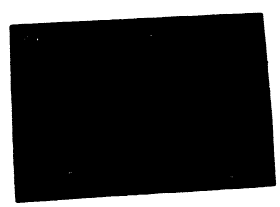

Fig. 124 - Curves Depicting Pressure Variation in a Bomb with a Nozzle.

A comparison of the $\omega\Gamma_{0n}$ and aa' diagrams characterizing the

size of the nozzle indicates the important difference between experimental and theoretical curves Γ, ψ which must be reflected on the nature of the pressure curves p, t obtained in the presence of a nozzle on the chamber.

Inasmuch as the test curves Γ, ψ show a sharp drop at the end of burning and tend toward zero, they must be intersected by the line aa'; the maximum must occur before the end of burning, whereas the end of burning will occur on the descending branch of the pressure curve.

The ascending portions of the Γ, ψ curves at the start of burning point at a gradual ignition at ignitor pressures of 20-40 kg cm^2, and if the Γ, ω curve lies a considerable distance below the corresponding straight line aa', ignition cannot take place and the powder will be extinguished because of lowered pressure. Such examples were obtained in testing cylindrical grains for ignition. When a pyroxyline igniter was used capable of developing a pressure of about 50 kg/cm^2, it was often found that after it was burned its gases would exit through the nozzle without igniting the powder charge. A subsequent examination of the powder grains would show that the latter were partly burned and became extinguished when the pressure dropped. Actual calculations for one such case show that

$$A \frac{s_m}{\omega} = 0.007 \frac{0.07}{0.005} = 0.098 \text{ cm}^2 \text{ kg} \cdot \text{sec},$$

and for this $\frac{7}{7}$ powder the theoretical $\Gamma_0 = 20 \cdot 0.0075 = 0.150$ cm^2/kg $\cdot$ sec.

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Fig. 125 - Relation Between Gas Input and Discharge per Second when Burning Strip Powder in Accordance with the Physical Combustion Law.

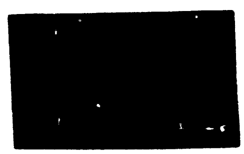

Fig. 126 - Relation Between Gas Input and Discharge per Second when Burning Powder with Many Perforations in Accordance with the Physical Combustion Law.

If the ignition were instantaneous, the powder would not have been extinguished because $\Gamma_{T.O.} > A \frac{s_m}{\omega}$ (*). But inasmuch as ignition is not instantaneous, and the initial Γ can actually equal 0.040-0.050 and then increase to 0.200, the value of Γ at the start of ignition is actually smaller than $A \frac{s_m}{\omega}$, the discharge through the nozzle exceeds the gas input, and the powder does not ignite.

(*) Subscript T.O. stands for "theoretical, initial." Editor.

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3. DERIVATION OF MAXIMUM PRESSURE FORMULA

According to the physical law of burning, all powders without exception, when burned in a chamber with a nozzle, due to the sharp surface area decrease at the end of burning, must develop maximum pressure before the end of burning, and hence a maximum $\frac{dp}{dt} = 0$ must occur on the pressure curve without fail. We shall derive the condition for obtaining maximum pressure and a formula for p_m, from the fundamental equation of pressure in a semi-closed space.

We had presented above the general pressure formula:

$$p = \frac{f\Delta(\psi - \eta)}{1 - \frac{\Delta}{\delta}(1 - \psi) - \alpha\Delta(\psi - \eta)} - \frac{a}{\delta}.$$

In order to determine the conditions for obtaining p_m, we differentiate p with respect to t, bearing in mind that

$$\frac{d\psi}{dt} = \Gamma p \text{ and } \frac{d\eta}{dt} = A\frac{s_m}{\omega}p.$$

We get:

$$\dot{p} = \frac{f\Delta\left(\frac{d\psi}{dt} - \frac{d\eta}{dt}\right)}{b} - p\,\frac{\left[-\alpha\Delta\left(\frac{d\psi}{dt} - \frac{d\eta}{dt}\right) + \frac{\Delta}{\delta}\frac{d\psi}{dt}\right]}{b} =$$

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$$= \left\{ \frac{f\Delta\left(\Gamma - A\,\frac{s_m}{\omega}\right) - p\left[\frac{\Delta}{\delta}\Gamma - \alpha\Delta\Gamma + \alpha\Delta A\,\frac{s_m}{\omega}\right]}{b} \right\} p.$$

Equating the derivative to zero, we obtain the condition necessary for obtaining p_m:

$$f\Delta\left(\Gamma_m - A\,\frac{s_m}{\omega}\right) - p_m\left[\alpha\Delta A\,\frac{s_m}{\omega} - \Delta\Gamma_m\left(\alpha - \frac{1}{\delta}\right)\right] = 0.$$

Eliminating Δ and dividing by f, we reduce similar terms:

$$\Gamma_m\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] = A\,\frac{s_m}{\omega}\left(1 + \alpha\,\frac{p_m}{f}\right),$$

whence

$$\omega\Gamma_m = A s_m n', \qquad \text{(N)}$$

where

$$n' = \frac{1 + \alpha\frac{p_m}{f}}{1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}} > 1.$$

At $f = 900{,}000$, $\alpha = 1$; the value of n' depends on pressure p_m and can be computed in advance:

$$n'_{200} = 1.014; \quad n'_{1000} = 1.066; \quad n'_{2000} = 1.126.$$

In the first approximation for rockets, where $p_m \leqslant 250$ atm, $n' = 1$; for chambers with nozzles $n' = 1.10$.

Thus, in order to obtain p_m, it is necessary that the inflow of gas per second at $p = 1$, i.e., $\omega\Gamma$, satisfy the condition (N), i.e., that it exceed somewhat the discharge of gas per second reduced to $p = 1$.

$$\Gamma_m = \frac{As_m}{\omega} n' \quad \text{or} \quad \omega\Gamma_m > As_m.$$

Having obtained from bomb tests or by means of theoretical calculations curves of ω and Γ as a function of I, and also the straight lines $\frac{As_m}{\omega} n'$ for various charges ω_i, the problem of determining p_m can be solved as follows.

Passing the straight line $A \frac{s_m}{\omega_i} n'$ for a given weight of charge through curve Γ, I in fig. 127, we find the point of intersection a_i, and dropping a vertical from this point we determine I_{mi} on the abscissa and the value Ψ_{mi} on curve Ψ. With $A\frac{s_m}{\omega_i}$ and I_{mi} known, we compute the relative gas discharge at the instant p_m is obtained:

$$\eta_{mi} = A \frac{s_m}{\omega_i} I_{mi}.$$

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Fig. 127 - Graph for Determining p_m During Gas Discharge.

Substituting the obtained values of ψ_m and η_m in the pressure formula, we find:

$$p_m = \frac{f\Delta(\psi_m - \eta_m)}{1 - \frac{\Delta}{\delta}(1 - \psi_m) - \alpha\Delta(\psi_m - \eta_m)},$$

where

$$\eta_m = Y : \omega = \frac{A s_m I}{\omega}.$$

It is known that the gas inflow $\psi = \int_0^I \Gamma dI$ is expressed by the area bounded by the Γ curve and the abscissa I, and the gas discharge $\eta = \frac{As_m}{\omega} I$ by the area of a triangle of altitude $A\frac{s_m}{\omega}$ and base I. The difference between these areas, cross-hatched in fig. 127, gives the gas residue in the chamber. The greater the charge ω_1, the lower will be the straight line $A\frac{s_m}{\omega_1}$, the greater will be the cross-hatched area $\psi_m - \eta_m$, the later will the maximum pressure occur, and the greater will be the maximum pressure.

It is of interest to note that the condition for obtaining maximum pressure obviously does not depend on the volume of the chamber and the charging density, but, rather, on the ratio between the oppositely reacting intensities of gas inflow $\omega\Gamma_m$ and gas discharge As_m, similarly to the condition in a gun wherein the maximum obtainable pressure depends on the ratio between the intensity of gas inflow $\omega\Gamma$ and the rate of increase of the volume of the bore sv.

The obtained derivations are valid for high pressures (above 1000 kg/cm^2) or for rapidly burned fine powders, when the burning rate law $u = u_1 p$ holds true.

At low pressures (up to 250 kg/cm^2) and for very thick powders the burning rate law $u = u_1 p$ no longer applies, as was shown in the chapter dealing with the burning rate law, and the relationships become somewhat different.

At small pressures and relatively slow burning of the powder, the mass of the latter succeeds in becoming heated to a considerable degree; the more so, the slower the process of burning. Therefore the rate of burning u_1 reduced to $p = 1$ increases at low pressures, and begins to decrease as the pressure increases.

Inasmuch as the true change of u_1 with heating and the degree to which the powder mass becomes heated at different rates of burning have not yet been determined experimentally, formally this phenomenon of burning reduced to $p = 1$, which is more intense at small pressures and less intense at high pressures, can be expressed by the burning rate law:

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$$u = u_1' p ,$$

where $u_1' > u_1$, whereby

$$u_1 = \frac{u_1'}{p^{1-\nu}}.$$

In such a case the rate of gas inflow $\omega \frac{d\psi}{dt}$ will be expressed by the formula:

$$\omega \frac{d\psi}{dt} = \omega \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1' p^{\nu} = \delta S u_1' p^{\nu},$$

and the intensity of discharge will be expressed by the previous relation

$$G_{sec} = \frac{dY}{dt} = A s_m p.$$

As was shown by Prof. Ya. M. Shapiro, this diversity of the exponents in the laws governing the input and discharge of the gases leads to a very interesting property of self-regulation and leveling-off of the p_m value, manifested during the burning of thick powders in bombs with nozzles at low pressures (10 to 200 kg/cm^2).

Indeed, if we were to depict the input and discharge of gases in fig. 128, the first process would be represented by a parabolic curve and the second process by a straight line passing through the origin of the coordinates, whose tangent equals $A\frac{s_m}{\omega}$ and can be chosen

at will.

Say, at point a the input and discharge of the gases become equalized and the pressure remains constant. Should the pressure be increased (to the right of point a), the intensity of gas discharge will become greater compared with the gas input and the pressure will drop, i.e., the process will reverse itself towards point a, maintaining p_m = const.

In exactly the same way, when the pressure drops (to the left of point a), the gas inflow process will be more intense, and this will cause the pressure to increase and to tend towards p_m = const.

Therefore, at low pressures, when the burning rate law is $u = u_1' p^{\nu}$, the process of maintaining the gas pressure at a specific level will be of the self-regulating kind; it will be more stable compared with the process of pressure change when powder is burned at high pressures of the order of 1000-2000 kg/cm^2.

This tendency towards leveling off of the pressure can be noted by comparing diagrams Γ, I and $A\frac{s_m}{\omega}$, I under different burning rate laws. It has been established by actual tests that at high pressures, at $\Delta > 0.10$-0.12, the integral curves I as a function of ψ do not depend on Δ and coincide at different charging densities. At small charging densities and low pressures the integral curves assume lower positions, which are the lower, the smaller the charging density.

Correspondingly, the Γ, ψ and I, ψ curves also coincide at high charging densities; at low charging densities the Γ, ψ curves are disposed the higher, the smaller the value of Δ; curves Γ, I, at the start, are likewise disposed higher and are then intersected by

Γ, I curves at higher values of Δ, because the total area $\int_0^{i_K} \Gamma dI = 1 = \text{const}$.

Fig. 128 - Diagram Depicting the Rate of Input and Discharge of Gases.

For high densities we will have the former graph (see fig. 127), where Γ, I and ψ, I are the same for different Δ (from 0.12 to 0.25). We shall construct an additional graph (fig. 129) for low charging densities taking into account the change of Γ and I obtained with the change of the charging density. Let $\Delta_1 < \Delta_2 < \Delta_3$; let us see what happens when the gas inflow with velocity Γ occurs simultaneously with a gas discharge at the rate of $\frac{As_m}{\omega}$ per second.

Fig. 129 - Rate of Gas Inflow and Discharge at Small Values of Δ

The smaller the value of Δ, the smaller will be the pressure developed by burning powder in a constant closed space, and the

higher will be the disposition of the Γ, I curve on the graph.

When the input Γ_m balances the discharge $A\frac{s_m}{\omega}n'$ at point a_3, the straight line $\frac{As_m}{\omega}n'$ will lie above Γ_3 and the pressure will begin to drop; however, at a lower pressure, use must be made of curve Γ_2 lying above Γ_3, and point a_2 can be intercepted at the same pressure. The same will occur in region a_2-a_1, whereby $\Psi_3 < \Psi_2 < \Psi_1$ - the burned portion of the charge grows, whereas the pressure remains constant, because the gas inflow equals the gas discharge. This will not occur at all at high pressures, at which curve Γ, I is the same even after it intercepts point a_1, point a_2 or point a_3. The Γ, I curve will be disposed below line $\frac{As_m}{\omega}n'$, and the pressure will continue to drop only.

K.E. TSIOLKOVSKY'S FORMULA

A rocket is propelled by the reaction force produced by the gases discharged from it. The Great Fatherland War has given us many examples of rocket application both in our country and in the countries of our allies and enemies. These may be exemplified by our famous "Katushas" or by the German multi-barreled rocket (mine) throwers.

We shall present here the derivation of the famous Tsiolkovsky formula for determining the velocity of a rocket on the basis of the relations presented above.

We shall designate by Q the total weight of the rocket, the charge included, by ω - the weight of the powder charge, and by q - the weight of the rocket less the charge, so that $Q = q + \omega$.

The total weight of the rocket will vary in flight from Q to q. Say, the weight of the gases discharged at a given time is Y_{kg}. The equation of quantity of motion (momentum) will be:

$$\frac{Q - Y}{g}\, dv = Rdt = \zeta\, s_m p dt = \zeta\, s_m dI,$$

but

$$dY = Gdt = As_m dI;$$

eliminating $s_m dI$, we get:

$$\frac{Q - Y}{g} dv = \frac{\zeta}{A} dY;$$

$$dv = \frac{\zeta g}{A}\,\frac{dY}{Q - Y} = -\frac{\zeta g}{A}\,\frac{d(Q - Y)}{Q - Y};$$

$$v = \frac{\zeta g}{A}\ln\frac{G}{Q - Y} = \frac{2.303\zeta g}{A}\log\frac{Q}{Q - Y}.$$

The greatest velocity will obtain after all the gas is discharged from the combustion chamber, i.e., $Y_{max} = \omega$:

$$Y_{max} = \frac{2.303\zeta g}{A}\log\frac{Q}{Q - \omega} \approx 32300 \log\frac{Q}{Q - \omega} \quad \text{dm/sec}.$$

This is Tsiolkovsky's formula derived without taking air resistance into consideration.

The values of ζ depending on the degree of expansion of the nozzle were presented above. The table given below gives these values with respect to the ratio between the discharge diameter d_a of the nozzle and its smallest cross section d_m (Table 32).

Table 32

$\frac{d_a}{d_m}$	1	2	2.5	3	4
ζ	1.24	1.61	1.675	1.73	1.80

Therefore, at $d_a/d_m = 2$ the gain in the reaction force produced compared with a straight nozzle is 30%. When d_a/d_m is increased to 3 and 4, the added increase in the reaction force amounts to only 7 and 4%, respectively.

Inasmuch as the enlargement of the discharge cross section of the nozzle is associated with increase of length and weight, which add to the weight of the rocket without offering any appreciable advantage, the value of d_a/d_m in actual practice is taken within the limits of 2-2.5.

CHAPTER 4 - A BRIEF DISCUSSION OF THE THEORY OF THE MUZZLE BRAKE

1. GENERAL CONSIDERATIONS

A muzzle brake is a device attached to the muzzle of the barrel. Its purpose is to deflect a portion of the discharged gases in the direction of the barrel recoil and thus reduce the velocity of recoil and the load imposed on the gun mount.

A portion of the gas entering the muzzle brake moves in the direction (behind) the projectile through the center opening of the brake, and the other portion of gas is discharged through side openings of the brake in the direction of recoil.

The deflection of the gases to the sides reduces the quantity of gas passing through the center opening of the brake behind the projectile, and this serves to reduce the maximum velocity of recoil. The reaction produced by a portion of the gases discharged through the side openings creates a force counteracting the power of recoil and also retards the latter.

Thus the main purpose of a muzzle brake is to reduce the energy of the recoiling parts.

Introducing the designations:

V_{max} - maximum velocity of free recoil without muzzle brake;

V_T - velocity of free recoil at the end of gas after-action in the presence of a muzzle brake,

then the efficiency of the muzzle brake may be called the "relative reduction of the kinetic energy of the recoiling masses," i.e.,

$$\gamma' = \frac{V_{max}^2 - V_T^2}{V_{max}^2}.$$

The corresponding relative reduction of the maximum velocity of recoil can be denoted thus:

$$r = \frac{V_{max} - V_T}{V_{max}},$$

whereby

$$\gamma' = r(2 - r).$$

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If the difference between the weights of the recoiling parts in the presence of the muzzle brake (Q_T) and in the absence of the latter (Q_0) is taken into account:

$$\gamma' = \frac{v_{max}^2 - \frac{Q_T}{Q_0} v_T^2}{v_{max}^2}.$$

The simplest types of muzzle brakes are the "active brakes," whose action is based on the impact of gases escaping in the wake of the projectile against a surface fastened in front of the barrel (fig. 130).

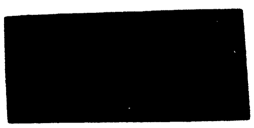

Fig. 130 - Diagram of an Active Brake.

Fig. 131 - Diagram of a Reaction Brake.

In reaction brakes the gases are discharged through curved passageways. The change of momentum along the bore axis will be equal to the reaction impulse of the stream against the deflecting brake surface (fig. 131).

2. GAS REACTION PRESSURE ON THE WALLS OF A CURVED BORE OF A MUZZLE BRAKE(*)

Let us not consider the flow of gas through a curved bore (fig. 132)

(*) D.A. Ventsel, "VNUTRENNIAYA BALLISTIKA" (Internal Ballistics) Part II, 1939.

whose entrance cross section is F_1 and exit cross section is F_2. The passageways are inclined at an angle a_1 with respect to the bore axis at the entrance opening and at an angle a_2 at the exit opening.

We shall apply the equation of the change of momentum to the volume of gas bounded by the curved walls of the bore and the two sections F_1 and F_2 normal to it.

The mass of gas entering the bore through section F_1 during the time interval dt is $\frac{G_T}{g}$ dt, whose component of the momentum along the axis of the gun barrel equals

$$\frac{G_T}{g} U_1 \cos \alpha_1 dt.$$

The same gas mass $\frac{G_T}{g}$dt will exit through section F_2, and will have a component of the momentum along the same axis equal to

$$\frac{G_T}{g} U_2 \cos \alpha_2 dt.$$

The increment of the projection of the momentum of the given volume on the x-axis equals the elementary impulse of time dt along the x-axis of the total pressure exerted by the bore on the gas and the pressures in the sections normal to it.

Inasmuch as the component of the pressure exerted by the bore on the gas along the x-axis equals the component of the gas reaction R_T on the bore with its sign reversed, we can write

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$$\frac{G_T}{g}(U_2 \cos \alpha_2 - U_1 \cos \alpha_1)dt = -R_{Tx}dt + F_1 p_1 \cos \alpha_1 dt - F_2 p_2 \cos\alpha_2 dt,$$

whence, bearing in mind that $\alpha_2 > \frac{\pi}{2}$ and $\cos \alpha_2 < 0$,

$$R_{Tx} = \frac{G_T}{g}(U_2 |\cos \alpha_2| + U_1 \cos \alpha_1) + F_2 p_2 |\cos \alpha_2| + F_1 p_1 \cos \alpha_1. \tag{99}$$

Fig. 132 - Diagram of Forces Acting in the Bore of the Muzzle Brake.

It follows that the component of the gas reaction on the bore along the x-axis is positive; it counteracts the recoil.

If the brake has several bores or passages inclined at the same angles α_1 and α_2, the expression for the reaction of the whole brake will remain exactly the same, where the designations R_{Tx}, G_T, F_1, F_2 relate to the sum of the areas of all the passages in the brake.

3. TOTAL REACTION R_x ON THE GUN BY GASES DISCHARGED THROUGH THE MUZZLE BRAKE

A gun equipped with a muzzle brake is subjected to the following forces acting along its axis during gas discharge.

1) The component along the x-axis of the reaction of gases discharged through the forward end of the brake (muzzle opening of the

barrel)

$$R_x = -(k+1)\left(\frac{2}{k+1}\right)^{k/k-1} sp = \zeta sp.$$

2) The component along the x-axis of the reaction of gases flowing into the muzzle brake through section F_1

$$R_{Tx} = -\frac{G_T}{g} U_1 \cos \alpha_1 - F_1 p_1 \cos \alpha_1 = -\left(\frac{G_T}{g} U_1 + F_1 p_1\right) \cos \alpha_1 .$$

3) The component along the x-axis of the gas reaction on the passageways of the muzzle brake (formula 99)

$$R_{Tx} = \frac{G_T}{g}(U_2 \left|\cos \alpha_2\right| + U_1 \cos \alpha_1) + F_2 p_2 \left|\cos \alpha_2\right| + F_1 p_1 \cos \alpha_1 .$$

We shall assume that the gas begins to flow simultaneously through all the openings after the base of the projectile has passed through the muzzle face. The component of the total reaction along the x-axis will be

$$R_{\Sigma x} = -\zeta sp + \left(\frac{G_T}{g} U_2 + F_2 p_2\right) \left|\cos \alpha_2\right| .$$

The first term is greater than the second, and the minus sign in front of the first term indicates that the total gas reaction on the gun acts in a direction opposite to that of the x-axis (opposite to the direction of the projectile's motion). The closer angle α_2 approaches π, the greater the values of G_T and F_2 and the greater the

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reaction force of the brake.

Obviously, the entry angle α_1 to the passageways in the brake does not enter into the expression for the total reaction R_Σ.

In computing the amount G_T and the velocity U_2 of the discharge from the passageways, we shall use the assumption that the pressure p_1 at the entrance to the brake passageways is critical with relation to the mean pressure in the bore of the gun at a given instant:

$$p_1 = x_{cr.}p = \left(\frac{2}{k+1}\right)^{k/k-1} p;$$

the incoming gas velocity U_1 may be disregarded.

By expanding in succession the values in R_Σ, Prof. D.A. Ventsel reduced this expression to the following general form:

$$R_\Sigma = \alpha_\Sigma \zeta s p,$$

where

$$\alpha_\Sigma = 1 - \frac{1}{k+1}\left[\Psi\chi \frac{2k}{\sqrt{k^2-1}}\left(\frac{2}{k+1}\right)^{1/k-1}\sqrt{1-\left(\frac{p_2}{p_1}\right)^{k-1/k}} + \frac{F_2 p_2}{F_1 p_1}\right]\frac{F_2}{s}\left|\cos\alpha_2\right|.$$

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In the last expression:

χ - a coefficient depending on the curvature of the passageways (χ = 0.75-1.0 at $\alpha_1 < 30^\circ$);

Ψ - a coefficient depending on the entrance angles α_1 and α_2 in terms of expression $\frac{\alpha_1 + \pi - \alpha_2}{2}$; its value is given in a table.

Table 33

$\frac{F_2}{F_1}$	$\frac{p_2}{p_1}$
1.01	0.5
1.06	0.4
1.19	0.3
1.46	0.2
2.25	0.1
3.61	0.05
11.8	0.01

The ratio of the pressures at the entrance and exit of the brake bore p_2/p_1 depends on the ratio F_2/F_1 and is determined from Table 33.

$\frac{\alpha_1 + \pi - \alpha_2}{2}$	15°	20°	25°	30°	40°	50°	60°	70°
Ψ	0.725	0.770	0.815	0.845	0.890	0.920	0.940	0.950

4. THE FULL IMPULSE OF THE TOTAL GAS REACTION

Similarly to a gun without a muzzle brake, we will have the following during the period of after-action between the gases and the brake:

$$\frac{Q_0}{g}(v_T - v_A) = \int_0^{t_n} R_\Sigma dt - \frac{\omega}{g}\frac{v_{A.a}}{2} = I_\Sigma - \frac{\omega}{g}\frac{v_{A.a}}{2},$$

whence

$$V_T = V_A + \frac{g}{Q_0} I_\Sigma - \frac{\omega}{Q_0} \frac{v_{A.a}}{2},$$

where

$$B_\Sigma = \alpha_\Sigma \zeta s p.$$

The pressure drop p as a function of time will be the same as in the usual case (in the absence of a muzzle brake), with the exception that the coefficient B' is replaced by the greater coefficient B_Σ, because the gases are discharged not only through the front opening of the brake, but also through the side passages:

$$p = \frac{p_A}{(1 + B_\Sigma t)^{2k/k-1}},$$

where

$$B_\Sigma = B'\left[1 + \chi \frac{F_1}{s} \left(\frac{2}{k+1}\right)^{(1/2)(k+1/k-1)}\right].$$

The period of after-action in the presence of a muzzle brake is

$$t_{ar} = \frac{1}{B_\Sigma}\left[\left(\frac{2}{k+1}\right)^{1/2}\left(\frac{p_A}{p_a}\right)^{k-1/2k} - 1\right].$$

Assuming $R_\Sigma = \alpha_\Sigma \xi s p$, substituting this expression for the impulse I_Σ and integrating, we get:

$$I_\Sigma = \alpha_\Sigma \xi s p_A \int_0^{t_n} \frac{dt}{(1 + B_\Sigma t)^{2k/k-1}} =$$

$$= \alpha_\Sigma (k - 1) \left(\frac{2}{k + 1}\right)^{k/k-1} \frac{s p_A}{B_\Sigma} \left[1 - \frac{1}{(1 + B_\Sigma t)^{k+1/k-1}}\right],$$

where

$$\xi = (k + 1) \left(\frac{2}{k + 1}\right)^{k/k-1};$$

$$B_\Sigma = B' \left[1 + \chi \frac{F_1}{s} \left(\frac{2}{k + 1}\right)^{(1/2)(k+1/k-1)}\right];$$

$$B' = \frac{k - 1}{2} \left(\frac{2}{k + 1}\right)^{(1/2)(k+1/k-1)} \frac{\sqrt{g k p_A v_A}}{l_0 + l_A}.$$

The function in brackets is close to unity.

Upon substituting $I_\Sigma = \frac{\omega}{g} \frac{v_{A.a}}{2}$ and $v_A = \frac{q + 0.5\omega}{Q_0} v_A$ in the expression for v_T, we get the final expression:

$$v_T = \frac{q}{Q_0} \left(1 + \frac{1}{2} \frac{\omega}{q}\right) v_A + \alpha_\Sigma \frac{2}{k} \left(\frac{2}{k + 1}\right)^{1/2} \frac{\omega}{Q_0} c_A -$$

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$$= \frac{\omega}{Q_0}\frac{v_{A.a}}{2} = \frac{q}{Q_0}\left(1 + \beta_\Sigma \frac{\omega}{q}\right)v_A,$$

where

$$\beta_\Sigma = \alpha_\Sigma \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{c_A}{v_A}.$$

At $k = 1.2$

$$\beta_\Sigma = 1.589\alpha_\Sigma \frac{c_A}{v_A} \text{ and } c_A = 10,85\sqrt{\frac{p_A(1 + \Lambda_A)}{\Delta}}.$$

Here p is in kg/cm^2, the velocity is in m/sec, Δ is in kg/dm^3.

Using the above formula, we can calculate the velocity v_T at the end of the period of gas after-action on the barrel and determine the efficiency of the brake:

$$\text{efficiency } \eta = \frac{v_{max}^2 - v_T^2}{v_{max}^2} = 1 - \left(\frac{1 + B_\Sigma \frac{\omega}{q}}{1 + \beta \frac{\omega}{q}}\right)^2.$$

The efficiency of modern muzzle brakes may be of the order of 40-50% and even 70 and 80% in exceptional cases.

Example. Calculate the efficiency of a muzzle brake. Say, the characteristics of the given gun are as follows:

$\Delta = 0.72$; $\frac{\omega}{q} = 0.453$; $\Lambda_A = 4.63$; $v_A = 1000$ m/sec; $p_A = 983$ kg/cm^2

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(subscript A represents muzzle-Translator).

In the absence of a muzzle brake (at $k = 1.2$)

$$\beta = 1.59 \frac{c_A}{v_A} = \frac{1.59}{v_A} \sqrt{gkp_A \frac{\Lambda_A + 1}{\Delta}} = \frac{1.59}{1000} \sqrt{117.7 \cdot 983 \cdot \frac{5.63}{0.72}} =$$

$$= 1.59 \frac{950}{1000} = 1.510.$$

$$1 + \beta \frac{\omega}{q} = 1 + 1.510 \cdot 0.453 = 1.684.$$

In the presence of a muzzle brake

$$\beta_\Sigma = 1.59 \alpha_\Sigma \frac{c_A}{v_A} = \alpha_\Sigma \beta;$$

$$\alpha_\Sigma = 1 - \frac{1}{k+1} \left[\psi \chi \frac{2k}{\sqrt{k^2 - 1}} \left(\frac{2}{k+1} \right)^{1/k-1} \sqrt{1 - \left(\frac{p_2}{p_1} \right)^{k-1/k}} + \right.$$

$$\left. + \frac{F_2}{F_1} \cdot \frac{p_2}{p_1} \right] \frac{F_1}{s} \cos \alpha_2 .$$

Say, the characteristics of the brake are as follows:

$$\alpha_1 = 30^\circ; \quad \alpha_2 = 120^\circ; \quad \frac{F_2}{s} = 1.5; \quad \frac{F_2}{F_1} = 1.01; \quad \frac{\alpha_1 + \pi - \alpha_2}{2} = 45^\circ.$$

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According to Table 33, $p_2/p_1 = 0.5$; $\psi = 0.905$; we shall assume that $\chi = 1$; $|\cos 120^\circ| = 0.50$.

For $k = 1.2$ $\frac{k-1}{k} = \frac{1}{6}$; $\frac{2k}{\sqrt{k^2 - 1}} \left(\frac{2}{k+1}\right)^{1/k-1} = 2.25$;

$$\left(\frac{p_2}{p_1}\right)^{k-1/k} = 0.5^{1/6} = 0.891.$$

$$\alpha_\Sigma = 1 - \frac{1}{2.2} \left[0.905 \cdot 1 \cdot 2.25 \sqrt{1 - 0.891} + 1.01 \cdot 0.50 \right] 1.5 \cdot 0.5 =$$

$$= 1 - \frac{1}{2.2} \left[0.672 + 0.505 \right] 0.75 = 1 - 0.40 = 0.60$$

$$\beta_\Sigma = 0.60 \cdot 1.510 = 0.906$$

$$1 + \beta_\Sigma \frac{\omega}{q} = 1 + 0.906 \cdot 0.453 = 1.410$$

$$\eta = 1 - \left(\frac{1 + \beta_\Sigma \frac{\omega}{q}}{1 + \beta \frac{\omega}{q}}\right)^2 = 1 - \left(\frac{1.410}{1.684}\right)^2 = 1 - 0.702 = 0.298.$$

Thus the given brake will absorb about 30% of the energy of free recoil.

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PART TWO - THE THEORY AND PRACTICE OF SOLVING PROBLEMS IN INTERNAL BALLISTICS

(THEORETICAL AND APPLIED PYRODYNAMICS)

INTRODUCTION

On the basis of the widespread study of the phenomena and processes occurring during a discharge, internal ballistics must establish the laws relating the conditions of loading to the quantities depending upon them - called ballistic elements of discharge — and must furnish the method of solving a large number of problems encountered in practice.

The establishment of such laws, providing the means for regulating a discharge, constitutes the general problem of internal ballistics.

The conditions of loading include the following: the dimensions of the powder chamber and those of the bore of the barrel, the weight of the latter, the arrangement of the rifling in the bore, the weight and arrangement of the projectile, the pressure necessary to overcome the inertia of the projectile, the weight of the charge, the make of powder, the physico-chemical and ballistic characteristics of the powder, the characteristics of the expansion of gases.

The ballistic elements of a discharge include the path of the projectile l, its velocity v, the pressure of the powder gases p, their temperature T, all values varying with time, and also the quantity of gas $\omega\psi$ formed at a given time.

In solving the above general problem of internal ballistics, [redacted]ne may distinguish two fundamental and most important problems of [redacted]yrodynamics, and a series of special problems.

The first fundamental problem consists in determining by calculation the change in gas pressure and the velocity of the projectile in the barrel as a function of the path of the projectile and of time, for given loading conditions. Together with the curves p, l-v, l-p, t-v, t two most important loading characteristics of the gun are determined: the maximum gas pressure p_m in the bore, and the muzzle velocity $v_д$ of the projectile, i.e., the velocity of the projectile at the instant it leaves the barrel of the gun. This problem may be called the direct problem of pyrodynamics.

For given conditions of loading it has a single solution - a single pressure curve with maximum p_m, a single velocity curve for the projectile, and a muzzle velocity $v_д$.

By varying the conditions of loading, it is possible to analyze the effect of these conditions on the variation of the gas pressure and projectile velocity curves, i.e., it is possible to solve a series of special problems related to the solution of the direct problem.

The second fundamental problem of pyrodynamics is the problem of the ballistic design of the gun; it consists in determining the design data of the barrel and conditions of loading necessary to impart some definite initial (muzzle) velocity to a projectile of a given caliber and weight. This velocity is determined from the tactical and technical requirements imposed upon the gun to be constructed.

In solving such a problem, the maximum gas pressure is usually given.

The design data and conditions of loading insuring that a

projectile of a given caliber and weight will attain the desired velocity, are obtained from the solution of the above problem. Once the conditions of loading are given, gas pressure and projectile velocity curves are drawn as a function of path and time, i.e., the direct problem of internal ballistics is solved for the selected type of gun and charge.

The obtained curve p, l is used by the engineers to calculate the strength of the gun barrel and projectile shell, while the curve p, t is used to design the carriage, the time fuzes and the igniters. At the same time, the necessary thickness and shape of the powder which must be prepared at the factory, are given.

Thus the further planning of the entire system of artillery and of the necessary ammunition depends to a considerable extent upon the feasibility and rationality of the selected form of the ballistic solution.

This is why the problem of the ballistic design of guns is the principal applied problem of interior ballistics.

The problem of ballistic design is broader than the first problem; it includes the latter as a final step and is in reality an inverse problem of interior ballistics. It admits of numerous solutions, numerous combinations of gun design data and loading conditions under which a projectile of a given caliber and weight will attain the required muzzle velocity.

Because of the indeterminate character of the solution, there arises the need of developing a definite method for obtaining the necessary answer in the shortest possible time, and for selecting from among this multiplicity of solutions the most efficient and desirable solution, satisfying the tactical and technical requirements

imposed upon the gun to be designed.

In this connection, special problems arise with regard to finding the most desirable solution, and for obtaining a gun of maximum power and minimum length or volume, the most suitable projectile, and the most desirable loading conditions.

The solution of these special problems permits in turn to pose the problem of the development of a general theory and method of ballistic design which would take into account the most desirable solutions and tactical and technical requirements.

Besides the indicated fundamental problems of internal ballistics, there is also a series of special and secondary problems introduced below.

For a given bore and a given projectile weight, calculate the weight ω of the projectile insuring a given muzzle velocity v_A, and the thickness $2e_1$ of powder giving the required maximum pressure p_m.

Because of the complexity of the phenomenon of discharge, not all of its details can be taken into account, even approximately; some of these details must be neglected and can not be introduced into the mathematical equations expressing the relations between the separate processes occurring during a discharge.

For this reason, the equations of internal ballistics give only approximate values of p, v, l, ψ, and t. But since in practice these equations must give results agreeing with experimental data, it is necessary, in order to insure this agreement, that the problem be solved by selecting certain constant characteristics. When these are substituted into the equations, they give values of p_m and v_A

for the gases and the projectile, respectively, which values correspond to the results of firing tests.

The very manner in which the problem is posed indicates that the processes taking place during a discharge are not yet all known and analyzed. For this reason one of the main problems of internal ballistics, that must be eventually solved, is the exact determination of constants, those of the gun powder in particular, as derived from its physical and chemical properties. The determination of the powder constants involves a more exact method of pressure determination by experimental means, because all the ballistic characteristics (f, α, u_1) are determined from the latter.

In addition to the problems enumerated above, one should note the problem of determining the variation in the maximum gas pressure and in the initial projectile velocity under specific changes in loading conditions, as well as a series of other problems.

The fundamental elements of a discharge - l, v, p, T, ψ and t - are interrelated by a series of equations expressing the fundamental processes taking place during a discharge, i.e.: the burning of the gun powder and the formation of gases, the transformation of the thermal energy of the gases into the kinetic energy of the system projectile - charge - barrel, and the movements of parts of this system.

The methods of solution of theoretical pyrodynamics must make it possible to compute and establish the dependence of gas pressure and of the velocity of the projectile on the path and the time it takes the projectile to move through the gun barrel, i.e., to solve the fundamental direct problem of internal ballistics.

The methods of solving problems in pyrodynamics may be divided into analytical, numerical, empirical and tabular methods.

The empirical methods were of definite advantage, so long as the theoretical concepts of internal ballistics had not been sufficiently developed.

They were based on some relatively simple empirical equations expressing in a simplified form the experimentally obtained interrelations of the elements of a discharge. Tables were used along with these equations, which tables offered the means for computing very rapidly the elements of the curves depicting gas pressure and projectile velocity.

The empirical methods were derived from the analysis of experimental data obtained in firing weapons under different conditions, with the characteristics and constants entering into these expressions determined from the conditions of the experiment.

The disadvantage of these methods (formulas and tables) consists in the fact that they fail to take into account certain very important factors and conditions of loading, and that such methods may be applied only under the conditions and within the limits established for the given case.

The number of empirical equations and tables is very large; prior to the development of analytical solutions, they were of primary value because of their simplicity. But the appearance of exact theoretical solutions, taking into account with sufficient completeness the influence of most of the conditions of loading and singularities of the processes occurring during a discharge, made it possible to solve all the fundamental problems of pyrodynamics by means of exact analytical relations. As a result, many empirical

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equations and tables have lost their significance and are now used only in certain auxiliary cases.

The analytical methods are based on a series of assumptions characterizing the conditions of powder burning and the motions of the gases, projectile and gun; these assumptions are based chiefly on experimental or theoretical data expressing the physical side of the process of discharge.

For this reason, analytical methods of solution give a more profound understanding of the real nature of the phenomenon than empirical methods, and approach more closely the essence of the processes taking place during a discharge.

In the analytical method the problem is reduced to the solution and integration of differential equations of different types. This solution can be obtained with greater accuracy (in which case the resulting equations become more complex), or approximately (which results in simpler relations).

Solutions may be given for the more complex cases obtained in practice, and also for simplified, admittedly schematic cases, in which case the analysis of the relationships is simplified.

Solutions may be based on the geometric and physical laws of powder burning.

Tables of auxiliary values or functions, necessary to calculate certain intermediate values, are prepared in order to expedite and simplify the computations involved in the solution of problems.

Numerical methods of integrating a system of differential equations are used along with the analytical methods. The integration is usually performed by the method of finite differences or by expansion in Taylor's series. These methods are resorted to in

especially difficult cases, when the value of one or several parameters varies throughout the process of discharge and their variation does not permit to solve the problem analytically in finite form. This happens, for example, when the cross section of a barrel bore varies (tapered bore), or when the parameter θ varies throughout the discharge accompanied by a varying gas temperature or by a change of the coefficient φ depicting to secondary work done in the process, etc.

On the basis of analytical or numerical solutions, it is possible to set up numerical tables of the fundamental elements (p, v, l, t) for different loading conditions and some general constants. These tables enable one to plot very rapidly the necessary curves p, l and v, l or p, t and v, t with a minimum number of calculations. In so doing, the process of solving the direct problem is greatly simplified and expedited. These tables are usually set up for certain average values of the constants (characteristics and shape of the powder), although in practice one may encounter a series of regressions from these average values. In that case it is necessary to introduce appropriate corrections into the results obtained.

Thus the ballistic tables for the solution of direct problems of pyrodynamics are in reality analytical equations reduced to numerical values in a series of concrete loading conditions.

However, the ballistic tables enable one to solve a series of problems which cannot be solved directly by means of analytical equations.

The basic difference between tabular values and analytical equations is the following: because of their complexity, the analytical equations do not give a direct relationship between

pressure or velocity and path length, for example; these variables are usually related through some auxiliary variable.

In tables, on the contrary, the basic elements of discharge are interrelated directly: the pressure, the projectile velocity, and the time of its travel through the barrel are given in function of the path traversed by the projectile; this simplifies considerably the analysis and permits the development of a special method for solving problems which cannot be solved by analytical means.

The development of the theory of ballistic design became possible only with the introduction of tables for the solution of internal ballistic problems.

For this reason the first tables prepared in our country by Prof. N.F. Drozdov on the basis of his exact solution given in 1910, are of great importance. These very tables simplified and expedited the calculations involved in the ballistic design of weapons, and gave the engineers a reliable means of solving rapidly inverse problems in pyrodynamics.

They also served as an example for a series of more detailed tables compiled subsequently.

SECTION SIX - ANALYTICAL METHODS OF SOLUTION OF THE DIRECT PROBLEM OF INTERNAL BALLISTICS.

BASIC ASSUMPTIONS

When we examined the phenomenon of a discharge, we had pointed out its extreme complexity and the fact that some of the factors influencing the results were still insufficiently known. For this reason, when solving theoretically the fundamental equation and deriving the relations between the physico-chemical and mechanical phenomena in a discharge, it is necessary to take recourse to certain simplifications and schemes.

The basic assumptions are as follows:

1) The burning of powder obeys the geometrical law of combustion.

2) The powder burns under an average pressure p.

3) The composition of the products of combustion does not change during burning, nor during the adiabatic expansion of the gases (f and α are constant) after the powder is burned.

4) The rate of burning is proportional to the pressure:

$$u = u_1 p.$$

5) The auxiliary work done is proportional to the principal work of the forward motion of the projectile, and is represented by the coefficient φ.

6) The projectile starts moving when the pressure developed in the chamber by the partial burning of the charge equals p_0, i.e., when the pressure is sufficient to force the driving hand completely into the rifling of the bore; the gradual forcing of the band and the increasing resistance encountered by it are not taken into account.

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7) The work done in forcing the driving band is not accounted for separately, nor the increasing velocity of the projectile during the gradual forcing of the band.

8) The expansion of the barrel during the discharge, the gases escaping through the clearance between the driving band and the walls of the gun, and the air resistance are disregarded.

9) The cooling of the gases through heat transfer to the walls of the barrel is not accounted for directly, and may be taken into account indirectly (for example, by decreasing the force $f = RT_1$ or increasing $\Theta = 1/(A + BT_{av.})$).

10) The motion of the projectile is considered only until it passes the muzzle face.

11) The quantity $\Theta = (c_p/c_w) - 1$ is taken as its average value, constant throughout the discharge.

The assumptions enumerated above make our representation of a discharge more schematic, and deviate the phenomenon to a greater or lesser degree from reality. For this reason the relations obtained in the solution will express the physico-mechanical nature of the discharge only with a certain degree of approximation. Thus the values of the fundamental elements (maximum gas pressure p_m, initial or muzzle velocity $v_\text{д}$ of the projectile) obtained from these equations may not coincide with the values obtained experimentally. Nevertheless, in order to solve practical problems, it is necessary to obtain analytical data which would agree with experiment. For this reason (keeping in mind the complexity of the discharge phenomenon, the incomplete knowledge of its elements, and the disagreement between our basic assumptions and reality) it is necessary to introduce coefficients of "agreement with experiment" into the constants obtained.

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Such a method is widely used in various scientific laws (hydrodynamics, aerodynamics, etc.) dealing with complex phenomena, whose details cannot be fully analyzed.

Eventually, as our knowledge is further developed, we will find it possible to render some of the assumptions with greater accuracy and take into account some of the conditions not yet understood. As new experimental data is accumulated and new methods are applied, the deductions arrived at may be modified and even replaced by others of a more complete and exact nature.

In solving the fundamental equation of pyrodynamics, one should strive to obtain the maximum possible mathematical accuracy. However, in that case some of the expressions become excessively cumbersome, so that even exact formulas will fail to represent the true phenomena of a discharge; and for this reason certain simplifications may be used with advantage in the process of solution.

A comparison of these simplified solutions with the exact ones may show the extent of mathematical error involved with the use of the same constants and conditions.

With an appropriate selection of constants, the somewhat simplified solutions may also yield results approaching experimental data as closely as those obtained by the use of more exact equations.

CHAPTER 1 - SOLUTION OF THE FUNDAMENTAL PROBLEM WHEN THE PRESSURE TO OVERCOME THE PROJECTILE INERTIA IS KNOWN, AND WHEN BURNING PROCEEDS ACCORDING TO THE GEOMETRIC LAW.

As we have shown above, the fundamental equation of pyrodynamics includes a large number of constants characterizing the projectile, charge and powder which determine the conditions of loading, and the four variables, ψ, v, l and p, which are called the elements of a shot.

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In order to establish the relation between the elements of a shot, new equations are added to the fundamental equation, which are the equations of powder burning and projectile motion; this leads to the appearance of a new variable, the time t, and to the appearance of quantity z when burning proceeds according to the geometrical law.

We obtain as a result the following system of equations:

The fundamental equation of pyrodynamics:

$$ps(\ell_\psi + \ell) = f\omega\psi - \frac{\theta}{2}\varphi m v^2. \tag{1}$$

The rate of powder burning:

$$u = \frac{de}{dt} = u_1 p. \tag{2}$$

The law of generation (inflow) of gases:

$$\psi = \varkappa z(1 + \lambda z) = \varkappa z + \varkappa\lambda z^2. \tag{3}$$

The law of motion of the projectile:

$$ps = \varphi m \frac{dv}{dt} \tag{4}$$

or

$$ps = \varphi m v \frac{dv}{dt}. \tag{5}$$

The totality of these equations affords the solution of the fundamental mathematical problem: of determining the curves p, ℓ

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and v, l and also p, t and v, t and of finding in particular the maximum gas pressure p_m and the muzzle velocity $v_д$ of the projectile.

We first solve the problem for regressive powder shapes, using the two-term formulas ($\varkappa > 1$, $\lambda < 0$, $\mu = 0$) and the assumptions enumerated above.

We shall solve the problem for all the periods of a shot in succession.

1. PRELIMINARY PERIOD

When establishing the relations for this period, we shall assume the simplest form of the phenomenon: the instantaneous forcing of the projectile band into the rifling.

Fundamental assumption. If the force necessary to overcome the resistance encountered by the driving band of the projectile in completely penetrating the rifling is Π_0, and the cross section of the bore is s, the quantity $p_0 = \Pi_0 s$ will be called "the pressure to overcome the inertia of the projectile" or the forcing pressure. We shall assume that the projectile is set in motion at the instant the gas pressure attains the value p_0.

Up to that moment the burning of the powder takes place in a constant volume. For this reason the preliminary period may be called pyrostatic and one may apply the already known equations of pyrostatics.

In this period, besides the forcing pressure p_0, we will be interested in the portion of the charge ψ_0 burned at the instant the projectile is set in motion, in the relative thickness of the powder $z_0 = e_0/e_1$, and in the relative surface area of the powder $S_0/S_1 = \sigma_0$.

These quantities, characterizing the end of the preliminary period, are simultaneously the initial values of the first period.

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Let us introduce the fundamental equations for the preliminary period.

The igniter is burned first and the pressure developed in the chamber is p_B, which may be computed by the following formula:

$$p_B = \frac{f_B \omega_B}{W_0 - \frac{\omega}{\delta} - \alpha_B \omega_B}, \tag{6}$$

where W_0 is the volume of the chamber; ω/δ is the volume of the charge proper; and f_B, α_B, ω_B are, respectively, the force, co-volume, and weight of the igniter. Under the usual conditions of ignition, $\alpha_B \omega_B$ may be neglected.

The charge proper will ignite when the pressure reaches p_B; at this instant the pressure is determined by the general equation of pyrostatics, which takes into account the effect of the igniter. At the instant the driving band is forced in the rifling, a certain portion of the charge ψ_0 will have burned, and:

$$p_0 = p_B + \frac{f \omega \psi_0}{W_0 - \frac{\omega}{\delta} - \frac{\omega}{\delta_1}\psi_0}, \tag{7}$$

where

$$\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}.$$

Inasmuch as the forcing pressure p_0 is known(*), we can determine

(*) p_0 varies between 250 and 400 kg/cm^2 for shells and between 300-500 kg/cm^2 for bullets when the entire side surface is forced into the rifling of the bore.

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what part ψ_0 of the charge will have been burned at the instant the projectile is set in motion. Solving equation (7) for ψ_0, we obtain:

$$\psi_0 = \frac{(p_0 - p_B)\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)}{f + (p_0 - p_B)\left(\alpha - \frac{1}{\delta}\right)} = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0 - p_B} + \frac{1}{\delta_1}}. \tag{8}$$

If we may neglect the pressure of the igniter, because p_0 is known only approximately, while p_B is small; we will obtain a simpler expression for computation:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}. \tag{9}$$

The quantity ψ_0 mainly depends on Δ and varies, in general, between 0.02 and 0.10.

If the amount ψ_0 of the burned portion of the powder is known, and the law of powder burning is in the form:

$$\psi = \varkappa z(1 + \lambda z) = \varkappa z + \varkappa \lambda z^2,$$

we can determine the relative thickness $z_0 = e_0/e_1$ of the powder burned at the start of motion, and the relative surface area σ_0.

We find σ_0 from the following formula:

$$\sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0},$$

and z_0, from the equation $\sigma_0 = 1 + 2\lambda z_0$:

$$z_0 = \frac{\sigma_0 - 1}{2\lambda} = \frac{(\sigma_0 - 1)(\sigma_0 + 1)}{2\lambda(\sigma_0 + 1)} = \frac{\sigma_0^2 - 1}{2\lambda(\sigma_0 + 1)} = \frac{2\psi_0}{(\sigma_0 + 1)\varkappa}.$$

Since usually $\sigma_0 \approx 1$, the following approximation is correct:

$$z_0 \approx \frac{\psi_0}{\varkappa}.$$

Besides these characteristics, we will also require the value l_{ψ_0} - the length of the free space in the chamber at the start of motion. This reduced value is determined from one of the following expressions:

$$l_{\psi_0} = \frac{W_{\psi_0}}{s} = \frac{1}{s}\left(W_0 - \frac{\omega}{\delta} - \frac{\omega\psi_0}{\delta_1}\right) = l_0\left(1 - \frac{\Delta}{\delta} - \frac{\Delta}{\delta_1}\psi_0\right) =$$

$$= l_0\Delta\left(\frac{1}{\Delta} - \frac{1}{\delta} - \frac{\psi_0}{\delta_1}\right),$$

where

$$l_0\Delta = \frac{W_0}{s}\,\frac{\omega}{W_0} = \frac{\omega}{s}.$$

2. FIRST PERIOD

In deriving the fundamental relationships for the first period, Prof. N.F. Drozdov was the first to propose the introduction of a new independent variable, $x = z - z_0$ (the relative thickness of

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the powder burned after the projectile is set in motion).

At the instant the projectile is set in motion $z = z_0$ and $x = 0$; at the ending of burning $z_K = 1$ and $x_K = 1 - z_0$.

Thus the limits of variation of the new argument are known in advance. Let us express all four fundamental elements, ψ, v, l, and p, as a function of this argument.

1. <u>Relation</u> $\psi = f_1(x)$. Substituting $z = z_0 + x$ in the formula $\psi = \varkappa z + \varkappa\lambda z^2$, we obtain:

$$\psi = \varkappa z_0 + \varkappa\lambda z_0^2 + \varkappa(1 + 2\lambda z_0)x + \varkappa\lambda x^2,$$

but

$$\varkappa z_0 + \varkappa\lambda z_0^2 = \psi_0; \; 1 + 2\lambda z_0 = \sigma_0.$$

Introducing, according to Drozdov, the additional designation $\varkappa\sigma_0 = k_1$, we obtain the desired relation:

$$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2. \tag{10}$$

2. <u>Relation</u> $v = f_2(x)$. The velocity v enters into the equation of motion:

$$sp = \varphi m \frac{dv}{dt}.$$

In order to eliminate p and t, we add the law governing the rate of burning:

$$u = \frac{de}{dt} = u_1 p.$$

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Multiplying these equations term by term and simplifying, we obtain:

$$dv = \frac{s}{\varphi m}\frac{de}{e_1} = \frac{se_1}{\varphi m u_1}dz = \frac{sI_K}{\varphi m}dz.$$

Integrating from 0 to v and from z_0 to z:

$$v = \frac{sI_K}{\varphi m}(z - z_0) = \frac{sI_K}{\varphi m}x. \tag{11}$$

Prior to the end of burning

$$v_K = \frac{sI_K}{\varphi m}(1 - z_0) = \frac{g}{\varphi}\frac{I_K}{q:s}(1 - z_0). \tag{12}$$

Consequently, the velocity of the projectile at the end of burning can be computed in advance, if the impulse of the powder pressure $I_K = e_1/u_1$ and the cross-sectional loading of the projectile are known.

Inasmuch as the quantities φ and $1 - z_0$ vary relatively little, the velocity of the projectile at the end of burning of the powder depends in the main on the ratio of the impulse I_K to the cross-sectional load q/s on the projectile, and during burning, the velocity of the projectile varies in proportion to x.

Equation (12) permits to compute v_K, but it does not tell us the point on the path the projectile at which the powder is burned, whether the speed v_K is properly chosen for the given gun, or whether the powder is fully burned before the projectile leaves the gun. For this reason, this equation alone is insufficient, and it is

necessary to find also the equation for the path traversed by the projectile at the end of burning.

Equation (12) is plotted in fig. 133; it shows the curve v, l and gives the value of v_K, but it does not show the position of the projectile at the end of burning.

Fig. 133 - Path of the Projectile at the End of Powder Burning.

3. Relation $l = f_3(x)$. In order to determine the path of the projectile, two equations must be used: the fundamental equation of pyrodynamics and the equation of motion of the projectile in the form of elementary work:

$$ps(l_\psi + l) = f\omega\psi - \frac{\theta}{2}\varphi m v^2 = f\omega\left(\psi - \frac{v^2}{v^2_{\Pi p}}\right);$$

$$psdl = \varphi m v dv,$$

where $v^2_{\Pi p} = 2f\omega/\varphi\theta m$.(*)

We eliminate p by dividing the second equation by the first:

(*) The Russian subscripts Пp denote: path traversed inside a barrel. Editor.

$$\frac{dl}{l_\psi + l} = \frac{\varphi m}{f\omega} \frac{v\,dv}{\psi - \dfrac{v^2}{v_{\Pi_p}^2}}.$$

Since v and ψ are functions of x according to equations (10) and (11), the right-hand side of this differential equation may be represented as a function of x.

Designating this function by dF(x) and substituting for v and ψ their expressions in x we obtain:

$$dF(x) = \frac{\varphi m}{fm} \frac{\dfrac{s^2 I_K^2}{\varphi^2 m^2} x\,dx}{\psi_0 + k_1 x + \varkappa\lambda x^2 - \dfrac{s^2 I_K^2 \varphi m \theta}{\varphi^2 m^2 2 f\omega} x^2} =$$

$$= \frac{s^2 I_K^2}{f\omega\varphi m} \frac{x\,dx}{\psi_0 + k_1 x - \left(\dfrac{s^2 I_K^2}{f\omega\varphi m} \dfrac{\theta}{2} - \varkappa\lambda\right) x^2}.$$

The same group $s^2 I_K^2 / f\omega\varphi m$ of constants and characteristics appears in the numerator and the denominator. As suggested by Prof. N.F. Drozdov, it is represented by B and is called "the parameter of the loading conditions" (Prof. N.F. Drozdov's parameter):

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$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{s^2 e_1^2}{u_1^2 f\omega\varphi m}.$$

The influence of this parameter will be established later.

Let us designate (also according to Drozdov):

$$\frac{B\theta}{2} - \varkappa\lambda = B_1.$$

Then

$$\frac{dl}{l_\psi + l} = \frac{Bxdx}{\psi_0 + k_1 x - B_1 x^2}. \qquad (13)$$

The expression obtained is the fundamental differential equation for the path of the projectile as a function of x. It is solved differently by various authors.

If we place outside the parenthesis $-B_1$ in the denominator of the right side of the equation (as suggested by Drozdov) in order to obtain a polynomial in descending powers of x, with the coefficient of x^2 equal to 1, we obtain:

$$\frac{dl}{l_\psi + l} = -\frac{B}{B_1}\,\frac{xdx}{x^2 - \frac{k_1}{B_1}x - \frac{\psi_0}{B_1}} = -\frac{B}{B_1}\,\frac{xdx}{f_1(x)}, \qquad (13')$$

where:

$$\xi_1(x) = x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}.$$

Prof. N.F. Drozdov was the first to solve this equation exactly, in 1903, by reducing it to the form of a linear equation of the first order:

$$\frac{dl}{dx} + \frac{B}{B_1}\,\frac{x}{\xi_1(x)}\,l = -\frac{B}{B_1}\,\frac{x}{\xi_1(x)}\,l_\Psi$$

or

$$\frac{dl}{dx} + P_x l = Q_x,$$

where P_x and Q_x are functions of x.

The full solution of this equation is presented later.

A simpler, but approximate solution is obtained if we assume $l_\Psi = l_{\Psi\,av.} = \text{const.}$

It will be presented later with the designation of the parameters, with some of the auxiliary functions derived according to Prof. Drozdov.

During burning of the powder at the start of the projectile's motion, l_Ψ varies within the limits of $l_{\Psi 0}$ and l_1:

$$l_{\Psi 0} > l_\Psi > l_1,$$

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where

$$l_{\psi} = l_0 \left[1 - \frac{\Delta}{\delta} - \Delta \left(\alpha - \frac{1}{\delta} \right) \psi \right].$$

The rate of change of l_{ψ} increases with Δ.

Assuming that $l_{\psi} = l_{\psi_{av.}}$ and integrating equation (13') we have:

$$\int_0^l \frac{dl}{l_{\psi_{av.}} + l} = - \frac{B}{B_1} \int_0^x \frac{x dx}{\xi_1(x)}.$$

The integral of the right-hand side is obtained by decomposing the integrand into the simplest fractions; it is a logarithmic function of x which we shall temporarily designate by ln Z_x. The left-hand side is integrated also:

$$\ln \left(1 + \frac{l}{l_{\psi_{av.}}} \right) = - \frac{B}{B_1} \ln Z_x,$$

whence:

$$l = l_{\psi_{av.}} \left(Z_x^{-\frac{B}{B_1}} - 1 \right). \qquad (14)$$

Thus the expression for the path l as a function of x is more complex than the expressions for ψ and v.

Substituting into it $x_K = 1 - z_0$, we can find the path l_K at

the end of powder burning. Comparing it with the full path $l_{д}$ traversed by the projectile within the bore, it is possible to determine whether the thickness of the powder and the velocity v_K of the projectile are correctly chosen for the given gun.

In computing $l_{\psi_{av.}}$ we may use in the equation $l_{\psi_{av.}} =$

$$= l_0 \left[1 - \frac{\Delta}{\delta} - \Delta \left(\alpha - \frac{1}{\delta} \right) \psi_{av.} \right]$$ the expression $\psi_{av.} = \frac{\psi_0 + \psi}{2}$.

The investigations of Prof. G.V. Oppokov in his book "O TOCHNOSTI NEKOTORYKH ANALITICHESKIKH SPOSOBOV RESHENIYA OSNOVNOI ZADACHI VNUTRENNEI BALLISTIKI DLIA PERVOGO PERIODA" (Concerning the Accuracy of Certain Analytical Methods of Solving the Fundamental Problem of Internal Ballistics for the First Period), 1932 [2] have shown the following. When the loading density is $\Delta = 0.5\text{-}0.7$, formula (14) is very accurate for evaluating p_m and $v_д$, if $l_{\psi_{av.}}$ is not taken to have the same value for all the values x from 0 to $1 - z_0$, and if a different value of $l_{\psi_{av.}}$ is taken for every value of x, assuming either of the following values for $l_{\psi_{av.}}$ in the formula: $\psi_{av.} = (\psi - \psi_0)\ 2$ (Oppokov) or $\psi_{av.} = (\psi_0 + \psi)\ 2$ (Serebryakov).

Inasmuch as x is directly proportional to v [equation (11)], equation (14) gives in fact the direct relation between the path l and the projectile velocity v.

The expression for Z_x presented below shows that this relation is expressed by a rather complex function.

4. __Relation__ $p = f_4(x)$. The pressure p is found from the fundamental equation of pyrodynamics:

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$$p = \frac{f\omega}{s} \frac{\psi - \frac{v^2}{v_{np}^2}}{l_\psi + l}.$$

If the quantities ψ, v, and l are replaced in the right-hand side by their expressions in function of x, then

$$p = \frac{f\omega}{s} \frac{\psi_0 + k_1 x - B_1 x^2}{l_\psi + l_{\psi_{av.}} (Z_x^{-\frac{B}{B_1}} - 1)} = \frac{f\omega}{s} \frac{\psi - \frac{B\theta}{2} x^2}{l_\psi + l}, \tag{15}$$

where l_ψ can be represented as a function of x as well.

Inasmuch as ψ, v, and l are already determined, it is no longer necessary in computing p to use equation (15) which is expressed in terms of x, and the numerical values of ψ, v, l_ψ, and l can be substituted in the preceding equation.

By attributing to x different values within the limits of 0 to $1 - z_0$, equations (10), (11), (14), and (15) permit one to find the values of all the elements ψ, v, l, and p of a shot entering into the fundamental equation of pyrodynamics, and to plot a curve showing the variation of p, v, ψ as a function of l, i.e., the curves p, l and v, l.

Consequently, the proposed problem concerning the solution of the fundamental equation of pyrodynamics has been resolved, and the relation between the elements has been found.

Substituting the value of $x_K = 1 - z_0$ in the above equations, we

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find all the elements v_K, p_K, and l_K corresponding to the instant the burning of the powder ends ($\psi = 1$). These values will be the initial values in the second period.

Note. The expression for the projectile velocity may be replaced by the following:

$$v^2 \frac{s^2 I_K^2}{\varphi^2 m^2} x^2 = \frac{s^2 I_K^2 \theta}{2 f \omega \varphi m} \frac{2 f \omega}{\varphi m \theta} x^2 = \frac{B\theta}{2} v_{\Pi p}^2 x^2,$$

whence,

$$v = v_{\Pi p} \sqrt{\frac{B\theta}{2} x}.$$

This expression brings out the effect of the limiting velocity of the projectile and that of the parameter of loading conditions, B. Since in most guns B varies within narrow limits, it follows that the velocity mainly depends upon the potential $f\,\theta$ of the powder and upon the relative weight $\omega\,q$ of the charge.

<u>Determination of the function</u> $Z_x = e^{\int_0^x \frac{x dx}{\xi_1(x)}}$. In order to evaluate the integral

$$\int_0^x \frac{x dx}{\xi_1(x)} = \int_0^x \frac{x dx}{x^2 - \frac{k_1}{B_1} x - \frac{\psi_0}{B_1}}$$

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we decompose the integrand into the simplest fractions, finding the roots of equation $\xi_1(x) = 0$ and introducing the designation:

$$b = \sqrt{1 + 4\frac{B_1\psi_0}{k_1^2}} = \sqrt{1 + 4\gamma} > 1;$$

$$x = \frac{k_1}{2B_1} \pm \sqrt{\frac{k_1^2}{4B_1^2} + \frac{\psi_0}{B_1}} = \frac{k_1}{2B_1}\left(1 \pm \sqrt{1 + 4\frac{B_1\psi_0}{k_1^2}}\right) =$$

$$= \frac{k_1}{2B_1}(1 \pm b); \qquad (16)$$

$$x_1 = \frac{k_1}{2B_1}(1 + b), \quad x_2 = \frac{k_1}{2B_1}(1 - b) < 0;$$

$$\xi_1(x) = (x - x_1)(x - x_2).$$

Let us write an equation to determine the numerators of the simplest fractions:

$$\frac{x}{\xi_1(x)} = \frac{A_1}{x - x_1} + \frac{A_2}{x - x_2};$$

Equating on both sides the coefficients of identical powers of x, we find:

$$A_1(x - x_2) + A_2(x - x_1) = x;$$

$$A_1 + A_2 = 1; \quad -A_1x_2 - A_2x_1 = 0,$$

whence,

$$A_1 = -\frac{x_1}{x_2 - x_1}, \quad A_2 = \frac{x_2}{x_2 - x_1},$$

but:

$$x_2 - x_1 = -\frac{k_1}{B_1} b,$$

and, consequently:

$$A_1 = \frac{b+1}{2b}, \quad A_2 = \frac{b-1}{2b},$$

$$\int_0^x \frac{x\,dx}{\xi_1(x)} = \frac{b+1}{2b}\int_0^x \frac{dx}{x - x_1} + \frac{b-1}{2b}\int_0^x \frac{dx}{x - x_2} =$$

$$= \ln\left(\frac{x - x_1}{-x_1}\right)^{\frac{b+1}{2b}}\left(\frac{x - x_2}{-x_2}\right)^{\frac{b-1}{2b}} = \ln\left(1 - \frac{x}{x_1}\right)^{\frac{b-1}{2b}}\left(1 - \frac{x}{x_2}\right)^{\frac{b-1}{2b}} = \ln Z,$$

where:

$$Z = \left(1 - \frac{x}{x_1}\right)^{\frac{b+1}{2b}}\left(1 - \frac{x}{x_2}\right)^{\frac{b+1}{2b}}. \tag{17}$$

x_1 and x_2 are expressed by equations (16).

Substituting here these values of x_1 and x_2, we get:

$$Z_x = \left(1 - \frac{2}{b+1}\frac{B_1}{k_1}x\right)^{\frac{b+1}{2b}}\left(1 + \frac{2}{b-1}\frac{B_1}{k_1}x\right)^{\frac{b-1}{2b}}.$$

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Inasmuch as the quantity $b = \sqrt{1 + 4B_1\Psi_0/k_1^2} = \sqrt{1 + 4\gamma}$ is itself a function of the parameter $\gamma = B_1\Psi_0/k_1^2$, the function Z_x actually depends only upon two quantities: the constant $\gamma = B_1\Psi_0/k_1^2$ and the variable $\beta = B_1x/k_1$.

From these data it is possible to set up a table. Since the equation of the path contains the expression Z_x^{-B/B_1}, the tables are set up for $\log Z_x^{-1}$ to make their use more convenient.

The quantities entered (introduced) are γ and β.

It is not difficult to show by another method that $\int_0^x \frac{x\,dx}{x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}} = \ln Z_x$ is a function of $\gamma = B_1\Psi_0/k_1^2$ and $\beta = B_1x/k_1$, if the numerator and denominator of the integrand are multiplied by B_1^2/k_1^2. Then,

$$\ln Z_x = \int_0^x \frac{\frac{B_1}{k_1}x\,d\frac{B_1}{k_1}x}{\left(\frac{B_1}{k_1}x\right)^2 - \frac{B_1}{k_1}x - \frac{B_1\Psi_0}{k_1^2}} = \int_0^\beta \frac{\beta\,d\beta}{\beta^2 - \beta - \gamma}.$$

This expression actually shows that $\ln Z_x$ is a function of γ and β.

The table of the logarithms of the function ($\log Z_x^{-1}$) is presented below (Table 1).

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Table 1 - Table of Logarithms of the Function log Z^{-1} (β, γ)

β \ γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.08
0	0	0	0	0	0	0	0	0	0	0	0	0
0.020	0.0088	0.0080	0.0075	0.0067	0.0057	0.0049	0.0044	0.0039	0.0023	0.0017	0.0011	0.00
0.040	0.0177	0.0168	0.0161	0.0150	0.0134	0.0123	0.0014	0.0106	0.0079	0.0054	0.0041	0.00
0.060	0.0269	0.0258	0.0250	0.0238	0.0219	0.0204	0.0192	0.0181	0.0144	0.0103	0.0081	0.00
0.080	0.0362	0.0351	0.0342	0.0329	0.0307	0.0290	0.0275	0.0262	0.0215	0.0161	0.0130	0.01
0.100	0.0458	0.0446	0.0436	0.0422	0.0398	0.0379	0.0362	0.0347	0.0292	0.0225	0.0185	0.01
0.120	0.0555	0.0543	0.0533	0.0517	0.0491	0.0471	0.0452	0.0435	0.0373	0.0294	0.0246	0.02
0.140	0.0655	0.0642	0.0632	0.0615	0.0588	0.0565	0.0545	0.0527	0.0458	0.0368	0.0311	0.02
0.160	0.0757	0.0744	0.0734	0.0716	0.0687	0.0663	0.0641	0.0622	0.0546	0.0446	0.0380	0.03
0.180	0.0862	0.0848	0.0838	0.0819	0.0789	0.0763	0.0740	0.0720	0.0637	0.0528	0.0453	0.03
0.200	0.0969	0.0955	0.0944	0.0925	0.0893	0.0867	0.0842	0.0820	0.0732	0.0613	0.0530	0.04
0.220	0.1079	0.1065	0.1053	0.1034	0.1001	0.0972	0.0947	0.0924	0.0830	0.0702	0.0611	0.05
0.240	0.1192	0.1177	0.1166	0.1145	0.1111	0.1081	0.1055	0.1031	0.0932	0.0794	0.0695	0.06
0.260	0.1308	0.1293	0.1281	0.1260	0.1224	0.1194	0.1166	0.1141	0.1037	0.0889	0.0783	0.07
0.280	0.1427	0.1411	0.1399	0.1378	0.1341	0.1309	0.1280	0.1254	0.1145	0.0988	0.0874	0.07
0.300	0.1549	0.1533	0.1521	0.1499	0.1461	0.1428	0.1398	0.1371	0.1256	0.1090	0.0969	0.08
0.320	0.1675	0.1659	0.1646	0.1624	0.1585	0.1551	0.1520	0.1491	0.1371	0.1196	0.1067	0.09
0.340	0.1805	0.1783	0.1775	0.1752	0.1712	0.1677	0.1645	0.1615	0.1490	0.1306	0.1169	0.10
0.360	0.1938	0.1922	0.1908	0.1884	0.1843	0.1807	0.1774	0.1743	0.1613	0.1419	0.1275	0.11
0.380	0.2076	0.2059	0.2046	0.2021	0.1979	0.1941	0.1907	0.1875	0.1740	0.1536	0.1385	0.12
0.400	0.2219	0.2201	0.2188	0.2163	0.2119	0.2080	0.2045	0.2012	0.1871	0.1659	0.1499	0.13
0.420	0.2366	0.2348	0.2335	0.2310	0.2264	0.2224	0.2187	0.2154	0.2007	0.1786	0.1617	0.14
0.440	0.2518	0.2500	0.2486	0.2461	0.2414	0.2373	0.2335	0.2301	0.2148	0.1917	0.1739	0.15
0.460	0.2676	0.2658	0.2643	0.2617	0.2569	0.2527	0.2488	0.2453	0.2294	0.2052	0.1866	0.17
0.480	0.2840	0.2822	0.2806	0.2779	0.2730	0.2687	0.2647	0.2610	0.2446	0.2193	0.1998	0.18
0.500	0.3010	0.2992	0.2976	0.2948	0.2898	0.2853	0.2812	0.2773	0.2604	0.2340	0.2136	0.19
0.520	0.3187	0.3169	0.3153	0.3124	0.3073	0.3026	0.2984	0.2943	0.2768	0.2493	0.2279	0.2
0.540	0.3372	0.3353	0.3337	0.3307	0.3255	0.3207	0.3163	0.3121	0.2939	0.2653	0.2428	0.2
0.560	0.3565	0.3545	0.3529	0.3498	0.3445	0.3396	0.3350	0.3307	0.3117	0.2819	0.2583	0.2
0.580	0.3767	0.3747	0.3730	0.3699	0.3614	0.3593	0.3545	0.3501	0.3304	0.2992	0.2745	0.2
0.600	0.3979	0.3959	0.3941	0.3909	0.3852	0.3799	0.3750	0.3704	0.3500	0.3174	0.2915	0.2

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Table 1 - Table of Logarithms of the Function log Z^{-1} (,)

01	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
	0	0	0	0	0	0	0	0	0	0	0	0
075	0.0067	0.0057	0.0049	0.0044	0.0039	0.0023	0.0017	0.0011	0.0009	0.0008	0.0006	0.0004
161	0.0150	0.0134	0.0123	0.0014	0.0106	0.0079	0.0054	0.0041	0.0033	0.0028	0.0020	0.0015
250	0.0238	0.0219	0.0204	0.0192	0.0181	0.0144	0.0103	0.0081	0.0067	0.0057	0.0042	0.0033
342	0.0329	0.0307	0.0290	0.0275	0.0262	0.0215	0.0161	0.0130	0.0109	0.0094	0.0070	[illegible]
436	0.0422	0.0398	0.0379	0.0362	0.0347	0.0292	0.0225	0.0185	0.0157	0.0137	0.0104	0.0084
533	0.0517	0.0491	0.0471	0.0452	0.0435	0.0373	0.0294	0.0246	0.0211	0.0185	0.0142	0.0116
632	0.0615	0.0588	0.0565	0.0545	0.0527	0.0458	0.0368	0.0311	0.0269	0.0238	0.0185	0.0152
734	0.0716	0.0687	0.0663	0.0641	0.0622	0.0546	0.0446	0.0380	0.0332	0.0295	0.0232	0.0191
838	0.0819	0.0789	0.0763	0.0740	0.0720	0.0637	0.0528	0.0453	0.0399	0.0356	0.0283	0.0234
944	0.0925	0.0893	0.0867	0.0042	0.0820	0.0732	0.0613	0.0530	0.0469	0.0421	0.0337	0.0281
053	0.1034	0.1001	0.0972	0.0947	0.0924	0.0830	0.0702	0.0611	0.0543	0.0490	0.0395	0.0381
166	0.1145	0.1111	0.1081	0.1055	0.1031	0.0932	0.0794	0.0695	0.0621	0.0562	0.0456	0.0384
281	0.1260	0.1224	0.1194	0.1166	0.1141	0.1037	0.0889	0.0783	0.0702	0.0638	0.0520	0.0440
399	0.1378	0.1341	0.1309	0.1280	0.1254	0.1145	0.0988	0.0874	0.0787	0.0717	0.0588	0.0499
521	0.1499	0.1461	0.1428	0.1398	0.1371	0.1256	0.1090	0.0969	0.0875	0.0799	0.0659	0.0561
646	0.1624	0.1585	0.1551	0.1520	0.1491	0.1371	0.1196	0.1067	0.0967	0.0885	0.0733	0.0626
775	0.1752	0.1712	0.1677	0.1645	0.1615	0.1490	0.1306	0.1169	0.1062	0.0974	0.0810	0.06[illegible]
908	0.1884	0.1843	0.1807	0.1774	0.1743	0.1613	0.1419	0.1275	0.1161	0.1067	0.0890	0.07[illegible]
046	0.2021	0.1979	0.1941	0.1907	0.1875	0.1740	0.1536	0.1385	0.1263	0.1163	0.0974	0.0840
188	0.2163	0.2119	0.2080	0.2045	0.2012	0.1871	0.1659	0.1499	0.1370	0.1264	0.1062	0.0917
35	0.2310	0.2264	0.2224	0.2187	0.2154	0.2007	0.1786	0.1617	0.1481	0.1369	0.1153	0.0998
86	0.2461	0.2414	0.2373	0.2335	0.2301	0.2148	0.1917	0.1739	0.1596	0.1478	0.1247	0.1082
43	0.2617	0.2569	0.2527	0.2488	0.2453	0.2294	0.2052	0.1866	0.1715	0.1589	0.1345	0.1169
06	0.2779	0.2730	0.2687	0.2647	0.2610	0.2446	0.2193	0.1998	0.1839	0.1705	0.1447	0.1260
76	0.2948	0.2898	0.2853	0.2812	0.2773	0.2604	0.2340	0.2136	0.1968	0.1827	0.1554	0.1354
53	0.3124	0.3073	0.3026	0.2984	0.2943	0.2768	0.2493	0.2279	0.2102	0.1954	0.1665	0.1452
37	0.3307	0.3255	0.3207	0.3163	0.3121	0.2939	0.2653	0.2428	0.2242	0.2086	0.1780	0.1555
29	0.3498	0.3445	0.3396	0.3350	0.3307	0.3117	0.2819	0.2583	0.2388	0.2223	0.1900	0.1662
90	0.3699	0.3614	0.3593	0.3545	0.3501	0.3304	0.2992	0.2745	0.2540	0.2366	0.2025	0.1773
41	0.3909	0.3852	0.3799	0.3750	0.3704	0.3500	0.3174	0.2915	0.2699	0.2516	0.2156	0.1889

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Procedure for Using the Table.

For every problem we will have one value for the entry parameter $\gamma = B_1\psi_0/k_1^2$ and a series of values $\beta = B_1 x/k_1$, where x varies between 0 and $1 - z_0$.

When determining $\log Z_x^{-1}$, write down the values from the columns containing the nearest smaller and larger tabular values of γ, so that the value of γ obtained from the solution would fall between them. The coefficient of interpolation will be the same along all the horizontal rows. For this reason, it is more convenient to interpolate first along the horizontal rows between which are contained the values of β selected in the problem, and then to interpolate along the columns (vertically) using the corresponding interpolation coefficients β.

In order to reduce the number of vertical interpolations (except such cases when $\log Z_x^{-1}$ is used for computing the values p_m and p_K), it is more convenient to assign tabular values of β for the intermediate values of x and to perform only the horizontal interpolation for γ, and then determine x by means of equation $x = k_1\beta/B_1$.

3. DETERMINATION OF THE MAXIMUM PRESSURE GENERATED BY THE POWDER GASES

The maximum gas pressure p_m in the barrel is the most important ballistic characteristic of a gun. Its value depends on the chosen conditions of loading, and the obtainment of the desired value of p_m serves as a criterion or control for the proper selection of the weight of the charge, the thickness of the powder, and other loading conditions.

For this reason, it is sometimes important to be able to compute the pressure p_m for the given loading conditions, without constructing

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the entire p, l pressure curve. In order to achieve this, it is necessary to derive first a formula for determining the value of x_m for which the gas pressure is maximum.

In this case the derivative dp/dl or dp/dt must be equated to zero. The expression for the derivative was derived earlier by differentiating the expression for p from the fundamental equation of pyrodynamics.

$$\frac{dp}{dl} = \frac{p}{l_\psi + l}\left\{\frac{f\omega}{s}\,\frac{\varkappa}{I_K}\,\frac{\sigma}{v}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p}{f}\right] - (1 + \theta)\right\}.$$

Equating the expression in braces to zero, and substituting for v and σ their expressions in terms of x:

$$v = \frac{sI_K}{\varphi m}\,x, \quad \sigma = 1 + 2\lambda z = \sigma_0 + 2\lambda x,$$

we obtain the possibility of determining x_m for which the pressure is maximum:

$$\frac{f\omega}{s}\,\frac{\varkappa}{I_K}\,\frac{(\sigma_0 + 2\lambda x_m)\varphi m}{sI_K x_m}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] - (1 + \theta) = 0$$

or

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$$\frac{\varkappa\sigma_0 + 2\varkappa\lambda x_m}{Bx_m}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] = 1 + \theta.$$

$$x_m = \frac{k_1}{\dfrac{B(1+\theta)}{1 + \left(\alpha - \dfrac{1}{\delta}\right)\dfrac{p_m}{f}} - 2\varkappa\lambda}. \tag{18}$$

If the powder has a constant burning area $\lambda = 0$, $k_1 = \varkappa\sigma_0 = 1$ and

$$x_m = \frac{1 + \left(\alpha - \dfrac{1}{\delta}\right)\dfrac{p_m}{f}}{B(1+\theta)}. \tag{19}$$

It is seen from these equations that in order to determine x_m it is necessary to know p_m, but inasmuch as we do not know it, we must find the real value of x_m by the method of successive approximations. First we assume a reference value $p_m^{(0)}$, substitute it in equation (18) or (19), and compute the value of x_m', following which we substitute the latter successively into all the fundamental equations

$$v = \frac{sI_K}{\varphi m}x;\quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;\quad l = l_{\psi\,av.}\left(Z_x^{-\frac{B}{B_1}} - 1\right);$$

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$$p = \frac{f\omega}{s} \frac{\psi - \frac{v^2}{v_{np}^2}}{l_\psi + l},$$

and find the values of v'_m, l'_m, ψ'_m, p'_m. If p'_m coincides with $p_m^{(0)}$, it is indeed the true maximum pressure. However, if $p'_m \neq p_m^{(0)}$, then p'_m must again be substituted in (18) or (19) and a new x''_m obtained; then the whole process is repeated and a new p''_m is obtained. If x''_m is chosen correctly, p''_m should not differ from p'_m by more than 10-20 kg/cm^2 (the accuracy of a slide rule).

It must be remembered that equations (18) and (19) are used for calculating x_m and can not be employed for calculating p_m.

When carrying out the approximations, the following should be kept in mind: the relation p, x is represented by a curve shown in fig. 134, which varies slowly in the neighborhood of the maximum.

The true value of x_m is not known, and we find by means of equations (18) and (19) only a certain approximate value, which, even upon substituting the value $p_0 = 300$ kg/cm^2 for p_m in (19) will give a value of x'_m differing from the real value by not more than 10%. The value of p'_m will then be sufficiently close to the true value of p_m, and at the next approximation x'_m will practically coincide with x_m.

Whatever the quantity of $p_m^{(0)}$ assigned in the first approximation, whether smaller or larger than the real value of p_m, the values of p'_m and p''_m will be smaller in both cases than the real p_m. In the subsequent approximations the pressure values must increase, tending toward the real p_m, i.e., $p'_m < p''_m < p'''_m \rightarrow p_{m\ real}$, regardless of the value of $x_m^{(0)}$. Lecturer Belenky proved analytically that in successive

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approximations the quantity x_m' is monotonic increasing, tending toward x_m as the limit, while x_m'' is monotonic decreasing and tends toward the same limit x_m.

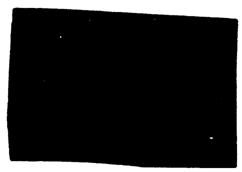

Fig. 134 - Determination of x_m and p_m (graph p, x).

This law must be used for controlling the accuracy of the calculations.

Since the pressure varies slowly in the neighborhood of the maximum, the p_m' obtained following the substitution of these values of x_m' in the working equations will be very close to the real p_m, and the second approximation will be adequate to obtain a value of p_m'' sufficiently close to the real value.

Having found x_m, we substitute it into (11) for v, (10) for ψ, (14) for l, and (15) for p_m, and obtain the elements of the projectile's motion, i.e., v_m, ψ_m, l_m, and p_m at the instant of greatest pressure.

Expression (18) gives the analytical expression for x_m at which the gas pressure becomes maximum. Can this equation always be used to determine the maximum pressure?

In most cases when the chosen loading conditions are normal, this formula will give the right answer. But there are cases when

it may yield a value x_m devoid of physical meaning. This occurs when:

$$x_m > x_K;$$

$x_K = 1 - z_0$ corresponds to the instant when the burning of the powder terminates, the instant when the inflow of gases ends. For this reason this formula will give realistic results while x_m is smaller than or at most is equal to $x_K (x_m \leq x_K)$.

When $x_m < x_K$, we have a normal case: the maximum pressure is reached before the end of burning. When $x_m = x_K$, the maximum pressure is reached at the end of burning. Finally, when $x_m > x_K$, we have the case of the so-called "unreal" maximum, i.e., a purely analytical case. In reality, when $x_m > x_K$, the powder, burning according to a definite law, stops burning on the upward branch of the pressure curve, the flow of gases stops, following which the pressure begins to drop, in spite of the fact that the analytic maximum had not yet been reached. In fact, the maximum pressure in this case will be the pressure p_K at the end of burning.

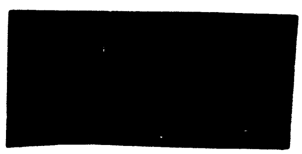

Fig. 135 - Pressure Curve with Normal Maximum.

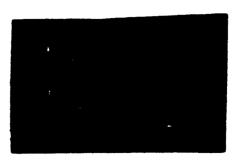

Fig. 136 - The Maximum Pressure Coincides with the End of Burning.

A large value of x_m may be obtained when the parameter of the loading

conditions, $B = s^2e_1^2/u_1^2 f\omega\varphi m$ is small; this happens when the powder is thin.

Fig. 137 - Unreal Maximum, $x_m > x_K$.

Such cases occur in practice when the firing is performed with thin powders for special purposes.

The appearance of the pressure curve in these three cases is shown in figs. 135, 136, 137.

At the end of the first period, we have:

$$\psi_K = 1; \quad l_{\psi_K} = l_1 = l_0(1 - \alpha\Delta);$$

$$x_K = 1 - z_0; \quad l_K = l_{av.K}\left(Z_{xK}^{-\frac{B}{B_1}} - 1\right);$$

$$v_K = \frac{sI_K}{\varphi m} x_K; \quad p_K = \frac{f\omega\left(1 - \frac{B\theta}{2} x_K^2\right)}{s(l_1 + l_K)}.$$

The same values will characterize the start of the second period - the period of adiabatic expansion of the gases.

4. SECOND PERIOD

The second period, starting at the end of burning of the charge and ending when the base of the projectile passes the muzzle face of the gun, constitutes a process of adiabatic expansion of the gases.

This period is considerably simpler than the first, because the whole process is reduced to the expansion of gases without the addition of energy and without heat losses.

In the second period $\psi = 1$, the number of variables is reduced, the independent variable is usually taken to be the path l of the projectile, and equations expressing the pressure p and the velocity v as a function of l are derived.

The beginning of the second period is characterized by the following data obtained at the end of the first period:

$$\psi = 1;\ v = v_K;\quad l = l_K;\ p = p_K;\quad l_\psi = l_1;\quad T = T_K.$$

The fundamental equation of the second period is:

$$ps(l_1 + l) = f\omega - \frac{\theta}{2}\varphi m v^2 = f\omega\left(1 - \frac{v^2}{v_{\Pi p}^2}\right). \tag{20}$$

where

$$\frac{2f\omega}{\varphi\theta m} = v_{\Pi p}^2.$$

Since the gas temperature is lower in the second period than in the first, θ should be made larger in the second period, but most authors take an average value of θ common to both periods.

A. Derivation of the Expression for Pressure in the Second Period

$[p = f_1(l)]$.

The equation for pressure is derived from the adiabatic equation:

$$pW^{1+\theta} = p_K W_K^{1+\theta}, \tag{21}$$

where p_K and p are the gas pressures at the beginning of the second period and at a given moment, respectively;

W_K and W are the free volumes of the initial air space at the same instants.

From equation (21), we have:

$$p = p_K \left(\frac{W_K}{W}\right)^{1+\theta}.$$

Expanding the quantities W_K and W, we obtain:

$$W_K = W_0 - \alpha\omega + sl_K = s(l_1 + l_K);$$

$$W = W_0 - \alpha\omega + sl = s(l_1 + l).$$

Substituting these values in the equation of p, we find:

$$p = p_K \left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta}. \tag{22}$$

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At the muzzle face, we will have:

$$p_{\text{д}} = p_{K}\left(\frac{l_1 + l_K}{l_1 + l_{\text{д}}}\right)^{1+\theta}.$$

B. Derivation of the Expression for Velocity in the Second Period, $v = f_2(l)$

Let us write the fundamental equation of pyrodynamics for any moment and for the beginning of the second period:

$$ps(l_1 + l) = f\omega\left(1 - \frac{v^2}{v_{\text{пр}}^2}\right);$$

$$p_K s(l_1 + l_K) = f\omega\left(1 - \frac{v_K^2}{v_{\text{пр}}^2}\right).$$

Dividing one equation by the other, term by term, and replacing the ratio p/p_K from (22), we obtain:

$$\left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta} = \frac{1 - \dfrac{v^2}{v_{\text{пр}}^2}}{1 - \dfrac{v_K^2}{v_{\text{пр}}^2}},$$

whence

$$v = v_{\text{пр}}\sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta}\left(1 - \frac{v_K^2}{v_{\text{пр}}^2}\right)}. \quad (23)$$

If we replace v_K by its expression $v_K = \frac{sI_K}{\varphi m}(1 - z_0)$,

$$\frac{v_K^2}{v_{\Pi p}^2} = \frac{s^2 I_K^2 (1 - z_0)^2}{\varphi^2 m^2} \frac{\varphi \theta m}{2 f \omega} = B \frac{\theta}{2}(1 - z_0)^2,$$

and then

$$v = v_{\Pi p} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta} \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]}. \tag{24}$$

When $l = l_Д$, we obtain an expression for the muzzle velocity $v_Д$:

$$v_Д = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q} \left\{1 - \left(\frac{l_1 + l_K}{l_1 + l_Д}\right)^{\theta} \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\}}. \tag{25}$$

This equation is of great importance for investigating the most desirable solutions when designing guns.

Equations (22) and (23) or (24) give the expressions for the gas pressure in the bore of the gun and for the projectile velocity in the second period as a function of the projectile path l.

Thus, on the basis of the assumptions made, the equations derived above express the relation between the conditions of loading and the ballistic elements of a gun discharge in both the first and the second periods. They enable one, for given loading conditions,

to compute the projectile velocity and the gas pressure at different points of the projectile's motion in the bore of the gun, and to determine the maximum pressure, the muzzle pressure, and the initial (muzzle) velocity of the projectile.

Curves of p and v as a function of l will usually have the form shown in fig. 138.

Fig. 138 - Normal p, l and v, l Curves.

1) Period I; 2) period II.

C. Equations for Calculating the Temperature of Powder Gases.

Having solved the fundamental equation of pyrodynamics and established the relation between the basic elements (p, v, l, and ψ) and the new independent variable x, and, consequently, also the relationship between these elements, an equation can be written for determining the temperature of the powder gases at any given instant, and, in particular, at the instant the projectile leaves the bore of the gun barrel.

The temperature of the gases flowing in the path of the projectile determines whether the discharge will be accompanied by a flash, or will be flashless, because according to the present concepts the flash accompanying a shot is a process involving the burning of

inflammable hydrogen and carbon oxide gases making up about 50% of the entire gas mixture.

If the temperature of these gases is very high, the gases will burst into flames when mixed with the oxygen in air, and produce a flash accompanying the shot.

In order to obtain the desired equation, let us make use of the energy balance equation in which Ec_w is replaced by $R\theta$:

$$\frac{RT_1\omega\psi}{\theta} - \frac{RT\omega\psi}{\theta} = \frac{\varphi m v^2}{2}.$$

Since $RT_1 = f$,

$$\frac{f\omega\psi}{\theta}\left(1 - \frac{T}{T_1}\right) = \frac{\varphi m v^2}{2},$$

or

$$\frac{T}{T_1} = 1 - \frac{\varphi m\theta}{2f\omega}\,\frac{v^2}{\psi} = 1 - \frac{1}{\psi}\,\frac{v^2}{v_{np}^2}. \qquad (26)$$

Knowing v and ψ from the first period, let us find T/T_1 and then T.

Inasmuch as

$$v = \frac{sI_K}{\varphi m}x\ldots$$

$$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2,$$

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then, bearing in mind that:

$$B = \frac{s^2 I_K^2}{f\omega\varphi m},$$

we get

$$\frac{T}{T_1} = 1 - \frac{B\theta}{2}\,\frac{x^2}{\psi} = 1 - \frac{B\theta}{2}\,\frac{x^2}{(\psi_0 + k_1 x + \varkappa\lambda x^2)}.$$

This equation shows that the variation in the temperature of the gases depends upon the conditions of loading (parameter B) and upon the shape of the grain (coefficients $\varkappa$ and λ).

At the end of burning ($\psi = 1$) we will have:

$$\frac{T_K}{T_1} = 1 - \frac{B\theta}{2}(1 - z_0)^2 = 1 - \varphi r_K. \tag{27}$$

In the second period $\psi = 1$ and we obtain from equation (26):

$$\frac{T}{T_1} = 1 - \frac{\varphi m\theta}{2f\omega}\,v^2 = 1 - \frac{v^2}{v_{np}^2}, \tag{28}$$

where

$$v_{np}^2 = \frac{2f\omega}{\varphi\theta m}.$$

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At the instant the projectile leaves the barrel

$$\frac{T_\Delta}{T_1} = 1 - \frac{v_\Delta^2}{v_{\Pi p}^2} = 1 - \varphi r_\Delta.$$

This value T_Δ / T_1 varies in artillery pieces between 0.65 and 0.75.

Comparing the value v from equation (28) with the values 1 T_1 and T_K / T_1 from equations (27) and (28), we obtain other expressions for T/T_1:

$$\frac{T}{T_1} = \left(\frac{l_1 + l_K}{l_1 + l}\right)^\theta \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] = \frac{T_K}{T_1}\left(\frac{l_1 + l_K}{l_1 + l}\right)^\theta;$$

$$\frac{T_\Delta}{T_1} = \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\left(\frac{l_1 + l_K}{l_1 + l_\Delta}\right)^\theta = \frac{T_K}{T_1}\left(\frac{l_1 + l_K}{l_1 + l_\Delta}\right)^\theta;$$

$$T_\Delta = T_1\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\left(\frac{l_1 + l_K}{l_1 + l_\Delta}\right)^\theta.$$

This equation proves that the temperature of the gases at the instant the base of the projectile passes the muzzle face depends on:

1) the temperature T_1 of the burning powder;

2) the temperature of the gases at the end of burning:

$$T_K = T_1\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right],$$

At the instant the projectile leaves the barrel

$$\frac{T_\text{д}}{T_1} = 1 - \frac{v_\text{д}^2}{v_{\text{пр}}^2} = 1 - \varphi r_\text{д}.$$

This value $T_\text{д}/T_1$ varies in artillery pieces between 0.65 and 0.75.

Comparing the value v from equation (28) with the values T/T_1 and T_K/T_1 from equations (27) and (28), we obtain other expressions for T/T_1:

$$\frac{T}{T_1} = \left(\frac{l_1 + l_K}{l_1 + l}\right)^\theta \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] = \frac{T_K}{T_1}\left(\frac{l_1 + l_K}{l_1 + l}\right)^\theta;$$

$$\frac{T_\text{д}}{T_1} = \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\left(\frac{l_1 + l_K}{l_1 + l_\text{д}}\right)^\theta = \frac{T_K}{T_1}\left(\frac{l_1 + l_K}{l_1 + l_\text{д}}\right)^\theta;$$

$$T_\text{д} = T_1\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\left(\frac{l_1 + l_K}{l_1 + l_\text{д}}\right)^\theta.$$

This equation proves that the temperature of the gases at the instant the base of the projectile passes the muzzle face depends on:

1) the temperature T_1 of the burning powder;

2) the temperature of the gases at the end of burning:

$$T_K = T_1\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right],$$

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this temperature decreases as B increases;

3) the ratio of free volumes $(l_1 + l_K)/(l_1 + l_\Delta)$, which depends upon the path traversed by the projectile at the end of burning and decreases as l_K increases.

D. Equations for Calculating the Time of Motion of the Projectile.

The time t does not appear directly in the solution of the fundamental problem of pyrodynamics; one may compute and draw the curves of the gas pressure p and the projectile velocity v as a function of the projectile path l, and by this means solve the fundamental problem of internal ballistics, yielding the design data of the gun (volume of powder chamber, length of projectile path).

But in order to fully clarify the phenomena taking place during a shot, it is also necessary to know the variation of the basic elements (p, v, l) as a function of the time t, particularly, because some of the existing devices permit determining the path l, the velocity v, and the gas pressure p as a function of the time t (velocimeter, piezoelectric manometer). Moreover, it is the pressures curves as a function of time which must be known when solving problems relating to the theory of gun mounts fuzes and firing devices.

The time of motion of the projectile in the barrel can be obtained most simply if the curve of the velocity v as a function of the path l is available, and by using the following equation of mechanics:

$$v = \frac{dl}{dt},$$

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whence:

$$dt = \frac{dl}{v}.$$

If, having the curve v, l, we plot the curve $\frac{1}{v}$, l, then by taking the integral:

$$\int_0^l \frac{1}{v} dl,$$

we could determine the time of motion of the projectile along the given path l. But inasmuch as at the lower limit, when $l = 0$, $v = 0$, and the integrand $1/v$ becomes infinite ($1/v = 1/0 = \infty$), it is impossible to perform the integration. Therefore, the time t is divided into two parts, t' and t'':

$$t = t' + t'', \qquad (29)$$

where the first time interval t' - from the start of motion up to a point representing a small length of the path l' - is calculated approximately, and the second interval t'', from l' to l along the path - is calculated by means of quadratic formulas:

$$t'' = \int_{l'}^{l} \frac{1}{v} dl.$$

The first time interval t' is found from the equation:

$$t' = \frac{l'}{v'_{av.}},$$

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where, in the first approximation:

$$v'_{av.} = \frac{0 + v'}{2} = \frac{v'}{2}$$

and v' is the velocity of the projectile at time t' and the path distance l'; consequently:

$$t' = \frac{2l'}{v'};$$

the smaller the distance l', the greater will be the accuracy of determining t.

Substituting t' and t" into (105), we obtain the equation giving the time of motion of the projectile in the bore in the form:

$$t = \frac{2l'}{v'} + \int_0^l \frac{1}{v} dl. \tag{30}$$

Inasmuch as the first interval of time for traversing the path l', as determined by (30), is very approximate, Prof. E.L. Bravin proposed a more exact expression for computing the average velocity of the projectile along the segment ot'. He assumed the acceleration, rather than the velocity, to be linear along this segment (fig. 139):

$$\frac{dv}{dt} = \frac{s}{\varphi m} p = \frac{s\alpha}{\varphi m}(p_0 + kt),$$

where $k = (p' - p_0)/t'$ is the angular coefficient of the straight line

p_0, p'; α is a factor, smaller than unity, determined from the condition that the areas bounded by the curve p, t and the straight line p_1p_2 replacing it along the segment ot' are equal. When determining $v'_{av.}$ in terms of v', the coefficient α is reduced.

$$dv = \frac{s\alpha}{\varphi m}\left(p_0 + \frac{p' - p_0}{t'}\,t\right) dt.$$

Fig. 139 - Curve p, t Along the Initial Path Segment (According to Bravin).

After integration, we obtain:

$$v = \frac{s}{\varphi m}\alpha\left(p_0 t + \frac{p' - p_0}{t'}\,\frac{t^2}{2}\right).$$

Assuming that t = t', we find v', the velocity of the projectile at the time t':

$$v' = \frac{s}{\varphi m}\alpha\left(p_0 t' + \frac{p' - p_0}{2}\,t'\right) = \frac{s}{\varphi m}\alpha\left(\frac{p_0 + p'}{2}\right)t'.$$

The average value of the projectile velocity along this segment is found from the following equation:

$$v'_{av.} = \frac{1}{t'} \int_0^{t'} v dt = \frac{1}{t'} \frac{s\alpha}{\varphi m} \int_0^{t'} \left(p_0 t + \frac{p' - p_0}{2t'} t^2 \right) dt =$$

$$= \frac{s\alpha}{\varphi m} \frac{1}{t'} \left(\frac{p_0 t'^2}{2} + \frac{p' - p_0}{6} t'^2 \right) = \frac{s\alpha}{\varphi m} \frac{2p_0 + p'}{6} t'$$

or, replacing $\frac{s\alpha}{\varphi m} t'$ from the preceding equation by v', we obtain:

$$v'_{av.} = \frac{2p_0 + p'}{p_0 + p'} \frac{v'}{3}.$$

The final expression for t' will be in the form:

$$t' = \frac{l'}{v_{av.}} = \frac{3l'}{v'} \frac{p_0 + p'}{2p_0 + p'}. \qquad (31)$$

This is the equation proposed by Prof. E.L. Bravin [3].

Comparing it with the previous expression for t', we note that the time t obtained by the first expression is shorter, and the difference between them increases as the length of the segment l' and the pressure p' increase. At the limit, when p' is reduced to p_0, the two expressions for t' become equal:

$$t' = \frac{3l'}{v'} \frac{2p_0}{3p_0} = \frac{2l'}{v'}.$$

If the relation p, t along the first segment is expressed by a second-degree equation, the resulting equation will be more exact:

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$$t'_2 = \frac{4l'}{v'} \cdot \frac{2p_0 + p'}{5p_0 + p'}.$$

Prof. Bravin also introduced equations for computing the time in segments measuring 0 to t_m, t_m to t_K, and t_K to t_Δ:

$$t_m = \frac{3l_m}{v_m} \frac{p_0 + p_m}{2p_0 + p_m}; \tag{a}$$

$$t_k - t_m = \frac{3(l_K - l_m)(p_m + p_K)}{v_m(p_m + 2p_K) + v_K(2p_m + p_K)}; \tag{b}$$

$$t_\Delta - t_K = \frac{3(l_\Delta - l_K)(p_K + p_\Delta)}{v_K(p_K + 2p_\Delta) + v_\Delta(2p_K + p_\Delta)}. \tag{c}$$

In order to compute t in the first period, the graph $\frac{1}{p}$, x can be used also.

Indeed, $x = z - z_0$;

$$\frac{dx}{dt} = \frac{dz}{dt} = \frac{1}{e_1}\frac{de}{dt} = \frac{u_1 p}{e_1} = \frac{p}{I_K};$$

whence

$$dt = I_K \frac{dx}{p};$$

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$$t = I_K \int_0^x \frac{dx}{p}, \tag{32}$$

The function under the integral sign does not become infinite when $p_0 > 0$.

When calculating the total time of the shot, it is necessary to consider not only the time of motion of the projectile, but also the burning time of the powder in the preliminary period before the start of this motion, which is computed by the following formula:

$$t_0 = 2.303\ \tau_0 \log \frac{p_0}{p_B},$$

where $\frac{1}{\tau_0} = \frac{f\Delta}{1 - \Delta\delta} \frac{\varkappa}{I_K}$, $I_K = e_1 u_1$, and p_B is the pressure of the igniter gases.

The time lapse between the instant the firing pin strikes the percussion cap and the end of burning of the igniter is usually not taken into account.

5. SAMPLE CALCULATION OF THE GAS PRESSURE CURVE AND OF THE PROJECTILE VELOCITY BY THE $\Psi_{av.}$ METHOD

The following data are given:

Barrel: 76 mm gun, 1936 model

Chamber capacity, W_0 in dm^3 .... 1.515

Cross-sectional area of the bore, including the rifling grooves, s, in dm^2 .... 0.4692

Path traversed by the projectile in the bore, l_A, in dm 33.91

Projectile:

Weight of projectile, q, in kg 6.2

Forcing pressure, p_0, in kg cm^2 300

Charge:

Weight of charge, ω, in kg 1.08

Powder constants:

Powder energy (force) f, in kg·dm kg 950,000

Co-volume, α, in dm^3 kg 0.98

Density of the powder, δ, in kg dm^3 .... 1.6

Burning rate, u_1, of the powder when p = 1, in dm sec:kg dm^2 .... 0.0000074

Dimensions of strip (thickness $2e_1$, in mm) 1.357

$$\varkappa = 1.06$$

$$\varkappa\lambda = -0.06$$

Polytropic index k 1.2

$\theta = k - 1$ 0.2

FUNDAMENTAL EQUATIONS

A. Preliminary Period

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}; \quad \sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0}; \quad z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)} \approx \frac{\psi_0}{\varkappa}.$$

B. First Period

$$v = \frac{sI_K}{\varphi m}x; \quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

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$$l = l_{\psi_{av.}} \left(Z_x^{-\frac{B}{B_1}} - 1 \right); \quad p = \frac{f\omega}{s} \, \frac{\psi - \frac{B\theta}{2} x^2}{l_\psi + l};$$

$$k_1 = \varkappa \sigma_0; \quad B = \frac{s^2 I^2}{f \omega \varphi m};$$

$$B = \frac{B\theta}{2} - \varkappa \lambda; \qquad l_\psi = l_\Delta - a\psi;$$

$$l_{\psi_{av.}} = l_\Delta - a\psi_{av.}; \qquad a = \frac{l_0 \Delta}{\delta_1} = \frac{\omega}{s\delta_1} = \frac{\omega}{s} \left(\Delta - \frac{1}{\varepsilon} \right);$$

$$l_\Delta = l_0 \Delta \left(\frac{1}{\Delta} - \frac{1}{\delta} \right) = \frac{\omega}{s} \left(\frac{1}{\Delta} - \frac{1}{\delta} \right).$$

Z_x is determined from the double-entry table:

$$\gamma = \frac{B_1 \psi_0}{k_1^2} \text{ and } \xi = \frac{B_1}{k_1} x;$$

$$x_m = \frac{k_1}{\dfrac{B(1 + \theta)}{1 + \dfrac{p_m}{f\delta_1}} - 2\varkappa\lambda}.$$

C. Second Period

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$$p = p_K \left(\frac{l_1 + l_K}{l_1 + l} \right)^{1+\theta};$$

$$l_1 = l_0 \Delta \left(\frac{1}{\Delta} - \alpha \right) = l_0 (1 - \alpha\Delta);$$

$$v = v_{np} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l} \right)^{\theta} \left(1 - \frac{v_K^2}{v_{np}^2} \right)}$$

or

$$v = v_{np} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l} \right)^{\theta} \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right]};$$

$$v_{np} = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q}}.$$

The calculation of the constants is effected first from the given data:

$$\Delta = 0.7128;$$

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{1}{0.7128} - \frac{1}{1.6}}{\frac{950000}{30000} + 0.98 - 0.625} = 0.02429;$$

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$$\sigma_0 = \sqrt{1 + 4\,\frac{\lambda}{\varkappa}\psi_0} = \sqrt{1 + \frac{4(-0.0566)0.02429}{1.06}} = 0.9972;$$

$$z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)} = 0.02294;$$

$$x_K = 1 - z_0 = 1 - 0.02294 = 0.97706;$$

$$k_1 = \varkappa\sigma_0 = 1.06\cdot 0.9972 = 1.057;$$

$$I_K = \frac{e_1}{u_1} = \frac{0.00678}{0.0000074} = 916.2;$$

$$\varphi = 1.03 + \frac{1}{3}\,\frac{\omega}{q} = 1.03 + \frac{1}{3}\cdot\frac{1.08}{6.2} = 1.088;$$

$$\frac{sI_K}{\varphi m} = \frac{0.4692\cdot 916.2\cdot 98.1}{1.088\cdot 6.2} = 6253;$$

$$l_0 = \frac{W_0}{s} = \frac{1.515}{0.4692} = 3.228;$$

$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{0.4692^2\cdot 916.2^2\cdot 98.1}{950000\cdot 1.08\cdot 1.088\cdot 6.2} = 2.617;$$

$$B_1 = \frac{B\theta}{2} - \varkappa\lambda = \frac{2.617\cdot 0.2}{2} + 0.06 = 0.3217;\quad \frac{B}{B_1} = 8.134;$$

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$$a = \frac{l_0 \Delta}{\delta_1} = 3.228 \cdot 0.7128 \cdot 0.355 = 0.8168;$$

$$l_\Delta = l_0 \Delta \left(\frac{1}{\Delta} - \frac{1}{\delta} \right) = 3.222 \cdot 0.7128 \left(\frac{1}{0.7128} - \frac{1}{1.6} \right) = 1.790;$$

$$\gamma = \frac{B_1 \psi_0}{k_1^2} = \frac{0.3217 \cdot 0.02429}{1.057^2} = 0.006995;$$

$$x_m = \frac{k_1}{\dfrac{B(1+\theta)}{1 + \dfrac{P_m}{f\delta_1}} - 2\gamma\lambda} = \frac{1.057}{\dfrac{2.617 \cdot 1.2}{1 + \dfrac{232000}{950000} \cdot 0.355} + 0.12} = 0.3512;$$

$$x_m \text{ 2nd approx.} = \frac{1.057}{\dfrac{2.617 \cdot 1.2}{1 + \dfrac{236000}{950000} \cdot 0.355} + 0.12} = 0.3517.$$

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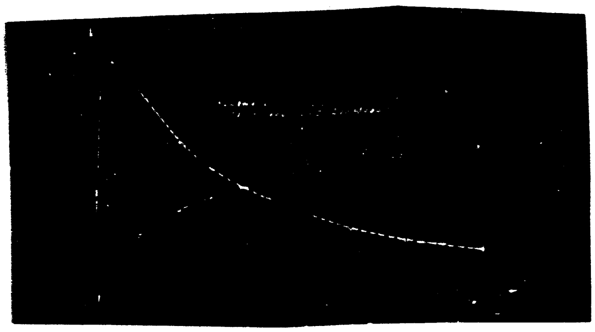

Fig. 140 - Curves p, l and v, l According to Prof. Drozdov and by the $l_{\Psi av.}$ Method.

1) v, in m sec; 2) p, in kg cm^2; 3) curves p(l) and v(l); 4) Drozdov's method; 5) $l_{\Psi av.}$ method;

6) l, in dm.

Table of the Ballistic Elements (ψ, v, ', and p) for the First Peri

Initial Formulas	No.	Operations		Max. Pressure	2nd Approx.	
$\frac{B_1}{k_1} = 0.3043$	1	x	0.1972	0.3512	0.3517	0.527
	2	$z = \frac{B_1}{k_1}x$	0.060	0.1069	0.1070	0.160
$v = \frac{sI_K}{\varphi m}x = 6253x$	3	v, in dm sec	1233	2196	2199	3295
	4	$k_1 x$	0.2084	0.3712	0.3717	0.557
$k_1 = 1.057$	5	$(+)\ \varkappa\lambda x^2$	-0.0023	-0.0074	-0.0074	-0.016
$\varkappa\lambda = -0.06$	6	ψ_0	0.0243	0.0243	0.0243	0.024
$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2$	7	ψ	0.2304	0.3881	0.3886	0.564
	8	$\psi + \psi_0$	0.2547	0.4124	0.4129	0.588
$\psi_0 = 0.0243$	9	$\psi_{av.}$	0.1273	0.2062	0.2064	0.294
$\psi_{av.} = \frac{\psi + \psi_0}{2}$	10	$(-)\ l_\Delta$	1.790	1.790	1.790	1.790
$l_{\psi av.} = l_\Delta - a\psi_{av.} = 1.79 - 0.186\psi_{av.}$	11	$a\psi_{av.}$	0.140	0.168	0.168	0.240
	12	$l_{\psi av.} = l_c$	1.650	1.622	1.622	1.550
	13	$(-)\ l_\Delta$	1.790	1.790	1.790	1.790
	14	$a\psi$	0.188	0.317	0.317	0.46

Table of the Ballistic Elements (ψ, v, l, and p) for the First Period

No.	Operations		Max. Pressure	2nd Approx.			End of Burning
1	x	0.1972	0.3512	0.3517	0.5270	0.723	0.9771
2	$\beta = \frac{B_1}{k_1}x$	0.060	0.1069	0.1070	0.160	0.220	0.29[illegible]
3	v, in dm sec	1233	2196	2199	3295	4521	6110
4	(+) $k_1 x$	0.2084	0.3712	0.3717	0.5570	0.7642	1.033
5	(+) $\varkappa\lambda x^2$	-0.0023	-0.0074	-0.0074	-0.0167	-0.0314	-0.0573
6	(+) ψ_0	0.0243	0.0243	0.0243	0.0243	0.0243	0.0243
7	ψ	0.2304	0.3881	0.3886	0.5646	0.7571	1.000
8	$\psi + \psi_0$	0.2547	0.4124	0.4129	0.5889	0.7814	1.0243
9	$\psi_{av.}$	0.1273	0.2062	0.2064	0.2944	0.3907	0.512[illegible]
10	(-) l_Δ	1.790	1.790	1.790	1.790	1.790	1.790
11	(-) $a\psi_{av.}$	0.140	0.168	0.168	0.240	0.319	0.418
12	$l_{\psi_{av.}} = l_c$	1.650	1.622	1.622	1.550	1.471	1.372
13	(-) $l\Delta$	1.790	1.790	1.790	1.790	1.790	1.790
14	(-) $a\psi$	0.188	0.317	0.317	0.461	0.618	0.317

$\kappa\lambda = -0.06$	5	(+) $\kappa\lambda x^2$				
	6	ψ_0	0.0243	0.0243	0.0243	0.0243
$\psi = \psi_0 + k_1 x + \kappa\lambda x^2$	7	ψ	0.2304	0.3881	0.3886	0.5646
	8	$\psi + \psi_0$	0.2547	0.4124	0.4129	0.5889
$\psi_0 = 0.0243$	9	$\psi_{av.}$	0.1273	0.2062	0.2064	0.2944
$\psi_{av.} = \dfrac{\psi + \psi_0}{2}$	10	(−) l_Δ	1.790	1.790	1.790	1.790
$l_{\psi av.} = l_\Delta - a\psi_{av.} = 1.79 - 0.186\psi_{av.}$	11	$a\psi_{av.}$	0.140	0.168	0.168	0.240
	12	$l_{\psi av.} = l_c$	1.650	1.622	1.622	1.550
	13	(−) l_Δ	1.790	1.790	1.790	1.790
	14	$a\psi$	0.188	0.317	0.317	0.461
$l_\psi = l_\Delta - a\psi$	15	l_ψ	1.602	1.473	1.473	1.329
colog Z from table	16	colog Z	0.0198	0.0402	0.04024	0.065
$\gamma = 0.006995$	17	$\frac{B}{B_1}$ colog Z	0.1610	0.3270	0.3273	0.530
$\frac{B}{B_1} = 8.134$	18	$Z_x^{-\frac{B}{B_1}}$	1.449	2.123	2.124	3.391
	19	$l = l_{\psi cp} \cdot$	0.7408	1.822	1.823	3.706
	20	$\cdot\ (Z_x^{-\frac{B}{B_1}} - 1)$	1.602	1.473	1.473	1.329
	21	$l_\psi + l$	2.343	3.295	3.296	5.03
$\frac{f\omega}{s} = 2,187,000$	22	(−) ψ	0.2304	0.3881	0 3886	0.56
	23	$\frac{B\theta}{2} x^2$	0.0102	0.0323	0.0324	0.07
$p = \dfrac{f\omega}{s} \dfrac{\psi - \frac{B\theta}{2} x^2}{l_\psi + l}$	24	$\psi - \frac{B\theta}{2} x^2$	0.2202	0.3558	0.3562	0.49
	25	p, in kg/cm^2	2054	2360	2362	2136

4	(+) $k_1 x$	0.2084	0.3712	0.3717	0.5570	0.7642	1.033
5	$\varkappa \lambda x^2$	-0.0023	-0.0074	-0.0074	-0.0167	-0.0314	-0.0573
6	ψ_0	0.0243	0.0243	0.0243	0.0243	0.0243	0.0243
7	ψ	0.2304	0.3881	0.3886	0.5646	0.7571	1.000
8	$\psi + \psi_0$	0.2547	0.4124	0.4129	0.5889	0.7814	1.0243
9	$\psi_{av.}$	0.1273	0.2062	0.2064	0.2944	0.3907	0.512[illegible]
10	(-) l_Δ	1.790	1.790	1.790	1.790	1.790	1.790
11	$a\psi_{av.}$	0.140	0.168	0.168	0.240	0.319	0.418
12	$l_{\psi_{av.}} - l_c$	1.650	1.622	1.622	1.550	1.471	1.372
13	(-) l_Δ	1.790	1.790	1.790	1.790	1.790	1.790
14	$a\psi$	0.188	0.317	0.317	0.461	0.618	0.317
15	l_ψ	1.602	1.473	1.473	1.329	1.172	0.973
16	colog Z	0.0198	0.0402	0.04024	0.06521	0.09596	0.1397
17	$\frac{B}{B_1}$ colog Z	0.1610	0.3270	0.3273	0.5304	0.7805	1.1369
18	$Z_x^{-\frac{B}{B_1}}$	1.449	2.123	2.124	3.391	6.033	13.69
19	$l - l_{\psi_{cp}}$.	0.7408	1.822	1.823	3.706	7.403	17.411
20	$\cdot\ (Z_x^{-\frac{B}{B_1}} - 1)$	1.602	1.473	1.473	1.329	1.172	0.973
21	$l_\psi + l$	2.343	3.295	3.296	5.035	8.575	18.384
22	(-) ψ	0.2304	0.3881	0 3886	0.5646	0.7571	1.000
23	$\frac{B\theta}{2} x^2$	0.0102	0.0323	0.0324	0.0727	0.1368	0.2498
24	$\psi - \frac{B\theta}{2} x^2$	0.2202	0.3558	0.3562	0.4919	0.6203	0.7502
25	p, in kg/cm^2	2054	2360	2362	2136	1580	892

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COMPUTATIONS FOR THE SECOND PERIOD

Calculation of the constants of the second period:

$$v_{\Pi p}^2 = \frac{2\cdot 950000\cdot 1.08\cdot 98.1}{1.088\cdot 0.2\cdot 6.2} = 149{,}300{,}000; \qquad \frac{v_K^2}{v_{\Pi p}^2} = 0.250;$$

$$C = 1 - \frac{v_K^2}{v_{\Pi p}^2} = 1 - 0.250 = 0.750.$$

From the first period, $l_1 + l_K = 18.384$.

$$p_K = 892\ \text{kg/cm}^2.$$

Table of the Elements of the Second Period

Initial Formulas	No.	Operations			Muzzle Face
$p = p_K\left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta} =$	1	$(+)\ l$	22.60	27.44	33.91
$= 892\left(\frac{18.384}{0.973 + l}\right)^{1.2}$	2	l_1	0.973	0.973	0.973
	3	$l + l_1$	23.573	28.413	34.883
	4	$\frac{l_1 + l_K}{l_1 + l} = \eta$	0.7799	0.6471	0.5270
	5	$\log \eta$	$\bar{1}.8920$ -0.1080	$\bar{1}.8110$ -0.1890	1.7218 -0.2782
$v = v_{\Pi p}\sqrt{1 - \eta^{0.2}\left(1 - \frac{v_K^2}{v_{\Pi p}^2}\right)} =$	6	$1.2 \log \eta$	-0.1296 $\bar{1}.8704$	-0.2268 $\bar{1}.7732$	-0.3338 $\bar{1}.6662$

Table (Cont'd.)

Initial Formulas	No.	Operations			Muzzle Face
$= 12{,}220\sqrt{1-0.750\eta^{0.2}}$	7	$\eta^{1.2}$	0.7420	0.5932	0.4636
	8	$p = p_K\eta^{1.2}$	662	529	414
	9	$0.2 \log \eta$	-0.0216 $\bar{1}.9784$	-0.0378 $\bar{1}.9622$	-0.0556 $\bar{1}.9444$
	10	$\eta^{0.2}$	0.9515	0.9166	0.8798
	11	$0.750\eta^{0.2}$	0.7136	0.6874	0.6598
	12	$1-0.750\eta^{0.2}$	0.2864	0.3126	0.3402
	13	v in dm sec	6530	6822	7117

The results of these calculations are shown in fig. 140 on p. 510 in the form of $p(l)$ and $v(l)$ curves.

CHAPTER 2 - PROF. N.F. DROZDOV'S EXACT METHOD [1]

(Written by Prof. G.V. Oppokov)

The assumption

$$l_\psi = l_{\psi_{av}},$$

made in the preceding chapter gives an approximate solution. Yet the differential equation of the projectile path in the first period

$$\frac{dl}{dx} = \frac{Bx(l_\psi + l)}{\psi - \frac{B\theta}{2}x^2} \tag{33}$$

can be integrated exactly.

Prof. N.F. Drozdov's great contribution to the field of internal ballistics lies in the very fact that he was the first to solve this equation exactly, without any additional assumptions or simplifications, as had been done before him by all the other authors without exception.

Namely, if we introduce for convenience the following designation:

$$M = \frac{Bx}{\psi - \frac{B\theta}{2}x^2}, \tag{34}$$

equation (33) takes on the form:

$$\frac{dl}{dx} - Ml = Ml_\psi. \tag{35}$$

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When this differential equation of the first order is integrated, the following relationship obtains:

$$l = e^{\int_0^x M dx} \int_0^x l_\psi \cdot e^{-\int_0^x M dx} M dx. \qquad (36)$$

The integral-exponent of e in the right-hand side of equation (36) is (see pp. 478-481) equal, as before, to:

$$\int_0^x M dx = \frac{B}{B_1} \int_0^\beta \frac{\beta d\beta}{\gamma + \beta - \beta^2} = \ln Z^{-\frac{B}{B_1}}. \qquad (37)$$

The main integral is:

$$Y = \int_0^x l_\psi e^{-\int_0^x M dx} M dx = \int_0^x l_\psi d(-e^{\int_0^x M dx}).$$

Noting this peculiarity, the author integrates by parts:

$$Y = \left| -l_\psi e^{-\int_0^x M dx} \right|_0^x + \int_0^x e^{-\int_0^x M dx} dl_\psi,$$

or, [see equation (37)]:

$$Y = -l_\psi Z^{\frac{B}{B_1}} + l_{\psi_0} + \int_{l_{\psi_0}}^{l_\psi} Z^{\frac{B}{B_1}} \, dl_\psi .$$

But

$$\psi = \psi_0 + k_1 x + \varkappa \lambda x^2; \qquad l_\psi = l_\Delta - a\psi .$$

where $a = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right)$.

$$dl_\psi = -ak_1 dx - 2a\varkappa\lambda x dx,$$

whence

$$Y = -l_\psi Z^{\frac{B}{B_1}} + l_\psi - ak_1 \int_0^x Z^{\frac{B}{B_1}} dx - 2a\varkappa\lambda \int_0^x Z^{\frac{B}{B_1}} x dx. \qquad (38)$$

It is now possible to proceed in two ways: eliminate from the equation either the first or the second integral in the right-hand side. The author selected the second course.

Namely, it follows from (34) and (37) that:

$$\int_0^x \frac{Bxdx}{\psi_0 + k_1 x - B_1 x^2} = -\frac{B}{B_1} \ln Z$$

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or

$$\int_0^x \frac{x\,dx}{x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}} = \ln Z,$$

whence

$$\frac{x\,dx}{x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}} = \frac{dZ}{Z},$$

and, consequently

$$x\,dx = \left(x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}\right)\frac{dZ}{Z}.$$

The desired integral is equal to:

$$\int_0^x Z^{\frac{B}{B_1}}\,x\,dx = \int_1^Z Z^{\frac{B}{B_1}-1}\left(x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}\right)dZ =$$

$$= \frac{B_1}{B}\int_1^z \left(x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}\right)dZ.$$

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Integrating by parts:

$$\int_0^x Z^{\frac{B}{B_1}} x dx = \frac{B_1}{B}\left(x^2 - \frac{k_1}{B}x - \frac{\psi_0}{B_1}\right) Z^{\frac{B}{B_1}} + \frac{B_1}{B}\frac{\psi_0}{B_1} -$$

$$- \frac{2B_1}{B}\int_0^x Z^{\frac{B}{B_1}} x dx + \frac{k_1}{B}\int_0^x Z^{\frac{B}{B_1}} dx,$$

because when x = 0, we will have:

$$Z^{\frac{B}{B_1}} = e^0 = 1.$$

We will find the desired integral from equation (38):

$$\int_0^x Z^{\frac{B}{B_1}} x dx = \frac{1}{B + 2B_1}(B_1 x^2 - k_1 x - \psi_0) Z^{\frac{B}{B_1}} +$$

$$+ \frac{\psi_0}{B + 2B_1} + \frac{k_1}{B + 2B_1}\int_0^x Z^{\frac{B}{B_1}} dx.$$

Now the obtained value of the integral must be substituted into (38), noting that:

$$x^2 - \frac{k_1}{B_1}x - \frac{\psi_0}{B_1} = \frac{1}{B_1}\left(\psi - \frac{B\theta}{2}x^2\right),$$

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and using Prof. N.F. Drozdov's designations:

$$a_1 = -\frac{2x\lambda}{B + 2B_1}; \quad b_1 = \frac{1}{l_0}(l_{\psi_0} + aa_1\psi_0); \quad c_1 = \frac{ak_1}{l_0}(1 - a_1). \qquad (36)$$

Then finally we have:

$$Y = -l_{\psi} Z^{\frac{B}{B_1}} - aa_1 \left(\psi - \frac{B\theta}{2}x^2\right) Z^{\frac{B}{B_1}} + l_0(b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx). \qquad (37)$$

Let us introduce the value of the integral from (37) and the value of the integral found immediately above:

$$Y = \int_0^x l_{\psi} e^{-\int_0^x M dx} M dx$$

into (36):

$$l = Z^{-\frac{B}{B_1}} \left[l_{\psi} Z^{\frac{B}{B_1}} - aa_1 \left(\psi - \frac{B\theta}{2}x^2\right) Z^{\frac{B}{B_1}} + l_0\left(b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx\right)\right].$$

Upon expanding the expression in brackets and replacing

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$$a = \frac{\omega}{s\delta_1}; \quad l_\psi = l_\Delta - \frac{\omega\psi}{s\delta_1},$$

after the transfer of l_ψ to the left-hand side, we obtain:

$$l + l_\Delta - \frac{\omega\psi}{s\delta_1} = -\frac{\omega}{s\delta_1} a_1 \left(\psi - \frac{B\theta}{2} x^2\right) +$$

$$+ l_0 (b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx) Z^{-\frac{B}{B_1}}.$$

Dividing both sides by l_0 and noting that:

$$\frac{\omega}{s\delta_1} : l_0 = \frac{\omega}{w_0 \delta_1} = \frac{\Delta}{\delta_1},$$

we obtain Drozdov's well-known equation:

$$\frac{l}{l_0} + \frac{l_\Delta}{l_0} - \frac{\psi\Delta}{\delta_1} = -\frac{a_1\Delta}{\delta_1} \left(\psi - \frac{B\theta}{2} x^2\right) +$$

$$+ (b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx) Z^{-\frac{B}{B_1}}. \qquad (40)$$

In this equation we have:

$$\int_0^x Z^{\frac{B}{B_1}}\,dx = \frac{k_1}{B_1}\int_0^\beta Z^{\frac{B}{B_1}}\,d\beta .$$

The table for the last integral with three entries, β, γ, and B/B_1, was prepared by Prof. D.A. Venttsel and is reproduced in the appendix.

Table 2 - Computational Formulas Used in Drozdov's Exact Method

$$\varphi = K + \frac{1}{3}\frac{\omega}{q};\quad \frac{\varphi m}{s};\quad I_K = \frac{e_1}{u_1};\quad v_{K,0} = I_K;\quad \frac{\varphi m}{s};$$

$$\frac{\omega}{s};\quad f\frac{\omega}{s};\quad l_\Delta = \frac{\omega}{s}\cdot\frac{1}{\delta_2};\quad a = \frac{\omega}{s}\cdot\frac{1}{\delta_1};$$

$$l_{\psi_0} = l_\Delta - a\psi_0;\quad \frac{\Delta}{\delta_1};\quad f\Delta;\quad l_0 = \frac{W_0}{s};\quad \frac{l_\Delta}{l_0}.$$

$$B = I_K^2;\quad \left(f\,\frac{\omega}{s}\cdot\frac{\varphi m}{s}\right);\quad B_1 = \frac{B\theta}{2} - x\lambda;$$

$$\frac{B}{B_1};\quad C = \frac{B_1}{k_1};\quad \gamma = \frac{C\psi_0}{k_1}.$$

$$a_1 = -\frac{2\varkappa\lambda}{B + 2B_1};\quad b_1 = \frac{1}{l_0}(l_{\psi_0} + a_\alpha a_1\psi_0);$$

$$\frac{c_1k_1}{B_1} = \frac{ak_1(1 - a_1)}{l_0C};\quad a_1\frac{\Delta}{\delta_1}.$$

$$\beta = Cx;\quad v = v_{K,0}x;\quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

$$\frac{l}{l_0} + \frac{l_\Delta}{l_0} - \psi\frac{\Delta}{\delta_1} = -a_1\frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) +$$

$$+\left(b_1 - \frac{c_1 K_1}{B_1}\int_0^{\beta} Z^{\frac{B}{B_1}}\, d\beta\right) Z^{-\frac{B}{B_1}};$$

$$p = f\Delta\left(\psi - \frac{B\theta}{2}x^2\right) : \left(\frac{l}{l_0} + \frac{l_\Delta}{l_0} - \psi\frac{\Delta}{\delta_1}\right).$$

The equation given by Drozdov for the maximum pressure is:

$$x_m = \frac{k_1}{B + 2B_1} + h,$$

where

$$h = \frac{c_1}{B + 2B_1}\,\frac{\psi_m - \frac{B\theta}{2}x_m^2}{\left(b_1 - c_1\int_0^x Z_x^{\frac{B}{B_1}}\,dx\right)Z_x^{-\frac{B}{B_1}}} \quad \text{and} \quad x_m = \frac{k_1}{B + 2B_1}.$$

h is a function of B and Δ for which a special table has been compiled and is presented below. This equation enables one to avoid approximations.

Table 3 - Approximate Values of the Function n

B \ Δ	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65
1.0	0.025	0.031	0.037	0.044	0.052	0.062	0.075	0.090	0.108	0.123
1.1	020	025	031	037	014	052	062	074	089	106
1.2	017	021	026	031	037	044	052	062	073	088
1.3	014	018	022	027	032	038	045	053	063	075
1.4	012	015	019	023	027	032	038	046	054	064
1.5	010	0.013	016	020	024	028	034	010	047	055
1.6	009	012	014	017	021	025	030	035	041	048
1.7	008	010	012	015	018	022	026	031	036	042
1.8	007	009	011	013	016	019	023	027	032	037
1.9	006	008	010	012	014	017	020	024	029	033
2.0	006	007	009	011	013	016	019	022	026	030
2.2	005	006	008	010	011	013	016	018	021	024
2.4	004	006	007	008	010	011	013	015	018	021
2.6	004	005	006	007	008	009	011	013	015	018
2.8	003	004	005	006	007	008	010	011	013	015
3.0	0025	0035	0045	0055	0065	0075	009	010	011	0125

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Table 3 - Approximate Values of the Function n

0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
0.037	0.044	0.052	0.062	0.075	0.090	0.108	0.123	0.151	0.177	0.208
031	037	014	052	062	074	089	106	125	147	173
026	031	037	044	052	062	073	088	104	122	144
022	027	032	038	045	053	063	075	088	103	122
019	023	027	032	038	046	054	064	075	088	104
016	020	024	028	034	010	047	055	065	076	0
014	017	021	025	030	035	041	048	056	066	077
012	015	018	022	026	031	036	042	049	057	066
011	013	016	019	023	027	032	037	043	050	058
010	012	014	017	020	024	029	033	039	045	051
009	011	013	016	019	022	026	030	035	040	046
008	010	011	013	016	018	021	024	028	032	037
007	008	010	011	013	015	018	021	024	027	030
006	007	008	009	011	013	015	018	020	022	025
005	006	007	008	010	011	013	015	017	019	021
0045	0055	0065	0075	009	010	011	0125	014	0155	0175

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The equations of the preceding table should be applied to the first period; the equations for the preliminary and second periods remain unchanged (Table 3).

6. EXAMPLE OF CALCULATING THE GAS PRESSURE CURVE AND THE PROJECTILE VELOCITY BY PROF. N.F. DROZDOV'S METHOD

The following data are given:

Barrel: 76 mm gun, model 1936

Chamber capacity, W_0, in dm^3 .... 1.515

Cross-sectional area of the bore, including rifling, s, in dm^2 .... 0.4692

Path traversed by projectile inside the bore, l_n, in dm 33.91

Projectile

Weight of projectile, q, in kg 6.2

Forcing pressure, p_0, in kg cm^2 300

Charge

Weight of charge, ω, in kg 1.08

Powder constants

Powder energy (force), f, in kg·dm kg 950000

Covolume, α, in dm^3 kg 0.98

Powder density, δ, in kg dm^3 .... 1.6

Burning rate of powder, u_1, when p = 1, in dm/sec : kg/dm^2 .... 0.0000074

Dimensions of strip (thickness $2e_1$ mm) 1.357

$$\varkappa = 1.06$$

$$-\varkappa\lambda = 0.06$$

Polytropic index k 1.2

$\theta = k - 1$ 0.2

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BASIC COMPUTATIONAL FORMULAS

A. Preliminary Period

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}; \quad \sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0}; \quad z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)}.$$

B. First Period

$$v = \frac{sI_K}{\varphi m}x; \quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

$$\Lambda_\psi + \Lambda = a_1 \frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) + \left(b_1 - \frac{c_1 k_1}{B_1}\int_0^x Z^{\frac{B}{B_1}}\,dx\right)Z^{-\frac{B}{B_1}}.$$

$$p = f\Delta\,\frac{\psi - \frac{B\theta}{2}x^2}{\Lambda_\psi + \Lambda}, \qquad (*)$$

where

$$B = \frac{s^2 I_K^2}{f\omega\varphi m}; \quad B_1 = \frac{B\theta}{2} - \varkappa\lambda;$$

$$k_1 = \varkappa\sigma_0; \quad a_1 = \frac{2\varkappa\lambda}{2 + 2B};$$

$$b_1 = \frac{l_\Delta}{l_0} - \frac{\Delta}{\delta_1}\Psi_0(1 + a_1); \quad c_1 = \frac{k_1}{\delta_1}\Delta(1 - a_1);$$

$$\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}; \quad \frac{l_\Delta}{l_0} = 1 - \frac{\Delta}{\delta};$$

$$\Lambda_\Psi = \frac{l_\Psi}{l_0} = 1 - \frac{\Delta}{\delta} - \frac{\Delta}{\delta_1}\Psi;$$

$$l_0 = \frac{W_0}{s}; \quad \Lambda = \frac{l}{l_0}.$$

Z and $\int_0^\beta Z^{B/B_1}\,d\beta$ are determined from tables with the two entries:

$$\gamma = \frac{B_1\Psi_0}{k_1^2}; \quad \beta = \frac{B_1}{k_1}x;$$

$$x_m = \frac{k_1}{B + 2B_1} + h,$$

where

$$h = \frac{c_1}{B + 2B_1} \; \frac{\Psi_m - \frac{B\theta}{2}x_m^2}{\left(b_1 - c_1\int_0^x Z^{\frac{B}{B_1}}dx\right)Z^{-\frac{B}{B_1}}}.$$

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C. Second Period

$$p = p_K \left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda} \right)^{1+\theta};$$

$$v = v_{np} \sqrt{1 - \left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda} \right)^{\theta} \left(1 - \frac{v_K^2}{v_{np}^2} \right)},$$

where

$$v_{np} = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q}}$$

or

$$v = v_{np} \sqrt{1 - \left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda} \right)^{\theta} \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right]};$$

$$\Lambda_1 = \frac{l_\Delta}{l_0} = \frac{\Delta}{\delta_1} = 1 - \alpha\Delta.$$

The computation of the constants is effected first from the known data:

$$\Delta = \frac{\omega}{W_0} = \frac{1.08}{1.515} = 0.7128;$$

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$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{1}{0.7128} - \frac{1}{1.6}}{\frac{950000}{30000} + 0.98 - 0.625} = 0.02429;$$

$$\sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0} = \sqrt{1 + \frac{4(-0.0566)0.02429}{1.06}} = 0.9972$$

$$z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)} = \frac{2 \cdot 0.02429}{1.06 \cdot 1.9972} = 0.02294;$$

$$x_K = 1 - z_0 = 1 - 0.02294 = 0.97706;$$

$$k_1 = \varkappa\sigma_0 = 1.06 \cdot 0.9972 = 1.057;$$

$$I_K = \frac{e_1}{u_1} = \frac{0.00678}{0.0000074} = 916.2;$$

$$\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q} = 1.03 + \frac{1}{3}\frac{1.08}{6.2} = 1.088;$$

$$\frac{sI_K}{\varphi m} = \frac{0.4692 \cdot 916.2 \cdot 98.1}{1.088 \cdot 6.2} = 6253;$$

$$l_0 = \frac{W_0}{s} = \frac{1.515}{0.4692} = 3.228\ \text{dm};$$

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$$\frac{l_\Delta}{l_0} = 1 - \frac{\Delta}{\delta} = 1 - \frac{0.7128}{1.6} = 0.5545;$$

$$\Lambda_\Delta = \frac{l_\Delta}{l_0} = \frac{33.91}{3.228} = 10.51;$$

$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{0.4692^2 \cdot 916.2^2 \cdot 98.1}{950000 \cdot 1.08 \cdot 1.088 \cdot 6.2} = 2.617;$$

$$B_1 = \frac{B\theta}{2} - \kappa\lambda = \frac{2.617 \cdot 0.2}{2} + 0.06 = 0.3217;$$

$$B + 2B_1 = 2.617 + 2 \cdot 0.3217 = 3.2604;$$

$$a_1 = \frac{2\kappa\lambda}{B + 2B_1} = \frac{2 \cdot (-0.06)}{2.617 + 2 \cdot 0.3217} = -0.0368.$$

$$b_1 = \frac{l_\Delta}{l_0} - \frac{\Delta\psi_0}{\delta_1}(1 + a_1) = 0.5545 - 0.7128 \cdot 0.02429 \cdot 0.355 \cdot 0.9632 = 0.5486;$$

$$c_1 = \frac{k_1}{\delta_1}\Delta(1 + a_1) = 1.057 \cdot 0.355 \cdot 0.7128 \cdot 0.9632 = 0.2576;$$

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$$\frac{c_1 k_1}{B_1} = \frac{0.2576 \cdot 1.057}{0.3217} = 0.8464;$$

$$\gamma = \frac{B_1 \psi_0}{k_1^2} = \frac{0.3217 \cdot 0.02429}{1.057^2} = 0.006995;$$

$$\frac{B}{B_1} = \frac{2 \cdot 617}{0.3217} = 8.134;$$

$$f \cdot \Delta = 950{,}000 \cdot 0.7128 = 677{,}200.$$

Computing x_m

$$x_m = \frac{k_1}{B + 2B_1} + h, \qquad (*)$$

where

$$h = \frac{c_1}{B + 2B_1} \frac{\psi_m - \frac{B\theta}{2} x_m^2}{\left(b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx\right) Z^{-\frac{B}{B_1}}}.$$

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Table for Computing x_m by Means of Equation (*)

Operations	
$x_m = \frac{k_1}{B + 2B_1}$	0.3242
$\frac{c_1}{B + 2B_1}$	0.07901
(+) $k_1 x_m$	0.3427
$\varkappa\lambda x_m^2$	-0.0063
Ψ_0	0.0243
$\Psi_m = k_1 x_m + \varkappa\lambda x_m^2 + \Psi_0$	0.3607
(−) $\frac{B\theta}{2} x_m^2$	0.0275
$\Psi_m - \frac{B\theta}{2} x_m^2$	0.3332
$\frac{c_1}{B + 2B_1} \cdot (\Psi_m - \frac{B\theta}{2} x_m^2)$	0.02633
$\beta = \frac{B_1}{k_1} x_m$	0.09865
colog Z	0.03646
$\frac{B}{B_1}$ colog Z	0.2966
$Z^{-\frac{B}{B_1}}$	1.98
$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.07438
	0.5486

$\xi_\beta = 0.9325$

Interpolation

$\longrightarrow \xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.08	0.0290	0.02826	0.0275
0.09865		0.03646	
0.100	0.0379	0.03706	0.0362

$\log Z^{-1} = 0.03646$

$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8.0$

β \ γ	0.006	0.006995	0.008
0.08	0.064	0.0645	0.065
0.09865		0.0747	
0.100	0.075	0.0755	0.076

$\int^\beta Z^{\frac{B}{B_1}}$ ... B ... 9.0

ψ_0	0.0243
$\psi_m = k_1 x_m + \kappa\lambda x_m^2 + \psi_0$	0.3607
$(-)\frac{B\theta}{2} x_m^2$	0.0275
$\psi_m - \frac{B\theta}{2} x_m^2$	0.3332
$\frac{c_1}{B + 2B_1} \cdot (\psi_m - \frac{B\theta}{2} x_m^2)$	0.02633
$\beta = \frac{B_1}{k_1} x_m$	0.09865
colog Z	0.03646
$\frac{B}{B_1}$ colog Z	0.2966
$Z^{-\frac{B}{B_1}}$	1.98
$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.07438
$(-) \begin{cases} b_1 \\ \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta \end{cases}$	0.5486 0.0629
$b_1 - c_1 \frac{k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.4857
$\left(b_1 - c_1 \frac{k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta\right) Z^{-\frac{B}{B_1}}$	0.9617

$$\log Z^{-1} = 0.03646$$

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 8.0$$

β \ γ	0.006	0.006995	0.008
0.08	0.064	0.0645	0.065
0.09865		0.0747	
0.100	0.075	0.0755	0.076

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 9.0$$

β \ γ	0.006	0.006995	0.008
0.08	0.062	0.0625	0.063
0.09865		0.0723	
0.100	0.072	0.0730	0.074

Continued

Operations	
$+\left\{ \dfrac{c_1}{B + 2B_1} \cdot \dfrac{\psi_m - \dfrac{B\theta}{2}x_m^2}{\left(b_1 - c_1 \dfrac{k_1}{B_1} \displaystyle\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta\right) Z^{\frac{B}{B_1}}} \right.$	0.0274
$\left. \dfrac{k_1}{B + 2B_1} \right\}$	0.3242
x_m	0.3516

$$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 8.134$$

$\frac{B}{B_1}$	8.0	8.134	9.0
$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$	0.0747	<u>0.07438</u>	0.0723

Knowing the value of x_m from equation (*) (p. 530), one may find the value of the maximum pressure p_m.

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Table for Computing the Ballistic Elements for the First Period
(Calculated on a 50-cm Slide-Rule)

Initial Formulas	No.	Operations	For Maximum Pressure				End of Burning
$\frac{B_1}{k_1} = 0.3043$	1	x	0.1972	0.3516	0.527	0.723	0.9771
$v = \frac{SI}{\varphi m} x = 6253x$	2	$\beta = \frac{B_1}{k_1} x$	0.060	0.1070	0.160	0.220	0.29[illegible]
	3	v in dm/sec	1233	2198	3295	4521	6110
$k_1 = 1.057$;	4	(+) $k_1 x$	0.2084	0.3716	0.5570	0.7642	1.033
$\varkappa\lambda = -0.06$	5	(+) $\varkappa\lambda x^2$	-0.0023	-0.0074	-0.0167	-0.0314	-0.0573
$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2$	6	(+) ψ_0	0.0243	0.0243	0.0243	0.0243	0.0243
	7	(−) ψ	0.2304	0.3885	0.5646	0.7571	1.00
$\frac{B\theta}{2} = 0.2617$	8	(−) $\frac{B\theta}{2} x^2$	0.0102	0.0323	0.0727	0.1368	0.2498
	9	$\psi - \frac{B\theta}{2} x^2$	0.2202	0.3562	0.4919	0.6203	0.7502
$a_1 \frac{\Delta}{\delta_1} = -0.00931$	10	$a_1 \frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2} x^2\right)$	-0.0020	-0.0033	-0.0046	-0.0058	-0.0070

Table for Computing the Ballistic Elements for the First Period
(Calculated on a 50-cm Slide-Rule)

Initial Formulas	No.	Operations	For Maximum Pressure				End of Burning
colog Z from Table 1, page 482	11	colog Z	0.0198	0.04024	0.06521	0.09596	0.1397
$\frac{B}{B_1} = 8.134$	12	$\frac{B}{B_1}$ colog Z	0.1610	0.3273	0.5304	0.7805	1.1363
$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$ from	13	$Z^{-\frac{B}{B_1}}$	1.449	2.124	3.391	6.033	13.69
table on page 662(*)	14	$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.05137	0.07829	0.09833	0.1120	0.1209
$\gamma = 0.006995$	15	$(-)\begin{cases} b_1 \\ \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta \end{cases}$	0.5486	0.5486	0.5486	0.5486	0.5486
	16		0.0435	0.0663	0.0832	0.0948	0.1023
	17	$b_1 - \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.5051	0.4823	0.4654	0.4538	0.4463
	18	$\left(b_1 - \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta\right) Z^{-\frac{B}{B_1}}$	0.7319	1.0244	1.5782	2.7378	6.1098
$\frac{\Delta}{\delta_1} = 0.2530$	19	$\Lambda_\psi + \Lambda = (10) + (18)$	0.7299	1.0211	1.5736	2.7320	6.1028
$\Lambda_\psi = \frac{l_\Delta}{l_0} -$	20	$(-)\begin{cases} \frac{l_\Delta}{l_0} = 1 - \frac{\Delta}{\delta} \\ \frac{\Delta}{\delta_1}\psi \end{cases}$	0.5545	0.5545	0.5545	0.5545	0.5545
$- \frac{\Delta}{\delta_1}\psi$	21		0.0583	0.0983	0.1428	0.1935	0.2530
			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ from table on page 662(*)	13	$Z^{-\frac{B}{B_1}}$	1.449	2.124	3.391	6.083	13.69
	14	$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$	0.05137	0.07829	0.09833	0.1120	0.1209
$\gamma = 0.006995$	15	(-) b_1	0.5486	0.5486	0.5486	0.5486	0 5486
	16	(-) $\frac{c_1 k_1}{B_1} \int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$	0.0435	0.0663	0.0832	0.0948	0.102[illegible]
	17	$b_1 - \frac{c_1 k_1}{B_1} \int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$	0.5051	0.4823	0.4654	0.4538	0.4463
	18	$\left(b_1 - \frac{c_1 k_1}{B_1} \int_0^{\beta} Z^{\frac{B}{B_1}} d\beta \right) Z^{-\frac{B}{B_1}}$	0.7319	1.0244	1.5782	2.7378	6.1098
$\frac{\Delta}{\delta_1} = 0.2530$	19	$\Lambda_{\psi} + \Lambda = (10) + (18)$	0.7299	1.0211	1.5736	2.7320	6.1028
$\Lambda_{\psi} = \frac{l_{\Delta}}{l_0} - \frac{\Delta}{\delta_1} \psi$	20	(-) $\frac{l_{\Delta}}{l_0} = 1 - \frac{\Delta}{\delta}$	0.5545	0.5545	0.5545	0.5545	0.5545
	21	(-) $\frac{\Delta}{\delta_1} \psi$	0.0583	0.0983	0.1428	0.1935	0.2530
	22	Λ_{ψ}	0.4962	0.4562	0.4117	0.3630	0.3015
	23	$\Lambda = (\Lambda_{\psi} + \Lambda) - \Lambda_{\psi}$	0.2337	0.5649	1.1619	2.3690	5.8013
$l_0 = 3.228$	24	$l = \Lambda l_0$	0.7544	1.828	3.751	7.647	18.73
$p = f\Delta \frac{\psi - \frac{B\theta}{2} x^2}{\Lambda_{\psi} + \Lambda}$	25	p, kg/cm^2	2043	2362	2116	1538	832.6

(*) This denotes the page number in the original manuscript. This portion has not been translated as yet.

COMPUTING THE SECOND PERIOD

Computation of the constants of the second period:

$$\sqrt{\frac{2gf\omega}{\varphi\theta q}} = \sqrt{\frac{2\cdot 98.1\cdot 950000\cdot 1.08}{1.088\cdot 0.2\cdot 6.2}} = 12{,}220 \text{ dm/sec};$$

$$\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] = 0.7502;$$

From the first period $\Lambda_1 + \Lambda_K = 6.1028$, $p_K = 832.6$ kg/cm^2.

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Table for Computing the Elements of the Second Period

Initial Formulas	No.	Operations				Muzzle Face
$p = p_K\left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda}\right)^{1+\theta} =$	1	(+)	Λ	7	8.5	10.51
$= 832.6\left(\frac{6.1028}{0.3015 + \Lambda}\right)^{1.2}$	2		Λ_1	0.3015	0.3015	0.3015
	3	$\Lambda_1 + \Lambda$		7.3015	8.8015	10.810
	4	$\eta = \frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda}$		0.8358	0.6933	0.5645
	5	$\log \eta$		$\bar{1}.9221$	$\bar{1}.8409$	$\bar{1}.7517$
				-0.0779	-0.1591	0.2483
	6	$1.2 \log \eta$		-0.09348	-0.1909	-0.2979
				$\bar{1}.9065$	$\bar{1}.8091$	$\bar{1}.702$
	7	$\eta^{1.2}$		0.8063	0.6443	0.5036
	8	$p = p_K\eta^{1.2}$		671	536	419
$v = v_{\Pi p} \times$	9	$0.2 \log \eta$		-0.01558	-0.03182	0.04966
				$\bar{1}.9844$	$\bar{1}.9682$	$\bar{1}.9503$
$\times\sqrt{1 - \eta^{0.2}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]} =$	10	$\eta^{0.2}$		0.9647	0.9294	0.8919
$= 12{,}220\sqrt{1 - 0.7502\eta^{0.2}}$	11	$0.7502\eta^{0.2}$		0.7237	0.6972	0.6691
	12	$1 - 0.7502\eta^{0.2}$		0.2763	0.3028	0.3309
	13	v, dm/sec		6424	6723	7030
	14	l, dm		22.60	27.44	33.91

The results of the calculations are presented graphically in fig. 140.

ADDITIONAL NOTES FOR THE SOLUTION OF THE PROBLEM OF INTERNAL BALLISTICS BY PROF. DROZDOV'S METHOD

In the equation for the path derived by Prof. Drozdov:

$$\Lambda_{\psi} + \Lambda = a_1 \frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) + \left(b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx\right) Z^{-\frac{B}{B_1}}$$

the function Z_x and the quantity $\int_0^x Z^{B/B_1} dx = \frac{k_1}{B_1}\int_0^{\beta} Z^{B/B_1} d\beta$ are found in the tables from the entries:

$$\gamma = \frac{B_1\psi_0}{k_1^2} \text{ and } \beta = \frac{B_1}{k_1}x.$$

For the sake of convenience all the calculations of $\log Z^{-1}$ and $\int_0^{\beta} Z^{B/B_1} d\beta$ are performed on another form for all values of x, i.e., for all combinations of β and γ.

$\gamma = 0.006995$; $\beta = 0.060$; $\beta_m = 0.1070$; $\beta = 0.160$; $\beta = 0.220$;

$$\beta_K = 0.2973.$$

The values of $\log Z^{-1}$ and of $\int_0^{\beta} Z^{B/B_1} d\beta$ are written for every combination of γ and β, as shown in the form. Then the interpolation factors ζ_{γ} and ζ_{β} are determined from the following equations:

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$$\zeta_\gamma = \frac{\gamma - \gamma_n}{\gamma_{n+1} - \gamma_n} \text{ for horizontal interpolation;}$$

$$\zeta_\beta = \frac{\beta - \beta_n}{\beta_{n+1} - \beta_n} \text{ for vertical interpolation.}$$

<u>Example</u>. Find $\log Z^{-1}$, if it is known that $\gamma = 0.006995$; $\beta = 0.2973$. These values of γ and β are not found in the table of the logarithms of function Z^{-1}, and we take the nearest values, i.e.:

$$\gamma = 0.006 \text{ and } \gamma = 0.008;$$

$$\beta = 0.28 \text{ and } \beta = 0.300,$$

and we find the values of $\log Z^{-1}$ for these combinations. We then calculate ζ_γ and ζ_β

$\zeta_\gamma = 0.4975$

$\zeta_\beta = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.1309	0.1295	0.1280
0.2973		0.1397	
0.300	0.1428	0.1413	0.1398

$$\zeta_\gamma = \frac{0.006995-0.006}{0.008-0.006} = 0.4975$$

$$\zeta_\beta = \frac{0.2973-0.28}{0.300-0.28} = 0.865$$

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We find:

$$\log Z^{-1}_{(1)} = 0.1309 - 0.4975(0.1309 - 0.1280) = 0.1295;$$

$$\log Z^{-1}_{(2)} = 0.1428 - 0.4975(0.1428 - 0.1398) = 0.1413.$$

Finally we obtain:

$$\log Z^{-1} = 0.1295 + 0.865\ (0.1413 - 0.1295) = 0.1397.$$

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WORK FORM FOR DETERMINING log Z^{-1} FROM THE TABLES.

→	$\xi_\gamma = 0.4975$		
β \ γ	0.006	0.006995	0.008
0.060	0.0204	0.0198	0.0192

$$\xi_\gamma = \frac{0.006995-0.006}{0.008-0.006} = 0.4975$$

$\log Z^{-1} = 0.0204-0.4975 \cdot$

$\cdot\ (0.0204-0.0192) = 0.0198$

→ $\xi_\gamma = 0.4975$

$\xi_\beta = 0.350$

β \ γ	0.006	0.006995	0.008
0.100	0.0379	0.03706	0.0362
0.1070	-	0.04024	-
0.120	0.0471	0.04616	0.0452

$\xi_\gamma = 0.4975$

$$\xi_\beta = \frac{0.1070-0.100}{0.120-0.100} = 0.350$$

$\log Z^{-1}_{(1)} = 0.0379-0.4975 \cdot$

$\cdot\ (0.0379-0.0362) = 0.03706$

$\log Z^{-1}_{(2)} = 0.0471-0.4975 \cdot$

$\cdot\ (0.0471-0.0452) = 0.04616$

$\log Z^{-1} = 0.03706 + 0.350 \cdot$

$\cdot\ (0.04616-0.03706) = 0.040$[illegible]

→ $\xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0663	0.06521	0.0641

$\log Z^{-1} = 0.06521$

→ $\xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.0972	0.09596	0.0947

$\log Z^{-1} = 0.09596$

→ $\xi_\gamma = 0.4975$

$\xi_\beta = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.1309	0.1295	0.1280
0.2973	-	0.1397	-
[illegible]		[illegible]	0.1398

$\log Z^{-1} = 0.1397$

$\log Z^{-1} = 0.0204 - 0.$[illegible] $\cdot$

$\cdot\ (0.0204-0.0192) = 0.0198$

$\rightarrow \xi_{\gamma} = 0.4975$

$\xi_{\beta} = 0.350$

β \ γ	0.006	0.006995	0.008
0.100	0.0379	0.03706	0.0362
0.1070	-	0.04024	-
0.120	0.0471	0.04616	0.0452

$\xi_{\gamma} = 0.4975$

$$\xi_{\beta} = \frac{0.1070-0.100}{0.120-0.100} = 0.350$$

$\log Z^{-1}_{(1)} = 0.0379 - 0.4975 \cdot$

$\cdot\ (0.0379-0.0362) = 0.03706$

$\log Z^{-1}_{(2)} = 0.0471 - 0.4975 \cdot$

$\cdot\ (0.0471-0.0452) = 0.04616$

$\log Z^{-1} = 0.03706 + 0.350 \cdot$

$\cdot\ (0.04616-0.03706) = 0.040$[illegible]

$\rightarrow \xi_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0663	0.06521	0.0641

$\log Z^{-1} = 0.06521$

$\rightarrow \xi_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.0972	0.09596	0.0947

$\log Z^{-1} = 0.09596$

$\rightarrow \xi_{\gamma} = 0.4975$

$\xi_{\beta} = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.1309	0.1295	0.1280
0.2973	-	0.1397	-
0.300	0.1428	0.1413	0.1398

$\log Z^{-1} = 0.1397$

DETERMINATION OF $\int_0^\beta Z^{B/B_1} d\beta$ FROM THE TABLES

(APPENDIX 3)

Prof. N.F. Drozdov recommends to calculate the quantity $\int_0^x Z^{B/B_1} dx$ by the trapezoid rule. In order to simplify the calculations, $\int_0^x Z^{B/B_1} dx$ may be found from the tables of the function:

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta,$$

where

$$\beta = \frac{B_1}{k_1} x,$$

whence

$$x = \frac{\beta k_1}{B_1}.$$

Consequently

$$\int_0^x Z^{\frac{B}{B_1}} dx = \frac{k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta.$$

The tables of the function $\int_0^\beta Z^{B/B_1} d\beta$ are computed for B/B_1 =

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= 5, 6, 7, 8, 9 and 10.

In our problem the ratio $B/B_1 = 8.134$, i.e., it is intermediate between $B/B_1 = 8.0$ and $B/B_1 = 9.0$. This requires an additional interpolation. Therefore the determination of $\int_0^{\beta} Z^{B/B_1} d\beta$ when $B/B_1 = 8.134$ is reduced to the calculation of $\int_0^{\beta} Z^{B/B_1} d\beta$ from the tables, first when $B/B_1 = 8.0$, then when $B/B_1 = 9.0$. Interpolating these values of the integrals along B/B_1, we finally obtain $\int_0^{\beta} Z^{B/B_1} d\beta$ for $B/B_1 = 8.134$ and the given values of β.

It should be remembered when performing these calculations that the interpolation of the intermediate integrals must be performed by the same procedure as that of the function $\log Z^{-1}$.

WORK FORM FOR DETERMINING $\int_0^{\beta} Z^{B/B_1} d\beta$ FROM THE TABLES

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8.0$

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.060	0.051	0.0515	0.052

$\longrightarrow \zeta_\gamma = 0.4975$

$\zeta_\beta = 0.350$ ↓

β \ γ	0.006	0.006995	0.008
0.100	0.075	0.0755	0.076
0.1070		0.0786	
0.120	0.084	0.0845	0.085

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.098	0.099	0.100

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.112	0.1130	0.114

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 9.0$

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.060	0.050	0.0505	0.05[illegible]

$\longrightarrow \zeta_\gamma = 0.4975$

$\zeta_\beta = 0.350$ ↓

β \ γ	0.006	0.006995	0.008
0.100	0.072	0.073	0.074
0.1070		0.07597	
0.120	0.080	0.0815	0.083

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0930	0.0940	0.095

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.104	0.1055	0.107

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8$

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.13
0.060	0.0515	0.05

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.13
0.1070	0.0786	0.07

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.13
0.160	0.099	0.09

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.13
0.220	0.113	0.11

WORK FORM FOR DETERMINING $\int_0^{\beta} Z^{B/B_1} d\beta$ FROM THE TABLES

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 9.0$ | $\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8.134$

$\rightarrow \zeta_{\gamma} = 0.4975$ | $\rightarrow \zeta_{\frac{B}{B_1}} = 0.134$

.008
.052

β \ γ	0.006	0.006995	0.008
0.060	0.050	0.0505	0.05?

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.060	0.0515	0.05137	0.0505

$\int_0^{\beta} Z^{8.134} d\beta =$

$= 0.5137$ when $\beta =$

$= 0.060$

$\rightarrow \zeta_{\gamma} = 0.4975$ | $\rightarrow \zeta_{\frac{B}{B_1}} = 0.134$

.008
.076
.085

$\downarrow$

$\zeta_{\beta} = 0.350$

β \ γ	0.006	0.006995	0.008
0.100	0.072	0.073	0.074
0.1070		0.07597	
0.120	0.080	0.0815	0.083

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.1070	0.0786	0.07829	0.07597

$\int_0^{\beta} Z^{8.134} d\beta =$

$= 0.07829$ when $\beta =$

$= 0.1070$

$\rightarrow \zeta_{\gamma} = 0.4975$ | $\rightarrow \zeta_{\frac{B}{B_1}} = 0.134$

0.008
0.100

β \ γ	0.006	0.006995	0.008
0.160	0.0930	0.0940	0.095

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.160	0.099	0.09833	0.0940

$\int_0^{\beta} Z^{8.134} d\beta =$

$= 0.09833$ when $\beta =$

$= 0.160$

$\rightarrow \zeta_{\gamma} = 0.4975$ | $\rightarrow \zeta_{\frac{B}{B_1}} = 0.134$

.008
.114

β \ γ	0.006	0.006995	0.008
0.220	0.104	0.1055	0.107

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.220	0.113	0.1120	0.1055

$\int_0^{\beta} Z^{8.134} d\beta =$

$= 0.1120$ when $\beta =$

$\longrightarrow \zeta_\gamma = 0.4975$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.160	0.098	0.099	0.100

$\longrightarrow \zeta_\gamma = 0.4975$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.160	0.0930	0.0940	0.095

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

$\beta \backslash \frac{B}{B_1}$	8.0	8.134
0.160	0.099	0.098

$\longrightarrow \zeta_\gamma = 0.4975$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.220	0.112	0.1130	0.114

$\longrightarrow \zeta_\gamma = 0.4975$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.220	0.104	0.1055	0.107

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

$\beta \backslash \frac{B}{B_1}$	8.0	8.134
0.220	0.113	0.112

$\longrightarrow \zeta_\gamma = 0.4975$

$\zeta_\beta = 0.865$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.28	0.119	0.1205	0.122
0.2973		0.1222	
0.300	0.121	0.1225	0.124

$\longrightarrow \zeta_\gamma = 0.4975$

$\zeta_\beta = 0.865$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.28	0.110	0.1115	0.113
0.2973		0.1124	
0.300	0.111	0.1125	0.114

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

$\beta \backslash \frac{B}{B_1}$	8.0	8.134
0.2973	0.1222	0.1209

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$= 0.07829$ when $\beta =$

$= 0.1070$

0.008
0.100

$\longrightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0930	0.0940	0.095

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.160	0.099	0.09833	0.0940

$\int_0^{\beta} z^{8.134} d\beta$ [illegible]

$= 0.09833$ when β

$= 0.160$

0.008
0.114

$\longrightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.104	0.1055	0.107

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.220	0.113	0.1120	0.10[illegible]

$\int_0^{\beta} z^{8.134} d\beta =$ [illegible]

$= 0.1120$ wh[illegible]

$= 0.220$

$\longrightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.28	0.110	0.1115	0.113
0.2973		0.1124	
0.300	0.111	0.1125	0.114

$\zeta_{\beta} = 0.865$

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.2973	0.1222	0.1209	0.1424

$\int_0^{\beta} z^{8.134}$ [illegible]

$= 0.1209$ wh[illegible]

$= 0.2973$

CHAPTER 3 - SOLUTION OF THE PROBLEMS OF INTERIOR BALLISTICS FOR THE SIMPLEST CASES

1. SOLUTION OF THE PROBLEM FOR THE CASE OF INSTANTANEOUS BURNING OF THE POWDER

The colloidal powders now used burn gradually, in parallel layers, and when the web thickness is properly selected, permit the regulation of the flow of gases during burning, so that the maximum pressure in the bore p_m would not exceed a given value (usually of the order of 2500-3500 kg cm^2).

The case of the instantaneous burning of the charge is anamalous and generally does not occur in practice. It can be achieved in practice only under special conditions, such as, for example, when burning a charge of dry pyroxylin in powder form, or of fine porous powder loaded very densely.

In that case, if the loading density were normal ($\Delta = 0.50$-0.75), the pressure prior to the projectile's displacement would reach a maximum value of the order of several tens of thousands of atmospheres (20,000 to 40,000 kg cm^2). The present ultimate strength of gun barrels is such that the walls of the barrel would burst when subjected to such pressures.

Nevertheless, the case of the instantaneous burning of a charge is very interesting; its examination has an important meaning when compared with gradual burning of powder because in so doing the importance of slow burning and of the shape and dimensions of the powder grains become evident. Moreover, the pressure curve p, l in the case of instantaneous burning becomes a sort of a "guide" for the curves depicting slow burning. These p, l curves arrange themselves with a certain regularity with respect to the instantaneous

burning curves.

The analytical solution of the problem is very simple in the case of instantaneous burning, because one of the four variables entering into the fundamental equation of pyrodynamics is transformed into a constant ($\psi = 1$).

Let the gun and the loading conditions be characterized as follows:

The chamber capacity is W_0, the cross sectional area of the bore, including the rifling is s, the path of the projectile is $l_\text{д}$, the weight of the charge is ω, and the weight of the projectile is q. The energy of the powder is f, and α is the covolume; the adiabatic index is $k = 1 + \theta$, and the secondary work done is taken into account by the coefficient $\varphi = a + b\,\frac{\omega}{q}$.

When $\psi = 1$, the fundamental equation of pyrodynamics is:

$$ps(l_1 + l) = f\omega - \frac{\theta}{2}\varphi m v^2; \tag{41}$$

the equation of the projectile motion is:

$$ps\,dl = \varphi m v\,dv, \tag{42}$$

where $l_1 = (W_0 - \alpha\omega)/s$ is the reduced length of the chamber at the end of burning.

When the powder in the chamber is burned instantaneously the maximum pressure is determined by means of the well known formula:

$$p_1 = \frac{f\Delta}{1 - \alpha\Delta} = \frac{f\omega}{W_0 - \alpha\omega} = \frac{f\omega}{s l_1} \tag{43}$$

The projectile will be set in motion when the following initial conditions obtain:

$$l = 0; \quad v = 0; \quad p = p_1.$$

Eliminating pressure p from equations (41) and (42), we obtain v as a function of l:

$$\frac{dl}{l_1 + l} = \frac{\varphi m v dv}{f\omega - \frac{\theta}{2}\varphi m v^2} = -\frac{1}{\theta}\,\frac{d\,\frac{\theta\varphi m v^2}{2}}{f\omega - \frac{\theta}{2}\varphi m v^2}.$$

We shall integrate this differential equation with the variables separable:

$$\frac{l_1 + l}{l_1} = \left(\frac{f\omega - \frac{\theta\varphi m v^2}{2}}{f\omega}\right)^{-\frac{1}{\theta}} = \left(1 - \frac{v^2}{v_{np}^2}\right)^{-\frac{1}{\theta}}, \tag{44}$$

whence

$$v^2 = v_{np}^2\left[1 - \left(\frac{l_1}{l_1 + l}\right)^{\theta}\right],$$

where $v_{\Pi p}^2 = 2gf\omega/\varphi\theta q$ is the limiting velocity of the projectile:

$$v = v_{\Pi p}\sqrt{1 - \left(\frac{l_1}{l_1 + l}\right)^{\theta}}. \tag{45}$$

This formula expresses the velocity v of the projectile as a function of its path l; the velocity increases when l increases.

In order to determine the dependence of p on l from (44) we determine:

$$f\omega - \frac{\theta}{2}\varphi m v^2 = f\omega\left(\frac{l_1}{l_1 + l}\right)^{\theta}$$

and include it into (41):

$$ps(l_1 + l) = f\omega\left(\frac{l_1}{l_1 + l}\right)^{\theta}.$$

Whence

$$p = \frac{f\omega}{sl_1}\left(\frac{l_1}{l_1 + l}\right)^{1+\theta} = p_1\left(\frac{l_1}{l_1 + l}\right)^{1+\theta}. \tag{46}$$

But

$$\frac{l_1}{l_1 + l} = \frac{W_1}{W_1 + sl} = \frac{W_0 - \alpha\omega}{W_0 - \alpha\omega + sl}$$

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is the ratio between the free volumes in the initial air space measured at the start of motion and at the given instant. Consequently, formula (46) is the equation of the adiabatic curve starting with the motion of the projectile under the pressure $p_1 = f\Delta/(1 - \alpha\Delta)$.

The change in temperature of the gases doing the work is expressed by the following relationship for the adiabatic process:

$$\frac{T}{T_1} = \left(\frac{l_1}{l_1 + l}\right)^{\theta}.$$

Consequently,

$$v = v_{np}\sqrt{1 - \frac{T}{T_1}}. \qquad (47)$$

If we divide the numerator and denominator in parentheses in formulas (45) and (47) by l_1, and designate l/l_1 by y, we will get:

$$v = v_{np}\sqrt{1 - \frac{1}{(1 + y)^{\theta}}}; \qquad (48)$$

$$p = p_1 \frac{1}{(1 + y)^{1+\theta}}. \qquad (49)$$

The quantity y is the ratio of the relative projectile path to the reduced length of the free volume in the chamber at the end of burning, and is called the "number of free volumes of gas expansion."

Equations (48) and (49) show that under the given loading conditions ($q, \omega, f, \alpha, W_0, s$) the pressure p and the velocity v depend only upon the number y of free volumes of expansion. The greater y, the higher is the projectile velocity and the smaller the pressure; the greater the reduced length l_1 of the free space in the chamber, the greater will be the gas pressure for a given projectile path. Consequently, the drop in pressure as a function of the projectile path will be slower in a large chamber than in a small one.

It can be proved that the velocity of the projectile computed by means of formula (48) for the case of instantaneous burning will be always greater than the true velocity for the case of slow burning, under the same charging conditions.

Indeed, the maximum work done by a powder charge ω of energy f in setting a projectile of mass m in motion, is determined by the expression $f\omega\,\theta$. This maximum work will be the same for both modes of burning (instantaneous and gradual) and is expressed by the areas under the curves sp as a function of l, when l varies between 0 and infinity. Consequently, in both cases the areas will be equal to:

$$s\int_0^\infty p\,dl = \frac{f\omega}{\theta}.$$

In the case of instantaneous burning the curve p, l starts from the maximum pressure p_1, then varies according to the adiabatic law, decreasing continuously (fig. 141, curve I). When burning is gradual, the curve II of the pressure p, l rises

gradually from p_0, losing a portion of area A; and inasmuch as the total area under the curve p, l in the second case, limited by $l = \infty$, must be the same as the first, curve II must necessarily cross curve I during burning of the powder when the pressure drops, and then continues to rise. The excess area B between the curves, when $l = \infty$ is the limit, must be equal to A. But inasmuch as the actual bore has a finite projectile path $l_{д}$, the portion of the area B on this finite length is always smaller than A, and consequently, for a given path length, the work done by the gases and the velocity of the projectile will be always smaller in the case of gradual burning than in the case of instantaneous burning.

Fig. 141 - Curves p, l and v, l Depicting the Instantaneous Burning of Powder.

The actual initial (muzzle) velocity of a projectile of a medium-caliber gun, measured experimentally, represents 80-90% of the velocity computed by formula (45).

Inasmuch as the work represented by the area under the curve p, l is larger in the case of instantaneous burning than in gradual burning, especially at the start of the motion, the corresponding velocity curve rises more steeply at first. Thereafter, because of

the addition of the area B, the velocity increase becomes greater in the case of gradual burning, and curve II gradually begins to approach curve I (see fig. 141), tending at $l = \infty$ to the common limit $v_{np} = \sqrt{2f\omega/\varphi\theta m}$.

2. SOLUTION OF THE PROBLEM FOR A POWDER WITH A CONSTANT BURNING AREA, WHEN THE FORCING PRESSURE IS ABSENT.

The solution of the problem of internal ballistics for degressive powders in the presence of forcing pressure results in equations which do not give an immediate relation between v, p, and l, and, therefore, exclude the possibility of an analytical examination of the basic relations. In order to obtain this possibility, it is necessary to introduce certain simplifications into the initial data, namely:

1) Consider a powder having a constant burning area

$$\varkappa = 1; \quad \lambda = 0; \quad \psi = z.$$

2) Consider the forcing pressure to be negligible; assume that the projectile is set in motion when the pressure equals the pressure of the igniter gases, and that the burning of the charge begins when the projectile is set in motion:

$$p_0 = p_B; \quad \psi_0 = 0.$$

3) Assume that $\alpha = 1/\delta$.

When these assumptions are made, the solution of the fundamental system of equations is greatly simplified.

The first assumption corresponds to the burning of long tubular

powder; the second corresponds to projectiles with pre-cut bands; the third assumption simplifies the solution and permits determining the qualitative effect of the loading conditions.

Under the assumptions made, the preliminary period does not exist. The motion of the projectile begins under the following conditions:

$$p_0 = p_B; \quad \psi_0 = 0; \quad z_0 = 0; \quad l = 0; \quad v = 0.$$

Inasmuch as $\alpha = 1\ \delta$,

$$l_\Delta = l_\psi = l_1 = l_0(1 - \alpha\Delta).$$

The law governing burning of powder, $\psi = f(z)$ will be expressed by the formula:

$$\psi = z = x, \tag{50}$$

and ψ may be taken as the independent variable. Then the equation of the projectile velocity will take on the form:

$$v = \frac{sI_K}{\varphi m}\psi. \tag{51}$$

The fundamental equation of pyrodynamics is:

$$ps(l_1 + l) = f\omega\psi - \frac{\theta}{2}\varphi m v^2 = f\omega\left(\psi - \frac{B\theta}{2}\psi^2\right).$$

The equation of the elementary work done is:

$$psdl = \varphi mvdv = \frac{s^2 I_K^2}{\varphi m} \psi d\psi;$$

the differential equation of the projectile path will be:

$$\frac{dl}{l_1 + l} = \frac{Bd\psi}{1 - \frac{B\theta}{2}\psi} = -\frac{2}{\theta} \frac{-\frac{B\theta}{2}d\psi}{1 - \frac{B\theta}{2}\psi}.$$

Integrating, we get:

$$1 + \frac{l}{l_1} = \left(1 - \frac{B\theta}{2}\psi\right)^{-\frac{2}{\theta}}. \qquad (52)$$

whence,

$$l = l_1 \left[\frac{1}{\left(1 - \frac{B\theta}{2}\psi\right)^{\frac{2}{\theta}}} - 1\right]. \qquad (53)$$

Designating, as in the case of instantaneous burning,

$$\frac{l}{l_1} = y,$$

we obtain from equation (52):

$$\psi = \frac{2}{B\theta}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right], \qquad (54)$$

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and inasmuch as

$$v = \frac{sI_K}{\varphi m}\psi,$$

$$v = \frac{2f\omega}{\theta s I_K}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right]. \qquad (55)$$

These equations give the direct dependence of v and ψ on the path of the projectile l, or $y = l : l_1$.

Let us write the pressure equation, taking into account the igniter pressure:

$$p_B = \frac{f_B\omega_B}{W_0 - \frac{\omega}{\delta}} \approx \frac{f_B\omega_B}{W_0 - \alpha\omega} = \frac{f_B\Delta_B}{s l_1};$$

$$p = \frac{f_B\omega_B + f\omega\psi - \frac{\theta}{2}\varphi m v^2}{s(l_1 + l)}. \qquad (56)$$

Let us designate the relative energy of the igniter gases by:

$$\frac{f_B\omega_B}{f\omega} = \chi_B.$$

Carrying $f\omega$ outside the parentheses in (56) and replacing v according to (52), we obtain:

$$p = \frac{f\omega\left[\chi_B + \psi\left(1 - \frac{B\theta}{2}\psi\right)\right]}{s l_1 (1 + y)} = p_1 \frac{\chi_B}{1 + y} + p_1 \frac{\psi}{(1 + y)^{1 + \frac{\theta}{2}}}, \tag{57}$$

where $p_1 = f\omega\, s l_1 = f\Delta\,(1 - \alpha\Delta)$ is the maximum pressure developed by the burning of the entire charge within the space of the chamber when the density of the loading is Δ;

$1 - \frac{B\theta}{2}\psi = \frac{1}{(1 + y)^{\frac{\theta}{2}}}$ on the basis of equation (52).

Substituting ψ by its expression in (54), we obtain the pressure p as a function of the path of the projectile:

$$p = p_1 \frac{\chi_B}{1 + y} + p_1 \frac{2}{B\theta}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right] \frac{1}{(1 + y)^{1 + \frac{\theta}{2}}}. \tag{58}$$

The quantity

$$p_1 \frac{\chi_B}{1 + y} = \frac{f_B \omega_B}{s l_1 (1 + y)} = \frac{p_{B,0}}{1 + y} = p_B$$

represents the pressure developed by the igniter gases in the variable space of the bore. At the start of motion $y = 0$, $p_B = p_{B,0}$; as the projectile moves forward and y increases, p_B decreases $(1 + y)$ times, and may be neglected when compared with the pressure developed by the

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gases of the powder charge.

At the beginning of motion, $y = 0$, the second term is equal to zero (equation 58); $p = p_{B,0}$. As the projectile moves and y increases, the factor in the brackets increases, while the factor $(1 + y)^{-(1+\theta/2)}$ decreases.

The maximum pressure p_m will occur at some value y_m.

Let us designate:

$$F(y) = \left[1 - \frac{1}{(1+y)^{\frac{\theta}{2}}} \right] \frac{1}{(1+y)^{1+\frac{\theta}{2}}} = (1+y)^{-\left(1+\frac{\theta}{2}\right)} - (1+y)^{-(1+\theta)}.$$

Differentiating F(y) with respect to y, and equating the derivative to zero, we find:

$$1 + y_m = \left(\frac{1+\theta}{1+\frac{\theta}{2}} \right)^{\frac{2}{\theta}} = F_1(\theta) = \text{const.} \tag{59}$$

[when $\theta = 0.2$; $F_1(\theta) = 2.387$]

whence,

$$l_m = l_1 [F_1(\theta) - 1] = l_0 (1 - \alpha\Delta) [F_1(\theta) - 1]. \tag{60}$$

Substituting (59) in (58), (54) and (55), we obtain the expressions for all the elements of motion at the instant $p = p_{max}$.:

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$$p_m = \frac{p_1}{B}\;\frac{2}{\theta}\left[\frac{\theta}{2(1+\theta)}\right]\left(\frac{1+\frac{\theta}{2}}{1+\theta}\right)^{\frac{2+\theta}{\theta}} =$$

$$= \frac{p_1}{B}\;\frac{1}{1+\theta}\left(\frac{1+\frac{\theta}{2}}{1+\theta}\right)^{\frac{2+\theta}{\theta}} = \frac{p_1}{B}F_2(\theta). \qquad (61)$$

When $\theta = 0.20$, $F_2(\theta) = 0.3200$.

$$\psi_m = \frac{1}{B(1+\theta)}; \qquad (62)$$

$$v_m = \frac{f\omega}{sI_K}\;\frac{1}{1+\theta}. \qquad (63)$$

Equations (60), (61), (62) and (63) give the direct relationship between the elements of motion and several characteristics and parameters at the instant of maximum pressure. Thus the path l_m is directly proportional to the reduced length of the chamber l_0 and to $1 - \alpha\Delta$. When Δ increases, l_m decreases. The pressure p_m is directly proportional to the pressure p_1 of instantaneous burning, determined by Nobel's equation; p_m is inversely proportional to Prof. Drozdov's parameter B. ψ_m is also inversely proportional to the parameter B.

When $\theta = 0.2$:

$$p_m = 0.320\,\frac{p_1}{B}, \quad \frac{l_m}{l_0} = \Lambda_m = 1.387(1 - \alpha\Delta).$$

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The equations are very simple and accessible to analysis.

At the end of burning, $\psi = 1$.

$$v_K = \frac{sI_K}{\varphi m}; \tag{64}$$

$$1 + y_K = \frac{1}{\left(1 - \frac{B\theta}{2}\right)^{\frac{2}{\theta}}}; \tag{65}$$

$$p_K = \frac{p_1}{(1 + y_K)^{1 + \frac{\theta}{2}}} = p_1 \left(1 - \frac{B\theta}{2}\right)^{\frac{2+\theta}{\theta}}. \tag{66}$$

In the second period we find the following relationships:

$$p = p_K \left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta} = p_K \left(\frac{1 + y_K}{1 + y}\right)^{1+\theta}.$$

Substituting for p_K and $1 + y_K$ their expressions from (65) and (66), we find:

$$p = \frac{p_1}{\left(1 - \frac{B\theta}{2}\right)} \frac{1}{(1 + y)^{1+\theta}}. \tag{67}$$

This is the equation of an adiabatic curve with initial ordinate $p_1/(1 - \frac{B\theta}{2})$; the latter increases with B, i.e., with the thickness

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of the powder.

From the general equation for the projectile velocity, we have:

$$v = v_{np}\sqrt{1 - \left(\frac{1 + y_K}{1 + y}\right)^{\theta}\left(1 - \frac{B\theta}{2}\right)} =$$

$$= v_{np}\sqrt{1 - \left(\frac{1}{1 - \frac{B\theta}{2}}\right)\frac{1}{(1 + y)^{\theta}}}. \qquad (68)$$

The Temperature of Powder Gases.

In the case of instantaneous burning we had:

$$\frac{T}{T_1} = \left(\frac{l_1}{l_1 + l}\right)^{\theta} = \frac{1}{(1 + y)^{\theta}}. \qquad (69)$$

When burning is gradual:

$$\frac{T}{T_1} = 1 - \frac{1}{\psi}\frac{v^2}{v_{np}^2} = 1 - \frac{v}{\psi}\frac{v}{v_{np}^2}.$$

Substituting the values of ψ, v from (51) and (55) and $v_{np}^2 =$ $= 2f\omega/\varphi\theta m$, we get:

$$\frac{T}{T_1} = 1 - \frac{sI_K}{\varphi m}\frac{\varphi\theta m}{2f\omega}\frac{2f\omega}{\theta s I_K}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right]$$

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or

$$\frac{T}{T_1} = \frac{1}{(1 + y)^{\frac{\theta}{2}}}. \tag{70}$$

A comparison of (69) with (70) will show that in the expression for relative gas temperature for gradual burning of a powder with a constant area, the value of the exponent is one-half of that of instantaneous burning. Consequently, the temperature drop in the case of gradual burning proceeds almost at half the rate of instantaneous burning, because the work done in traversing a given path is considerably smaller.

The expression for the temperature T/T_1 may be written as a function of ψ only:

$$\frac{T}{T_1} = 1 - \frac{v_K^2 \psi^2}{\psi v_{\Pi_p}^2} = 1 - \frac{v_K^2}{v_{\Pi_p}^2}\psi = 1 - \frac{B\theta}{2}\psi, \tag{71}$$

i.e., the temperature of the gases inside the barrel during burning of powder with constant area is a linear decreasing function of ψ.

At the end of burning ($\psi = 1$)

$$\frac{T_K}{T_1} = 1 - \frac{B\theta}{2}.$$

The thicker the powder, the larger is B, and the lower is the temperature T_K.

In the second period, from expression (68):

$$\frac{T}{T_1} = 1 - \frac{v^2}{v^2_{n_p}} = \frac{1}{1 - \frac{B\theta}{2}} \frac{1}{(1 + y)^\theta} \tag{72}$$

or

$$\frac{T}{T_1} = \left(1 - \frac{B\theta}{2}\right) \left(\frac{1 + y_K}{1 + y}\right)^\theta = \frac{T_K}{T_1} \frac{T}{T_K}. \tag{73}$$

These relatively simple equations enable one to perform an analysis of the variation of the elements of a shot (p, v, ψ, T) as a function of y - the relative path of the projectile - and to establish a series of relations and properties of the variation curves of these elements.

CHAPTER 4 - ANALYSIS OF THE BASIC RELATIONS FOR THE SIMPLEST CASE

$$\left(\kappa = 1,\ \psi_0 = 0,\ \alpha = \frac{1}{\delta}\right)$$

1. ANALYSIS OF THE FUNDAMENTAL CURVES p, v, T, ψ.

An analysis of the equations obtained, and of the curves represented by them, shows that they represent certain simple combinations of two types of curves (fig. 142):

a) Two polytropic curves with exponents $k = 1 + \theta$ and $k' = 1 + \theta/2$, starting from the point (1, 0), first dropping steeply and then more gradually;

b) Two curves analogous to the polytropic curves but with exponents smaller than unity ($k - 1 = \theta$ and $k' - 1 = \theta/2$) also starting from point (1, 0) and descending much more slowly with the convex side directed downward.

Fig. 142 - Basic Types of Curves for the Simplest Case.

Indeed, in the case of instantaneous burning:

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$$\frac{T}{T_1} = \frac{1}{(1 + y)^{\theta}}; \qquad (69)$$

in the case of gradual burning:

$$\frac{T}{T_1} = \frac{1}{(1 + y)^{\frac{\theta}{2}}} \qquad (70)$$

$$\psi = \frac{2}{B\theta}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right] = \frac{2}{B\theta}\left[1 - \frac{T}{T_1}\right]; \qquad (54')$$

$$\frac{v}{v_{np}} = \sqrt{\frac{B\theta}{2}}\,\psi = \sqrt{\frac{2}{B\theta}}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right] =$$

$$= \sqrt{\frac{2}{B\theta}}\left[1 - \frac{T}{T_1}\right]; \qquad (55')$$

$$\frac{p}{p_1} = \frac{\psi}{(1 + y)^{1+\frac{\theta}{2}}}; \qquad (57')$$

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$$\frac{p_K}{p_1} = \frac{1}{(1 + y_K)^{1+\frac{\theta}{2}}}. \qquad (66')$$

In the second period:

$$\frac{p}{p_1} = \frac{1}{\left(1 - \frac{B\theta}{2}\right)} \frac{1}{(1 + y)^{1+\theta}}; \qquad (67')$$

$$\frac{T}{T_1} = \frac{1}{1 - \frac{B\theta}{2}} \frac{1}{(1 + y)^{\theta}}; \qquad (72)$$

$$\frac{v}{v_{np}} = \sqrt{1 - \frac{T}{T_1}}. \qquad (68')$$

Each of these equations contains one of the polytropics indicated above in the form of a variable component.

The curves $\frac{T}{T_1}$, y for instantaneous and gradual burning are expressed directly in the first period by curves with exponents θ and $\theta/2$, according to equations (69) and (70).

The ordinates of the curves ψ (54') and v (55') are obtained from the ordinates of the curves $\Delta T/T_1 = 1 - T/T_1$ (fig. 142), measured from the horizontal 1-1, multiplied by the coefficients $2/B\theta$ and $2f\omega/I_K\theta s$ respectively, and laid off upwards along the abscissa from the origin

of coordinates to y_K which corresponds to the end of burning (fig. 143). The curves ψ, y and $\frac{v}{v_{np}}$, y are inverted with respect to the curves $\frac{\Delta T}{T_1}$, y.

The ordinates of the relative pressure curve (57') are obtained by multiplying the ordinates of the ψ curve (54') by those of the auxiliary polytropic curve $y = 1/(1 + y)^{1+\theta/2}$. This same polytropic curve is the geometrical locus of the pressure p_K/p_1 at the end of burning, expressed as a function of y (66).

Fig. 143 - Curves ψ and v/v_{np} in the First Period.

It is seen from equation (66) that when B (powder thickness) increases, p_K decreases, while y_K increases [according to (65)].

The curves of the second period elements start to the right of the abscissa y_K; the curve T/T_1 represents the curve $T/T_1 = 1/(1 + y)^{\theta}$ of instantaneous burning, multiplied by the quantity $1/(1 - B\theta/2) > 1$, which means that the gradual burning curve is higher than the instantaneous burning curve, the difference in height increasing with the parameter B, i.e., the difference being greater for thicker powders.

From equation (60) it is seen that the maximum pressure is independent of the parameter B, of the thickness of the powder, and

of the weight of the projectile. For a given Δ, the length l_m is proportional to the length l_0 of the chamber; when Δ increases the maximum is shifted toward the start of motion.

The real maximum pressure [equation (61)] is proportional to the maximum pressure in the case of instantaneous burning, $p_1 = f\Delta/(1 - \alpha\Delta)$, and is inversely proportional to the parameter B, or to the powder density squared. When the energy of the powder, entering in the expressions for the pressure p_1 and the denominator B, is changed, the maximum pressure varies proportionally to the energy of the powder squared, and to the weight of the projectile, because

$$B = \frac{s^2 I_K^2 g}{f\omega\varphi q},$$

and

$$p_m = \frac{p_1}{B} F_2(\theta) = F_2(\theta) \frac{f\Delta}{1 - \alpha\Delta} \frac{f\omega\varphi m}{s^2 I_K^2}.$$

When B increases, ψ_m and v_m decrease also [equations (62) and (63)].

The adiabatic pressure curve in the case of instantaneous burning, $p/p_1 = 1/(1 + y)^{1+\theta}$, acts as "guide" for the adiabatic curves of the second period when the powder burns gradually. The ordinates of these curves are obtained by multiplying the ordinates of the first curve by $1/(1 - B\theta/2) > 1$ [equation (67)]. The thicker the powder, the larger is B, the larger is this value, and the higher will the adiabatic curve of the second period lie above the

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adiabatic curve of instantaneous burning. The ratio of the ordinates of these adiabatics for the same value of y (or l) is constant and equal to $1/(1 - B\theta/2)$ = const.

In the case of instantaneous burning and in the second period, the projectile velocity is proportional to the square root of the temperature drop, rather than to the first power of this factor, as is the case for the first period.

For a given charging density and during the burning of the powder, the projectile velocity in a given section does not depend upon the weight of the projectile. Indeed, from equation (55)

$$v = \frac{2f\omega}{I_K\theta s}\left[1 - \frac{1}{(1+y)^{\frac{\theta}{2}}}\right]$$

it is seen that for a given value of $y = l/l_0(1 - \alpha\Delta)$ and for one and the same powder (f, α, I_K) the projectile velocity is independent of the weight q of the projectile. The same may be said of the temperature of the gases [according to formula (70)].

If, all other conditions being equal, we vary only q which enters into parameter B, the pressure p and ψ from equations (54) and (56) and the maximum pressure p_m and ψ_m increases in proportion with q, while the location of the maximum and of the value v_m does not change.

Consequently, the velocity curves coincide point by point when superimposed on each other(*), and only when the projectile

(*) The result obtained (when $\alpha = 1/\delta$) can be confirmed by comparing it with the GAU tables (ordnance tables), compiled for $p_0 = 300$, $\alpha = 1$, and $\varkappa = 1.06$.

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is heavier will the velocity v_K [from equation (64)] be attained earlier.

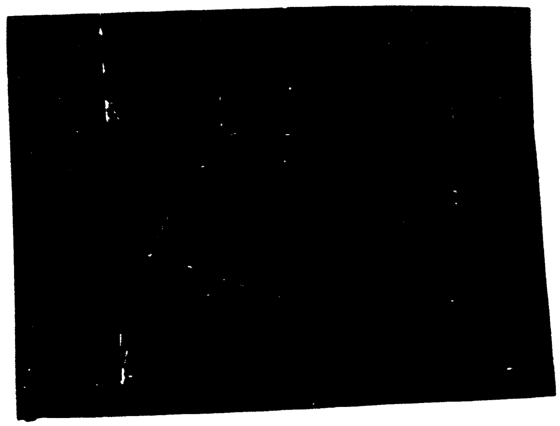

Fig. 144 - Ballistic Curves for the Simplest Case.

The fact that the velocity of the projectile does not vary with its weight may be used to verify the complete burning of the powder in the gun. If a gun using the same charge and type of powder is used to fire two projectiles of different weights and the velocity remains unchanged, it is proof that the combustion of the powder was incomplete in both cases.

Figure 144 illustrates the basic curves of the elements of a shot (ψ, v, T, p) as a function of l or y for the cases of instantaneous and gradual burning of the powder.

All the curves on the graph are marked with the number of the equation they represent.

There are two basic points on the ordinate axis, one is at -1,

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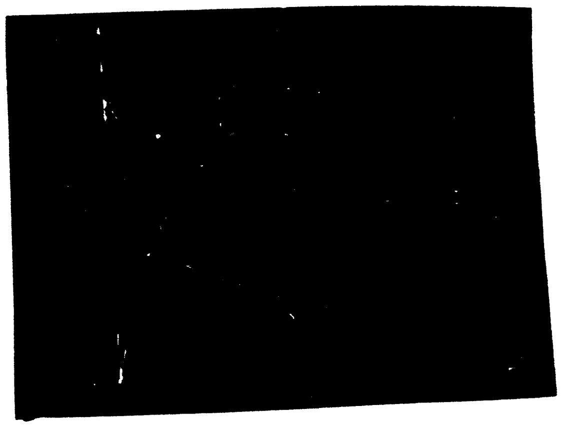

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All the curves on the graph are marked with the number of the equation they represent.

There are two basic points on the ordinate axis, one is at -1,

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and the other at $-1/(1 - B\theta/2)$.

The two pressure polytropic curves with exponents k and k', and the two "polytropics" of temperature, with exponents θ for instantaneous burning (69), and $\theta/2$ for gradual burning (70) issue from the first point; the ordinate at $y = -1$ is a common asymptote for all of these curves.

The curves p/p_1 (67') and T/T_1 (72) for the second period, issue from the second point, whose ordinate is $1/(1 - B\theta/2)$. These curves are real only to the right of the ordinates p_K/p_1 and $T_K/T_1 = 1 - B\theta/2$ with abscissa y_K.

Both of these curves lie above the corresponding curves for instantaneous burning, the ratio between the two sets being constant and equal to $1/(1 - B\theta/2) > 1$. The horizontal line whose ordinate equals unity is the origin for the curves p/p_1(1) and T/T_1 (69) and (70), the terminal point for ψ and v/v_K (54'), and is an asymptote for $v/v_{\Pi p}$ for both instantaneous and gradual burning of powder.

The curves ψ, y and $\frac{v}{v_{\Pi p}}$, y in the first period are similar to the curve $\Delta T/T$ which is measured from the horizontal along the ordinate equal to unity.

The curve p/p_1 for gradual burning is obtained by multiplying the ordinates of curve ψ(54') and of the adiabatic curve 2 with the exponent $k' = 1 + \theta/2$.

2. THE CONDITIONS FOR MAINTAINING THE MAXIMUM PRESSURE CONSTANT.

This question is very important in the ballistic design of guns, because the condition generally imposed is that p_m must not exceed a certain given value. For this reason the designer must

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know how to vary the loading conditions in order to keep the maximum pressure constant.

The relations derived above permit one to establish the analytical conditions under which the maximum pressure p_m will remain constant when the weight of the charge or its density are varied in a given gun.

Indeed,

$$p_m = \frac{p_1}{B} F_2(\theta) = \frac{f\Delta}{1 - \alpha\Delta} \frac{F_2(\theta) f \omega \varphi q}{s^2 I_K^2 g}. \qquad (61')$$

For a given type of powder (f, α, u_1) and a given projectile weight q, p_m can be changed by varying either Δ or $I_K = e_1/u_1$. If Δ is increased simultaneously with $2e_1$ or I_K, p_m can be kept constant. The condition of the constancy of the pressure is obtained in the form:

$$p_m = \frac{F_2(\theta)}{B} \frac{f\Delta}{1 - \alpha\Delta} = \text{const}.$$

Let us group together the constants:

$$B\left(\frac{1}{\Delta} - \alpha\right) = \frac{f}{p_m} F_2(\theta) = \text{const}.$$

Designating:

$$\frac{f}{p_m} F_2(\theta) = a_m \approx \text{const},$$

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we obtain the condition for maintaining p_m constant:

$$B\left(\frac{1}{\Delta} - \alpha\right) = a_m = \text{const.} \tag{74}$$

In the case when $\alpha \neq 1/\delta$, when $\psi_0 = 0$ and $\kappa = 1$, the condition p_m = const. has an analogous form:

$$B\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) = \text{const.}$$

Knowing a_m and given Δ, one may find the quantity B insuring the obtainment of the given p_m, and, knowing B, one may find the corresponding value of $2e_1$ or I_K.

Condition (74) shows that in order to keep p_m constant when Δ is increased, it is necessary to increase the thickness $2e_1$ of the powder in order to offset the decrease of B obtained from increasing the weight ω of the charge together with the increase of Δ.

From the condition (74) of the constancy of the maximum pressure for a given gun, projectile and powder of definite physico-chemical properties (f, α, δ, u_1), a direct relation may be established between the weight ω of the charge, the thickness $2e_1$ of the powder or its pressure impulse I_K, and the reduced length of the free space in the chamber at the end of burning:

$$l_1 = l_0(1 - \alpha\Delta) = \frac{W_0}{s}(1 - \alpha\Delta) = \frac{W_1}{s}.$$

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Indeed, substituting the value of B in (74) and replacing Δ in the denominator by ω/W, we obtain:

$$\frac{s^2 I_K^2 W_0 (1 - \alpha\Delta)}{f\omega\varphi m\omega} - \frac{s^2 I_K^2 W_1}{f\omega^2 \varphi m} - a_m.$$

Transposing all the constants to the right side, and designating them by K_m, we obtain:

$$\frac{I_K^2 W_1}{\omega^2} - \frac{a_m f\varphi m}{s^2} - \frac{f^2 F_2(\theta)\varphi m}{p_m s^2} - K_m. \tag{75}$$

Computing first K_m from the loading conditions by the following equation:

$$K_m - \frac{F_2(\theta) f^2 \varphi m}{p_m s^2}$$

and calculating the value ω of the charge necessary to insure a given initial (muzzle) velocity v_A, we can determine the full pressure impulse $I_K - e_1/u_1$

$$I_K - \frac{\sqrt{K_m \omega}}{\sqrt{W_0 - \alpha\omega}}. \tag{76}$$

This equation shows that in order that the pressure in a given gun remain constant when the weight of the charge is increased, the full pressure impulse I_K or the thickness $2e_1$ of the powder must be

increased at a somewhat higher rate than the weight of the charge.

Let us apply equation (76) to calculate the powder thickness for a 76 mm gun, 1902 model.

The conditions of loading are:

$W_0 = 1.654$; $s = 0.4693$; $\omega = 0.930$; $q = 6.5$; $f = 9 \cdot 10^5$, $\alpha = 1$;

$u_1 = 7.5 \cdot 10^{-6}$; $\varphi = 1.08$; $\theta = 0.2$; $p_m = 2320 \cdot 10^2$.

$$\sqrt{K_m} = \sqrt{\frac{F_2(\theta) f^2 \varphi m}{s^2 p_m}} = \sqrt{\frac{0.32 \cdot 9^2 \cdot 10^{10} \cdot 1.08 \cdot 0.0663}{0.4693^2 \cdot 2.32 \cdot 10^5}} =$$

$$= \sqrt{36.4 \cdot 10^4} = 603;$$

$$I_K = \frac{e_1}{u_1} = \frac{\sqrt{K_m}\,\omega}{\sqrt{W_0 - \alpha\omega}} = \frac{603 \cdot 0.930}{\sqrt{1.654 - 0.930}} = \frac{603 \cdot 0.930}{0.85} = 660 \text{ kg} \cdot \text{sec dm}^2;$$

$$2e_1 = 2u_1 I_K = 2 \cdot 7.5 \cdot 10^{-6} \cdot 660 = 0.0099 \text{ dm} = 0.99 \text{ mm} \approx 1 \text{ mm}(*).$$

We have obtained the thickness of strip or tubular powder of grade SP which was used in this gun, and developed a velocity v_A =

(*) This thickness of tubular powder corresponds to the grade 7/7, while the thickness of 1.28 mm corresponds to grade 9/7.

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= 588 m/sec when the charge ω was 0.930 kg.

The same gun may be fired with a charge ω = 1.08 kg and a velocity of v_0 = 620 m/sec can be obtained at the same p_m.

Find from equation (75) the thickness of the powder in this case.

Inasmuch as $\sqrt{K_m}$ remains the same as in the first case,

$$I_{K2} = \frac{603 \cdot 1.08}{\sqrt{1.654 - 1.08}} = \frac{603 \cdot 1.08}{0.757} = 860,$$

the thickness of tubular powder for the same u_1 will be

$$2e_1 = 2 \cdot 7.5 \cdot 10^{-6} \cdot 860 = 0.0129 \text{ dm} = 1.29 \text{ mm},$$

and this is the thickness of our previous powder C_{42}.

The ratio

$$\frac{I_{K2}}{I_{K1}} = \frac{860}{660} = 1.3 \approx \frac{9}{7}.$$

Calculations show that the equations derived from approximate relations yield results which are close to experimental data, and may be used to calculate the variation in the powder thickness concomitant with variations in the charge, if the maximum pressure is kept the same.

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3. THE POSITION OF MAXIMUM PRESSURE p_m IN THE BORE OF THE GUN, OR THE PATH l_m TRAVERSED BY THE PROJECTILE AT THE INSTANT OF MAXIMUM PRESSURE.

$$l_m = l_1 [F_1(\theta) - 1] = l_0(1 - \alpha\Delta)[F_1(\theta) - 1]. \tag{77}$$

For a given loading density in a gun employing a given type of powder, the maximum pressure p_m is developed at the same distance from the starting point of the projectile (l_m = const.), regardless of the powder thickness and the projectile weight. That is, the path traversed by the projectile up to the instant of maximum pressure does not depend on the powder thickness $2e_1$ nor on the weight q of the projectile.

In a given gun (W_0, s, l_0) using a given type of powder (f, α, u_1, θ) the position (l_m) of maximum pressure depends only on the density of the charge. The larger Δ, the smaller will be $(1 - \alpha\Delta)$ and the nearer to the starting point of motion will be p_m(*).

Equation (77) shows that under the condition of constancy of the maximum pressure p_m, when Δ is increased with the simultaneous increase of B and of the powder thickness $2e_1$, the maximum p_m shifts toward the origin of the projectile motion. Because the parameter B increases thereby, then, on the basis of formula (62):

(*) This conclusion is also confirmed by the GAU tables for the case of p_0 = 300 kg/cm2, $\varkappa$ = 1.06, and $\alpha \neq 1/\delta$.

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$$\psi_m = \frac{1}{B(1 + \theta)}$$

the portion of the charge burned at the instant of maximum pressure decreases.

4. END OF BURNING AND PATH l_K TRAVERSED BY PROJECTILE IN THE BORE OF THE GUN.

The location of the projectile at the end of burning is determined by equation (65), while the corresponding pressure p_K is found from equation (66).

From (65) we get:

$$l_K = l_0(1 - \alpha\Delta)\left[\frac{1}{\left(1 - \frac{B\theta}{2}\right)^{\frac{2}{\theta}}} - 1\right].$$

The equation shows that for a given loading density Δ, the quantity l_K increases with the increase of parameter B, i.e., mainly with the increase in powder thickness. When the weight of the projectile is diminished while Δ remains the same, the path traversed by the projectile at the end of burning is shifted toward the muzzle face.

Equation (66) shows that for a given loading density the values of p_K at the end of burning, when B is varied (i.e., when the powder thickness and the projectile weight are varied) lie on the curve p, y whose equation is:

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$$p_K = p_1 \frac{1}{(1 + y_K)^{1+\frac{\theta}{2}}} . \qquad (66)$$

This curve is of the same type as the adiabatic curve for instantaneous burning whose exponent is however $1 + \theta/2$. This curve is known as the pressure curve of completely burned powder.

The above statements are clarified by the graph of fig. 145.

The curves а, б, в and г depict the pressure variation during the burning of powders of different thicknesses when Δ is the same.

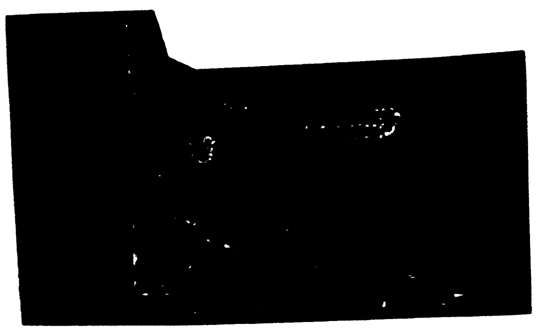

Fig. 145 - Pressure Curves for Different Powder Thicknesses.

а - thick powder; б - normal powder; в - thin powder; г - very thin powder.

2... Δ - curve p_K, y_K; 1... o - curve p, y for instantaneous burning.

Curve 1 – $\frac{p}{p_1} = \frac{1}{(1 + y)^{1,2}}$; Curve 2 – $\frac{p_K}{p_1} = \frac{1}{(1 + y_K)^{1,1}}$.

They are disposed in such a way that their maxima are on the same ordinate at a distance y_m from the origin. The end of burning occurs at a distance which is governed by the powder thickness, the distance being the greater the thicker the powder ($y_{Ka} > y_{Kб} > y_{Kв}$); the pressure values at the end of burning increase as the powder thickness decreases ($p_{Ka} < p_{Kб} < p_{Kв}$). The points corresponding to the end of burning lie on curve 2-2 calculated from equation (66) [when $\theta = 0.2$, $1 + \theta/2 = 1.1$].

Curve 1-1 corresponds to the adiabatic variation of the pressure at instantaneous powder burning ($1 + \theta = 1.2$).

The disposition of curves 1-1 and 2-2 shows that the pressure curves for gradual burning (a, б, в, г) intersect the curve 1-1 depicting instantaneous burning. The second period curves for the cases a, б, в, and г, which are not represented on the diagram, are all disposed below the curve 2-2 and above the curve 1-1.

At the same time, since for the given powder $p_1/(1 - B\theta/2) = \text{const.}$, the nature of the pressure change p in the second period depends upon the variation of the variable factor $1/(1 + y)^{1+\theta}$, the latter varying as in the case of instantaneous burning, i.e., along the adiabatic curve with initial pressure

$$p_1' = p_1 \frac{1}{\left(1 - \frac{B\theta}{2}\right)} > p_1.$$

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When the powder thickness is decreased, B and p_1' decrease also, and inasmuch as the adiabatic curves with the same exponent $1 + \theta$ do not cross, the adiabatics in the second period are disposed the lower, the thinner the powder, i.e., inversely to the disposition of the pressure curves in the first period.

If we compare the expressions for pressure in the second period and at instantaneous burning, keeping the value of y the same, we will get the following:

In the case of gradual burning in the second period

$$p'' = p_1 \frac{1}{\left(1 - \frac{B\theta}{2}\right)} \frac{1}{(1 + y)^{1+\theta}};$$

in the case of instantaneous burning

$$p' = p_1 \frac{1}{(1 + y)^{1+\theta}}$$

or

$$\frac{p''}{p'} = \frac{1}{1 - \frac{B\theta}{2}} = \text{const},$$

i.e., <u>when the loading density is the same, the ratio of the pressure in the second period to the pressure at instantaneous burning remains constant for any path length of the projectile</u> (greater than l_K). This

ratio decreases when B decreases and the projectile weight increases.

The graphs and equations presented above for the simplest case ($\varkappa = 1$, $\psi_0 = 0$, $\alpha = 1/\delta$) permit one to estimate directly the appearance and the form of the basic relations between the ballistic elements of a shot. They depict the location and magnitude of maximum pressure, its dependence on the loading conditions (Δ, B) the position of the projectile at the end of burning and the gas pressure developed thereby, the condition of maintaining the maximum pressure constant when the weight of the charge and the powder thickness are varied, and the independence of the curve y, l in the first period of the weight of the projectile.

Such simple relations are not obtained for the more complex cases ($\varkappa \neq 1$, $\psi_0 \neq 0$, $\alpha \neq 1/\delta$). In such a case it becomes necessary to analyze the effect of the individual elements by computing a series of variations or by using the data found in ballistic tables.

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CHAPTER 5 - A SURVEY OF CERTAIN OTHER METHODS OF SOLUTION

(Written by Prof. G.V. Oppokov)

1. A VARIATION OF PROF. G.V. OPPOKOV'S SOLUTION

In order to integrate equation (13), Chapter 1, (p. 473):

$$\frac{dl}{l_{\psi} + l} = \frac{Bxdx}{\psi_0 + k_1 x - B_1 x^2} \tag{78}$$

it is convenient to apply the usual method of classical mathematical analysis - the method of substitution, namely, of temporarily introducing into the process a new variable, ζ, so that:

$$l = \zeta + a \frac{B\theta}{2} x^2 - l_{\Delta}, \tag{79}$$

where a is the difference between the lengths of the free volumes of the chamber at the start and end of burning:

$$a = l_{\Delta} - l_1 = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right). \tag{80}$$

When this substitution of variables is effected in the new equation, it will become presently apparent that "the last term" does not contain in the denominator the difference:

$$\psi_0 + k_1 x - B_1 x^2.$$

This obviates the need for mathematical transformations in the course of integration for the purpose of replacing the obtained

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This obviates the need for mathematical transformations in the course of integration for the purpose of replacing the obtained

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integral by other, more simple ones.

Indeed, it follows from equation (79) that:

$$\frac{dl}{dx} = \frac{d\zeta}{dx} + B\theta ax,$$

because l_Δ is a constant in every concrete case. Moreover:

$$l_\psi + l = l_\Delta - a\psi + l = l_\Delta - a(\psi_0 + k_1 x + \varkappa\lambda x^2) + l.$$

Substituting in the above the value of l from (79):

$$l_\psi + l = \zeta + a\frac{B\theta}{2}x^2 - a(\psi_0 + k_1 x + \varkappa\lambda x^2)$$

or

$$l_\psi + l = \zeta - a(\psi_0 + k_1 x - B_1 x^2).$$

We shall substitute into the differential equation (78) the obtained values of dl/dx and $(l_\psi + l)$:

$$\frac{d\zeta}{dx} + B\theta ax = \frac{Bx}{\psi_0 + k_1 x - B_1 x^2}\left[\zeta - a(\psi_0 + k_1 x - B_1 x^2)\right].$$

We now remove the brackets and effect the necessary simplifications:

$$\frac{d\zeta}{dx} + B\theta ax = \frac{Bx}{\psi_0 + k_1 x - B_1 x^2}\zeta - Bax.$$

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Grouping the terms:

$$\frac{d\zeta}{dx} - \frac{Bx}{\Psi_0 + k_1 x - B_1 x^2}\zeta = -Ba(1 + \theta)x.$$

The common integral of this linear differential equation of the first order including the last term can be represented in the following form:

$$\zeta = e^{\int \frac{Bxdx}{\Psi_0 + k_1 x - B_1 x^2}} \left[C_1 - Ba(1 + \theta) \int e^{-\int \frac{Bxdx}{\Psi_0 + k_1 x - B_1 x^2}} xdx \right], \qquad (81)$$

where e is the base of natural logarithms, and C_1 is still an arbitrary constant.

We must introduce into the analysis Prof. Drozdov's function:

$$e^{\int \frac{Bxdx}{\Psi_0 + k_1 x - B_1 x^2}} = Z^{-\frac{B}{B_1}}.$$

Then the partial integral of the last equation with respect to the derivative $d\zeta/dx$ will be:

$$\zeta = Z^{-\frac{B}{B_1}} \left[C_1 - Ba(1 + \theta) \int_0^x Z^{\frac{B}{B_1}} xdx \right].$$

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We shall now return to the desired path l of the projectile, and after substituting the obtained value of ζ, get:

$$l = Z^{-\frac{B}{B_1}} \left[C_1 - Ba(1 + \theta) \int Z^{\frac{B}{B_1}} x\,dx \right] + a\,\frac{B\theta}{2}\,x^2 - l_\Delta .$$

Let us determine now the constant C_1 from the initial conditions, at which:

$$l = 0; \quad x = 0; \quad Z^{-\frac{B}{B_1}} = 1; \quad \int_0^x Zx\,dx = 0 .$$

We have from the latter equation for the path of the projectile:

$$0 = 1(C_1 + 0) + 0 - l_\Delta ,$$

whence

$$C_1 = l_\Delta .$$

Thus the desired path of the projectile is defined by the following expression:

$$l = Z^{-\frac{B}{B_1}} \left[l_\Delta - aB(1 + \theta) \int_0^x Z^{\frac{B}{B_1}} x\,dx \right] + a\,\frac{B\theta}{2}x^2 - l_\Delta . \qquad (82)$$

2. PARTICULARS OF PROF. I.P. GRAVE'S SOLUTION

This method of solution was developed by Prof. I.P. Grave in order to perfect Bianchi's method, which was the first variant (in time) of the $l_{\psi_{av.}}$ method. In Bianchi's original equations the effect of the variation of $l_{\psi_{av.}}$ is discounted and in integrating he considers the quantity l_ψ as a certain incompletely determined constant. Bianchi divides the curve p, l into three segments, for which instead of l_ψ he takes l_Δ, $l_\Delta - \frac{1}{2}a$ and l_1, respectively, which corresponds to the following conditions:

$$\psi_{av.} = 0; \quad \psi_{av.} = 0.5; \quad \psi_{av.} = 1.$$

But in this case the curves p, l and v, l are not smooth; they have angular points corresponding to the beginning of the second and third segments.

In order to take into account the effect of the variation of l_ψ and obtain smooth p, l and v, l curves, Prof. I.P. Grave, in integrating the equation for the path of the projectile, considers l_ψ as a variable, defined by the average value of its derivative with respect to l. This average value of the derivative must be negative, because l_ψ decreases during the burning of the powder. Consequently:

$$\frac{dl_\psi}{dl} = -k,$$

whence, after integration, we obtain:

$$l_\psi = l_{\psi_0} - kl. \tag{83}$$

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At the end of burning (when $\psi = 1$), we obtain from the above

$$l_1 = l_{\psi_0} - kl_K,$$

from which the constant k is determined:

$$k = \frac{l_{\psi_0} - l_1}{l_K} = \frac{l_{\psi_0} - l_1}{l_1} : \frac{l_K}{l_1}. \qquad (84)$$

If we substitute the value of l_ψ obtained from equation (83) into the differential equation (78) for the projectile path, we will get:

$$\frac{dl}{dx} = \frac{Bx(l_{\psi_0} - kl + l)}{\psi - \frac{B\theta}{2}x^2},$$

whence, after separating the variables, we have:

$$\frac{dl}{l_{\psi_0} + (1 - k)l} = \frac{Bxdx}{\psi - \frac{B\theta}{2}x^2}.$$

Integrating this equation:

$$\frac{1}{1 - k} \ln \frac{l_{\psi_0} + (1 - k)l}{l_{\psi_0}} = \int_0^x \frac{Bxdx}{\psi - \frac{B\theta}{2}x^2}$$

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and, consequently, the following must obtain:

$$(1 - k)\, l = l_{\psi_0} Z^{-(1-k)\frac{B}{B_1}} - l_{\psi_0}.$$

Using a second time equation (83), we obtain the following equation from the above:

$$l = l_{\psi_0} Z^{-(1-k)\frac{B}{B_1}} - l_{\psi}. \qquad (85)$$

A certain difficulty arises from the fact that in order to apply equation (85) it is necessary to know the constant k, for which, in turn, it is necessary to know l_K l_1 [see equation (84)]; but the path l_K of the projectile at the end of the period is unknown beforehand.

In order to overcome this difficulty, a nomograph is given [see I.P. Grave, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics), Pyrodynamics, No. 1, p. 58] which enables one to determine the ratio l_K / l_1 if

$$\frac{l_{\psi_0}}{l_1} \text{ and } \frac{\Phi(x_K)}{H\varkappa\lambda} = \frac{B}{B_1} \ln Z_K^{-1} = 2.303\, \frac{B}{B_1} \log Z_K^{-1}.$$

are known.

Having found from the conditions of loading:

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CHAPTER VI - SOLUTION OF THE FUNDAMENTAL PROBLEM OF INTERNAL BALLISTICS ON THE BASIS OF THE PHYSICAL LAW OF BURNING

(M.Ye. Serebriakov's Method)

As was shown in Part I of this text, the actual burning of powders deviates from the geometric law under the influence of a number of factors. An analysis obtained by the aid of the progressivity curves Γ, ψ and Γ, t has shown that certain anomalies actually occur even during the burning of powders of simple shapes: non-instantaneous ignition, accelerated burning of the outside layers (ballooning), etc. The burning law cannot be established at all on the basis of the geometric law for adulterated and porous powders used in pistol cartridges.

The actual burning law can be established only by burning powder in a test bomb at different loading densities and by obtaining pressure-time curves reflecting all the deviations and peculiarities of a given sample.

The variation in the intensity of gas formation Γ and $I = \int_0^t p\,dt$ as a function of ψ and t can be established from the obtained p, t curve.

Both graphs Γ, ψ and $\int_0^t p\,dt$, ψ in conjunction with the fundamental p, t curve obtained from the bomb test enable us to solve the fundamental problem of pyrodynamics, i.e., to compute the gas-pressure and projectile velocity variation curves under conditions of actual burning of powder in the bore of a gun.

These graphs also enable us to establish the individual behavior of powder lots of different grades occasionally differing considerably

$$\frac{l_{\Psi 0}}{l_1} \text{ and } 2.303\,\frac{B}{B_1} \log Z_K^{-1},$$

we can determine l_K/l_1 from this graph; this will enable us to calculate k from equation (84).

If it is necessary to find k more accurately, the obtained value of k may be rendered more exact by successive approximations. The value k_1 found from the graph is substituted into (85), $l_{K_1 2}/l_1$ is found in the second approximation, and a new value k_2 is then determined from (84) representing a second approximation, and so on, until two consecutive values of l_K/l_1 coinciding with the required degree of accuracy are obtained.

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as to their properties, which behavior could not be disclosed by any method other than by bomb tests.

There have been actual cases where powder lots of the same grade and the same manufacturer having identical chemical composition and dimensions produced a difference of 6-8% in the charging weights when fired at the same values of v_A and p_{max}.

Bomb tests had shown that the burning rate u_1 of these powders varied as much as 15-20%. This variation could not have been disclosed by any other means except the bomb test.

The fundamental problem of internal ballistics for adulterated or porous powders can be solved in exactly the same manner only on the basis of the experimental (physical) law of burning.

We are presenting below the method of solving the fundamental problem on the basis of the physical law of burning, when the burning rate law is $u = u_1 p$, which corresponds to the coincidence of the curves I, ψ or $\int pdt, \psi$ at various loading densities.

The basic assumption made here is that both in a bomb at different loading densities Δ and in a weapon with a variable space in the case of a continuously decreasing loading density, the value of $\int pdt$ is a single-valued function of ψ only, and does not depend on the loading density. This condition, which has been proved by bomb tests at different loading densities, is being extrapolated in the given case for considerably higher values of Δ in a weapon.

1. DERIVATION OF BASIC RELATIONSHIPS AS APPLIED TO THE PHYSICAL LAW OF BURNING.

The solution is based on applying the pressure curve obtained from bomb tests to the computation of curves depicting the gas pressure and velocity of the projectile in a weapon.

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As the projectile moves through the bore and the initial air space becomes larger, the pressure will depend on the current value of the loading density $\Delta = \omega/(W_0 + sl)$.

We shall introduce the following designations:

Δ_1 - loading density of powder in test bomb;

Δ_0 - initial loading density in weapon.

P and τ - gas pressure and time corresponding to the given value of ψ at constant loading density Δ_1, at which the bomb test was conducted and at which the curve P,τ was obtained;

p and t - gas pressure and time corresponding to the same value of ψ when Δ is variable, which condition applies to a given disposition of the projectile in the bore of the barrel.

We shall designate the corresponding integral values as follows:

a) In bomb

$$\int_0^{\psi} Pd\tau = I$$

$$\int_0^{\psi_0} Pd\tau = I_0$$

$$\int_0^{1} Pd\tau = I_K$$

b) In weapon

$$\int_0^{\psi} pdt = i$$

$$\int_0^{\psi_0} pdt = i_0$$

$$\int_0^{1} pdt = i_K.$$

I is obtained from a table or graph as a function of ψ or τ, on the basis of bomb tests.

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Inasmuch as the pressure impulse does not depend on Δ, we will have the equalities

$$\int_0^{\psi} p\,dt = \int_0^{\psi} P\,d\tau \quad \text{or } i = I \tag{86}$$

and, correspondingly,

$$I_0 = i_0; \quad I_K = i_K.$$

Differentiating (86), we get:

$$p\,dt = P\,d\tau. \tag{87}$$

Here $d\tau$ - an elementary time lapse during which the portion of change ψ burned up to a given instant under pressure P will receive the increment $d\psi$ when the powder is burned in a constant volume at a loading density Δ_1;

dt - time lapse during which the same portion ψ of the burned powder will receive the same increment $d\psi$ when the powder is burned in the gun barrel at pressure p at loading density Δ determined by the current disposition of the projectile in the bore of the barrel:

$$\Delta = \frac{\omega}{W_0 + sl}.$$

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We shall consider henceforth the value of ψ as the independent variable.

2. DETERMINING THE PROJECTILE VELOCITY AS THE FUNCTION OF ψ

On the basis of the impulse theorem

$$\varphi m dv = spdt.$$

Integrating from the start of motion:

$$\varphi m v = s \int_{\psi_0}^{\psi} pdt = s(i - i_0),$$

where ψ_0 is the portion of the charge burned in the gun at the start of motion:

$$\psi_0 = \frac{\frac{1}{\Delta_0} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}.$$

Determining v:

$$v = \frac{s}{\varphi m}(i - i_0) \tag{88}$$

or on the basis (86)

$$v = \frac{s}{\varphi m} \int_{\psi_0}^{\psi} Pd\tau = \frac{s}{\varphi m}(I - I_0). \tag{89}$$

The values of I and I_0 for ψ and ψ_0 are known from the bomb test, $s/\varphi m$ is known from the gun data. Thus the velocity of the projectile is determined from equation (89) as the function of ψ.

At the end of burning when $\psi = 1$

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$$v_K = \frac{s}{\varphi m}(I_K - I_0). \qquad (90)$$

In contrast to the analogous formula in the case of the geometric law of burning, the value of I_K corresponds not to the average thickness of the powder but to the maximum thickness, which may considerably exceed the average thickness of powders having a variable thickness. A diagram of the pressure impulse of tabular powder usually obtained in bomb tests is offered in fig. 146.

The value of the impulse $I_1 = e_1$ av. u_1 corresponds to the burning of powder of average thickness; the value of $I_K > I_1$ corresponds to the burning of the thickest element of the charge.

Fig. 146 - Pressure Impulse of Tubular Powder

Therefore also v_K in the case of actual powder burning assumes a considerably greater value than in the case of the geometric law, but in that case the projectile will also traverse a considerably longer path at the end of powder burning, so that:

$$\underset{\text{(Physical law of burning)}}{l_K} > \underset{\text{(Geometric law of burning)}}{l_K}$$

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3. DETERMINING THE PATH OF THE PROJECTILE AS A FUNCTION OF ψ

In the method outlined the relation between l and ψ is established by means of the auxiliary function L, ψ determined from the same bomb test at a loading density Δ_1, by the additional analysis of the test curve P, τ. And if the value of the pressure impulse I, ψ does not depend on the choice of Δ_1, then the function L, ψ depends on the value of Δ_1 chosen at the test.

As we shall see later, the function L has the dimensionality of the path and a definite physical meaning.

We will have from equation (88):

$$dl = vdt = \frac{s}{\varphi m}(i - i_0)dt,$$

where $(i - i_0)$ is a function of ψ.

Upon integrating, the path of the projectile will be determined by the formula

$$l = \frac{s}{\varphi m}\int_{\psi_0}^{\psi}(i - i_0)dt. \tag{91}$$

Here the element dt corresponds to the element $d\psi$ when the powder is burned under conditions of variable volume (space) and depends on the value of pressure p at any given instant which is still unknown.

We can obtain an expression from the bomb test analogous to expression (91):

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$$L_{\psi_0}^{\psi} = \frac{s}{\varphi m}\int_{\psi_0}^{\psi}(I - I_0)d\tau = \frac{s}{\varphi m}\iint_{\psi_0}^{\psi} Pd\tau d\tau = \frac{s}{\varphi m}\cdot G. \qquad (92)$$

The value of L - function of ψ - is obtained by the second integration of curve $\int_{\psi_0}^{\psi} Pd\tau$ with respect to τ and by multiplying same by the coefficient $s\ \varphi m$.

If l is the path traversed by the projectile at the instant the portion ψ of the charge is burned, then L is the path the projectile would have traversed if the pressure behind it developed according to the same law as in a bomb with a constant loading density Δ_1, at the instant the same portion of the charge ψ is burned.

L has a definite physical meaning. For example, it is obtained in practice in a bomb of considerable capacity with a free piston. Thus in a bomb of capacity $W_0 = 300$ cm^3 the piston displacement (with the piston usually having a cross-sectional area of $s = 1$ cm^2) is about 3 cm. The change in volume amounts to only 1%, and hence the piston is displaced by a pressure which increases in almost a constant volume.

The value of L as a function ψ is found from the bomb test using the procedure given in the table below.

It is necessary to establish the relation between l and L, and hence between l and ψ, because L is function of ψ.

Differentiating equations (91) and (92) and taking their ratio and reducing, we get:

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$$dl = DL\frac{dt}{d\tau}. \tag{93}$$

From equation (87)

$$\frac{dt}{d\tau} = \frac{P}{p}, \tag{94}$$

whereas the ratio P/p is replaced by the ratio of the free volumes on the basis of the equation of state:

$$P = \frac{RT_1\omega_1\psi}{(W_\psi)} = \frac{f\Delta_1\psi}{1 - \frac{\Delta_1}{\delta} - \Delta_1\left(\alpha - \frac{1}{\delta}\right)\psi} = \frac{f\psi}{\frac{1}{\Delta_1} - \frac{1}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\psi}.$$

The expression in the denominator represents the free specific gas volume at loading density Δ_1.

An analogous expression will obtain also in the formula for p. We shall replace them with average values, because the last term is small in comparison with the first two, and, moreover, the ratios of the free volumes will enter them all.

$$W_{\psi_{av.}} = W_0\left[1 - \frac{\Delta_1}{\delta} - \Delta_1\left(\alpha - \frac{1}{\delta}\right)\psi_{av.}\right] = W_0\left(1 - \frac{\alpha + \frac{1}{\delta}}{2}\Delta_1\right) =$$

$$= W_0(1 - \alpha'\Delta_1),$$

where $\alpha' = \dfrac{\alpha + \frac{1}{\delta}}{2}$.

We shall introduce the designations:

$$\frac{1}{\delta} + \left(\alpha - \frac{1}{\delta}\right) \cdot \frac{1}{2} = \frac{1}{2}\left(\frac{1}{\delta} + \alpha\right) = \alpha';$$

$$\frac{1}{\Delta_1} - \alpha' = a_1; \quad \frac{1}{\Delta_0} - \alpha' = a_0.$$

Then

$$P = \frac{f\psi}{a_1};$$

$$p = \frac{RT\omega\psi}{W_{\psi_{av.}} + sl} = \frac{f\Delta_0\psi}{1 - \alpha'\Delta_0 + \frac{l}{l_0}} \frac{T}{T_1} = \frac{f\psi}{a_0 + \frac{l}{l_0\Delta_0}} \frac{T}{T_1} =$$

$$= \frac{f\psi}{a_0\left(1 + \frac{l}{l_0 a_0 \Delta_0}\right)} \frac{T}{T_1},$$

but

$$l_0 a_0 \Delta_0 = l_0(1 - \alpha'\Delta_0) = l_{\psi_{av.}} = l_c.$$

For a gradually burned powder

$$\frac{T}{T_1} = \left(\frac{W_0 - \alpha'\omega}{W_0 - \alpha'\omega + sl}\right)^{\frac{\theta}{2}} = \left(\frac{a_0}{a_0 + \frac{l}{l_0\Delta_0}}\right)^{\frac{\theta}{2}} =$$

$$= \frac{1}{\left(1 + \frac{l}{l_c}\right)^{\frac{\theta}{2}}}.$$

The ratio P/p, following substitution of the proper expressions and simplification, will take on the form:

$$\frac{P}{p} = \frac{a_0}{a_1}\left(1 + \frac{l}{l_c}\right)^{1 + \frac{\theta}{2}}. \tag{95}$$

Upon incorporating this expression in (94) and then in (93), we get:

$$dl = dL\,\frac{a_0}{a_1}\left(1 + \frac{l}{l_c}\right)^{k'}, \quad \text{where } k' = 1 + \frac{\theta}{2}.$$

Dividing the variables and integrating:

$$\int_0^l \frac{dl}{\left(1 + \frac{l}{l_c}\right)^{k'}} = \frac{a_0}{a_1}\int_{\psi_0}^{\psi} dL.$$

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Designating $1 + l/l_c = x$, we get in the left side

$$\int_0^l \frac{dl}{\left(1 + \frac{l}{l_c}\right)^{k'}} = \frac{2}{\theta} l_c \left(1 - \frac{1}{x^{\frac{\theta}{2}}}\right);$$

and in the right side

$$\frac{a_0}{a_1} L^*_0.$$

We then obtain:

$$1 - \frac{1}{x^{\frac{\theta}{2}}} = \frac{\theta}{2} \frac{a_0}{a_1 l_c} L^*_0 = B' L^*_0. \qquad (96)$$

where

$$B' = \frac{\theta}{2} \frac{a_0}{a_1 l_c} = \frac{\theta}{2} \frac{\frac{1}{\Delta_0} - \alpha'}{\frac{1}{\Delta_1} - \alpha'} \frac{1}{l_0(1 - \alpha'\Delta_0)} = \frac{\theta}{2} \frac{1}{\Delta_0 l_0} \frac{1}{\frac{1}{\Delta_1} - \alpha'},$$

but

$$\Delta_0 l_0 = \frac{\omega}{s},$$

and

$$B' = \frac{\theta}{2}\frac{S}{\omega}\frac{1}{\frac{1}{\Delta_1} - \alpha'}. \tag{97}$$

Solving (96) for l, we get:

$$l = l_c\left[\frac{1}{(1 - B'L^{\Psi}_{\Psi_0})^{\frac{2}{\theta}}} - 1\right]. \tag{98}$$

As in the solution of the problem of internal ballistics by the average l_Ψ method, the value l_c - average for the entire burning process - can be replaced in this formula by the current value $l_{\Psi av.}$ by means of the usual formula:

$$l_{\Psi av.} = l_0\left[1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\frac{\Psi_0 + \Psi}{2}\right], \tag{99}$$

and then

$$l = l_{\Psi av.}\left[\frac{1}{(1 - B'L^{\Psi}_{\Psi_0})^{\frac{2}{\theta}}} - 1\right] \tag{100}$$

Formula (100) gives the path l as a function of ψ by means of the auxiliary function $L^{\psi}_{\psi_0}$, determined in bomb tests at loading density $\Delta = \Delta_1$, which function reflects (depicts) the true burning law.

Comparing this formula with the analogous formula used in the method fo solution in which $l_\psi = l_{\psi\,av.}$, we will note that in place of Prof. Drozdov's function $Z_x^{-B\,B_1}$ formula (100) contains the function $(1 - B'L^{\psi}_{\psi_0})^{-2\theta}$. For the case where $\varkappa = 1$, $\lambda = 0$,

$$\frac{B}{B_1} = \frac{2}{\theta}.$$

Therefore the expression in parentheses $(1 - B'L^{\psi}_{\psi_0})$ has replaced in this solution the function Z_x in the case of the geometric law of burning.

Formulas (89) and (100) enable us to calculate and plot the projectile velocity curve as a function of path l.

Pressure p is found from the fundamental equation of pyrodynamics:

$$p = \frac{f\omega\psi - \frac{\theta}{2}\varphi m v^2}{s(l_\psi + l)}, \qquad (101)$$

wherein the variable quantities as the ψ functions are already known.

To determine the maximum pressure p_m and the corresponding value ψ_m, we differentiate the equation (101) with respect to t:

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$$\frac{dp}{dt} = \frac{p}{l_\psi + l}\left\{\frac{f\omega}{s}\Gamma\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p}{f}\right] - (1 + \theta)v\right\},$$

whereby $v = S\,\varphi m(I - I_0)$ (89) and Γ are given in the table as a function of ψ.

Equating the expression in braces to zero and replacing v by its expression in (89), we get:

$$\frac{f\omega}{s}\Gamma_m\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] - (1 + \theta)\frac{s}{\varphi m}(I_m - I_0) = 0,$$

whence

$$I_m - I_0 = \frac{f\omega\varphi m}{s^2(1 + \theta)}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right]\cdot\Gamma_m. \qquad (102)$$

Denoting the factor of Γ_m by D, we get:

$$I_m - I_0 = D\cdot\Gamma_m.$$

The value of ψ_m is found as the point of intersection of curves $I - I_0$ and $D\cdot\Gamma$ as a function of ψ.

The point of intersection gives the values of $(I_m - I_0)$, ψ_m and Γ_m.

The diagram in fig. 147 clarifies the above.

It is not difficult to see that if $I_m - I_0$ and Γ_m are replaced by theoretical expressions in terms of z and x on the basis of

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the geometric law, we will obtain the usual relationship for x_m.

At the end of burning when $\psi = 1$, we will have:

$$v_K = \frac{s}{\varphi m} (I_K - I_0);$$

$$l_K = l_c \left[\frac{1}{(1 - B'L'\psi_0)^{\frac{2}{\theta}}} - 1 \right];$$

$$p_K = \frac{f\omega}{s} \frac{1 - \frac{v_K^2}{v_{np}^2}}{l_1 + l_K}.$$

The usual formulas apply to the second period:

$$p = p_K \left(\frac{l_1 + l_K}{l_1 + l} \right)^{1+\theta}; \tag{103}$$

$$v = v_{np} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l} \right)^{\theta} \left(1 - \frac{v_K^2}{v_{np}^2} \right)}. \tag{104}$$

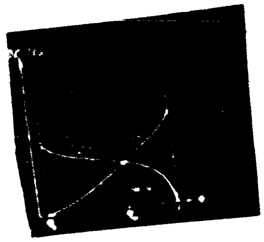

Fig. 147 - Determining Ψ_m for Maximum Pressure.

4. GRAPHICAL CLARIFICATION OF THE METHOD OF SOLUTION

In order to solve the problem on the basis of the physical law of burning, it is first necessary to perform the ballistic analysis of the given powder. To do so, bomb tests are conducted at two loading densities Δ_1 and Δ_2, the ballistic characteristics - propellent force of powder f and covolume α - are determined, and also the test characteristic of the intensity of gas formation Γ, ψ and the impulse of pressure increase $\int_0^{\psi} P d\tau = I$ (fig. 148).

Knowing the loading conditions of the weapon, we determine

$$\psi_0 = \frac{\frac{1}{\Delta_0} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}};$$

from graph I, ψ we find the corresponding value of $I_0 = \int_0^{\psi_0} P d\tau$, and upon subtracting this value of I_0 from all the values of I, obtain the dependence of $I - I_0$ on ψ and τ (fig. 149). Integrating numerically the curve $I - I_0, \tau$ with respect to τ, we find the integral $\int_{\psi_0}^{\psi} (I - I_0) d\tau$ as a function of ψ.

Introducing the designation:

$$G_{\psi_0}^{\psi} = \int_{\psi_0}^{\psi} \int P d\tau d\tau = \int_{\psi_0}^{\psi} (I - I_0) d\tau .$$

Multiplying $I - I_0$ and $G_{\psi_0}^{\psi}$ by s φm, we get

$$v = \frac{s}{\varphi m} (I - I_0) ; \tag{89}$$

$$L_{\psi_0}^{\psi} = \frac{s}{\varphi m} G_{\psi_0}^{\psi} = \frac{s}{\varphi m} \int_{\psi_0}^{\psi} (I - I_0) d\tau . \tag{92}$$

v does not depend on Δ (Δ_1 or Δ_2) and is a function of ψ only for all the loading densities. The function $L_{\psi_0}^{\psi}$, being a function of ψ, depends at the same time on Δ, because the time element $d\tau$ during which a definite portion of charge $d\psi$ is burned decreases with the increase of Δ.

Indeed, from the equality

$$\Gamma = \frac{d\psi}{P d\tau}$$

it follows that

$$d\tau = \frac{d\psi}{\Gamma P},$$

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where

$$P = \frac{f\Delta_1\psi}{1 - \frac{\Delta_1}{\delta} - \Delta_1\left(\alpha - \frac{1}{\delta}\right)\psi} \approx \frac{f\Delta_1\psi}{1 - \alpha'\Delta}$$

and hence $d\tau$ varies inversely with the change of Δ_1.

For this reason the curves $G^*_{\psi_0}$ and $L^*_{\psi_0}$ as a function of ψ will also be disposed the lower the greater Δ_1 when testing the powder in a bomb (see fig. 149).

Fig. 148 - Basic Curves Γ, ψ and I, ψ.

Fig. 149 - Auxiliary Curves for Determining the Elements of a Shot.

The arrangement of a table for analyzing bomb tests as a means of obtaining all the auxiliary functions is presented below; this table serves to clarify the graphs in figs. 148 and 149.

Procedure for Analyzing a Bomb Test at Loading Density Δ_1 as a Means of Obtaining the Basic Functions

$$G = \int_{\psi_0}^{\psi} (I - I_0)\,d\tau = \int\int_{\psi_0}^{\psi} P\,d\tau\,d\tau \quad \text{and} \quad L_{\psi_0}^{\psi} = \frac{s}{\varphi m} G_{\psi_0}^{\psi}.$$

ψ_0 - from the preliminary period;

I_0 - according to curve I, ψ.

τ	P	ψ	$I = \int P d\tau$	$I - I_0$	$I_{av.} - I_0$	$(I - I_0)_{av.}\Delta\tau = \Delta G$	$G_{\psi_0}^{\psi} = \Sigma(I - I_0)\Delta\tau$	$L = \frac{s}{\varphi m} G_{\psi_0}^{\psi}$
0 τ_B	5-7 P_B	– 0	– 0	– –	– –	– –	– – 0	– – 0
τ^I	P^I	$\psi^I = \psi_0$	$I^I = I_0$	0	$I_{av.}^{II} - I_0$	$(I_{av.}^{II} - I_0)\Delta\tau$	G^{II}	L^{II}
τ^{II}	P^{II}	ψ^{II}	I^{II}	$I^{II} - I_0$	$I_{av.}^{III} - I_0$	$(I_{av.}^{III} - I_0)\Delta\tau$	G^{III}	L^{III}
τ^{II}	P^{III}	ψ^{III}	I^{III}	$I^{III} - I_0$	⋮	⋮	G^{IV}	L^{IV}
⋮	⋮	⋮	⋮	⋮	–	–	⋮ $G_K = G'_{\psi_0}$	⋮ $L_K = L'_{\psi_0}$
τ_K	P_m	1	I_K	$I_K - I_0$				

We shall construct, according to equations (89) and (92) for the same values of ψ, a curve showing the dependence of v on L when $\Delta = \Delta_1$ [fig. 150, curve v, $L(\Delta_1)$]. If the test were analyzed for $\Delta_2 > \Delta_1$, the relationship v, $L(\Delta_2)$ would obtain, which relationship curve has a larger slope angle and in which the shorter path of the projectile: $L_K(\Delta_2) < L_K(\Delta_1)$ corresponds to the end of powder burning. Both curves have a smaller curvature than the true v,l curve of the velocities of the projectile in the bore.

If we were to extrapolate the function $L_{\psi_0}^{\psi}$ for the initial loading density Δ_0 in the gun, we would have obtained curve v, $L(\Delta_0)$ (150), which at the start of motion has a common point of tangency with the true v,l curve. The latter is obtained when Δ decreases continuously, and as ψ and v increase curve v,l (heavy dotted line) gradually goes over from curve v, $L(\Delta_0)$ to curves corresponding to ever smaller Δ, which family of curves includes also curves v, $L(\Delta_2)$ and v, $L(\Delta_1)$.

This transition is the one given by the fundamental formula (100):

$$l = l_{\psi_{av.}}\left[(1 - B'L_{\psi_0}^{\psi})^{-\frac{2}{\theta}} - 1\right]$$

together with formula (89):

$$v = \frac{s}{\varphi m}(I - I_0).$$

The difference between the method of solving the fundamental problem outlined above and other such methods lies in the fact that

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in deriving the dependence of l on ψ, use is made not of the fundamental equation of pyrodynamics, but, rather, of the equation of the state of powder gases for different positions of the projectile in the bore of the barrel.

In solving this problem use is made of the gas pressure curve p, τ obtained from bomb tests, which expresses the true burning law with all the deviations from the geometric law.

An analogous result can be obtained only by the numerical integration of Taylor's series or from finite differences.

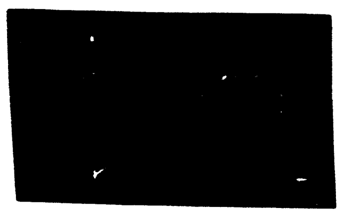

Fig. 150 - Relation Between Auxiliary Curves v, L and Actual Curve v, l.

The method outlined here permits the solution of the problem also in the case of the geometric law, by assuming the following theoretical relationship for Γ, ψ:

$$\Gamma = \frac{\varkappa\sigma}{I_K} = \frac{\varkappa}{I_K}\sqrt{1 + 4\frac{\lambda}{\varkappa}\psi} = \frac{1}{I_K}\sqrt{\varkappa^2 + 4\varkappa\lambda\psi}.$$

Therefore, this method is a more general one than the methods based on the geometric law of burning.(*)

(*) For a more detailed explanation see: Serebriakov, M.Ye. "FIZICHESKY ZAKON GORENIA VO VNUTRENNEY BALLISTIKE" (The Physical Law of Burning in Internal Ballistics). "OBORONGIZ" (State Publishers of Defense Literature), 1940.

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5. ANALYSIS OF THE OBTAINED CURVES p, l AND v, l.

Analysis of curves p, l and v, l obtained on the basis of the physical law of burning indicates that:

Fig. 151 - Curves p, l and v, l Obtained on the Basis of the Physical and Geometric Laws of Burning.

ф. з. г. - physical law of burning.

г. з. г. - geometric law of burning.

1) Due to ballooning - accelerated burning of outer layers - the maximum pressure is attained earlier, and the pressure curve is disposed higher at the start than in the case of the geometric law;

2) The beginning and the first half of the velocity curve v, l obtained on the basis of the physical law, are disposed above the corresponding v, l curve in the case of the geometric law of burning; the curves merge at the end;

3) The same value of $p_{max.}$ is obtained at a smaller propellant force of powder than in the case of the geometric law;

4) Due to after-burning of the thicker elements of the charge,

the end of burning is transposed nearer the muzzle face, and the velocity v_K exceeds the theoretical value for the average powder thickness;

5) Due to the gradual decrease of the intensity of gas formation Γ, ψ at the end of burning, the transition of the pressure curve from the first period to the second proceeds without a jump and forms no turning points on the curve, as it does in the case of the geometric law.

The graph in fig. 151 clarifies the above.

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SECTION SEVEN - NUMERICAL METHODS OF SOLUTION

USE OF NUMERICAL ANALYSIS IN INTERNAL BALLISTICS

Various variable quantities possessing definite physical significance usually take part in processes which occur in nature or are considered in technology. In this connection, numerical variations of one or more quantities are accompanied by or associated with variations of other variables. Thus there always exists a definite functional relation between the variable quantities under consideration. This functional relation may be expressed by means of three methods, such as tabulations, diagrams, and formulas. In the vast majority of processes, especially those encountered in technology, this relation is expressed by the aid of tables or diagrams obtained directly from experiment or from observation of the process, whereas the formulas appear only after subsequent analysis of the results obtained, and then only in the case of the simplest processes. It is thus apparent that the most natural means of expressing a functional relation between variable quantities representing in their totality the process under investigation is a tabulation; this is especially true when such a process is being utilized directly for technological purposes, in which case a formula or even a diagram will not serve the purpose, and only numerical values of the variable quantity considered to be of primary importance on the basis of practical considerations must be had.

Numerical analysis must thus serve as a means for studying and making practical utilization of the functional relations between the

variable quantities involved.

Ordinary differential equations may be approximately integrated by means of any of the known methods of which there are a great many. These methods include the following:

1) Expansion in a Taylor's series in powers of the argument.

2) Integration by the method of successive approximations.

3) Expansion in a series in powers of small parameters entering into the equation.

4) Expansion in a series in powers of the initial values of the unknown function and its derivatives.

5) Method of successive approximations applied to equations for vibrational motion.

6) Methods of Euler, E.L. Bravin, and others.

7) Method of numerical integration.

The first six methods do not require the use of finite differences; all variants of the method of numerical integration are based on the use of such differences.

The principal variants of the method of numerical integration are discussed in the book by Academician A.N. Krylov, "Lectures on Approximate Computations". [4]

Fundamental information on the theory of finite differences and on the technical features of their application to the engineering of artillery may be found in the book by Professor G.V. Oppokov, "Numerical Analysis Applied to the Science of Artillery". [5]

Internal ballistics is an applied science possessing a perfectly definite technical content (the study of the motion of a projectile in the bore of a gun and of the laws of burning of powder)

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and a perfectly well-defined technical objective: the creation of means for plotting the curve of the speed of the projectile in the bore and for plotting the curve of the pressure of the powder gases in the bore as functions of the path of the projectile and of time.

These curves can be plotted after obtaining suitable tabulations giving the functional relation between the various variable quantities participating in the phenomenon of a shot. The necessary tabulations are obtained by analyzing primary experimental data and those formulas which, in their simplest form, express the relation existing between the initial variable quantities.

The use of numerical analysis constitutes the subject treated in this section of the book. The essence of numerical analysis, its specific features and its principal operations will also be considered here in the proper degree.

CHAPTER 1 - NUMERICAL INTEGRATION BY FINITE DIFFERENCES

(Written by Professor G.V. Oppokov)

1. APPLICATION OF NUMERICAL INTEGRATION TO THE DETERMINATION OF FUNCTIONS

1) Concept of Tabular Functions

That variable function whose numerical values can be chosen arbitrarily is usually designated as the argument or as the independent variable quantity. The remaining variable quantities taking part in the process under consideration are then designated as functions.

Let us assume that a certain independent variable quantity takes on a series of particular values:

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$$x_0,\ x_1,\ x_2, \ldots,\ x_i, \ldots,\ x_n,$$

which are separated from one another by invariable equal intervals, so that:

$$x_1 - x_0 = x_2 - x_1 = \ldots = x_{i+1} - x_i = \ldots = x_n - x_{n-1} = h.$$

This interval is called the step of the argument and is designated by h.

In addition to the step h, the limits of the region x_0 and x_n must be stated in the form of finite numerical values.

Cases in which the step h is variable, or in which the variable x assumes infinitely large values in the region under consideration from x_0 to x_n, including the limits of the region themselves, are not considered at all.

Furthermore, let some other variable quantity y also assume a series of particular values:

$$y_0,\ y_1,\ y_2, \ldots,\ y_i, \ldots,\ y_n,$$

each of these particular values corresponding to one of the particular values of the argument x, so that it is always true that:

$$y_i = f(x_i),$$

where:

$$i = 0,\ 1,\ 2, \ldots,\ n - 1,\ n.$$

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It follows that the functional relation between the variables y and x is established in the form of a tabulation.

An example of such a relation is presented in Table 4.

Table 4 - Pressure Curve as a Function of Time t.

$t \cdot 10^3$ sec	0	4	8	12	16	18	20	21	22	23	23.5	24	24.5	25
p $\frac{kg}{cm^2}$	21	23	26	32	48	63	88	105	128	175	223	274	330	394
$t \cdot 10^3$ sec	25.5	26	26.5	27.0	27.5	28	28.5	29	29.5	0	30.5	30.75	31	-
p $\frac{kg}{cm^2}$	466	546	636	739	857	990	1137	1312	1516	1743	1983	2097	2175	-

This table is the result of analysis of experimental data obtained by burning in a bomb a weighed sample of strip powder grade SP (1 x 18 x 40 mm) at $\Delta = 0.201$. The values of t were chosen when evaluating the experimental data, so that t is the argument; the values of pressure p were taken from the experimental curve for the chosen values of t. Consequently, the pressure of the powder gases is a function of the time t.

Use has been made in Table 4 of the so-called vertical notation, which we shall adopt henceforth. In this notation the particular values of each of the variable quantities are invariably placed in one row, while each column contains one particular value of each of these variables. This notation is found to be most convenient for the various operations to which the functions stated by the tabulation are subjected.

2) Finite Differences of Various Orders.

Once a function has been given in table form, it is not difficult to find the so-called finite differences of this function or, more

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simply, its table of differences, these differences being of various orders. Thus, for a portion of Table 4, we obtain the following table of differences.

Table 4-a.

$t \cdot 10^3$	23	23.5	24	24.5	25	25.5
p	175	223	274	330	394	466
Δp	48	51	56	64	72	-
$\Delta^2 p$	3	5	8	8	-	-
$\Delta^3 p$	2	3	0	-	-	-

Thus on the basis of the table of particular function values it is possible to compute the following differences:

$$\Delta p_0 = p_1 - p_0 = 223 - 175 = 48$$
$$\Delta p_1 = p_2 - p_1 = 274 - 223 = 51$$
$$\Delta p_2 = p_3 - p_2 = 330 - 274 = 56$$
....

These differences are called differences of the first order or, more briefly, as the first differences.

From the first differences, the following new differences can be easily found:

$$\Delta^2 p_0 = \Delta p_1 - \Delta p_0 = 51 - 48 = 3$$
$$\Delta^2 p_1 = \Delta p_2 - \Delta p_1 = 56 - 51 = 5$$
$$\Delta^2 p_2 = \Delta p_3 - \Delta p_2 = 64 - 56 = 8$$
....

These new differences are now called differences of the second order, or second differences.

From the second differences it is also possible to compute in a similar manner the third differences:

$$\Delta^3 p_0 = \Delta^2 p_1 - \Delta^2 p_0 = 5 - 3 = 2$$
$$\Delta^3 p_1 = \Delta^2 p_2 - \Delta^2 p_1 = 8 - 5 = 3$$
$$\Delta^3 p_2 = \Delta^2 p_3 - \Delta^2 p_2 = 8 - 8 = 0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

followed by the fourth differences, etc. Generally, by definition, the k^{th} difference equals:

$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i .$$

Rule. In formulating a table of differences, the number at the left must be subtracted from the number at the right in the same row, and the result recorded in the next lower row under the number at the left.

It is useful to point out that in following this rule the differences with the same subscript (the function itself also being considered as a zero order difference) are automatically recorded in one and the same column.

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Table 4-b.

x	x_0	x_1	x_2	x_3	x_4
y	y_0	y_1	y_2	y_3	y_4
Δy	Δy_0	Δy_1	Δy_2	Δy_3	-
$\Delta^2 y$	$\Delta^2 y_0$	$\Delta^2 y_1$	$\Delta^2 y_2$	-	-
$\Delta^3 y$	$\Delta^3 y_0$	$\Delta^3 y_1$	-	-	-

In the table of differences, in the column for $i = 0$, for example, are found differences such as:

$$p_0 = 175, \quad \Delta p_0 = 48,$$
$$\Delta^2 p_0 = 3, \quad \Delta^3 p_0 = 2$$

etc.

3) Certain Properties of Finite Differences.

1. All differences of any order can be expressed only in terms of particular functions of the function itself.

By definition of the first differences:

$$\Delta y_0 = y_1 - y_0, \quad \Delta y_1 = y_2 - y_1, \quad \Delta y_2 = y_3 - y_2.$$

Then, by definition of the second differences:

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 = (y_2 - y_1) - (y_1 - y_0) = y_2 - 2y_1 + y_0;$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1 = (y_3 - y_2) - (y_2 - y_1) = y_3 - 2y_2 + y_1$$

etc. Generally:

$$\Delta^2 y_i = y_{i+2} - 2y_{i+1} + y_i.$$

The same procedure can also be applied to differences of higher orders.

2. A constant number can be taken outside the difference symbol of any order and, conversely, can be brought inside this difference symbol.

Let:

$$y = c \cdot f(x).$$

where c is a constant number. On the basis of the definition of differences, we have:

$$\Delta y = c \cdot f(x + h) - c \cdot f(x) = c \cdot [f(x + h) - f(x)] = c \cdot \Delta f(x).$$

Thus,

$$\Delta [c \cdot f(x)] = c \cdot \Delta f(x).$$

By rewriting this relation from right to left, we obtain:

$$c\Delta f(x) = \Delta [cf(x)].$$

This is a mathematical formulation of the second part of the assertion under consideration.

3. For an entire function of the k^{th} degree:

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$$y = A_0(x - x_i)^k + A_1(x - x_i)^{k-1} + \ldots + A_{k-1}(x - x_i) + A_k;$$

the k^{th} differences are identical.

Let us confine ourselves to the case of $k = 2$, so that:

$$y = A_0(x - x_i)^2 + A_1(x - x_i) + A_2.$$

Taking:

$$x_{i+1} = x_i + h;$$

$$x_{i+2} = x_i + 2h.$$

$$x_{i+3} = x_i + 3h.$$

we obtain from the formula for the entire function:

$$y_i = A_2;$$

$$y_{i+1} = A_0h^2 + A_1h + A_2.$$

$$y_{i+2} = 4A_0h^2 + 2A_1h + A_2;$$

$$y_{i+3} = 9A_0h^2 + 3A_1h + A_2;$$

$$\Delta y_i = y_{i+1} - y_i = A_0h^2 + A_1h.$$

$$\Delta y_{i+1} = y_{i+2} - y_{i+1} = 3A_0h^2 + A_1h;$$

$$\Delta y_{i+2} = y_{i+3} - y_{i+2} = 5A_0h^2 + A_1h;$$

and finally:

$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i = 2A_0h^2;$$

$$\Delta^2 y_{i+1} = \Delta y_{i+2} - \Delta y_{i+1} = 2A_0h^2.$$

The differences $\Delta^2 y_i$ and $\Delta^2 y_{i+1}$ are found to be identical (because A_0 and h are constant numbers), which is what we set out to prove.

4) Determination of Coefficients of an Entire Function.

Let us undertake the task of expressing the coefficients of an entire function in terms of differences of this function. To do this for the case of $k = 2$, we shall make use of the following relations:

$$y_i = A_2; \quad \Delta y_i = A_0 h^2 + A_1 h; \quad \Delta^2 y_i = 2A_0 h^2.$$

Substituting the value of A_0 from the last relation, namely:

$$A_0 = \frac{\Delta^2 y_i}{2h^2},$$

into the equation for Δy_i, we find:

$$\Delta y_i = \frac{1}{2} \Delta^2 y_i + A_1 h,$$

whence

$$A_1 = \frac{1}{2h} (2\Delta y_i - \Delta^2 y_i).$$

Thus, finally:

$$A_0 = \frac{\Delta^2 y_i}{2h^2}; \qquad A_1 = \frac{2\Delta y_i - \Delta^2 y_i}{2h^2}; \qquad A_2 = y_i.$$

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The coefficients for an entire third degree function can be found in a similar manner:

$$A_0 = \frac{\Delta^3 y_i}{6h^3}; \qquad A_1 = \frac{\Delta^2 y_i - \Delta^3 y_i}{2h^2};$$

$$A_2 = \frac{6\Delta y_i - 3\Delta^2 y_i + 2\Delta^3 y_i}{6h}; \qquad A_3 = y_i.$$

These coefficients will be needed subsequently for deriving the formula of the interpolation function. It is of interest to note the fact that the coefficients of the entire function are here expressed in terms of differences with the same subscript i.

The same method may be used to determine these coefficients in terms of differences with different subscripts. Thus, for example, by taking t_i, t_{i-1}, t_{i-2}, t_{i-3} (instead of t_i, t_{i+1}, t_{i+2}, t_{i+3}), we shall find the following relations:

$$A_0 = \frac{\Delta^3 y_{i-3}}{6h^3}; \qquad A_1 = \frac{\Delta^2 y_{i-2} + \Delta^3 y_{i-3}}{2h^2};$$

$$A_2 = \frac{6\Delta y_{i-1} + 3\Delta^2 y_{i-2} + 2\Delta^3 y_{i-3}}{6h}; \qquad A_3 = y_i.$$

These relations, which the reader himself will be able to derive directly by using the same method, will find an application in the derivation of working formulas for the numerical integration of equations.

5) The Practical Value of Differences.

Differences are employed for the following purposes: STAT

a) Determination of intermediate values of a function.

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b) Computation of intermediate values of the argument.

c) Factual determination of derivatives of various orders of a function.

d) Determination of definite integrals.

e) Numerical integration of ordinary differential equations.

The universal character of these differences used in the operations enumerated above is reflected in the fact that it is perfectly immaterial whether the function has been stated in the form of a tabulation, a diagram, or a formula.

It has already been established that the k^{th} difference of an entire function of the k^{th} degree is constant. It is not difficult to show that, conversely, if the differences of the k^{th} order of a certain function are constant, the latter is an entire function of the k^{th} degree.

This proposition constitutes the cornerstone of the utilization of differences in all of the operations indicated above, for the following reason. If the formulation of the table of differences of a certain function shows that the k^{th} differences are almost constant, we have the right to replace our function by an entire function of the k^{th} degree and then subject the latter function to all the necessary operations.

It is this entire function which is designated as the interpolation function. A general expression for it must be derived. When k = 3, we have:

$$y = A_0(x - x_1)^3 + A_1(x - x_1)^2 + A_2(x - x_1) + A_3.$$

Let us introduce such a variable ξ that:

$$x = x_1 + \xi h,$$

where h is the step. Then:

$$\xi = \frac{x - x_1}{h}.$$

The quantity ξ is called the coefficient of interpolation.

Using the coefficient of interpolation and the general relations for the coefficients A_0, A_1, A_2, A_3 of the entire function as derived in Subsection 4, we shall rewrite the formula for the interpolation function as follows:

$$y = \frac{\Delta^3 y_1}{6}\xi^3 + \frac{\Delta^2 y_1 - \Delta^3 y_1}{2}\xi^2 + \frac{6\Delta y_1 - 3\Delta^2 y_1 + 2\Delta^3 y_1}{6}\xi + y_1,$$

or, finally, after regrouping the terms:

$$y = y_1 + \xi \cdot \Delta y_1 - \frac{1}{2}\xi(1 - \xi)\Delta^2 y_1 + \frac{1}{6}\xi(1 - \xi)(2 - \xi)\Delta^3 y_1. \qquad (105)$$

The interpolation function is thus found to be expressed in terms of differences of the given function up to and including those of the third order, with all these differences having one and the same subscript.

In replacing any function (regardless of the manner in which it is stated) by this interpolation function, it is categorically

imperative to direct attention, on the basis of the table of differences of the given function, to the character of the variation of these differences; it is necessary, and triply necessary, that the k^{th} differences be nearly constant.

Obviously this condition will be satisfied the better the smaller the absolute values of the differences of any order.

Rule. In order to reduce the differences of any order, the step must be reduced.

This rule results from the following relation:

$$\Delta^k y = y^{(k)} \cdot h^k + h^k \cdot \varepsilon = h^k(y^{(k)} + \varepsilon),$$

where $y^{(k)}$ is the k^{th}-order derivative of x, and ε is an infinitely small quantity of the highest order. The relation itself, which is derived at the proper stage of the differential computation, represents a generalization of the better-known relation between the increment of a function and its differential:

$$\Delta y = dy + h\varepsilon$$

or:

$$\Delta y = y'h + h\varepsilon = h(y' + \varepsilon).$$

Thus, as the step is halved, the first differences decrease approximately by a factor of two, the second differences decrease by a factor of four, and the third differences decrease by a factor of eight.

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6) Determination of Intermediate Values of a Given Function.

To determine the intermediate values of a given function, regardless of how it is stated, it is necessary to formulate a table of differences of this function and, having made certain that the second differences are constant or are very small in comparison with the particular values of the function itself, replace it by an entire function of the second degree, i.e., by an interpolation function (105):

$$y = y_i + \xi \cdot \Delta y_i - \frac{1}{2}\xi(1-\xi)\Delta^2 y_i. \qquad (106)$$

where y_i designates the tabular value of the given function corresponding to the next smaller value of the argument nearest to that value of x for which the intermediate value of the function is sought.

Knowing the intermediate value of x for which the intermediate value of the given function is sought, and having obtained from the table of differences of this function the nearest value of x_i, we can determine the value of the coefficient of interpolation ξ and the unknown value of y by means of formula (106).

Example 1. Given the following table of pressure impulses:

Table 4-c

$t \cdot 10^3$	20	21	22
I	71.0	80.6	92.1

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where the impulses are expressed in $kg \cdot dm^{-2} \cdot sec$, determine the pressure impulse for $t = 0.020556$.

Solution. Let us formulate the table of differences:

Table 4-d.

$t \cdot 10^3$	20	21	22
I	71.0	80.6	92.1
ΔI	9.6	11.5	
$\Delta^2 I$	1.9		

Let us find the coefficient of interpolation:

$$\xi = \frac{t - t_1}{h} = \frac{0.020556 - 0.020}{0.001} = 0.556.$$

The desired intermediate value of the function is:

$$I = I_1 + \xi \cdot \Delta I_1 - \frac{1}{2}\xi(1 - \xi)\Delta^2 I_1 = 71.0 + 0.556 \cdot 9.6 - \frac{1}{2}0.556(1 - 0.556)1.9,$$

since

$$\xi = 0.556; \quad I_1 = 71.0; \quad \Delta I_1 = 9.6 \text{ and } \Delta^2 I_1 = 1.9$$

We finally obtain:

$$I = 71.0 + 5.33 - 0.25 \approx 76 \ kg \cdot dm^{-2} \cdot sec$$

7) Computation of Intermediate Values of the Argument.

Cases are sometimes encountered in practice of a contrary nature in which an intermediate value y of the function is given and it is required to find the coefficient of interpolation ξ and the intermediate value of the argument x. Such an operation is sometimes called an

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inverse interpolation.

Then, discarding the last term in formula (106), we obtain as a first approximation:

$$\xi_1 = \frac{y - y_1}{\Delta y_1}.$$

As a second approximation, we find for ξ from the same formula (106):

$$\xi = \frac{y - y_1}{\Delta y_1 - \frac{1}{2}(1 - \xi)\Delta^2 y_1}.$$

Let us assume in the right-hand side of the above expression $\xi = \xi_1$, then in the second approximation:

$$\xi_2 = \frac{y - y_1}{\Delta y_1 - \frac{1}{2}(1 - \xi_1)\Delta^2 y_1}.$$

where ξ_1 is already known from the first approximation.

Sometimes the following expression is used as the third approximation:

$$\xi_3 = \frac{y - y_1}{\Delta y_1 - \frac{1}{2}(1 - \xi_2)\Delta^2 y_1 + \frac{1}{6}(1 - \xi_2)(2 - \xi_2)\Delta^3 y_1}.$$

This formula can be easily obtained from the general expression for the interpolation function (105) by placing ξ outside the parentheses, determining ξ, and substituting in the right-hand side $\xi = \xi_2$ known from the second approximation.

The desired value of x will be:

$$x = x_1 + \xi_2 h \text{ or } x = x_1 + \xi_3 h.$$

Example 2. We have the following table for the relative portion ψ of the charge:

Table 4-e.

$t \cdot 10^3$	20	21	22
ψ	0.034	0.042	0.054

It is desired to find the value of t corresponding to the inflow of gases $\psi = 0.038$.

Solution. Let us formulate the following table of differences:

Table 4-f.

$t \cdot 10^3$	20	21	22
ψ	0.034	0.042	0.054
$\Delta\psi$	0.008	0.012	-
$\Delta^2\psi$	0.004	-	-

We obtain as a first approximation:

$$\xi_1 = \frac{\psi - \psi_1}{\Delta\psi_1} = \frac{0.038 - 0.034}{0.008} = \frac{1}{2}.$$

We find as a second approximation:

$$\xi_2 = \frac{\psi - \psi_i}{\Delta\psi_i - \frac{1}{2}(1 - \xi_1)\Delta^2\psi_i} = \frac{0.038 - 0.034}{0.008 - \frac{1}{2}\cdot\frac{1}{2}0.004} = \frac{4}{7} = 0.556.$$

The desired value of the argument is:

$$t = t_i + \xi_2 h = 0.020 + 0.556(0.024 - 0.023) = 0.020556.$$

8) Numerical Differentiation of Functions.

For this operation it is necessary, first of all, to formulate a table of differences of the given function and to make absolutely certain that the differences of the third order are nearly constant. This fact makes it possible to replace the given function (regardless of the manner in which it is stated) by the interpolation function (105):

$$y = y_i + \xi \cdot \Delta y_i - \frac{1}{2}\xi(1 - \xi)\Delta^2 y_i + \frac{1}{6}\xi(1 - \xi)(2 - \xi)\Delta^3 y_i.$$

From this we find the desired derivative:

$$\frac{dy}{dx} = \frac{d\xi}{dx}\Delta y_i + \frac{1}{2}\left(2\xi\frac{d\xi}{dx} - \frac{d\xi}{dx}\right)\Delta^2 y_i + \frac{1}{6}\left(2\frac{d\xi}{dx} - 6\xi\frac{d\xi}{dx} + 3\xi^2\frac{d\xi}{dx}\right)\Delta^3 y_i,$$

keeping in mind that the differences y_i, Δy_i, $\Delta^2 y_i$, and $\Delta^3 y_i$ are certain constant numbers.

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We place the derivative $\frac{d\xi}{dx}$ outside the parentheses:

$$\frac{dy}{dx} = \frac{d\xi}{dx}\left[\Delta y_i + \frac{1}{2}(2\xi - 1)\Delta^2 y_i + \frac{1}{6}(2 - 6\xi + 3\xi^2)\Delta^3 y_i\right];$$

But

$$\xi = \frac{x - x_i}{h};$$

$$\frac{d\xi}{dx} = \frac{1}{h}.$$

Therefore, finally:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_i + \left(\xi - \frac{1}{2}\right)\Delta^2 y_i + \left(\frac{1}{2}\xi^2 - \xi + \frac{1}{3}\right)\Delta^3 y_i\right].$$

Such is the general working formula for numerical differentiation by means of differences of the given function. It permits finding the value of the derivative at any value of the independent variable x (i.e., at any ξ). This formula assumes its simplest form at $\xi = 0$:

$$\boxed{\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_i - \frac{1}{2}\Delta^2 y_i + \frac{1}{3}\Delta^3 y_i\right].}$$

This relation permits determining the derivative only for those values of the argument which appear in the table of differences of the given function (i.e., for $\xi = 0$), by applying the subscript i successively to each column of this table.

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If the step h is not a prime number (for example, $\frac{557}{738}$), the constant h may be introduced under the sign of the difference of any desired order. Therefore, upon introducing into the process the auxiliary function:

$$\Phi = \frac{1}{h} y,$$

we obtain

$$\Delta\Phi_1 = \frac{1}{h}\Delta y_1; \quad \Delta^2\Phi_1 = \frac{1}{h}\Delta^2 y_1; \quad \Delta^3\Phi_1 = \frac{1}{h}\Delta^3 y_1,$$

and the working formula for numerical differentiation acquires its final form:

$$\left(\frac{dy}{dx}\right)_1 = \Delta\Phi_1 - \frac{1}{2}\Delta^2\Phi_1 + \frac{1}{3}\Delta^3\Phi_1 .$$

Example 3. We have the following table for ψ, the relative portion of the charge:

Table 4-g.

$t \cdot 10^3$	21	21.5	22.0	22.5
$\psi \cdot 10^3$	42	47	54	64

It is desired to find the rate of gas formation $\frac{d\psi}{dt}$ at the instant $t = 0.0210$.

Solution. Let us formulate the following table of differences:

Table 4-h.

$t \cdot 10^3$	21	21.5	22.0	22.5
$\psi \cdot 10^3$	42	47	54	64
$\Delta\psi \cdot 10^3$	5	7	10	
$\Delta^2\psi \cdot 10^3$	2	3		
$\Delta^3\psi \cdot 10^3$	1			

Since the given values are $t_1 \cdot 10^3 = 21$ and $\psi_1 \cdot 10^3 = 42$, it follows that:

$$\Delta(\psi_1 \cdot 10^3) = 5; \quad \Delta^2(\psi_1 \cdot 10^3) = 2; \quad \Delta^3(\psi_1 \cdot 10^3) = 1.$$

The first of the working formulas for the derivative $\frac{dy}{dx}$ will make it possible to determine the rate of gas formation ($h = 0.5 \cdot 10^3$):

$$\frac{d(\psi \cdot 10^3)}{dt} = \frac{1}{h}\left[\Delta(\psi_1 \cdot 10^3) - \frac{1}{2}\Delta^2(\psi_1 \cdot 10^3) - \frac{1}{3}\Delta^3(\psi_1 \cdot 10^3)\right],$$

from which:

$$\left(\frac{d\psi}{dt}\right)_{t=0.021} = \frac{5-1}{0.5} = 8\ \frac{1}{\sec}.$$

9) Computation of Definite Integrals.

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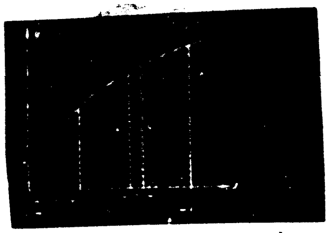

Fig. 152 - Determination of $\int_a^b ydx$ as a function of x from the curve $y = f(x)$.

If it is necessary to find any definite integral:

$$Y = \int_a^b ydx,$$

where the limits of integration a and b are finite, this operation is equivalent to computing the shaded area shown in fig. 152, which is limited at the top by a curve representing the function whose integral is to be found. To compute this area Y, the usual method is applied first: the interval of integration (b - a) must be divided into n equal portions, lines normal to the O-x axis are then erected at the points of division, and the unknown area is divided into elementary areas:

$$\Delta Y_0, \quad \Delta Y_1, \ldots, \quad \Delta Y_i, \ldots, \quad \Delta Y_{n-1}.$$

The problem is then reduced to the determination of these elementary areas. For this purpose it is sufficient to replace the portion of the curve $f(x_i)$ corresponding to the elementary area

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ΔY_i by a cubic parabola, keeping in mind that:

$$\Delta Y_i = \int_{x_i}^{x_{i+1}} y dx.$$

This replacement is accomplished by means of the interpolation function:

$$y = y_i + \xi \cdot \Delta y_i - \frac{1}{2}\xi(1 - \xi)\Delta^2 y_i + \frac{1}{6}\xi(1 - \xi)(2 - \xi)\Delta^3 y_i,$$

where

$$x = x_i + \xi h,$$

so that:

$$dx = h d\xi; \quad \xi_i = 0; \quad \xi_{i+1} = 1.$$

Consequently, after replacing the variables, we have:

$$\Delta Y_i = h \int_0^1 \left[y_i + \xi \cdot \Delta y_i - \frac{1}{2} \xi(1 - \xi)\Delta^2 y_i + \frac{1}{6}\xi(1 - \xi)(2 - \xi)\Delta^3 y_i \right] d\xi.$$

We integrate the right-hand side of this relation keeping in mind the fact that y_i, Δy_i, $\Delta^2 y_i$ and $\Delta^3 y_i$, being particular values of the corresponding differences, are constant numbers:

$$\Delta Y_i = h \left| y_i \xi + \frac{1}{2}\xi^2 \cdot \Delta y_i - \frac{1}{2}\left(\frac{\xi^2}{2} - \frac{\xi^3}{3}\right)\Delta^2 y_i + \frac{1}{6}\left(\xi^2 - \xi^3 + \frac{\xi^4}{4}\right)\Delta^3 y_i \right|_0^1,$$

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from which, after substituting the extreme values of ξ, we obtain:

$$\boxed{\Delta Y_i = h\left(y_i + \frac{1}{2}\Delta y_i - \frac{1}{12}\Delta^2 y_i + \frac{1}{24}\Delta^3 y_i\right).}$$

This is the working formula employed in finding a definite integral by the method of numerical integration.

To determine the unknown integral, it remains necessary to summate gradually and successively the individual elementary areas:

$$Y_1 = Y_0 + \Delta Y_0; \qquad Y_i = Y_{i-1} + \Delta Y_{i-1};$$

$$Y_2 = Y_1 + \Delta Y_1; \qquad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots \qquad Y_n = Y_{n-1} + \Delta Y_{n-1}.$$

Example 4. We have the following table for a function to be integrated:

Table 4-i.

$t \cdot 10^3$	8	12	16	20
p, $kg \cdot cm^{-2}$	26	32	48	88

It is desired to find the elementary area ΔI_i, the subscript i being applied to the first column.

Solution. Let us formulate the following table of differences:

Table 4-j.

$t \cdot 10^3$	8	12	16	20
p	26	32	48	88
Δp	6	16	40	-

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Table 4-j (Cont'd.)

$\Delta^2 p$	10	24	-	-
$\Delta^3 p$	14	-	-	-

Since:

$$h = 4 \cdot 10^{-3}; \quad y_i = 26; \quad \Delta y_i = 6; \quad \Delta^2 y_i = 10; \quad \Delta^3 y_i = 14,$$

the elementary area under consideration will be:

$$\Delta Y_i = h\left(y_i + \frac{1}{2}\Delta y_i - \frac{1}{12}\Delta^2 y_i + \frac{1}{24}\Delta^3 y_i\right) =$$

$$= 4 \cdot 10^{-3}\left(26 + 3 - \frac{10}{12} + \frac{14}{24}\right) \approx 4 \cdot 10^{-3}(26 + 3 - 1 + 1) \approx$$

$$\approx 4 \cdot 10^{-3} \cdot 29 \approx 0.116 \text{ kg} \cdot \text{sec} \cdot \text{cm}^{-2} \approx 11.6 \text{ kg} \cdot \text{sec} \cdot \text{dm}^{-2}.$$

10) Determination of Pressure Impulse from a Pressure-Bomb Test.

As is already known from the preceding course in internal ballistics, the pressure impulse is expressed by the following relation:

$$I = \int_0^t p\,dt.$$

It is clear that in order to determine it from a bomb test yielding directly the (p, t) curve, it is necessary to compute a definite integral, for which the pressure of the powder gases p is the function being integrated and the time t is the independent

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variable. This can be accomplished of course by means of one of the usual methods of quadratures. However, it is usually necessary to have a curve for the pressure impulse as a function of the gas inflow ψ; in other words, in addition to the pressure impulse I_K at the end of the burning of the powder, it also becomes necessary to find a series of intermediate values for this quantity, which will correspond to intermediate values of the gas inflow. This problem is solved most simply by the method of numerical integration.

Example 5. Given the pressure curve (p, t) represented by Table 4, find the correlation (I, ψ) by means of the table correlating ψ with t.

Table 4-k - Experimental Table of ψ as a Function of t.

$t \cdot 10^3$	0	4	8	12	16	18	20	21	22	23	23.5	24	24.5
$\psi \cdot 10^3$	0	2	3	6	14	22	34	42	54	79	101	127	155
$t \cdot 10^3$	25	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0
$\psi \cdot 10^3$	186	221	260	304	354	409	470	539	619	711	813	919	1000

Solution. We perform all computations on the working form in Table 5, without considering third differences, and designating:

$$\Sigma = y + \frac{1}{2}\Delta y - \frac{1}{12}\Delta^2 y = p + \frac{1}{2}\Delta p - \frac{1}{12}\Delta^2 p;$$

from which it follows that:

$$\Delta I_i = \Delta Y_i = h\left(y_i + \frac{1}{2}\Delta y_i - \frac{1}{12}\Delta^2 y_i\right) = h \cdot \Sigma.$$

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Table 5 - Computation of Pressure Impulse I.

1	$t \cdot 10^3$	0	4	8	12	16	20	16	18	20	22	20	21	22	23	24	23	23.5	24
2	p	21	23	26	32	48	88	48	63	88	128	88	105	128	175	274	175	223	274
3	Δp	2	3	6	16	40	-	15	25	40	-	17	23	47	99	-	48	51	56
4	$\Delta^2 p$	1	3	10	24	-	-	10	15	-	-	6	24	52	-	-	3	5	8
5	p	21	23	26	32	-	-	48	63	-	-	88	105	128	-	-	175	223	274
6	$\frac{1}{2}\Delta p$	1	2	3	8	-	-	8	12	-	-	8	12	24	-	-	24	26	28
7	$-\frac{1}{12}\Delta^2 p$	0	0	-1	-2	-	-	-1	-1	-	-	0	-2	-4	-	-	0	0	-1
8	Σ	22	25	28	38	-	-	55	74	-	-	96	115	148	-	-	199	249	301
9	$\Delta I = h\Sigma$	8.8	10.0	11.2	15.2	-	-	11.0	14.8	-	-	9.6	11.5	14.8	-	-	10.0	12.4	15.0
10	I	0	8.8	18.8	30.0	45.2	-	45.2	56.2	71.0	-	71.0	80.6	92.1	106.9	-	106.9	116.9	129.3

Continued

1	$t \cdot 10^3$	24.5	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5	31	30.5	30.75	31
2	p	330	394	466	546	636	739	857	990	1137	1312	1516	1743	1983	2175	1983	2097	2175
3	Δp	64	72	80	90	103	118	133	147	175	204	227	240	192	-	114	78	-
4	$\Delta^2 p$	8	8	10	13	15	15	14	28	29	23	13	-18	-	-	-36	-	-
5	p	330	394	466	546	636	739	857	990	1137	1312	1516	1743	-	-	1983	2097	-
6	$\frac{1}{2}\Delta p$	32	36	40	45	52	59	66	74	88	102	114	120	-	-	57	39	-
7	$-\frac{1}{12}\Delta^2 p$	-1	-1	-1	-1	-1	-1	-1	-2	-2	-2	-1	4	-	-	3	(3)	-
8	Σ	361	429	505	590	687	797	922	1062	1223	1412	1629	1867	-	-	2043	2139	-
9	$\Delta I = h\Sigma$	18.0	21.4	25.2	29.5	34.4	39.8	46.1	53.1	61.2	70.6	81.4	93.4	-	-	51.1	53.5	-
10	I	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-	[illegible]	[illegible]	823.0

Table 5 - Computation of Pressure Impulse I.

1	$t \cdot 10^3$	0	4	8	12	16	20	16	18	20	22	20	21	22	23	24	23	23.5	24
2	p	21	23	26	32	48	88	48	63	88	128	88	105	128	175	274	175	223	274
3	Δp	2	3	6	16	40	-	15	25	40	-	17	23	47	99	-	48	51	56
4	$\Delta^2 p$	1	3	10	24	-	-	10	15	-	-	6	24	52	-	-	3	5	8
5	p	21	23	26	32	-	-	48	63	-	-	88	105	128	-	-	175	223	274
6	$\frac{1}{2}\Delta p$	1	2	3	8	-	-	8	12	-	-	8	12	24	-	-	24	26	28
7	$-\frac{1}{12}\Delta^2 p$	0	0	-1	-2	-	-	-1	-1	-	-	0	-2	-4	-	-	0	0	-1
8	Σ	22	25	28	38	-	-	55	74	-	-	96	115	148	-	-	199	249	301
9	$\Delta I = h\Sigma$	8.8	10.0	11.2	15.2	-	-	11.0	14.8	-	-	9.6	11.5	14.8	-	-	10.0	12.4	15.0
10	I	0	8.8	18.8	30.0	45.2	-	45.2	56.2	71.0	-	71.0	80.6	92.1	106.9	-	106.9	116.9	129.3

Continued

1	$t \cdot 10^3$	24.5	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5	31	30.5	30.75	31
2	p	330	394	466	546	636	739	857	990	1137	1312	1516	1743	1983	2175	1983	2097	2175
3	Δp	64	72	80	90	103	118	133	147	175	204	227	240	192	-	114	78	-
4	$\Delta^2 p$	8	8	10	13	15	15	14	28	29	23	13	-18	-	-	-36	-	-
5	p	330	394	466	546	636	739	857	990	1137	1312	1516	1743	-	-	1983	2097	-
6	$\frac{1}{2}\Delta p$	32	36	40	45	52	59	66	74	88	102	114	120	-	-	57	39	-
7	$-\frac{1}{12}\Delta^2 p$	-1	-1	-1	-1	-1	-1	-1	-2	-2	-2	-1	4	-	-	3	(3)	-
8	Σ	361	429	505	590	687	797	922	1062	1223	1412	1629	1867	-	-	2043	2139	-
9	$\Delta I = h\Sigma$	18.0	21.4	25.2	29.5	34.4	39.8	46.1	53.1	61.2	70.6	81.4	93.4	-	-	51.1	53.5	-
10	I	144.3	162.3	183.7	208.9	238.4	272.8	312.6	358.7	411.8	473.0	543.6	625.0	718.4	-	718.4	769.5	823.0

1	[illegible]	0	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22	23	24	23	23.5	24
2	p	21	23	26	32	48	88	48	63	88	128	88	105	128	175	274	175	223	274
3	Δp	2	3	6	16	40	-	15	25	40	-	17	23	47	99	-	48	51	56
4	$\Delta^2 p$	1	3	10	24	-	-	10	15	-	-	6	24	52	-	-	3	5	8
5	p	21	23	26	32	-	-	48	63	-	-	88	105	128	-	-	175	223	274
6	$\frac{1}{2}\Delta p$	1	2	3	8	-	-	8	12	-	-	8	12	24	-	-	24	26	28
7	$-\frac{1}{12}\Delta^2 p$	0	0	-1	-2	-	-	-1	-1	-	-	0	-2	-4	-	-	0	0	-1
8	Σ	22	25	28	38	-	-	55	74	-	-	96	115	148	-	-	199	249	301
9	$\Delta I = h\Sigma$	8.8	10.0	11.2	15.2	-	-	11.0	14.8	-	-	9.6	11.5	14.8	-	-	10.0	12.4	15.0
10	I	0	8.8	18.8	30.0	45.2	-	45.2	56.2	71.0	-	71.0	80.6	92.1	106.9	-	106.9	116.9	129.3

Continued

1	$t \cdot 10^3$	24.5	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5	31	30.5	30.75	31
2	p	330	394	466	546	636	739	857	990	1137	1312	1516	1743	1983	2175	1983	2097	2175
3	Δp	64	72	80	90	103	118	133	147	175	204	227	240	192	-	114	78	-
4	$\Delta^2 p$	8	8	10	13	15	15	14	28	29	23	13	-18	-	-	-36	-	-
5	p	330	394	466	546	636	739	857	990	1137	1312	1516	1743	-	-	1983	2097	-
6	$\frac{1}{2}\Delta p$	32	36	40	45	52	59	66	74	88	102	114	120	-	-	57	39	-
7	$-\frac{1}{12}\Delta^2 p$	-1	-1	-1	-1	-1	-1	-1	-2	-2	-2	-1	4	-	-	3	(3)	-
8	Σ	361	429	505	590	687	797	922	1062	1223	1412	1629	1867	-	-	2043	2139	-
9	$\Delta I = h\Sigma$	18.0	21.4	25.2	29.5	34.4	39.8	46.1	53.1	61.2	70.6	81.4	93.4	-	-	51.1	53.5	-
10	I	144.3	162.3	183.7	208.9	238.4	272.8	312.6	358.7	411.8	473.0	543.6	625.0	718.4	-	718.4	769.5	823.0

The computations in Table 5 are conducted in rows, proceeding from left to right in each row and downward from one row to the next, in the following manner.

1) The first row is filled with values of the argument, $t \cdot 10^3$.

2) The numbers for the pressure are taken from Table 4 (page 616).

3) From each number in the second row there is subtracted the preceding number in the same row, and the result is written under the left-hand number of this pair:

$$23 - 21 = 2; \; 26 - 23 = 3; \text{ etc.}$$

4) The numbers in the fourth row are obtained in the same manner as in the preceding row:

$$3 - 2 = 1; \; 6 - 3 = 3; \text{ etc.}$$

5) The numbers of the second row are repeated.

6) The numbers in the third row are halved.

7) The numbers in the fourth row are divided by 12 and written with the opposite sign.

8) The numbers in the three preceding rows are added.

9) The numbers in the eighth row are multiplied by the step h, the decimal point being correctly placed (cf. Example 4) to convert $kg \cdot sec \cdot cm^{-2}$ into $kg \cdot sec \cdot dm^{-2}$.

10) To obtain the next succeeding value of I_i, it is necessary to add to the preceding value of I_{i-1} the corresponding increment ΔI_{i-1}:

$$I_i = I_{i-1} + \Delta I_{i-1},$$

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or, in other words, it is necessary to add the two numbers in the ninth and tenth rows of the preceding column:

$$0 + 8.8 = 8.8;$$

$$8.8 + 10.0 = 18.8, \text{etc.}$$

By taking the values for the gas inflow ψ and the pressure impulse I corresponding to the same instants t, it is easy to obtain the desired correlation between I and ψ (cf. Table 6).

Table 6 - Correlation between I and ψ for Test.

I $kg \cdot sec \cdot dm^{-2}$	0	9	19	30	45	56	71	81	92	107	117	129	144	162
$\psi \cdot 10^3$	0	2	3	6	14	22	34	42	54	79	101	127	155	186
I $kg \cdot sec \cdot dm^{-2}$	184	209	238	273	313	359	412	473	544	625	718	770	823	
$\psi \cdot 10^3$	221	260	304	354	409	470	539	619	711	813	919	967	1000	

Table 6 will be needed subsequently for the solution of the principal problem of internal ballistics (determination of the curves for the speed of the projectile and for the pressure of the powder gases as a function of the path of the projectile). This solution must be prepared by a discussion of the necessary theory, which will be undertaken in the next section.

2. NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS

11) Numerical Integration of First-Order Equation

We have such an equation in its general form:

$$F(y, y', x) = 0.$$

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It is composed of the following elements: the independent variable x, the unknown function y, and its first derivative y'. It is necessary, first of all, to determine this derivative from the general equation:

$$y' = f(y, x).$$

It is this equation which is solved by the method of numerical integration, regardless of the form of the function which constitutes its right-hand side. In practice, what is usually found is not the total integral of the last equation (in which case, of course, numerical integration cannot be employed), but, rather, a partial integral, in consequence of which it is necessary to indicate the "initial conditions", i.e., the values of y_0 and x_0. In addition, the value of x_n at the end of the finite region of integration must also be known. Under these conditions the desired function will be:

$$y = \int_{x_0}^{x_n} y'dx$$

or

$$y = \int_{x_0}^{x_n} f(y, x)dx.$$

The problem is consequently reduced to the finding of a definite integral whose characteristic feature is such that the part of the function to be integrated is played by the derivative of the function.

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It would seem that, as on the previous occasion involving the finding of a definite integral, it might be possible to perform computations by rows and to make use of a working formula containing differences of an auxiliary function with the same subscript i. In actual practice, however, this procedure cannot be adopted. As a matter of fact, the last formula shows that the function to be integrated includes the original function itself, and we thus obtain a "vicious circle;" in order to determine the particular value y_i of the function, we must know the particular value Φ_i, and in order to determine this particular value of the auxiliary function:

$$\Phi_i = f(y_i, x_i)h$$

we must have the value of y_i which is a constituent of the auxiliary function.

It is now clear that in performing numerical integration of the equations it is not possible to perform computations by rows, it being necessary instead to proceed gradually, step by step, performing the required computations downward along each column and to the right from one column to the next.

Let us further assume that the values Φ_{i-3}, Φ_{i-2}, Φ_{i-1} and Φ_i of the auxiliary function have already been found. With their aid it is possible to compute the following differences.

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Table 6-a.

Φ	Φ_{i-3}	Φ_{i-2}	Φ_{i-1}	Φ_i
$\Delta\Phi$	$\Delta\Phi_{i-3}$	$\Delta\Phi_{i-2}$	$\Delta\Phi_{i-1}$	
$\Delta^2\Phi$	$\Delta^2\Phi_{i-3}$	$\Delta^2\Phi_{i-2}$		
$\Delta^3\Phi$	$\Delta^3\Phi_{i-3}$			

The above table shows that in order to determine Δy_i, it is not possible to make use of the following formula for determining definite integrals:

$$\Delta y_i = \Phi_i + \frac{1}{2}\Delta\Phi_i - \frac{1}{12}\Delta^2\Phi_i + \frac{1}{24}\Delta^3\Phi_i.$$

because the differences $\Delta\Phi_i$, $\Delta^2\Phi_i$ and $\Delta^3\Phi_i$ are not yet contained in the table. Consequently, it becomes necessary to derive an additional formula containing the differences:

$$\Delta\Phi_{i-1}, \quad \Delta^2\Phi_{i-2}, \quad \Delta^3\Phi_{i-3},$$

already contained in the table.

For this purpose, as on the previous occasion, we shall replace the function to be integrated, $y' = f(y, x)$, by the following interpolation function:

$$y' = A_0(x - x_i)^3 + A_1(x - x_i)^2 + A_2(x - x_i) + A_3,$$

and then:

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$$\Delta y_i = \int_{x_i}^{x_{i+1}} f(y, x)dx = \int_{x_i}^{x_{i+1}} \left[A_0(x - x_i)^3 + A_1(x - x_i)^2 + A_2(x - x_i) + A_3 \right] dx =$$

$$= \left| \frac{1}{4}A_0(x - x_i)^4 + \frac{1}{3}A_1(x - x_i)^3 + \frac{1}{2}A_2(x - x_i)^2 + A_3(x - x_i) \right|_{x_i}^{x_{i+1}},$$

whence

$$\Delta y_i = \frac{1}{4} A_0 h^4 + \frac{1}{3} A_1 h^3 + \frac{1}{2} A_2 h^2 + A_3 h,$$

because

$$x_{i+1} - x_i = h.$$

In contrast with the preceding case, we shall substitute here the following values for the coefficients:

$$A_0 = \frac{\Delta^3 y'_{i-3}}{6h^3}; \qquad A_1 = \frac{\Delta^2 y'_{i-2} + \Delta^3 y'_{i-3}}{2h^2};$$

$$A_2 = \frac{6\Delta y'_{i-1} + 3\Delta^2 y'_{i-1} + 2\Delta^3 y'_{i-3}}{6h}; \quad A_3 = y'_i,$$

which are expressed in terms of differences of the function y under the integral sign provided with the required subscripts.

Following obvious transformations, we obtain:

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$$\boxed{\Delta y_i = h\left(y'_i + \frac{1}{2}\Delta y'_{i-1} + \frac{5}{12}\Delta^2 y'_{i-2} + \frac{3}{8}\Delta^3 y'_{i-3}\right).}$$

It is usually convenient to make use of the following auxiliary function:

$$\Phi = hy',$$

the derivative y' being computed in accordance with the given equation:

$$y' = f(x, y).$$

Then:

$$hy'_i = \Phi_i; \quad h\Delta y'_{i-1} = \Delta\Phi_{i-1}; \quad h\Delta^2 y'_{i-2} = \Delta^2\Phi_{i-2}; \quad h\Delta^3 y'_{i-3} = \Delta^3\Phi_{i-3},$$

and the working formula for the numerical integration of ordinary differential equations of the first order will assume the following form:

$$\boxed{\Delta y_i = \Phi_i + \frac{1}{2}\Delta\Phi_{i-1} + \frac{5}{12}\Delta^2\Phi_{i-2} + \frac{3}{8}\Delta^3\Phi_{i-3}.}$$

This formula now includes differences of the auxiliary function Φ provided with precisely those subscripts which are required in accordance with the table of difference presented above.

It will be seen from the same table that in order to determine Δy_i, it becomes necessary to take differences from different columns, which, of course, is inconvenient. For this reason it is desirable

to modify the system of writing the differences in such a manner as to have in the same column differences with different subscripts, namely:

Table 6-b.

Φ	Φ_{i-3}	Φ_{i-2}	Φ_{i-1}	Φ_i
$\Delta\Phi$		$\Delta\Phi_{i-3}$	$\Delta\Phi_{i-2}$	$\Delta\Phi_{i-1}$
$\Delta^2\Phi$			$\Delta^2\Phi_{i-3}$	$\Delta^2\Phi_{i-2}$
$\Delta^3\Phi$				$\Delta^3\Phi_{i-3}$

In such a procedure each column will contain those differences which are necessary for the application of the new working formula for numerical integration.

Rule for writing new differences. From the right-hand number in a row there is subtracted the left-hand number in the same row, but the result is written under the right-hand number.

Finally, from the last table of differences there follows the most disagreeable characteristic feature of the numerical integration of equations: in order to determine Δy_i, it is necessary to know the differences:

$$\Delta\Phi_{i-1}, \quad \Delta^2\Phi_{i-2}, \quad \Delta^3\Phi_{i-3},$$

i.e., to have the following particular values of the function:

$$\Phi_{i-1}, \quad \Phi_{i-2}, \quad \Phi_{i-3}.$$

These will become known in the course of the process; but at the start of the integration only one particular value of this function

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namely Φ_0, is known from the originally stated equation for y'.

In order to find

$$\Phi_{-1}, \quad \Phi_{-2}, \quad \Phi_{-3}$$

it is necessary to employ special supplementary methods, of which the most commonly used is the method of successive approximations. The essence of this method resides in the fact that, at the start of the computations, there is adopted a gradual and stepwise advance, each new approximation permitting one additional step, in which those differences that have already appeared earlier are utilized. There exist several variants of this method, one of which, the most exact one, can be best studied by the aid of a concrete example.

Example 6. Solve by the method of numerical integration the equation:

$$y' = x - y$$

over the interval 0; 0.5, with a step of $h = 0.1$, if $y = 1$ when $x = 0$.

Solution. All the computations are presented in Table 7.

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Solution. All the computations are presented in Table 7.

Table 7 – Integration of Equation $y' = x - y$.

1	x	0	0.1	0	0.1	0.2	0	0.1	0.2	0.3	0
2	$-y$	-1.0000	-0.9000		-0.9100	-0.8385		-0.9096	-0.8373	-0.7815	
3	y'	-1.0000	-0.8000		-0.8100	-0.6385		-0.8096	0.6373	-0.4815	
4	$\Phi = hy'$	-0.1000	-0.0800	-0.1000	-0.0810	-0.0638	-0.1000	-0.0810	-0.0637	-0.0482	-0.1000
5	$\Delta\Phi$	(200)	200	(208)	190	172	(206)	190	173	155	(-206)
6	$\Delta^2\Phi$			(-18)	(-18)	-18	(-15)	(-16)	-17	-18	(-15)
7	$\Delta^3\Phi$						(-1)	(-1)	(-1)	-1	(-1)
8	Φ	-0.1000		-0.1000	-0.0810		-0.1000	-0.0810	-0.0637		-0.1000
9	$\frac{1}{2}\Delta\Phi$			100	95		104	95	86		103
10	$\frac{5}{12}\Delta^2\Phi$						-8	-8	-7		-6
11	$\frac{3}{8}\Delta^3\Phi$										0
12	$\Delta y = \Sigma$	-0.1000		-0.0900	-0.0715		-0.0904	-0.0723	-0.0558		-0.0903
13	y	1.0000	0.9000	1.0000	0.9100	0.8385	1.0000	0.9096	0.8373	-0.7815	1.0000
		First approximation		Second approximation			Third approximation				

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Table 7 - Integration of Equation y' = x - y.

	0	0.1	0.2	0	0.1	0.2	0.3	0	0.1	0.2	0.3	0.4	0.5
		-0.9100	-0.8385		-0.9096	-0.8373	-0.7815		0.9097	0.8375	-0.7815	-0.7400	
		-0.8100	-0.6385		-0.8096	0.6373	-0.4815		-0.8097	0.6375	-0.4815	-0.3400	
	-0.1000 (208) (-18)	-0.0810 190 (-18)	-0.0638 172 -18	-0.1000 (206) (-15) (-1)	-0.0810 190 (-16) (-1)	-0.0637 173 -17 (-1)	-0.0482 155 -18 -1	-0.1000 (-206) (-15) (-1)	-0.0810 190 (-16) (-1)	-0.0638 172 -18 (1)	-0.0482 156 -16 1	-0.0340 142 -14 0	
	-0.1000 100	-0.0810 95		-0.1000 104 -8	-0.0810 95 -8	-0.0637 86 -7		-0.1000 103 -6 0	-0.0810 95 -7 0	-0.0638 86 -8 0	-0.0482 78 -7 0	-0.0340 71 -6 0	
	-0.0900 1.0000	-0.0715 0.9100	0.8385	-0.0904 1.0000	-0.0723 0.9096	-0.0558 0.8373	-0.7815	-0.0903 1.0000	-0.0722 0.9097	-0.0560 0.8375	-0.0411 0.7815	-0.0275 0.7404	0.7129
	Second approximation			**Third approximation**				**Normal computations**					

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2) $-y_1 = -0.9000$ in accordance with the thirteenth row.

3) $y_1' = -0.8000$; in accordance with the given equation for y' we add the numbers in the first and second rows of this column (above).

4) $\Phi_1 = -0.0800$; the number in the third row is multiplied by the step $h = 0.1$.

Thus a number appears in the fifth row; $\Delta\Phi_0 = 200$, for which we subtract from the number (-0.0800) in the fourth row above the number (-1.0000) at the left in the same row, omitting the zeros at the left for greater convenience.

Wedge of First Approximation.

It consists merely of a single number $\Delta\Phi_{-1}$ and is surrounded by a heavy line. To obtain it, we postulate that

$$\Delta\Phi_{-1} = \Delta\Phi_0 = 200.$$

Second Approximation

First Column.

13) Leading row: $\Phi_0 = 1$; 1) $x_0 = 0$.

The second and third rows are not filled, as this is no longer necessary (cf. first column of first approximation). Then:

4) $\Phi_0 = -0.1000$, as in the same row of the first approximation.

Omitting the fifth, sixth, and seventh rows, we write:

8) $\Phi_0 = -0.1000$; the number in the fourth row is repeated.

9) $\frac{1}{2}\Delta\Phi_{-1} = 100$; we take the difference $\Delta\Phi_{-1}$ from the wedge of the first approximation.

The tenth and eleventh rows are not filled for lack of necessary data.

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12) $\Delta y_0 = -0.0900$; we add the numbers in the eighth and ninth rows above.

The second column of this approximation is formed in an analogous manner, with the only difference that numbers appear in the first and second rows (cf. the number in the thirteenth row of this column) and in the fifth row, for which we subtract from the number (-0.0810) in the fourth row the number (-0.1000) in the same row, but on the left. The number in the leading row is obtained by adding the numbers in the twelfth and thirteenth rows of the preceding column.

The third column is distinguished from second in that a number (-17) appears in the sixth row, for which we subtract from the number 173 in the fifth row the number 190 at the left, and another number (-8) appears in the tenth row, for which the number in the sixth row is multiplied by $\frac{5}{12} = \frac{10}{24}$.

Filling in the Wedge of the Second Approximation.

In the sixth line we repeat twice the number (-18), noting that:

$$\Delta^2\Phi_{-2} = \Delta^2\Phi_{-1} = \Delta^2\Phi_0 = -18.$$

The number 208 in the fifth row of the first column of the second approximation will appear in accordance with the definition of the second differences:

$$\Delta^2\Phi_{-1} = \Delta\Phi_0 - \Delta\Phi_{-1},$$

from which:

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$$\Delta\Phi_{-1} = \Delta\Phi_{-0} - \Delta^2\Phi_1 = 190 - (-18) = 208.$$

Third Approximation.

In the first column, after the thirteenth and first rows are filled, the places in the second, third, fifth, sixth, and seventh rows are left blank. Then:

8) The number in the fourth row is repeated.

9) $\frac{1}{2}\Delta\Phi_{-1} = 104$; the number 208 in the wedge of the second approximation is halved.

10) $\frac{5}{12}\Delta^2\Phi_{-2} = -8$; the number (-18) in the wedge of the second approximation is multiplied by $\frac{10}{24}$.

The eleventh row is omitted. Thereupon:

12) The numbers in the eighth, ninth, and tenth rows above are added:

$$-0.1000 + 0.0104 - 0.0008 = -0.0904.$$

In the second column, after the thirteenth, first, second, third, fourth and fifth rows are filled, the sixth and seventh rows are omitted. Then:

8) The number in the fourth row is repeated.

9) $\frac{1}{2}\Delta\Phi_0 = 95$; the number in the fifth row of this column is halved.

10) $\frac{5}{12}\Delta^2\Phi_{-2} = -8$; the number (-18) in the second column of the preceding wedge is multiplied by $\frac{10}{24}$.

The eleventh row is likewise omitted. Thereupon:

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12) The numbers in the eighth, ninth, and tenth rows above are added.

In the third column, only the thirteenth, first, second, third, fourth, fifth and sixth rows are filled. There appears for the first time a number in the seventh row:

$$-18 - (-17) = -1.$$

Filling in the Wedge of the Third Approximation.

We make:

$$\Delta^3\Phi_{-3} = \Delta^3\Phi_{-2} = \Delta^3\Phi_{-1} = \Delta^3\Phi_0 = -1.$$

The number (-16) in the sixth row of the second column of the wedge will appear from the definition of the third differences:

$$\Delta^2\Phi_{-1} = \Delta^2\Phi_0 - \Delta^3\Phi_{-1} = -17 -(-1) = -16.$$

In an analogous manner, the blanks in the rows of the first column of the wedge will be filled as follows:

$$\Delta^2\Phi_{-2} = \Delta^2\Phi_{-1} - \Delta^3\Phi_{-2} = -16 - (-1) = -15;$$

$$\Delta^2\Phi_{-1} = \Delta\Phi_0 - \Delta^2\Phi_{-1} = 190 - (-16) = 206.$$

For convenience of computation, the numbers of this wedge are transferred to their places in the columns of the subsequent normal computations. The approximations are considered to be completed because of the closeness of the results of the third and second

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approximations, respectively.

Normal Computations.

First Column.

13) As before, $y_0 = 1.0000$; 2) $x_0 = 0$.

The second and third rows need not be filled. The numbers in the fifth, sixth, and seventh rows are already in place. Thereupon:

8) $\Phi_0 = -0.1000$; the number in the fourth row is repeated.

9) $\frac{1}{2}\Delta\Phi_{-1} = 103$; the number 206 in the fifth row is halved.

10) $\frac{5}{12}\Delta^2\Phi_{-2} = -6$; the number (-15) in the sixth row is multiplied by $\frac{10}{24}$.

11) $\frac{3}{8}\Delta^2\Phi_{-3} = 0$; the number (-1) in the seventh row is multiplied by $\frac{3}{8}$.

12) $\Delta y_0 = -0.0903$; the numbers in the eighth, ninth, tenth, and eleventh rows are added.

The subsequent columns are filled in exactly the same manner.

Attention is once again directed to the order of computation in each column of normal computations.

a) First of all, the leading thirteenth row is filled by adding the numbers in the twelfth and thirteenth rows of the preceding column, because

$$\Delta y_i = y_{i-1} + \Delta y_{i-1};$$

b) Thereupon, the spaces in each column are filled from top to bottom without omissions.

12) Use of Numerical Integration of the First-Order Equation with Argument v.

The method of numerical integration must be applied to the solution of the following equation:

$$\frac{dl}{dv} = \frac{1}{v_{np}^2} \frac{2v}{\psi - \frac{v^2}{v_{np}^2}} (l_\psi + l),$$

where:

$$l_\psi = l_\Delta - l_\alpha \psi.$$

In the physical law of powder burning the correlation between ψ and v is given by a table, whereas in the geometric law:

$$\psi = \psi_0 + \frac{\varkappa \sigma_0 \varphi m}{s I_k} v + \frac{\varkappa \lambda \varphi^2 m^2}{s^2 I_k^2} v^2.$$

Since it is also necessary as a rule to find the pressure curve, it is preferable to make use of the following equation:

$$\varphi m v \frac{dv}{dl} = ps,$$

whence

$$\frac{dl}{dv} = \frac{\varphi m}{s} \frac{v}{p},$$

where

$$p = \frac{f\omega}{s}\,\frac{\psi - \frac{v^2}{v_{np}^2}}{l_\psi + l}.$$

It is not difficult to see that the auxiliary function Φ is found in the given case from the following relations:

$$l_\psi = l_\Delta - l_\alpha\psi; \quad p = \frac{f\omega}{s}\,\frac{\psi - \frac{v^2}{v_{np}^2}}{l_\psi + l}; \qquad \Phi = \frac{\varphi m h}{s}\,\frac{v}{p}. \qquad (*)$$

As for the working formula of numerical integration, it has the usual form:

$$\Delta l_i = \Phi_i + \frac{1}{2}\Delta\Phi_{i-1} + \frac{5}{12}\Delta^2\Phi_{i-2} + \frac{3}{8}\Delta^3\Phi_{i-3}.$$

The purpose of the preliminary computations is to determine all the constants:

$$1)\ \frac{\omega}{s};\quad 2)\ \frac{\varphi m}{s} = \frac{q\varphi}{gs};\quad 3)\ v_{np}^2 = 2\,\frac{f}{\theta}\,\frac{\omega}{s} : \frac{\varphi m}{s};$$

$$4)\ l_\Delta = \frac{\omega}{s}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right);\quad 5)\ l_\alpha = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right);$$

$$6)\ \psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}};\quad 7)\ \frac{f\omega}{s}.$$

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To determine the step of integration h, the speed of the projectile v_K at the end of powder burning must be known, and then:

$$h = \frac{v_K}{n},$$

where n - the number of sections - is taken in the range of 10-40, depending upon the density of loading (as this density increases, the number n must also be increased).

While determining l and p, the same working form may be used to find the time of motion of the projectile in accordance with the following relation:

$$\frac{dt}{dv} = \frac{\varphi m}{s} \frac{1}{p},$$

in the form of a definite integral.

<u>Example 7.</u> Determine the projectile velocity and gas pressure curves for a 76 mm gun, given the following conditions: $W_0 = 1.654$; $s = 0.4693$; $l_\Delta = 18.44$; $q = 6.5$; $p_0 = 30,000$; $f = 900,000$; $\alpha = 1$; $\delta = 1.6$; $\theta = 0.2$; $\omega = 0.930$; $\varphi = 1.05$; $g = 98.1$, using strip powder type SP (1 x 18 x 40 mm). It is assumed that the powder burns in conformity with the physical law derived from the pressure-bomb test presented in Table 5 (correlation between I and ψ).

To start with, we find:

$$\log \frac{\omega}{s} = 0.2970; \quad \log \frac{\varphi m}{s} = \bar{1}.1709; \quad \log v_{np}^2 = \log\left(\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q}\right) = 8.0803;$$

$$l_\Delta = l_0\left(1 - \frac{\Delta}{\delta}\right) = 2.284; \ \log l_a = \bar{1}.8710; \ \psi_0 = 0.038; \ \log \frac{f\omega}{s} = 6.2512.$$

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From the value ψ_0 found we compute the value of t_0 by inverse interpolation with the aid of the data in Table 2 (cf. Example 2, Subsection 7)

$$t_0 = 0.020556 \text{ sec}$$

For this value of t_0 we determine the initial impulse I_0 by direct interpolation with the aid of Table 4 (cf. Example 1, Subsection 6):

$$I_0 = 76 \text{ kg}\cdot\text{sec}\cdot\text{dm}^{-2}$$

We find the velocity of the projectile at the end of burning of the powder, keeping in mind that $I_K = 823$ (cf. Table 5):

$$v_K = \frac{s}{\varphi m}(I_K - I_0) = 5040 \text{ dm}\cdot\text{sec}^{-1}$$

Bt choosing $n = 20$ as the number of sections, we obtain the step as being:

$$h = \frac{v_K}{20} = 252 \text{ dm}\cdot\text{sec}^{-1}$$

so that

$$\log\left(\frac{\varphi m}{s}h\right) = 1.5723; \quad \log\left(\frac{\varphi m}{s}\, h\cdot 10^5\right) = 6.5723.$$

On the basis of Table 5, we plot the (ψ, I) curve to the following scale: for ψ, 1 mm = 0.001; for I, 1 mm = 1 $\text{kg}\cdot\text{sec}\cdot\text{dm}^{-2}$.

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We read on it the values of the gas inflow ψ corresponding to equal intervals

$$\frac{823 - 76}{20} = 37.35 \text{ kg}\cdot\text{sec}\cdot\text{dm}^{-2}$$

for the pressure impulse, or, what is the same thing, to equal intervals $h = 252 \text{ dm}\cdot\text{sec}^{-1}$ for the velocity of the projectile.

These values of ψ are given subsequently in the working form used for performing the computations (in the fifth row).

Form for Computations with Argument v.

1	v	0	252	0	252	504	7
2	2 log v		4.8028			5.4048	5.7
3	$-\log v^2_{np}$		9.9197			9.9197	9.9
4	$\log \frac{v^2}{v^2_{np}}$		$\bar{4}$.7225			$\bar{3}$.3245	3.6
5	ψ	0.038	0.092			0.166	0.2
6	$-\frac{v^2}{v^2_{np}}$		-0.001			-0.002	-0.0
7	$\psi - \frac{v^2}{v^2_{np}}$		0.091			0.164	0.2
8	$\log l_\alpha$	1.8710	1.8710			1.8710	1.
9	$\log \psi$	2.5793	2.9638			1.2201	1.
10	$\log l_\alpha \psi$	2.4503	2.8348			1.0911	1.
11	l_Δ	2.284	2.284			2.284	2.
12	$-l_\alpha \psi$	-0.028	-0.068			-0.123	-0.
13	l_ψ	2.256	2.216		2.216	2.161	2.
14	l	0.000	0.000		0.064	0.262	0.
15	$l_\psi + l$	2.256	2.216		2.280	2.423	2.
16	$\log \frac{f\omega}{s}$	6.2512	6.2512		6.2512	6.2512	6.
17	$\left(\log \psi - \frac{v^2}{v^2_{np}}\right)$	2.5793	2.9590		2.9590	1.2148	1.
18	$-\log(l_\psi + l)$	1.6466	1.6544		1.6421	1.6257	1.
19	log p	4.4771	4.8646		4.8523	5.0817	5.
20	p	300	732		711	1207	
21	$\log\left(\frac{f\omega}{s} h\right)$		1.5723		1.5723	1.5723	1
22	log v		2.4014		2.401	2.7024	2
23	-log p		5.1354		5.1477	6.9183	6
24	log Φ		1.1091		1.121	1.1930	1
25	Φ	0	0.129	0.000	0.132	0.156	0
26	$\Delta\Phi$		129		132	24	
27	$\Delta^2\Phi$	$\boxed{(129)}$				-108	-
28	$\Delta^3\Phi$						

Form for Computations with Argument v.

0	252	0	252	504	756	1008	1260
	4.8028			5.4048	5.7570	6.0068	6.2008
	9.9197			9.9197	9.9197	9.9197	9.9197
	$\bar{4}$.7225			3.3245	3.6767	3.9265	2.1205
0.038	0.092			0.166	0.227	0.284	0.339
	-0.001			-0.002	-0.005	-0.009	-0.013
	0.091			0.164	0.222	0.275	0.326
1.8710	1.8710			1.8710	1.8710	1.8710	1.8710
2.5793	2.9638			1.2201	1.3560	1.4533	1.5302
2.4503	2.8348			1.0911	1.2270	1.3243	1.4012
2.284	2.284			2.284	2.284	2.284	2.284
-0.028	-0.068			-0.123	-0.169	0.211	0.252
2.256	2.216		2.216	2.161	2.115	2.073	2.032
0.000	0.000		0.064	0.262	0.385	0.573	0.789
2.256	2.216		2.280	2.423	2.500	2.646	2.821
6.2512	6.2512		6.2512	6.2512	6.2512	6.2512	6.2512
2.5793	2.9590		2.9590	1.2148	1.3464	1.4393	1.5132
1.6466	1.6544		1.6421	1.6257	1.6021	1.5774	1.5496
4.4771	4.8646		4.8523	5.0817	5.1997	5.2679	5.3150
300	732		711	1207	1586	1853	2065
	1.5723		1.5723	1.5723	1.5723	1.5723	1.5723
	2.4014		2.401[illegible]	2.7024	2.8785	[illegible].0034	3.1004
	5.1354		5.1477	6.9183	6.8003	6.7321	6.6850
	1.1091		1.121[illegible]	1.1930	1.2511	1.3078	1.3577
0	0.129	0.000	0.132	0.156	0.178	0.203	0.228
	129		132	24	22	25	25
(129)				-108	-2	3	0

20	p	300	732		711	1207	
21	$\log\left(\frac{4m}{s}h\right)$		1.5723		1.5723	1.5723	
22	$\log v$		2.4014		2.401[illegible]	2.7024	2
23	$-\log p$		5.1354		5.1477	6.9183	6
24	$\log \Phi$		1.1091		1.121[illegible]	1.1930	1
25	Φ	0	0.129	0.000	0.132	0.156	0
26	$\Delta\Phi$	$\boxed{(129)}$	129		132	24	
27	$\Delta^2\Phi$					-108	-
28	$\Delta^3\Phi$						
29	Φ	0		0.000	0.132	0.156	0
30	$\frac{1}{2}\Delta\Phi$			64	66	12	
31	$\frac{5}{12}\Delta^2\Phi$					-45	-
32	$\frac{3}{8}\Delta^3\Phi$						
33	$\Delta l = \Sigma$	0		0.064	0.198	0.123	0.
34	l	0	0	0.000	0.064	0.262	0.
35	$\log \frac{4m}{s} h \cdot 10^5$			6.5723	6.5723	6.5723	6.
36	$-\log p$			5.5229	5.1477	6.9183	6.
37	$\log F$			2.0952	1.7200	1.4906	1.
38	F			124	52	31	
39	ΔF			-72	-21	-7	-4
40	$\Delta^2 F$			51	14	3	
41	$\Delta^3 F$			-37			
42	F			124	52	31	
43	$\frac{1}{2}\Delta F$			-36	-10	-4	-2
44	$-\frac{1}{12}\Delta^2 F$			-4	-1		
45	$\frac{1}{24}\Delta^3 F$			-2			
46	$\Delta t \cdot 10^5 = \Sigma$			82	41	27	
47	$t \cdot 10^5$			0	82	123	

300	732		711	1207	1586	1853	2065
	1.5723		1.5723	1.5723	1.5723	1.5723	1.5723
	2.4014		2.401[illegible]	2.7024	2.8785	[illegible].0034	3.1004
	5.1354		5.1477	6.9183	6.8003	6.7321	6.6850
	1.1091		1.121[illegible]	1.1930	1.2511	1.3078	1.3577
0	0.129	0.000	0.132	0.156	0.178	0.203	0.228
89)	129		132	24	22	25	25
				-108	-2	3	0
		0.000	0.132	0.156	0.178	0.203	0.228
		64	66	12	11	12	12
				-45	-1	1	0
		0.064	0.198	0.123	0.188	0.216	0.240
0	0	0.000	0.064	0.262	0.385	0.573	0.789
		6.5723	6.5723	6.5723	6.5723	6.5723	6.5723
		5.5229	5.1477	6.9183	6.8003	6.7321	6.6850
		2.0952	1.7200	1.4906	1.3726	1.3044	1.2573
		124	52	31	24	20	18
		-72	-21	-7	-4	-2	-1
		51	14	3			
		-37					
		124	52	31	24	20	18
		-36	-10	-4	-2	-1	0
		-4	-1				
		-2					
		82	41	27	22	19	18
		0	82	123	150	172	191

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The working form can be broken down into three main sections. The upper section contains lines 1-24 and is reserved for computations necessary for determining the logarithm of the auxiliary function in accordance with the relations (*). Consequently, the pressure of the powder gases is also found at the same time. The values for the gas inflow in line 5 are first read from the (ψ, I) curve in the case of the physical law of burning of the powder or computed in accordance with the corresponding formula in the case of the geometric law of burning.

The upper section is the most involved part of the work and makes the use of four-place logarithms obligatory.

In the middle section, which consists of lines 25-34, are performed computations necessary for the use of the working formula for the numerical integration of a first-order equation:

$$\Delta l_i = \Phi_i + \frac{1}{2}\Delta\Phi_{i-1} + \frac{5}{12}\Delta^2\Phi_{i-2} + \frac{3}{8}\Delta^3\Phi_{i-3},$$

and for finding the path of the projectile: $l_i = l_{i-1} + \Delta l_{i-1}$.

Finally, the lower section, consisting of lines 35-47, is reserved for finding the time of motion of the projectile in the bore in the form of a definite integral, using the following working formula:

$$\Delta t_i = F_i + \frac{1}{2}\Delta F_i - \frac{1}{12}\Delta^2 F_i + \frac{1}{24}\Delta^3 F_i.$$

This section is best filled after completion of the numerical integration of the equation for the path of the projectile:

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$$\frac{dl}{dv} = \left(\frac{\varphi m}{s} h\right) \frac{v}{p},$$

i.e., after the two sections above it are filled.

It is useful to point out some of the characteristic features of the computations.

a) The path of the projectile is determined with an accuracy of 0.001 dm; the time t, within 0.00001 seconds.

b) To facilitate computations, only one approximation is made, and only one wedge is filled out (cf. line 26, first column of the working form).

c) At first, the differences of the auxiliary function up to and including the second order are introduced.

d) The third differences of this function are utilized only after p_m is passed.

e) The second difference of the function Φ appears only in the second column of the normal computations.

f) The rows which are not filled are: lines 2, 3, 4, 6, 7, 21, 22, 23, 24, 27, 28, 30, 31, 32, and 35-47 in the first column, lines 27-33 and 35-47 in the second column of the approximation, lines 2-24, 26, 27, 28, 31, and 32 of the first column of normal computations, and, finally, lines 2-12, 27, 28, 31, and 32 of the second column of normal computations.

g) Line 34 is the leading row and is filled first in each column.

For a more definite conception of the character of the work involved when filling in the two upper sections of the working form it would be desirable to describe in detail the computations performed for

one of the columns, for example, for the column of normal computations with $v = 504$.

34) The numbers in lines 33 and 34 of the preceding column are added:

$$l = 0.198 + 0.064 = 0.262.$$

since:

$$l_i = l_{i-1} + \Delta l_{i-1};$$

1) We add the integration step h:

$$v = 252 + h = 252 + 252 = 504;$$

2) The number in the preceding row, $v = 504$, gives:

$$2 \log v = 5.4048;$$

3) The complement of the logarithm of $v_{\Pi p}^2$ is taken:

$$-\log v_{\Pi p} = \overline{9}.9197;$$

4) The logarithms in lines 2 and 3 are added:

$$g \frac{v^2}{v_{\Pi p}^2} = 5.4048 + \overline{9}.9197 = \overline{3}.3245;$$

5) The value of ψ is taken from the (ψ, I) curve:

$$\psi = 0.166;$$

6) The logarithm in line 4 is used to find the number within 0.001:

$$\frac{v^2}{v^2_{np}} = 0.002;$$

7) The number in line 6 is subtracted from the number in line 5:

$$\psi - \frac{v^2}{v^2_{np}} = 0.166 - 0.002 = 0.164;$$

8) The logarithm of l_α has been found by preliminary computations:

$$\log l_\alpha = 1.8710;$$

9) The number in line 5 gives the logarithm:

$$\log \psi = \bar{1}.2201;$$

10) The logarithms in lines 8 and 9 above are added:

$$\log l_\alpha \psi = \bar{1}.8710 + \bar{1}.2201 = \bar{1}.0911;$$

11) The reduced length l_Δ has been found by preliminary computations:

$$l_\Delta = 2.284;$$

12) The logarithm in line 10 is used to find the number within 0.001:

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$$l_\alpha \psi = 0.123;$$

13) The number in line 12 is subtracted from the number in line 11:

$$l_\psi - l_\Delta - l_\alpha \psi = 2.284 - 0.123 = 2.161;$$

14) The number in the leading row, line 34, is taken:

$$l = 0.262,$$

15) The numbers in lines 13 and 14 are added:

$$l_\psi + l = 2.161 + 0.262 = 2.423;$$

16) This logarithm has been obtained by preliminary computations:

$$\log \frac{f\omega}{s} = 6.2512;$$

17) The logarithm of the number in line 7 is found:

$$\log \left(\psi - \frac{v^2}{v^2_{np}} \right) = \bar{1}.2148;$$

18) The complement of the logarithm of the number in line 15 is taken:

$$-\log(l_\psi + l) = -\log 2.423 = \bar{1}.6257;$$

19) The logarithms in lines 16, 17, and 18 are added:

$$\log p = 6.2512 + \bar{1}.2148 + \bar{1}.6257 = 5.0817;$$

20) The logarithm in line 19 is used to find the pressure at $v = 504$:

$$p = 1207 \ \mathrm{kg \cdot cm^{-2}}$$

21) This logarithm has been obtained by preliminary computations:

$$\log \left(\frac{\Psi D}{s} h \right) = 1.5723;$$

22) From the number in line 1 we have:

$$\log v = 2.7024$$

23) The complement of the logarithm in line 19 is taken:

$$-\log p = -5.0817 = \bar{6}.9183$$

24) The logarithms in lines 21, 22, and 23 are added:

$$\log \Phi = 1.5723 + 2.7024 + \bar{6}.9183 = \bar{1}.1930.$$

We now proceed to the next, middle, section.

25) The logarithm in line 24 is used to obtain the number with an accuracy of 0.001:

$$\Phi_2 = 0.156;$$

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26) From the number 0.156 in this column we subtract the number 0.132 in the same row of the preceding column (keeping in mind the rule governing the notation of differences):

$$\Delta\Phi_1 = 0.156 - 0.132 = 0.024.$$

and the notation is facilitated by omitting the zeros.

27) From the number 24 in this column we subtract the number 132 in the same row of the preceding column:

$$\Delta^2\Phi_0 = 24 - 132 = -108;$$

29) The number in line 25 is repeated:

$$\Phi_2 = 0.156;$$

30) The number in line 26 is halved:

$$\frac{1}{2}\Delta\Phi_1 = 12;$$

31) The number in line 27 is multiplied by $\frac{10}{24}$ (i.e., it is multiplied by ten and the result divided by 24):

$$\frac{5}{12}\Delta^2\Phi_0 = \frac{5}{12}(-108) = -45;$$

33) The numbers in lines 29, 30, and 31 are added:

$$\Delta l_2 = \Phi_2 + \frac{1}{2}\Delta\Phi_1 + \frac{5}{12}\Delta^2\Phi_0 = 0.123.$$

It is now possible to take the next step, i.e., proceed to the next column with v = 756, starting the computations therein, as always, from the leading row, line 34.

At the muzzle, we have: v_μ = 583 m/sec; p_μ = 625 kg/cm^2.

13) Numerical Integration of the Second-Order Differential Equation.

In the general case, a differential equation of the second order contains the following components: the independent variable x, the function y itself, the first derivative y' of the function y with respect to x, and the second derivative y" of the same function with respect to x. Consequently, this equation can be represented in the following form:

$$F(x, y, y', y") = 0;$$

where the symbol F represents an elementary function or a combination of such elementary functions.

Numerical integration makes it possible to solve this equation in any form, provided only that the given equation permits the determination of the second derivative as an explicit function of all the remaining variable quantities:

$$y" = f(x, y, y').$$

In this numerical integration the increment of the derivative $\Delta y'$ is determined by the aid of the following formula:

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$$\Delta y'_i = h\left(y''_i + \frac{1}{2}\Delta y''_{i-1} + \frac{5}{12}\Delta^2 y''_{i-2} + \frac{3}{8}\Delta^3 y''_{i-3}\right)$$

or

$$\Delta y'_i = \Phi_i + \frac{1}{2}\Delta\Phi_{i-1} + \frac{5}{12}\Delta^2\Phi_{i-2} + \frac{3}{8}\Delta^3\Phi_{i-3},$$

where use is made of the following new auxiliary function:

$$\Phi = hy'',$$

expressed in terms of the second derivative of the desired function. It is not necessary to derive the fundamental formula for $\Delta y'_i$ by means of y" and differences of this second derivative, inasmuch as this formula is already obtained from the previously derived relation

$$\Delta y_i = h\left(y'_i + \frac{1}{2}\Delta y'_{i-1} + \frac{5}{12}\Delta^2 y'_{i-2} + \frac{3}{8}\Delta^3 y'_{i-3}\right)$$

by simply replacing y_i by y'_i and y'_i by y''_i.

On the other hand it is necessary to derive a working formula for the increment Δy of the function itself in terms of differences of the second derivative rather than of the first. Such a formula should simplify the work, because in computing Δy it will be possible to make use of the already available differences of the auxiliary function Φ, and it will not be necessary to find the differences of still another auxiliary function:

$$F = hy'$$

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For this purpose, we shall utilize the equality

$$\Delta y_i = \int_{x_i}^{x_{i+1}} y'dx.$$

But in all cases

$$y' = y'_i + \Delta y' = y'_i + \int_{x_i}^{x} y''dx.$$

Let us replace here the derivative y" by the following interpolation function:

$$y'' = A_0(x - x_i)^3 + A_1(x - x_i)^2 + A_2(x - x_i) + A_3,$$

and then

$$y' = y'_i + \int_{x_i}^{x} \left[A_0(x - x_i)^3 + A_1(x - x_i)^2 + A_2(x - x_i) + A_3 \right] dx =$$

$$= y'_i + \frac{1}{4} A_0(x - x_i)^4 + \frac{1}{3} A_1(x - x_i)^3 + \frac{1}{2} A_2(x - x_i)^2 + A_3(x - x_i),$$

so that, consequently

$$\Delta y_i = \int_{x_i}^{x_{i+1}} \Big[y'_i + \frac{1}{4} A_0(x - x_i)^4 + \frac{1}{3} A_1(x - x_i)^3 +$$

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$$+ \frac{1}{2} A_2(x - x_i)^2 + A_3(x - x_i)\Big] dx = \Big| y_i'(x - x_i) + \frac{1}{20} A_0(x - x_i)^5 +$$

$$+ \frac{1}{12} A_1(x - x_i)^4 + \frac{1}{6} A_2(x - x_i)^3 + \frac{1}{2} A_3(x - x_i)^2 \Big|_{x_i}^{x_{i+1}},$$

whence

$$\Delta y_i = h\left[y_i' + \frac{1}{20} A_0 h^4 + \frac{1}{12} A_1 h^3 + \frac{1}{6} A_2 h^2 + \frac{1}{2} A_3 h\right],$$

because

$$x_{i+1} - x_i = h.$$

We shall substitute into the last expression the following values for the coefficients:

$$A_0 = \frac{\Delta^3 y''_{i-3}}{6h^3}; \qquad A_1 = \frac{\Delta^2 y''_{i-2} + \Delta^3 y''_{i-3}}{2h^2};$$

$$A_2 = \frac{6\Delta y''_{i-1} + 3\Delta^2 y''_{i-2} + 2\Delta^3 y''_{i-3}}{6h}; \quad A_3 = y''_i,$$

because the part of the function is here played by the derivative y", which has been replaced by the interpolation function.

After obvious transformations this substitution gives:

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$$\Delta y_i = h\left[y'_i + h\left(\frac{1}{2}\,y''_i + \frac{1}{6}\,\Delta y''_{i-1} + \frac{1}{8}\Delta^2 y''_{i-2} + \frac{19}{180}\,\Delta^3 y''_{i-3}\right)\right]$$

or

$$\Delta y_i = h\left[y'_i + h\left(\frac{1}{2}\,y''_i + \frac{1}{6}\Delta y''_{i-1} + \frac{1}{8}\,\Delta^2 y''_{i-2} + \frac{1}{10}\,\Delta^3 y''_{i-3}\right)\right],$$

if, instead of the inconvenient number $\frac{19}{180}$ we take the closely approching it number:

$$\frac{18}{180} = \frac{1}{10}$$

It remains necessary to make use once again of the auxiliary function:

$$\Phi = hy'',$$

and then, finally, in the numerical integration of the second-order differential equation, we shall have to deal with working formulas of the following type:

$$\Delta y'_i = \Phi_i + \frac{1}{2}\Delta\Phi_{i-1} + \frac{5}{12}\,\Delta^2\Phi_{i-2} + \frac{3}{8}\Delta^3\Phi_{i-3};$$

$$\Delta y_i = h\left(y'_i + \frac{1}{2}\Phi_i + \frac{1}{6}\Delta\,\Phi_{\,i-1} + \frac{1}{8}\,\Phi_{\,i-2} + \frac{1}{10}\Delta^3\Phi_{i-3}\right).$$

To simplify the computations, it is better, however, to conduct the numerical integration with so small a step h as would enable us

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to do without third differences.

At the start of the numerical integration, as before, most frequent use is made of the method of successive approximations.

The number of rows in the lower part of the form will increase (in comparison with the number of rows in the lower part of the form for the integration of the first-order equation) by at least seven rows, because in addition to the ten rows:

$$\Phi, \Delta\Phi, \Delta^2\Phi, \Delta^3\Phi, \Phi, \frac{1}{2}\Delta\Phi, \frac{5}{12}\Delta^2\Phi, \frac{3}{8}\Delta^3\Phi, \Sigma, \quad \Delta y' = h\Sigma, \; y',$$

corresponding to the lower part of the form for the numerical integration of the first-order equation, there will also appear the following additional rows:

$$\frac{1}{2}\Phi, \frac{1}{6}\Delta\Phi, \frac{1}{8}\Delta^2\Phi, \frac{1}{10}\Delta^3\Phi, \Sigma_1, \quad \Delta y = h\Sigma_1, \; y.$$

In the method under consideration, the computations themselves are in no way different from similar computations for the solution of the first-order equation, so that the need for citing a special example is obviated.

14) Use of Numerical Integration of Second-Order Equations with Argument t.

In this process the leading part is played by the equation for the forward motion of the projectile:

$$\varphi = \frac{dv}{dt} - sp,$$

from which it follows that

$$l_t'' = \frac{s}{\varphi m}\, p, \qquad (107)$$

because

$$\frac{dv}{dt} = v_t' = \left(\frac{dl}{dt}\right)_t' = l_t'',$$

if the phenomenon of recoil is not considered in the explicit form.

Equation (107) is subject to numerical integration by means of the following relations:

$$p = \frac{f\omega}{s}\,\frac{\psi - \dfrac{v^2}{v_{np}^2}}{l_\psi + l} \quad \text{and} \quad l_\psi = l_\Delta - l_\alpha \psi,$$

which have already been employed earlier. The auxiliary function Φ is equal in this case to:

$$\Phi = h l_t'' = \frac{sh}{\varphi m}\, p.$$

where the step h is already a certain time interval of the motion of the projectile in the bore. This step is chosen in advance.

The working formulas for the numerical integration will be:

$$\Delta v_i = \Phi_i + \frac{1}{2}\Delta\Phi_{i-1} + \frac{5}{12}\Delta^2\Phi_{i-2} + \frac{3}{8}\Delta^3\Phi_{i-3};$$

$$\Delta l_i = h\left(v_i + \frac{1}{2}\Phi_i + \frac{1}{6}\Delta\Phi_{i-1} + \frac{1}{8}\Delta^2\Phi_{i-2} + \frac{1}{10}\Delta^3\Phi_{i-3}\right),$$

because the function y to be determined represents in this case the relative path l of the projectile in the bore, and its derivative y' is the velocity of the projectile v.

The preliminary computations are reduced to the determination of constants:

1) $\log \frac{\omega}{s}$; 2) $\log \frac{\varphi m}{s}$; 3) $\log v^2_{\Pi p}$; 4) l_Λ ; 5) $\log l_\alpha$; 6) ψ_0;

7) $\log \frac{f\omega}{s}$,

as in the case of numerical integration of a first-order equation.

The values of I_0, I_K and v_K are found in an analogous manner. The gas inflow ψ is read off the (I, ψ) curve, but, in contrast with the preceding case, it is necessary here to find for each point, during the process of integration itself:

$$I = I_0 + \frac{\varphi m}{s} v,$$

because the integration gives values of the velocity v which are separated from one another by unequal intervals. Once the value of the pressure impulse is had, we can obtain the required value of ψ from the I, ψ curve. Consequently, this curve cannot be utilized in advance, but it must be available in the course of the entire work of numerical integration.

In the case of the geometric law of burning, the I, ψ curve is replaced by the following relation:

$$\psi = \psi_0 + \frac{\varkappa \sigma_0 \varphi m}{s I_K} v + \frac{\varkappa \lambda \varphi^2 m^2}{s^2 I_K^2} v^2.$$

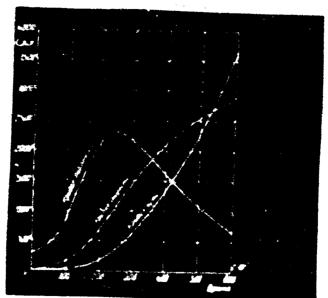

Fig. 153 - Velocity, Path, and Pressure Curves.

1) Time; 2) pressure curve; 3) velocity curve;
4) path curve.

Figure 153 contains curves for p, v, and l as functions of time t, obtained by numerical integration with respect to the argument t.

Computation Form with Argument t.

1	$t.10^5$	0	25	0	25	50	0	25	50
2	$\log \frac{\varphi m}{s}$		1.1709		1.1709	1.1709			1.17
3	$\log v$		1.7076		1.7404	2.0755			2.08
4	$\log \frac{\varphi m}{s} v$		0.8785		0.9113	1.2464			1.25
5	$\frac{\varphi m}{s} v$		8		8	18			18
6	I_0		76		76	76			
7	I	76	84		84	94			
8	$2 \log v$		3.4152		3.4808	4.1510			
9	$-\log v^2_{np}$		9.9197		9.9197	9.9197			
	$\log \frac{v^2}{v^2_{np}}$		5.3349		5.4005	4.0707			
11	ψ		0.045		0.045	0.057			
12	$-\frac{v^2}{v^2_{np}}$		0.000		0.000	0.000			
13	$\psi - \frac{v^2}{v^2_{np}}$	0.038	0.045		0.045	0.057			
14	$\log l_\alpha$	1.8710	1.8710		1.8710	1.8710			
15	$\log \psi$	2.5793	2.6532		2.6532	2.7559			
16	$\log l_\alpha \psi$	2.4503	2.5242		2.5242	2.6269			
17	l_Δ	2.284	2.284		2.284	2.284			
18	$-l_\alpha \psi$	-0.028	-0.033		-0.033	-0.042			
19	l_ψ	2.256	2.251		2.251	2.242			
20	l	0	0.006		0.007	0.029			
	$l_\psi + l$	2.256	2.257		2.258	2.271			
	$\log \frac{f\omega}{s}$	6.2512	6.2512		6.2512	6.2512			
23	$\log\left(\psi - \frac{v^2}{\ldots}\right)$	[illegible]							

Computation Form with Argument t.

25	0	25	50	0	25	50	75	100	125
1.1709		1.1709	1.1709			1.1709	1.1709	1.1709	1.1709
1.7076		1.7404	2.0755			2.0864	2.3160	2.5211	2.7152
0.8785		0.9113	1.2464			1.2575	1.4869	1.6920	1.8861
8		8	18			18	31	49	77
76		76	76				76	76	76
84		84	94				107	125	153
3.4152		3.4808	4.1510				4.6320	5.0422	5.4304
9.9197		9.9197	9.9197				9.9197	9.9197	9.9197
5.3349		5.4005	4.0707				4.5517	4.9619	3.3501
0.045		0.045	0.057				0.079	0.119	0.172
0.000		0.000	0.000				0.000	-0.001	-0.002
0.045		0.045	0.057				0.079	0.118	0.170
.8710		1.8710	1.8710				1.8710	1.8710	1.8710
.6532		2.6532	2.7559				2.8976	1.0755	1.2355
2.5242		2.5242	2.6269				2.7686	2.9465	1.1065
2.284		2.284	2.284				2.284	2.284	2.284
-0.033		-0.033	-0.042				-0.059	-0.088	-0.128
2.251		2.251	2.242				2.225	2.195	2.156
0.006		0.007	0.029				0.069	0.135	0.239
2.257		2.258	2.271				2.294	2.330	2.395
6.251		6.2512	6.2512				6.2512	6.2512	6.2512

			0.006		0.007	0.029			
21	$l_\psi + l$	2.256	2.257		2.258	2.271			
22	$\log \frac{f\omega}{s}$	6.2512	6.2512		6.2512	6.2512			
23	$\log\left(\psi - \frac{v^2}{v^2_{\text{II}p}}\right)$	2.5793	2.6532		2.6532	2.7559			
24	$-\log(l_\psi + l)$	1.6466	1.646[illegible]		1.6463	1.6438			
25	$\log p$	4.4771	4.5508		4.5509	4.6509			
26	$\log\left(\frac{sh}{\varphi m}\right)$	3.2270	3.2270		3.2370	3.2270			
27	$\log \Phi$	1.7041	1.7778		1.7779	1.8779			
28	p kg·cm^{-2}	300	355	300	355	448	300	355	
29	Φ	51	60	51	60	76	51	60	
30	$\Delta\Phi$	$\boxed{(9)}$	9	(2)	9	16	(2)	9	
31	$\Delta^2\Phi$			(7)	(7)	7	(7)	(7)	
32	$\Delta^3\Phi$								
33	Φ	51		51	60		51	60	
34	$\frac{1}{2}\Delta\Phi$			4	4		1	4	
35	$\frac{5}{12}\Delta^2\Phi$						2	2	
36	$\frac{3}{8}\Delta^3\Phi$								
37	$\Delta v = \Sigma$	51		55	64		54	66	
38	v	0	51	0	55	119	0	54	
39	$\frac{1}{2}\Phi$	26		26	30		26	30	
40	$\frac{1}{6}\Delta\Phi$			2	2		0	2	
41	$\frac{1}{8}\Delta^2\Phi$						1	1	
42	Σ_1	26		28	87		27	87	
43	$\Delta l = h\Sigma_1$	0.006		0.007	0.0022		0.007	0.022	0
44	l	0.000	0.006	0.000	0.007	0.029	0.000	0.007	0

		[illegible]	0.029				0.069	0.135	[illegible]
		2.258	2.271				2.294	2.330	2.395
		6.2512	6.2512				6.2512	6.[illegible]512	6.2512
		2.6532	2.7559				2.8976	1.0719	1.2304
		1.6463	1.6438				1.6394	1.[illegible]26	1.6209
8		4.5509	4.6509				4.7882	4.9557	5.1025
0		3.2370	3.2270				3.2270	3.2270	3.2270
8		1.7779	1.8779				2.0150	2.1827	2.3295
	300	355	448	300	355	448	614	902	1266
	51	60	76	51	60	76	104	152	214
	(2)	9	16	(2)	9	16	28	48	62
	(7)	(7)	7	(7)	(7)	7	12	20	14
							5	8	-6
	51	60		51	60	76	104	152	214
	4	4		1	4	8	14	24	31
				2	2	3	5	8	6
							2	3	-2
	55	64		54	66	87	125	187	249
	0	55	119	0	54	120	207	332	519
	26	30		26	30	38	52	76	107
	2	2		0	2	3	5	8	10
				1	1	1	2	2	2
	28	87		27	87	162	266	418	638
	0.007	0.0022		0.007	0.022	0.040	0.066	0.104	0.159
	0.000	0.007	0.029	0.000	0.007	0.029	0.069	0.135	0.239

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CHAPTER 2. SOLUTION BY EXPANSION IN TAYLOR'S SERIES

Integration of equations of internal ballistics describing the relations existing between the fundamental elements of a shot leads to rather complex integrals, which can be solved only by means of quadratures with any desired degree of accuracy (Professor Drozdov's solution). In this connection, in order to make integration possible, some of the variable parameters in the fundamental equations (θ, u_1, p_0, etc.) are usually assumed to be constant.

The method of numerical integration makes it possible to solve a system of differential equations not only without simplifying the functions entering same, but even by assigning variable values to those quantities which are usually assumed to be constant. This makes it possible to solve the problem on the basis of any desired hypotheses concerning the character of burning of the powder, the law of resistance to motion, the design of the bore, etc.

In solving the problem by the method of numerical integration, it is necessary to proceed successively from one value of the argument to another by the addition of its finite differences, starting at the very beginning. For this reason, for example, it is not possible to determine in advance the values of p_m or the values of the variables v, l and p corresponding to the end of burning of the powder, it being necessary, instead, to compute successively, point by point, the elements of the curves of pressure p, the path of the projectile l, its velocity v, etc. This constitutes the disadvantage of this method. Moreover, the method of numerical integration gives the relation between the individual variables only in the form of numerical tables, rather than in the form of analytical formulas.

In spite of these disadvantages, the method of numerical integration may serve as a means for indirect verification of the degree of accuracy obtained by the aid of the various approximate analytical methods available.

In this connection, when developing new theoretical problems, numerical integration may be utilized for determining the errors involved in the transition from exact equations and relations to others that are less exact but more convenient from the analytical point of view. Numerical integration may be employed with equal success both in the case of the geometric law of burning and in the case of the more complex physical law of burning; it may also be applied to barrels having a bore of variable cross section.

In the USSR the method of numerical integration was first applied to the solution of ballistic problems by V.M. Trofimov in 1918. This method has been developed in great detail by G.V. Oppokov, who employed the method of finite differences (1931-1938) discussed above.

In 1932 P.V. Melentyev proposed to apply the method of expansion in Taylor's series for the numerical solution of equations in ballistics, and this method, after being subjected to certain modifications, has been found to be sufficiently convenient.

Investigation of the fundamental relations expressing the conditions accompanying a shot shows that all the principal elements (l, v, ψ, and p) can be expressed in one form or another as functions of the path l and of its derivatives with respect to time up to the third order inclusive.

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As a matter of fact, taking the fundamental system of equations expressing the relationship between the elements of a shot, we obtain the following.

1) The fundamental equation of pyrodynamics or the equation of transformation of energy:

$$ps(l_\psi + l) = f\omega\psi - \frac{\theta}{2}\varphi m v^2.$$

2) The equations expressing the law of burning of the powder:

$$\psi = \varkappa z(1 + \lambda z);$$

$$\frac{de}{dt} = u = u_1 p;$$

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\sigma u_1 p = \frac{\varkappa}{e_1}\sigma u_1 p.$$

3) The equation of motion:

$$ps = \varphi m\frac{dv}{dt} = \varphi m\frac{d^2 l}{dt^2} = \varphi m l''_t,$$

where

$$v = \frac{dl}{dt} = l'_t,$$

v being related to z by the following equation:

$$v = \frac{sI_K}{\varphi m}(z - z_0),$$

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whence

$$z = z_0 + \frac{\varphi m}{sI_K} v.$$

All the variables entering into the fundamental system of equations can be expressed in terms of the path l and of its derivatives, since $v = l'_t$, $z = z_0 + \frac{\varphi m}{sI_K} l'_t$, ψ - being a function of z - will also be expressed as a function of l'_t, the pressure is proportional to l''_t, and the derivative $\frac{dp}{dt}$ is proportional to l'''_t. Consequently, if the time t of travel of the projectile through the bore is taken as the independent variable, and the path of the projectile l is taken as the function to be expanded in a series, it becomes possible to employ Taylor's series for finding the value of the path l_{n+1} and of its derivatives for the neighboring segment corresponding to the time $t_{n+1} = t_n + \Delta t = t_n + h$, provided that the values of the path l_n and of its derivatives for the preceding instant t_n are known. It is thus possible to find all the elements of burning of the powder and of the motion of the projectile during a shot, i.e., z, ψ, v, p, l, and t.

Let it be assumed that for a certain instant of time t_n the path l_n and its derivatives with respect to time l'_n, l''_n, l'''_n,... are known; if a sufficiently small increment of time $\Delta t = h$ is assumed and consideration is limited to derivatives up to and including the third order, then, in accordance with Taylor's formula, we shall have for $t_{n+1} = t_n + h$:

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$$l_{n+1} = l_n + h l'_n + \frac{h^2}{2} l''_n + \frac{h^3}{2.3} l'''_n . \qquad (108)$$

Differentiating with respect to t and rejecting the terms containing l^{IV}_n, i.e., considering that l'''_n is constant over the given interval Δt and equals its mean value, we obtain:

$$l'_{n+1} = l'_n + h l''_n + \frac{h^2}{2} l'''_n ;$$

$$l''_{n+1} = l''_n + h l'''_{n\ av.} = l''_n + h \frac{l'''_n + l'''_{n+1}}{2},$$

where $\frac{l'''_n + l'''_{n+1}}{2}$ is the mean value of the third derivative in the interval under consideration (Fig. 154). From the last equation we obtain the following value for l'''_{n+1}: $l'''_{n+1} = \frac{2}{h} l''_{n+1} - \frac{2}{h} l''_n - l'''_n$.

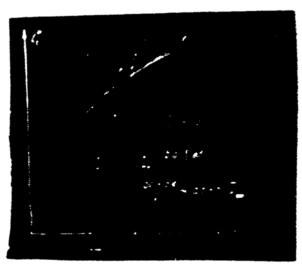

Fig. 154 - Diagram for l''_t and l'''_t.

As has been shown by P.V. Melentyev, it is more convenient to compute not the derivatives themselves, but, rather, the quantities proportional to them, namely: hl', $h^2 l''$ and $\frac{h^3}{2} l'''$. Therefore, by multiplying l'_{n+1} by h and l'''_{n+1} by $\frac{h^3}{2}$, we will obtain the following equations:

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$$h\,l'_{n+1} = h\,l'_n + h^2 l''_n + \frac{h^3}{2}\,l'''_n; \tag{109}$$

$$\frac{h^3}{2}\,l'''_{n+1} = h^2 l''_{n+1} - h^2 l''_n - \frac{h^3}{2}\,l'''_n. \tag{110}$$

Comparing these two equations with the initial equation (108), it will be seen that l enters everywhere without a coefficient, l' enters with the coefficient h, l'' enters with the coefficient h^2, and l''' enters with the coefficient $\frac{h^3}{2}$. This considerably accelerates the subsequent computations.

The expression for the second derivative l''_t will be:

$$l''_t = \frac{s}{\varphi m}\,p$$

or, multiplying by h^2:

$$h^2 l''_t = h^2\,\frac{s}{\varphi m}\,p. \tag{111}$$

By combining the resulting values for the path l and its derivatives with the equations of the fundamental system, we obtain the totality of formulas necessary for the solution in the following form and sequence, which corresponds to the order of their application, the constants encountered being designated below as follows:

$$\frac{\varphi m}{sI_K} = k_1, \quad \frac{f\omega}{s} = k_2, \quad \frac{\theta\varphi m}{2f\omega} = \frac{1}{v^2_{np}} = k_3, \quad \frac{s}{\varphi m} = k_4, \quad \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right) = a.$$

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$$\begin{cases} h l'_{n+1} = h l'_n + h^2 l''_n + \frac{h^3}{2} l'''_n ; & \text{(I)} \\ v_{n+1} = \frac{h l'_{n+1}}{h} ; & \text{(II)} \\ z_{n+1} = z_0 + \frac{\varphi m}{s I_K} v_{n+1} = z_0 + k_1 v_{n+1} ; & \text{(III)} \\ \psi_{n+1} = \varkappa z_{n+1} + \varkappa \lambda z^2_{n+1} ; & \text{(IV)} \\ l_{n+1} = l_n + h l'_n + \frac{1}{2} h^2 l''_n + \frac{1}{3} \frac{h^3}{2} l'''_n ; & \text{(V)} \\ p_{n+1} = \frac{f\omega}{s} \frac{\psi_{n+1} - \frac{v^2_{n+1}}{v^2_{np}}}{l_{\psi n+1} + l_{n+1}} - k_2 \frac{\psi_{n+1} - k_3 v^2_{n+1}}{l_\Delta - a\psi_{n+1} + l_{n+1}} ; & \text{(VI)} \\ h^2 l''_{n+1} = h^2 \frac{s}{\varphi m} p_{n+1} = h^2 k_4 p_{n+1} ; & \text{(VII)} \\ \frac{h^3}{2} l'''_{n+1} = h^2 l''_{n+1} - h^2 l''_n - \frac{h^3}{2} l'''_n . & \text{(VIII)} \end{cases}$$

The subscript (n+1) designates those values of the derivatives at the end of the given interval of time which, in the process of computation, are transferred from the column being computed into the corresponding rows of the right-hand neighboring column; however, the transferred values now bear the index n because they characterize the initial value of the given quantity in the next column.

In order to perform the computation by means of this totality of formulas, the values of l and of its derivatives at the start of the projectile's motion, i.e., at the instant $t = 0$, must be known. Since at the start of motion the path l and the speed v are equal to zero, we obtain:

$$(l)_0 = 0, \quad (l')_0 = (v)_0 = 0, \quad (l'')_0 = k_4 p_0, \; h^2(l'')_0 = h^2 k_4 p_0,$$

where p_0 is the forcing pressure usually specified beforehand. As regards the third derivative $(l''')_0$, we shall first find an expression for it at the present instant in the form of l'''.

To determine l''', we differentiate the equation $l_t'' = k_4 p$ with respect to t:

$$l_t''' = k_4 p_t'.$$

But the quantity p_t' has already been derived:

$$p_t' = \frac{p}{l_\psi + l}\left[\frac{f\omega}{s}\,\frac{\varkappa s}{I_K}\left(1 + \frac{p}{f\delta_1}\right) - v(1 + \theta)\right],$$

for the start of motion when $l = 0$, $v = 0$, $p = p_0$, and $\psi = \psi_0$; we will therefore obtain: $(p_t')_0 = \frac{p_0}{l_{\psi_0}}\,\frac{f\omega}{s}\,\frac{\varkappa s_0}{I_K}\left(1 + \frac{p_0}{f\delta_1}\right) = k_2\,\frac{\varkappa\sigma_0}{I_K}\left(1 + \frac{p_0}{f\delta_1}\right)\frac{p_0}{l_{\psi_0}}$.

The quantity $\left(1 + \frac{p_0}{f\delta_1}\right) = \frac{l_\Delta}{l_{\psi_0}}$, and this expression is therefore

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sometimes given in the following form:

$$(p_t')_0 = k_2 \frac{\varkappa\sigma_0}{I_K} \frac{l_\Delta}{l_{\psi_0}^2} p_0.$$

Consequently, the value of the third derivative for the initial instant is likewise known:

$$\frac{h^3}{2} l_0''' = \frac{h^3}{2} k_4 (p_t')_0 = \frac{h^3}{2} k_4 k_2 \frac{\varkappa\sigma_0}{I_K} \frac{l_\Delta}{l_{\psi_0}^2} p_0,$$

and it is possible to begin the successive solution of System (I) first for the first column corresponding to the interval of time $\Delta t = h$, then for the second column, etc., thus obtaining a successive series of values for l, v, z, ψ and p as functions of t.

The quantity $h = \Delta t$ must be so chosen as to obtain 10-15 columns for the period of burning of the powder, which will give a corresponding number of points for each of the quantities p, v, l, and ψ.

Since the time of burning is fundamentally determined by the thickness of the powder, the interval of time $h = \Delta t$ may be taken approximately according to the formula:

$$h \approx 0.001\ e_1,$$

where $e_1 = \frac{1}{2}$ the thickness of the powder in millimeters, h being rounded off to one or two significant figures (to 5 in the second significant figure). For example, if $2e_1 = 1.28$ mm:

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$h = 0.001\ (0.64) = 0.00064 = 0.0006$ or 0.00065

Since v_Δ and l_Δ are known at least approximately in advance, it is possible, after computing the average time of motion of the projectile $t_{av.} = \frac{2l_\Delta}{v_\Delta}$, to take for the value of the time step (increment) $\Delta t = h \approx \frac{t_{av.}}{15}$, rounded off to two significant figures.

Sequence of Computation. All the constants are computed first:

$$\varkappa, \lambda, \varkappa\lambda, \Lambda, \psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \frac{1}{\delta_1}}; \quad \sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0};$$

$$z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)}; \quad I_K = \frac{e_1}{u_1}; \quad l_0 = \frac{W_0}{s}; \quad l_\Delta = \frac{1}{s}\left(W_0 - \frac{\omega}{\delta}\right) = l_0\left(1 - \frac{\Delta}{\delta}\right);$$

$$a = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right) = \frac{\omega}{s\delta_1}; \quad l_{\psi_0} = l_\Delta - a\psi_0; \quad \varphi m = \frac{\varphi q}{98.1}; \quad h \approx 0.001\ e_1;$$

$$k_1 = \frac{\varphi m}{sI_K}; \quad k_2 = \frac{f\omega}{s}; \quad k_3 = \frac{\theta\varphi m}{2f\omega} = \frac{1}{v_{np}^2} \text{ (small quantity)}; \quad k_4 = \frac{s}{\varphi m};$$

$$h^2 l_0'' = h^2 k_4 p_0; \quad \frac{h^3}{2} l_0''' = \frac{h^3}{2} k_4 k_2 \frac{\varkappa\sigma_0}{I_K} \frac{l_\Delta}{l_{\psi_0}^2} p_0 = \frac{h^3}{2} k_4 \frac{\varkappa\sigma_0}{I_K}\left(1 + \frac{p_0}{f\delta_1}\right)\frac{p_0^2}{\psi_0}.$$

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The sequence of computation is not affected regardless of whether the computation is performed for a degressive or a progressive powder.

In computing the segment after the decomposition of the progressive powder, it is necessary to substitute for the usual formula the following previously derived formula:

$$\psi = \psi_s + \varkappa_z(z - 1)[1 + \lambda_2(z - 1)] = \psi_s + \varkappa_2(z - 1) + \varkappa_2\lambda_2(z - 1),$$

where z varies from 1 to $1 + \frac{\rho}{e_1}$, and $\varkappa_2$ and λ_2 are characteristics of the powder form after decomposition.

A form for conducting such computations is presented on pages 691-692.

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Table 7-c - Computation Form for the Solution of Problems in Internal Ballistics by Expansion in Taylor's Series.

Subscript n: 0 1 2 3

Computation Formulas		Column No.	1	2	3	Remarks
		Time $t_{n+1} = (n + 1)h$	0.0008	0.0016	0.0024	
$h = 0.0008$	1	$h\ell'_n$	0	1.1322	0.3856	From line 4 of preceding column
$h\ell'_{n+1} = h\ell'_n + h^2\ell''_n + \frac{h^3}{2}\ell'''_n$	2	$h^2\ell''_n$	0.0958	0.1928	0.3546	From line 25 of preceding column. Into line 26 of the given column.
$h\ell''_0 = 0$	3	$\frac{h^3}{2}\ell'''_n$	0.0364	0.0606	0.1012	From line 28 of preceding column. Into line 27 of the given column.
$h^2\ell''_0 = 0.0958$	4	$h\ell'_{n+1}$	0.1322	0.3856	0.8414	Into line 1 of next column. Into line 16 of next column.
$\frac{h^3}{2}\ell'''_0 = 0.0364$						
$v_{n+1} = \frac{h\ell'_{n+1}}{h}$	5	$v_{n+1} = \frac{h\ell'_{n+1}}{h}$	165.3	482		
$z_{n+1} = z_0 + k_1v_{n+1}$	6	k_1v_{n+1}	0.0308	0.0899		
$k_1 = 0.0001864$	7	$+ z_0$	0.0297	0.0297	0.0297	In all columns.
	8	z_{n+1}	0.0605	0.1196		
$\psi_{n+1} = \varkappa z_{n+1} + \varkappa\lambda z^2_{n+1}$	9	$\varkappa z_{n+1}$	0.0641	0.1268		
$\varkappa = 1.06$		+				
$\varkappa\lambda = -0.06$	10	$\varkappa\lambda z^2_{n+1}$	0.0002	0.0009		
	11	ψ_{n+1}	0.0639	0.1259		
		−				
	12	$k_3v^2_{n+1}$	0.0002	0.0016		

$l_0'' = 0.0958$ $l_0''' = 0.0364$	4	$h l'_{n+1}$	[illegible]			Into line 16 of next column.
$_{n+1} = \frac{h l'_{n+1}}{h}$	5	$v_{n+1} = \frac{h l'_{n+1}}{h}$	165.3	482		
$_{n+1} = z_0 + k_1 v_{n+1}$ $_1 = 0.0001864$	6	$k_1 v_{n+1}$	0.0308	0.0899		
	7	$+ \; z_0$	0.0297	0.0297	0.0297	In all columns.
	8	z_{n+1}	0.0605	0.1196		
$\psi_{n+1} = \varkappa z_{n+1} + \varkappa\lambda z^2_{n+1}$ $\varkappa = 1.06$ $\varkappa\lambda = -0.06$	9	$\varkappa z_{n+1}$	0.0641	0.1268		
	10	$+ \; \varkappa\lambda z^2_{n+1}$	0.0002	0.0009		
	11	ψ_{n+1}	0.0639	0.1259		
	12	$- \; k_3 v^2_{n+1}$	0.0002	0.0016		
$k_3 = 0.087030$ $k_2 = 2,850,000$	13	$\psi_{n+1} - k_3 v^2_{n+1}$	0.0637	0.1243		
	14	$k_2(\psi_{n+1} - k_3 v^2_{n+1}) = A_{n+1}$	181500	354300		

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Table 7-c (Cont'd.)

Computation Formulas		n	0	1	2	3 Remarks
		Column No.	1	2	3	Remarks
		Time $t_{n+1} = (n + 1)h$	0.0008	0.0016	0.0024	
	15	l_n	0	0.0600	0.3088	From line 19 of preceding column
$p_{n+1} = k_2 \dfrac{\psi_{n+1} - k_3 v^2_{n+1}}{l_\Delta - a\psi_{n+1} + l_{n+1}}$	16	$h l'_n$	0	0.1322	0.3856	From line 4 of preceding column.
$l_{n+1} = l_n + h l'_n + \frac{1}{2}h^2 l''_n + \frac{1}{3}h^3 l'''_n$	17	$+ \frac{1}{2} h^2 l''_n$	0.0479	0.0964	0.1773	$\frac{1}{2}$ (of line 2 of the given column)
$(l)_0 = 0;\ h l'_0 = 0$	18	$\frac{1}{3} \frac{h^3}{2} l'''_n$	0.0121	0.0202	0.0337	$\frac{1}{3}$ (of line 3 of the given column)
	19	l_{n+1}	0.0600	0.3088	0.9054	Into line 15 of next column
	20	$+ l_\Delta$	3.016	3.016	3.016	In all columns
$a = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right) = 1.065$	21	$l_{n+1} + l_\Delta$	3.076	3.325		
	22	$- a\psi_{n+1}$	0.068	0.134		
	23	$B_{n+1} = l_{n+1} + l_\Delta - a\psi_{n+1}$	3.008	3.191		
	24	$p_{n+1} = \frac{A_{n+1}}{B_{n+1}}$ kg/cm^2	604	1100		
$h^2 l''_{n+1} = k_4 h^2 p_{n+1}$	25	$h^2 l''_{n+1}$	0.1928	0.3546	-	Into lines 2 and 26 of next column
	26	$h^2 l''_n$	0.0958	0.1928	0.3546	From line 25 of preceding

		$\frac{}{3}\ \frac{}{2}l'''_n$	0.0121	0.0202	0.0337	$\frac{1}{3}$ (of line 3 of the given column)
	19	l_{n+1}	0.0600	0.3088	0.9054	Into line 15 of next column
	20	$+\ l_\Delta$	3.016	3.016	3.016	In all columns
$a = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right) = 1.065$	21	$l_{n+1} + l_\Delta$	3.076	3.325		
	22	$-\ a\psi_{n+1}$	0.068	0.134		
	23	$B_{n+1} = l_{n+1} + l_\Delta - a\psi_{n+1}$	3.008	3.191		
	24	$p_{n+1} = \frac{A_{n+1}}{B_{n+1}}$ kg/cm^2	604	1100		
$h^2 l''_{n+1} = k_4 h^2 p_{n+1}$	25	$h^2 l''_{n+1}$	0.1928	0.3546	-	Into lines 2 and 26 of next column
$k_4 h^2 = 0.0531192$	26	$-h^2 l''_n$	-0.0958	-0.1928	-0.3546	From line 25 of preceding column
$\frac{h^3}{2} l'''_{n+1} = h^2 l''_{n+1} - h^2 l''_n - \frac{h^3}{2} l'''_n$	27	$-\frac{h^3}{2} l'''_n$	-0.0364	-0.0606	-0.1012	From line 28 of preceding column
	28	$\frac{h^3}{2} l'''_{n+1}$	0.0606	0.1012		Into lines 3 and 27 of next column

Formulas for the second period:

$$v = v_{np}\sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta}(1 - k_3 v_K^2)}; \quad p = p_K\left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta}$$

when $l = l_A$ $\quad v = v_A \quad p = p_A$.

p_K and l_K are determined from first period when $\psi = 1$.

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The extreme left column contains the "computation formulas" and constants of System (I); in the next column to the right these formulas are broken down into individual operations, which are followed in the computations.

To start with, the first column (No. 1) corresponding to the time interval 0 to h is filled in first. In this column the subscript n relates to the start of the interval, and the subscript n+1 relates to its end; for this column n = 0 and n+1 = 1.

For the next (second) column, n = 1, n+1 = 2, etc. For the first column, computation of the constants gives us at n = 0 $l_n = 0$ (the path at the start of the motion), which we write on line 15; $hl'_n = 0$ (the velocity at the start of the motion) is written on lines 1 and 16; $h^2 l''_n = h^2 l''_0 = 0.0958$ is written on lines 2 and 26; $\frac{h^3}{2} l'''_0$ is written on lines 3 and 27. Line 17 is filled with $\frac{1}{2}(h^2 l''_0)$, and line 18 with $\frac{1}{3}(\frac{h^3}{2} l'''_0)$. Thus, all the quantities with the subscript n = 0 are inserted in the first column. We now subject them to the necessary operations. The sum of the first three rows gives the fourth $hl'_{n+1} = hl'_{0+1}$, which is immediately transferred to lines 1 and 16 of the neighboring column wherein, provided with the subscript n, it characterizes the value of this quantity at the start of the next interval; we then determine v_{n+1}, z_{n+1}, Ψ_{n+1}, and A_{n+1}. By adding the four rows from line 15 through line 18, we obtain in line 19 l_{n+1} - the path of the projectile at the end of the given interval of time - and this quantity, provided with the subscript n, is transferred to line 15 of the neighboring column. After determining $p_{n+1} = \frac{A_{n+1}}{B_{n+1}}$ and multiplying it by $k_4 h^2$, we obtain $h^2 l''_{n+1}$, which we write on line 25 of the first column and on lines 26 and 2 of the

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next column, where this quantity acquires the subscript n, as does also $\frac{1}{2}(h^2 l''_{n+1})$, which is written in line 17 of the next column. After performing the operations indicated in the form with lines 25, 26, and 27, we obtain in line 28 of the first column $\frac{h^3}{2} l'''_{n+1}$, which we immediately transfer to lines 3 and 27 of the next column, while $\frac{1}{3}(\frac{h^3}{2} l'''_{n+1})$ is written in line 18 of the same column.

Thus, all operations with the quantities bearing the subscript n in the second column are already prepared, and the second column is then treated in the same manner as was the first.

Constants such as z_0 in line 7 and l_Δ in line 20 are inserted in the series of columns in advance.

By applying the same rules to the neighboring second column, we shall gradually, step by step, obtain values for v, z, ψ, l, and p as functions of t = (n+1)h, and this is continued to the end of burning or to the instant of emergence of the projectile from the bore, it being necessary to use $\psi = 1$ after the end of burning.

In performing the computations it is necessary to exercise extreme care not to commit any errors, because an error in one of the preceding columns will distort the results obtained in the succeeding columns.

It is best to follow up the computation of the data in each column by plotting them on graph paper as a function of t. In so doing an error in the given column will cause a deviation from the regular disposition of the points derived from the preceding columns, and such an error can be detected and corrected.

As a criterion of accuracy, it is also useful to plot on the diagram the third derivative (or $\frac{h^3}{2} l'''_{n+1}$ in the last row), which

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should first increase, then pass through a maximum, then become zero ($p'_t = 0$) at the instant p_m is attained, and thereupon acquire a negative value, fluctuating slightly in either direction.

The instant of time cut off on the diagram at $p'_t = 0$ or $\frac{h^3}{2} l''' = 0$ corresponds to the instant of maximum pressure, and all elements for it are best taken from the diagram.

The time t_K corresponding to the end of burning of the powder is determined from the diagram on the basis of the ψ, t curve at $\psi = 1$; thereupon, the elements corresponding to the end of burning of the powder for this time are found by interpolation. If, without changing the segments $\Delta t = h$, the second period is computed as a direct continuation of the first, assuming $\psi = 1$ and $l_\psi = l_1$ throughout, the third derivative l'''_t usually begins to fluctuate, sometimes entering the region of positive values, which contradicts the physical nature of the process of pressure change.

For this reason, once the elements of the end of burning t_K, v_K, l_K, and p_K have been obtained from the computation of the first period, the procedure is continued by adopting the same step $\Delta t = h$ with t_K as the starting point by first computing the values of the path and of its derivatives for the start of the second period in accordance with the following formulas:

$$l_{(0)} = l_K; \quad hl'_{(0)} = hv_K; \quad h^2 l''_{(0)} = h^2 \frac{s}{\varphi m} p_K = h^2 k_4 p_K;$$

$$\frac{h^3}{2} l'''_{(0)} = -\frac{h^3}{2} \frac{s}{\varphi m} \frac{(1 + \theta) v_K p_K}{(l_1 + l_K)} = -\frac{h^3}{2} k_4 (1 + \theta) \frac{v_K p_K}{l_1 + l_K};$$

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whereupon they are written in the initial column for computating the data of the second period and subjected to the same operations as in the first period, with the sole exceptions that $\psi = 1$ is assumed in line 11 and that lines 6-10 are omitted.

The computation is continued in this manner until $l_{n+1} \geqslant l_{\text{A}}$.

If $l_{n+1} = l_{\text{A}}$, the remaining elements v_{A} and p_{A} are obtained automatically in the same column for the subscript n+1; if $l_{n+1} > l_{\text{A}}$, the computations in this column must be carried as far as line 24, with lines 6-10 omitted, whereupon the value of l_{A} is used to obtain t_{A} by interpolation in the last column for the purpose of subsequently obtaining the elements p_{A} and v_{A}.

Instead of expanding in a series after obtaining the elements corresponding to the end of burning of the powder, it is possible to compute v_{A} and p_{A} by means of the usual second-period formulas, but without determining the time t.

The solution by expansion in a series is applicable to both the geometric and the physical law of burning of powder. In the latter case:

$$v = \frac{s}{\varphi m} \int_{\psi_0}^{\psi} p\,dt = \frac{s}{\varphi m}(I - I_0),$$

from which we have the following expression for I:

$$I = I_0 + \frac{\varphi m}{s} v,$$

and the correlation between ψ and z is replaced by the graphical dependence of I upon ψ, which is found from the bomb test.

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The equation for $(p'_t)_0$ is replaced by the following expression:

$$(p'_t)_0 = k_2 \Gamma_0 \frac{l_\Delta}{l^2_{\psi_0}} p_0$$

and:

$$\frac{h^3}{2} l'''_0 = \frac{h^3}{2} k_4 k_2 \Gamma_0 \frac{l_\Delta}{l^2_{\psi_0}} p_0,$$

where Γ_0 is the value of the experimental function Γ corresponding to the quantity $\psi = \psi_0$.

In the case of ballooning powders, Γ_0 is greater than in the case of the geometric law of burning, and therefore the values of both the first derivative p'_t and of p itself will increase more rapidly, and the maximum pressure will occur earlier. If the propellant force of the powder is the same, the maximum pressure will be greater in the case of the physical law of burning with ballooning than in the case of the geometric law of burning.

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SECTION EIGHT - EMPIRICAL METHODS OF SOLUTION

Even with a certain schematization in the assumptions made, the analytical solution of problems in internal ballistics leads to rather complex correlations, which require time-consuming and complicated computations to obtain pressure and velocity curves. For this reason, many investigators have approached the solution of problems in internal ballistics either on the basis of simple algebraic correlations with coefficients determined by reference to experimental data or on the basis of very simple tables or formulas resulting from the treatment of experimental data obtained in firing tests.

Such simple formulas and tables, which leave out of account the great complexity of the phenomenon of the shot, and which coordinate their data with experiment with the aid of certain coefficients, form the basis of empirical methods of solution.

We shall briefly consider some of the most widely known formulas and tables employed in practice.

CHAPTER 1 - MONOMIAL AND DIFFERENTIAL FORMULAS

1. MONOMIAL EMPIRICAL FORMULAS

Monomial formulas usually express the dependence of the initial velocity of the projectile and the maximum gas pressure upon various loading conditions. Like other empirical formulas, they were widely employed prior to the development of exact analytical methods and of tables derived on the basis of these methods.

Such formulas include the monomial formulas of N. A. Zabudsky, which are derived in his works "On the Pressure of Smokeless-Powder Gases in Gunbarrels" [7] and "On the Pressure of Powder Gases in the Bore of the Three-Inch Gun and on the Remaining Velocity" [8]

on the basis of treatment of a large number of firing tests. These formulas are:

$$v_0 = H_1 \frac{\omega^{\frac{3}{4}}}{d_{km}^{\frac{1}{2}} \, \ell_0^{\frac{1}{4}} \, q^{\frac{5}{16}}} \; ; \; p_m = K_1 \frac{\omega^2 q^{\frac{3}{4}}}{d_{km}^2 \, \ell_0}$$

for the 3-inch, 4.2-inch, and 6-inch guns, and:

$$v_0 = H \frac{\Delta^{\frac{1}{4}} \omega^{\frac{1}{2}}}{(2e_1)^{\frac{1}{3}} q^{\frac{1}{4}}} \; ; \; p_m = K \frac{\Delta^{\frac{9}{10}} \omega^{\frac{9}{10}} q^{\frac{4}{5}}}{(2e_1)^{\frac{7}{5}}}$$

for the 1902 model 3-inch gun.

Here, H_1, H, K_1, and K are numerical coefficients, which are determined from the results of firing tests under known loading conditions.

Taking into account the influence of the increase in pressure p_m upon the change in the exponents, N. A. Zabudsky proposed that, at high pressures (2200 atm and higher at that time), the exponents of q and ℓ_0 be taken as unity instead of $\frac{4}{5}$ and $\frac{9}{10}$:

$$p_{max} = K \frac{\omega^{\frac{9}{5}} q}{(2e_1)^{\frac{7}{5}} \ell_0} \quad \text{or} \quad p_{max} = K \frac{\Delta \omega^{\frac{4}{5}} q}{(2e_1)^{\frac{7}{5}}} .$$

Closely related to monomial formulas are differential formulas. By taking the logarithm of a monomial formula and differentiating it term by term, it is possible to obtain a dependence of the relative change in initial velocity and of the maximum gas pressure upon the change in loading conditions.

For example:

$$\frac{dv_0}{v_0} = a_1 \frac{d\omega}{\omega} - a_2 \frac{dq}{q} - a_3 \frac{d\ell_0}{\ell_0} + \ldots$$

2. EMPIRICAL DIFFERENTIAL FORMULAS OF IKOPZ

Wide acceptance and practical use have been accorded in this country to the formulas of the Test Commission of the Okhta Powder Works (IKOPZ), which were derived empirically on the basis of a large number of firing tests conducted during the development and adoption into service of smokeless powders between 1895 and 1910. G. P. Kisnemsky and N. A. Zabudsky took an important part in these tests.

The IKOPZ differential formulas, which are also known as correction formulas, give the dependence of the relative change in maximum pressure and initial velocity upon changes in the weight of the charge, the thickness of the powder, the volume of the chamber, the weight of the projectile, and the volatile content and temperature of the powder in the following form:

$$\frac{\Delta p_m}{p_m} = 2\frac{\Delta\omega}{\omega} - \frac{4}{3}\frac{\Delta e_1}{e_1} - \frac{4}{3}\frac{\Delta W_0}{W_0} + \frac{3}{4}\frac{\Delta q}{q} - 0.15\,(\Delta H\%) + 0.0036\,(\Delta t^\circ);$$

$$\frac{\Delta v_0}{v_0} = \frac{3}{4}\frac{\Delta\omega}{\omega} - \frac{1}{3}\frac{\Delta e_1}{e_1} - \frac{1}{3}\frac{\Delta W_0}{W_0} - \frac{2}{5}\frac{\Delta q}{q} - 0.04\,(\Delta H\%) + 0.0011(\Delta t^\circ).$$

If any one of the loading conditions does not change, its change is equal to zero, and the corresponding term in the right-hand part drops out; if only one factor changes, the right-hand part contains only one term, which characterizes the influence of this factor alone.

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Coefficients in excess of unity show that the relative change in pressure is greater than the change in the given factor; coefficients smaller than unity show that the pressure or the velocity vary less than the given factor.

A plus sign indicates that the pressure and the velocity change in the same direction, i.e., increase or decrease as the factor increases or decreases; a minus sign indicates that p_m and $v_Д$ change in the direction opposite to the direction of the change in the given factor.

Inspection of the formulas shows that changes in all factors affect the change in pressure much more than the change in the velocity of the projectile.

The formulas presented above find widespread practical use in the selection of the charge and thickness, in applying corrections for the volume of the crusher gage, and in firing at a powder temperature other than $15^{\circ}C$, which is considered to be normal, and to which the results of firing must be reduced in determining the initial velocity, since the firing tables are computed at $t = 15^{\circ}C$.

<u>Example 1.</u> In firing a 1902 model 76-mm gun with an inserted crusher gage and at a powder temperature of $+12^{\circ}C$, the following results were obtained: $p_m = 2380$ kg/cm^2 and $v_Д = 593$ m/sec. To determine p_m and $v_Д$ at $t = +15^{\circ}C$, without an inserted crusher gage, with normal loading, if the volume of the chamber is $W_0 = 1654$ cm^3 and the volume of the crusher gage is $W_{cr.} = 35$ cm^3.

We shall consider $W_{cr.} = \Delta W_0$; consequently, the firing was conducted with a chamber volume $W_0' = W_0 - \Delta W_0 = 1654 - 35 = 1619$ cm^3 and at $t = +12^{\circ}C$.

Reduction to the normal chamber volume requires the following correction:

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$$\Delta w_0 = +35 \text{ a} \cdot \frac{\Delta w_0}{w_0} = \frac{35}{1619} = 0.022 = 2. \;\; , \;\; \Delta \vartheta = 15 - 12 = +3^\circ.$$

We introduce the following corrections:

$$\frac{\Delta p_m}{p_m} = -\frac{4}{3}\, 0.022 + 0.0036 \cdot 3 = -0.029 + 0.011 = -0.018 = -1.8\%;$$

$$\frac{\Delta v_0}{v_0} = -\frac{1}{3}\, 0.022 + 0.0011 \cdot 3 = -0.007 + 0.003 = -0.004 = -0.4\%.$$

Since all the coefficients are approximate, the corrections are also computed with a precision of only two significant figures.

Introduction of the corrections gives:

$$\Delta p_m = -0.018 \cdot 2380 = -43; \; p_m = 2380 - 43 = 2337.$$

or, rounded off to the nearest 5 kg

$$p_m = 2335 \text{ kg/cm}^2.$$

$$\Delta v_0 = -0.004 \cdot 593 = -2.4 \text{ m/sec}; \; v_0 = 593 - 2.4 = 590.6 \text{ m/sec}.$$

The formulas presented above make it possible to solve not only direct, but also inverse problems, for example: by what amounts is it necessary to change the thickness of the powder and the weight of the charge in order that the pressure be changed by so many per cent and the initial velocity by so much; or by how many per cent is it necessary to change the volatile content of the powder in order that the pressure and velocity be changed by the required amounts if the weight of the charge is changed in a certain manner.

Example 2. In firing a 1910 model 107-mm gun, a regulation charge containing 2.050 kg of new powder gave a (regulation) p_m = 232 kg/cm^2 and a velocity v_0 = 570.5 m/sec instead of v_0 = 579 m/sec, whi was required in accordance with the technical conditions. The questi is whether the charge can be corrected by changing the volatile cont and, since both the pressure and the velocity will change as a resul of this, how should the charge be changed so as to retain the same

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pressure?

Consequently, the problem is to determine $\frac{\Delta\omega}{\omega}$ and $\Delta H\%$ under such conditions that $\frac{\Delta p_m}{p_m} = 0$ and $\frac{\Delta v_0}{v_0} = \frac{3.5}{570.5} = 0.015 = 1.5\%$.

We formulate two equations:

$$\frac{\Delta p_m}{p_m} = 2\,\frac{\Delta\omega}{\omega} - 0.15\ (\Delta H\%),\quad \frac{\Delta v_0}{v_0} = \frac{3}{4}\,\frac{\Delta\omega}{\omega} - 0.04\ (\Delta H\%).$$

By substituting the values $\frac{\Delta p_m}{p_m} = 0$ and $\frac{\Delta v_0}{v_0} = 0.015 = 1.5\%$, converting 0.15 and 0.04 into percentages (i.e., 15 and 4), and designating $\frac{\Delta\omega}{\omega} = x$ and $\Delta H\% = y$, we obtain:

$$0 = 2x - 15y;\quad 1.5 = \frac{3^{2}}{4}\,x - 4y.$$

Upon solving this system, we find

$$x = \frac{15}{2}\,y;$$

$$1.5 = \frac{3}{4}\,\frac{15}{2}y - 4y = \left(\frac{45}{8} - 4\right)y = \frac{13}{8}\,y;$$

$$y = \frac{3}{2}\ \frac{8}{13} = \frac{12}{13} \approx 0.9\%;\quad x = \frac{12}{13}\,\frac{15}{2} = \frac{90}{13} \approx 7\%.$$

Consequently, the volatile content ΔH must be increased by 0.9%, and the charge must be increased by 7%.

Example 3. By how many per cent is it necessary to change the thickness of the powder and the weight of the charge in order that the pressure remain unchanged and the velocity may be increased by 2%?

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$$\frac{\Delta p_m}{p_m} = 0 = 2\,\frac{\Delta\omega}{\omega} - \frac{4}{3}\,\frac{\Delta e_1}{e_1}\ \text{or}\ 0 = 2x - \frac{4}{3}y;$$

$$\frac{\Delta v_0}{v_0} = 2\% = \frac{3}{4}\frac{\Delta\omega}{\omega} - \frac{1}{3}\frac{\Delta e_1}{e_1}, \quad 2 = \frac{3}{4}x - \frac{1}{3}y;$$

$$x = \frac{2}{3}y; \quad 2 = \frac{3}{4}\,\frac{2}{3}y - \frac{1}{3}y = \frac{1}{6}y;$$

$$y = 12\%; \quad x = \frac{2}{3}\,12 = 8\%.$$

Consequently, to satisfy the imposed requirements, the thickness of the powder must be increased by 12%, and the charge must be increased by 8%.

The expression $x = (2/3)y$ or $\Delta\omega/\omega = (2/3)(\Delta e_1/e_1)$ obtained from the first equation shows that, in order that the pressure remain unchanged, the thickness of the powder and the charge must be changed in such a manner that:

$$\frac{\Delta\omega}{\omega} = \frac{2}{3}\,\frac{\Delta e_1}{e_1}.$$

As is seen from the examples presented above, empirical differential formulas make it possible to solve very rapidly and simply many of the problems that are continually encountered in firing-ground or powder-works practice. It is only necessary to keep in mind that these formulas were originally derived for medium-power guns ($v_0 \approx$ 400-600 m/sec), and that, in individual cases, the coefficients may deviate in either direction from the average values given in the formulas. Nevertheless, these formulas are entirely suitable for estimates and tentative computations.

As has been shown by N. A. Zabudsky, the values of some coefficients of p_m and v_0 increase with increasing pressure.

The same is noted in the French literature, where the coefficient

change as the density of loading increases. For example, in the formulas

$$\frac{\Delta p_m}{p_m} = m_\omega \frac{\Delta\omega}{\omega} \text{ and } \frac{\Delta v_0}{v_0} = \ell_\omega \frac{\Delta\omega}{\omega},$$

the coefficients

$$m_\omega = \frac{1}{1 - 0.9\Delta} \text{ and } \ell_\omega \approx \log 10\Delta,$$

i.e., it is thereby taken into account that both the pressure and the velocity change more sharply with increasing Δ. At $\Delta = 0.55$, $m_\omega \approx 2$, and $\ell_\omega = 0.74 \approx \frac{3}{4}$, i.e., the values of the coefficients coincide with the values of the coefficients of the Test Commission of the Okhta Powder Works.

3. CORRECTION FORMULAS AND TABLES OF PROFESSOR V.E. SLUKHOTSKY

The influence of the density of loading and of the relative length of the gun upon the coefficients of differential formulas has been considered in greater detail by V. E. Slukhotsky [9].

The correction formulas may be represented in the following form as functions of the parameters X:

$$\frac{\Delta p_m}{p_m} = m_x \frac{\Delta X}{X} \text{ and } \frac{\Delta v_Д}{v_Д} = \ell_x \frac{\Delta X}{X},$$

where m_x and ℓ_x are numerical coefficients, as in the IKOPZ formulas.

There are presented below excerpts from the tables of V. E. Slukhotsky. The coefficients m_x are presented for the maximum pressure p_m within the limits of 2000-4500 kg/cm^2 and for values of Δ in the range of 0.50-0.80 kg/dm^3.

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	m_{I_K}				m_ω				m_f			
p_m \ Δ	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8
2000	1.49	1.40	1.32	1.24	2.04	2.17	2.29	2.38	1.80	1.78	1.72	1.64
2500	1.50	1.46	1.40	1.33	2.14	2.28	2.43	2.57	1.81	1.81	1.76	1.67
3000	1.50	1.50	1.46	1.40	2.22	2.39	2.56	2.74	1.78	1.81	1.78	1.69
3500	1.45	1.51	1.50	1.44	2.30	2.49	2.69	2.90	1.73	1.78	1.78	1.70
4000	1.36	1.48	1.50	1.46	2.38	2.59	2.82	3.05	1.65	1.73	1.76	1.71
4500	1.24	1.42	1.48	1.47	2.45	2.69	2.94	3.19	1.58	1.68	1.74	1.71
	m_q				m_{W_0}				ℓ_{W_0}			
2000	0.69	0.73	0.76	0.78	1.36	1.45	1.52	1.59	$\Lambda_Д = 4$	6	8	10
2500	0.72	0.78	0.81	0.83	1.48	1.58	1.67	1.74	0.34	0.23	0.16	0.14
3000	0.72	0.80	0.84	0.86	1.57	1.68	1.78	1.86				
3500	0.70	0.80	0.86	0.88	1.63	1.75	1.86	1.96				
4000	0.66	0.79	0.87	0.89	1.66	1.80	1.92	2.03				
4500	0.59	0.76	0.86	0.89	1.68	1.83	1.96	2.08				

The correction coefficients m_x and ℓ_x are given for the cases of corrections of the following quantities: I_K - pressure impulse of powder gases for the period of burning of the powder; ω - weight of the charge; f - propellant force of the powder; q - weight of the projectile; and W_0 - volume of the chamber.

Since the values of the coefficients ℓ_x depend not only upon p_m and Δ, but also upon the quantity $\Lambda_Д = \ell_Д/\ell_0$, tables for various $\Lambda_Д$ have been formulated for determining the values of the coefficient ℓ_x. In the tables presented below, the values of 4, 6, 8, and 10 have been taken for $\Lambda_Д$. For each value of $\Lambda_Д$, there is given its own table of values of ℓ_x as a function of p_m and Δ(p_m 2000-4500 kg/cm^2, Δ = 0.5, 0.6, 0.7, and 0.8).

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$\Lambda_Д$		4				6				8				10			
	Δ / P_m	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8
l_{I_K}	2000	0.38	0.55	-	-	0.30	0.45	0.49	-	0.25	0.38	0.46	-	0.22	0.33	0.46	-
	2500	0.24	0.39	0.53	-	0.18	0.29	0.44	0.48	0.16	0.26	0.37	0.46	0.14	0.22	0.32	[illegible]
	3000	0.17	0.28	0.41	0.50	0.12	0.21	0.32	0.46	0.10	0.17	0.27	0.39	0.09	0.15	0.23	[illegible]
	3500	0.12	0.20	0.31	0.43	0.09	0.15	0.23	0.35	0.07	0.12	0.19	0.29	0.07	0.11	0.17	0.26
	4000	0.09	0.15	0.23	0.33	0.07	0.11	0.17	0.25	0.06	0.09	0.14	0.21	0.05	0.08	0.13	0.19
	4500	0.07	0.12	0.18	0.26	0.05	0.09	0.13	0.18	0.05	0.08	0.11	0.15	0.04	0.07	0.10	0.14
l_ω	2000	0.86	0.97	-	-	0.76	0.87	0.95	-	0.73	0.83	0.92	-	0.72	0.80	0.89	0.93
	2500	0.76	0.86	0.97	-	0.68	0.77	0.86	0.92	0.66	0.73	0.81	0.88	0.65	0.71	0.77	0.84
	3000	0.68	0.77	0.86	0.94	0.63	0.69	0.75	0.82	0.61	0.66	0.71	0.77	0.60	0.65	0.69	0.74
	3500	0.63	0.70	0.77	0.84	0.59	0.63	0.68	0.73	0.58	0.61	0.65	0.62	0.56	0.60	0.63	0.67
	4000	0.60	0.65	0.71	0.76	0.56	0.59	0.63	0.66	0.55	0.58	0.60	0.62	0.54	0.56	0.58	0.61
	4500	0.58	0.62	0.67	0.71	0.54	0.56	0.59	0.62	0.53	0.55	0.57	0.58	0.52	0.54	0.55	0.57
l_r	2000	0.69	0.77	-	-	0.66	0.72	0.73	-	0.63	0.69	0.72	-	0.62	0.67	0.72	0.69
	2500	0.63	0.69	0.75	-	0.61	0.66	0.71	0.72	0.59	0.64	0.69	0.71	0.57	0.62	0.66	[illegible]
	3000	0.59	0.64	0.69	0.72	0.57	0.61	0.66	0.71	0.56	0.60	0.64	0.68	0.54	0.57	0.61	0.66
	3500	0.57	0.60	0.64	0.69	0.55	0.58	0.62	0.66	0.54	0.57	0.60	0.64	0.53	0.55	0.58	[illegible]
	4000	0.55	0.58	0.61	0.64	0.54	0.56	0.59	0.62	0.53	0.55	0.57	0.60	0.52	0.54	0.56	[illegible]
	4500	0.54	0.56	0.59	0.62	0.53	0.55	0.57	0.59	0.52	0.54	0.56	0.57	0.52	0.53	0.55	0.57
l_q	2000	0.28	0.18	-	-	0.32	0.26	0.19	-	0.34	0.29	0.21	-	0.36	0.31	0.26	0.21
	2500	0.34	0.29	0.20	-	0.37	0.32	0.27	0.22	0.39	0.34	0.29	0.23	0.40	0.36	0.31	0.26
	3000	0.38	0.33	0.28	0.22	0.40	0.36	0.32	0.27	0.42	0.38	0.34	0.29	0.43	0.39	0.35	0.30
	3500	0.41	0.37	0.33	0.28	0.42	0.39	0.35	0.32	0.44	0.41	0.37	0.33	0.44	0.41	0.38	0.34
	4000	0.43	0.39	0.36	0.32	0.44	0.41	0.38	0.35	0.45	0.43	0.40	0.37	0.45	0.43	0.40	0.37
	4500	0.44	0.41	0.38	0.35	0.45	0.43	0.40	0.33	0.46	0.44	0.42	0.40	0.46	0.44	0.42	0.40

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4. IDEA OF FORMULAS AND TABLES OF KISNEMSKY

While working in the Test Commission of the Okhta Powder Works and investigating the question of the applicability of the tables of Heidenreich to our guns and powders, G. P. Kisnemsky arrived at the necessity of substantially changing these tables and formulated his own tables on the basis of tests of our powders and guns.

Since, as a rule, no account is taken in empirical formulas of the influence of the weight of the charge and of the thickness of the powder, he proposed several formulas to eliminate this disadvantage.

For example, to establish the efficiency of the charge in the gun, Kisnemsky proposed the following formula:

$$v_0 = h\,(\omega - \omega_0)^{\frac{1}{2}},$$

where h is a proportionality factor determined by the system of the gun, and ω_0 is that part of the charge whose energy is consumed in the production of harmful work during the shot.

To determine this part of the charge, Kisnemsky gave two formulas, which took into account the influence of the thickness and propellant force of the powder and of some other data relating to the design of the gun and the loading conditions:

$$\omega_0 = 0.001\,(s\ell_{\text{д}} q)^{\frac{1}{2}}\,(2e_1)^{\frac{1}{3}}$$

or

$$\omega_0 = \omega - (W_0 + s\ell_{\text{д}})\,\frac{p_{\text{д}}}{f + \alpha p_{\text{д}}}\,.$$

CHAPTER 2 - EMPIRICAL FORMULAS AND TABLES

1. IDEA OF FORMULAS OF LEDUC.

The emprical formulas of Leduc (1903) were employed for rapid computation of pressure and velocity curves and were utilized for the

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solution of various problems in internal ballistics. The advantage of these tables consisted in their simplicity. At present, however, in view of the availability of tables formulated on the basis of more exact analytical formulas, the empirical formulas of Leduc, like the tables of Heidenreich, have lost their importance and possess some interest merely from the point of view of the method on which they are based.

On the basis of a study of extensive experimental and computational data, Leduc assumed for the velocity of the projectile a hyperbolic correlation of the following type:

$$v = \frac{a\ell}{b + \ell},$$

where a and b are constants, which depend upon the loading conditions.

It follows from this formula that, as ℓ approaches infinity, v approaches a, and the constant a expresses the limiting velocity of the projectile. As a matter of fact:

$$v_{np} = \left(\frac{a\ell}{b + \ell}\right)_{\ell = \infty} = \left(\frac{a}{\frac{b}{\ell} + 1}\right)_{\ell = \infty} = a.$$

The constant a has the dimension of velocity, and the constant b has the dimension of length.

To establish the dependence of pressure upon the path of the projectile, use is made of the usual equation of motion of the projectile in the following form:

$$ps = \varphi mv \frac{dv}{d\ell};$$

By substituting therein the expression for the velocity of the projectile and for its derivative with respect to the path, $dv/d\ell = ab/(b + \ell)^2$, we obtain the following formula for the pressure as a

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function of the path:

$$p = \frac{\varphi m a^2 b}{s} \frac{\ell}{(b + \ell)^3} .$$

To find the maximum pressure p_m and the path ℓ_m traversed by the projectile prior to that instant, we equate to zero the derivative of the pressure with respect to the path traversed by the projectile:

$$\frac{dp}{d\ell} = \frac{m}{s} a^2 b \frac{(b + \ell)^3 - 3\ell(b + \ell)^2}{(b + \ell)^6} = \frac{m}{s} a^2 b \frac{b - 2\ell}{(b + \ell)^4} ;$$

$$\frac{dp}{d\ell} = 0 \text{ at } b = 2\ell_m ; \quad \ell_m = \frac{b}{2} .$$

Upon substituting this value for ℓ_m into the equation, we obtain:

$$p_m = \frac{4}{27} \frac{\varphi m a^2}{sb} .$$

We obtain the velocity of the projectile in the instant of maximum pressure from the equation at $\ell_m = b/2$:

$$v_m = \frac{a\ell_m}{b + \ell_m} = \frac{a \frac{b}{2}}{b + \frac{b}{2}} = \frac{a}{3} .$$

Consequently, the constant a (or the limiting velocity) equals three times the velocity of the projectile in the instant of maximum pressure ($a = 3v_m$).

To find the correlation between the path and time, we start out on the basis of the fact that:

$$dt = \frac{d\ell}{v} = \frac{b + \ell}{a\ell} d\ell .$$

Integration within the limits from t_1 to t and from ℓ_1 to ℓ gives:

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$$t = t_1 + \frac{1}{a} (b \ln \frac{\ell}{\ell_1} + \ell - \ell_1),$$

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where ℓ_1 generally stands for a certain sufficiently small segment of the path and t_1 stands for the time corresponding to it.

Taking $\ell_1 = \ell_m$ and consequently $t_1 = t_m$, and moreover assuming that the projectile moves along this segment with a constant acceleration equal to the arithmetic mean between the initial acceleration (which equals zero) and the final acceleration (which equals $4/27 \cdot a^2/b$), we can write $\ell_m = b/2 = (1/2)(2/27)(a^2/b)(t_m^2)$, from which $t_1 = t_m =$ $= \sqrt{27/2}\,(b/a)$.

By substituting this value for t_1 into the equation for t, and taking into account that $\ell_1 = \ell_m = b/2$, we find the time necessary for the projectile to traverse the path ℓ through the bore:

$$t = \sqrt{\frac{27}{2}}\,\frac{b}{a} + \frac{1}{a}\left[b\,\ln\frac{2\ell}{b} + \ell - \frac{b}{2}\right] =$$

$$= \frac{b}{a}\left[\left(\sqrt{\frac{27}{2}} - \frac{1}{2}\right) + \frac{\ell}{b} + \ln\left(\frac{2\ell}{b}\right)\right].$$

Completion of the above operations and transformation of the result in terms of decimal logarithms finally gives us:

$$t = \frac{b}{a}\left[3.174 + \frac{\ell}{b} + 2.303\,\log\frac{2\ell}{b}\right].$$

To utilize the formulas of Leduc, it is necessary to know the constants a and b. If the values of v_ℓ and p_m are known from experiment or have been computed on the basis of exact formulas, the constants a and b are defined by the following system of equations:

$$v_\ell = \frac{a\,\ell_\ell}{b + \ell_\ell};$$

$$p_m = \frac{4}{27}\,\frac{\varphi m}{s}\,\frac{a^2}{b},$$

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from which:

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$$a = \frac{27 p_m s \ell_\ell}{8 \varphi m v_\ell} \left(1 \pm \sqrt{1 - \frac{16 \varphi m v_\ell^2}{27 p_m s \ell_\ell}} \right).$$

In this formula, only the minus sign need be retained before the expression under the radical, since the plus sign gives for ℓ a value beyond the limits of the bore.

By designating $\eta_\ell = \frac{p_{av.}}{p_m} = \frac{\varphi m v_\ell^2}{2 p_m s \ell_\ell}$ and introducing this expression into the equation for a, we obtain:

$$a = \frac{27}{16 \eta_\ell} \left(1 - \sqrt{1 - \frac{32}{27} \eta_\ell} \right) v_\ell .$$

Knowing a, we find the quantity b from the following equation:

$$b = \left(\frac{a}{v_\ell} - 1 \right) \ell_\ell .$$

The formulas of Leduc have the disadvantage that, in order to utilize them and to determine the constants a and b, it is necessary to know in advance p_m and v_ℓ; this reduces their value considerably. For this reason, Leduc made the attempt to predetermine the constants a and b in advance in conformity with the conditions of loading and the characteristic properties of the powder employed.

The formulas for a and b have the following form:

$$a = \alpha \left(\frac{\omega}{q} \right)^{\frac{1}{2}} \Delta^{\frac{1}{12}} ; \qquad (112)$$

$$b = \beta \left(\frac{W_0}{q} \right)^{\frac{3}{8}} \left(1 - \frac{3}{4} \Delta \right) . \qquad (113)$$

The quantities α and β characterize the powder; the quantity α characterizes the potential of the powder, depends principally upon its nature, and fluctuates within narrow limits; on the other hand,

the quantity β characterizes the rate of burning of the powder, depends principally upon the thickness of the powder grain, and may fluctuate within rather wide limits (2-65).

In the case of pyroxylin powders, the value of α may be assumed to be equal to 2080 kg · m · sec, and consequently:

$$a = 2080 \left(\frac{\omega}{q}\right)^{\frac{1}{2}} \Delta^{\frac{1}{12}} .$$

Knowing a, the value of b can be determined from:

$$b = \left(\frac{a}{v_д} - 1\right) \ell_д ,$$

and, in case of necessity, the quantity β may be found from Equation (113).

The author has conducted a treatment of the results of firing tests and powder tests in a pressure bomb for the purpose of determining the dependence of the coeffic'ent β upon the thickness of the powder or upon the pressure impulse. The following relations were established.

For powders with one perforation:

$$\beta = 2.15 \int_{\psi = 0.05}^{\psi = 0.95} p\,dt ;$$

For powders with seven performations:

$$\beta = 4.25 \int_{0.05}^{0.85} p\,dt ,$$

where the integrals $\int$ pdt were obtained by treatment of bomb tests.

In addition, the author has proposed the following simplified relations for a and b:

$$a = 0.16\sqrt{2gf\frac{\omega}{q}};\quad b = 2\ell_0\Delta\ (*).$$

2. IDEA OF TABLES OF HEIDENREICH

The tables of Heidenreich (1900) were formulated on the basis of a treatment of a large number of velocimetric recoil curves obtained by firing various guns under a variety of loading conditions.

They consist of two separate tables.

Table 8 presents values which make it possible to determine the elements of a shot for the instant of maximum pressure and for the instant of passage of the projectile through the muzzle face (p_m and $\ell_Д$).

Table 8

$\eta = \frac{p_{av.}}{p_m}$	$\Sigma(\eta) = \frac{\ell_m}{\ell_Д}$	$\Pi(\eta) = \frac{p_Д}{p_{av.}}$	$\Phi(\eta) = \frac{v_m}{v_Д}$	$\Theta(\eta) = \frac{t_m}{t_{av.}}$	$T(\eta) = \frac{t_Д}{t_{av.}}$
0	0	-	-	0	-
0.05	0.0046	-	-	0.033	-
0.10	0.0104	0.200	0.288	0.069	0.646
0.15	0.0177	0.240	0.306	0.108	0.695
0.20	0.0262	0.274	0.322	0.150	0.744
0.25	0.0360	0.306	0.337	0.196	0.792
0.30	0.0471	0.338	0.352	0.246	0.842
0.35	0.0597	0.368	0.367	0.300	0.893
0.40	0.0740	0.400	0.383	0.358	0.946
0.45	0.0903	0.432	0.399	0.420	1.000

(*)For a more detailed description of the application of Leduc's formulas to various cases encountered in practice, cf. M. E. Serebryakov, G. V. Oppokov, and K. K. Greten, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics), 1939, pp. 333-341./6/

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Table 8 (Cont'd.)

$\eta = \frac{p_{av.}}{p_m}$	$\Sigma(\eta) = \frac{\ell_m}{\ell_A}$	$\Pi(\eta) = \frac{p_A}{p_{av.}}$	$\Phi(\eta) = \frac{v_m}{v_A}$	$\Theta(\eta) = \frac{t_m}{t_{av.}}$	$T(\eta) = \frac{t_A}{t_{av.}}$
0.50	0.1090	0.465	0.416	0.487	1.056
0.55	0.132	0.501	0.435	0.560	1.116
0.60	0.160	0.541	0.457	0.642	1.180
0.65	0.192	0.585	0.482	0.734	1.249
0.70	0.231	0.635	0.511	0.835	1.322
0.75	0.283	0.697	0.546	0.958	1.406
0.80	0.360	0.779	0.592	1.115	1.507
0.825	0.422	0.838	0.636	1.225	1.575
0.85	0.605	1.000	0.747	1.485	1.715
0.825	0.855	1.181	0.908	1.735	1.815
0.80	0.980	1.254	0.987	1.835	1.845
0.79	1.000	1.266	1.000	1.850	1.850

In using Table 8, the initial quantity is $\eta = p_{av.}/p_m$, where $p_{av.}$ is defined by the following formula:

$$p_{av.} = \frac{q\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_A^2}{2\ gs\ \ell_A},$$

where $(1 + (1/2)\omega/q)$ is a coefficient which takes into account the work required to move the projectile.

Table 8 presents the following functions of η:

$$\Sigma(\eta) = \frac{\ell_m}{\ell_A};\quad \Phi(\eta) = \frac{v_m}{v_A};\quad \Theta(\eta) = \frac{t_m}{t_{av.}}\quad \Pi(\eta) = \frac{p_A}{p_{av.}};\quad T(\eta) = \frac{t_A}{t_{av.}},$$

where $t_{av.} = 2\ell_A/v_A$ is the time of motion of the projectile through

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the bore with the average velocity $v_\Lambda + 0/2 = v_\Lambda/2$.

Once the numerical value of:

$$\eta_\Lambda = \frac{\varphi\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_\Lambda^2}{2gs\ell_\Lambda p_m}$$

for a given gun is known, it is found in the first column, and the values for all the remaining functions are written out as indicated in the same line.

With ℓ_Λ, $p_{av.}$, v_Λ, and $t_{av.}$ known, these numbers are used to find the quantities ℓ_m, v_m, t_m, p_Λ, and t_Λ, i.e., the elements of the shot for the instant of p_m and v_Λ.

In order to use the tables, it is necessary first to know p_m and v_Λ, as well as q, s, ℓ_Λ, and ω q, which also constitutes a disadvantage of these tables.

Table 9 presents data for finding intermediate values for the pressure, velocity, and time as a function of the relative path of the projectile $\Lambda = \ell/\ell_m$.

Table 9 contains numerical values of the following functions:

$$H(\lambda) = \eta \ ; \ \Psi(\lambda) = \frac{p}{p_m} \ ; \ \Omega(\lambda) = \frac{v}{v_m} \text{ and } Z(\lambda) = \frac{t}{t_m} ,$$

which represent curves for the pressure, velocity, and time of motion of the projectile as functions of the path ℓ/ℓ_m.

For $\lambda = 1$ ($\ell = \ell_m$, $p = p_m$), the values of the last three functions equal unity, and this line corresponds to the pressure maximum on the pressure curve. The lines above this line give values for p, v, and t on the ascending branch of the pressure curve; the lines below this line give values for these quantities on the descending branch of the curve.

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In this connection, the limit of descent in Table 9 is the line

for which $\lambda = \ell_\Omega / \ell_m$ or $\eta = \eta_\Omega$.

Table 9

$\lambda = \frac{\ell}{\ell_m}$	$H(\lambda) = \eta$	$\Psi(\lambda) = \frac{p}{p_m}$	$\Omega(\lambda) = \frac{v}{v_m}$	$Z(\lambda) = \frac{t}{t_m}$
0.25	0.445	0.690	0.375	0.689
0.50	0.615	0.890	0.624	0.830
0.75	0.723	0.970	0.828	0.924
1.00	0.790	1.000	1.000	1.000
1.25	0.833	0.966	1.145	1.063
1.50	0.848	0.893	1.268	1.119
1.75	0.849	0.828	1.372	1.170
2.00	0.843	0.769	1.460	1.218
2.5	0.818	0.668	1.609	1.306
3.0	0.786	0.590	1.726	1.387
3.5	0.753	0.527	1.824	1.463
4.0	0.721	0.475	1.909	1.536
4.5	0.691	0.433	1.981	1.606
5.0	0.663	0.397	2.046	1.672
6	0.614	0.340	2.158	1.801
7	0.572	0.297	2.250	1.923
8	0.536	0.263	2.328	2.042
9	0.504	0.236	2.395	2.156
10	0.476	0.214	2.453	2.267
11	0.451	0.195	2.504	2.376
12	0.429	0.179	2.551	2.483
13	0.409	0.166	2.592	2.588
14	0.391	0.154	2.630	2.692
15	0.375	0.144	2.665	2.79

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Table 9 (cont'd.)

$\lambda = \frac{\ell}{\ell_m}$	$H(\lambda) = \eta$	$\Psi(\lambda) = \frac{p}{p_m}$	$\Omega(\lambda) = \frac{v}{v_m}$	$Z(\lambda) = \frac{t}{t_m}$
16	0.360	0.135	2.698	2.895
17	0.347	0.127	2.730	2.994
18	0.335	0.120	2.760	2.092
19	0.323	0.114	2.787	3.189
20	0.312	0.108	2.812	3.286
25	0.270	0.086	2.921	3.758
30	0.238	0.071	3.004	4.214
35	0.213	0.060	3.070	4.659
40	0.194	0.052	3.132	5.095
50	0.164	0.041	3.220	5.946
75	0.120	0.027	3.373	7.995
100	0.096	0.020	3.480	9.966

By copying from the tables the values of the relative quantities p/p_m, ℓ/ℓ_m, v/v_m, and t/t_m and multiplying them by p_m, ℓ_m, v_m, and t_m, respectively, we obtain the current values for p, ℓ, v, and t, with the aid of which we can plot curves for $p(\ell)$, $v(\ell)$, $p(t)$, and $v(t)$.

<u>Example of Computation of Pressure and Velocity Curves</u>

The following conditions are given:

Caliber $d = 76.2$ mm;

Weight of projectile $q = 6.5$ kg;

Weight of charge $\omega = 0.905$ kg;

Cross-sectional area of bore $s = 0.4693$ dm^2;

Muzzle velocity $v_{\text{д}} = 5880$ dm/sec;

Maximum pressure $p_m = 2320$ kg/cm^2;

Length of path of projectile $\ell_{\text{д}} = 18.44$ dm.

We find $p_{av.}$ $(g = 98.1 \text{ dm/sec})$:

$$p_{av.} = \frac{q\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_{A}^{2}}{2\, gs\ell_{A}} = \frac{6.5\left(1 + \frac{1}{2}\,\frac{0.905}{6.5}\right) 5880}{298.1 \cdot 0.4693 \cdot 18.44} = 134500 \text{ kg/dm}^2 = 1345 \text{ kg/cm}^2$$

We compute $\eta = p_{av.}/p_m = 1345/2320 = 0.58$. From Table 8, interpolating for $\eta = p_{av.}/p_m = 0.58$, we find:

Table 10

η	$\Sigma(\eta) = \frac{\ell_m}{\ell_A}$	$\Phi(\eta) = \frac{v_m}{v_A}$	$\Theta(\eta) = \frac{t_m}{t_{av.}}$
0.55	0.132	0.435	0.560
0.58	0.149	0.448	0.609
0.60	0.160	0.457	0.642

Having the values of $\Sigma(\eta)$, $\Phi(\eta)$, and $\Theta(\eta)$, we compute ℓ_m, v_m, and t_m:

$$\ell_m = \ell_A \Sigma(\eta) = 18.44 \cdot 0.149 = 2.75 \text{ dm};$$

$$v_m = v_A \Phi(\eta) = 588 \cdot 0.448 = 264 \text{ m/sec};$$

$$t_{av.} = \frac{2\ell_A}{v_A} = \frac{2 \cdot 18.44}{5880} = 0.00628 \text{ sec};$$

$$t_m = t_{av.}\Theta(\eta) = 0.00628 \cdot 0.609 = 0.00382 \text{ sec}.$$

Having these values, we compute p, v, and t with the aid of the following formulas:

$$p = p_m \Psi(\lambda); \quad v = v_m \Omega(\lambda); \quad t = t_m Z(\lambda).$$

The value of λ for the muzzle face is:

$$\frac{\ell_A}{\ell_m} = \frac{18.44}{2.75} = 6.71.$$

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We find from Table 9:

$$\psi(\lambda); \quad \Omega(\lambda); \quad Z(\lambda).$$

In conformity with this table, we compute the current values of ℓ, p, v, and t.

Table 11

ℓ, dm	p, kg cm^2	v, m sec	t, sec.
0.688	1600	99	0.00263
1.375	2060	164	0.00317
2.06	2250	218	0.00353
2.75	2320	263	0.00382
3.44	2240	301	0.00406
4.12	2070	334	0.00427
4.82	1920	361	0.00447
5.50	1785	384	0.00465
6.87	1550	423	0.00498
8.25	1370	454	0.00530
9.62	1220	480	0.00559
11.00	1100	502	0.00587
12.37	1005	521	0.00613
13.75	920	538	0.00638
16.50	790	568	0.00687
18.44	715	588	0.00722

The results of the computation are plotted in figs. 155 and 156 (the computations were performed with the aid of a slide rule).

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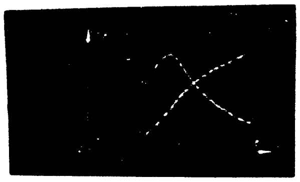

Fig. 155

p(t) and v(t) Curves.

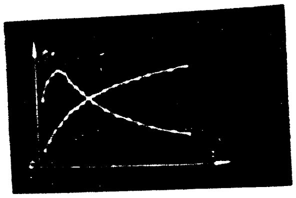

Fig. 156

p(ℓ) and v(ℓ) Curves.

SECTION NINE - TABULAR METHODS OF SOLUTION OF PROBLEMS IN INTERNAL BALLISTICS

CHAPTER 1 - IMPORTANCE OF TABULAR METHODS OF SOLUTION IN ARTILLERY PRACTICE

In adopting the analytical method for the solution of the principal problem of pyrodynamics, i.e., for the determination of the gas-pressure curve in the bore and of the velocity of the projectile as a function of its path, it is necessary to perform a large number of computations, which require a considerable expenditure of time. Moreover, the analytical formulas do not make it possible to solve inverse problems connected with the design of the system or with determining the thickness of the powder. For this reason, in solving such problems, it has been necessary to go through a large number of variants of the direct problem and then to choose from among them by interpolation the variant suitable for the case under consideration. This introduced extraordinary complications into the solution of problems connected with the design of guns and the selection of powders, making it necessary to resort to tables and simple formulas of empirical origin which do not take into account all the circumstances surrounding the phenomenon of the shot. For example, the formulas of Leduc do not take cognizance of the weight of the charge and the thickness of the powder, just as the thickness of the powder also fails to be reflected in the tables of Heidenreich.

Neither the tables nor the formulas mentioned above make it possible to determine the position of the end of burning of the powder and to find out whether or not it burns entirely in the barrel. While this may not be essential for computing the strength of the barrel or for designing the gun carriage, it is of decisive importance in choosing the thickness of the powder to assure attainment of the necess

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ballistic data.

For this reason, when, in 1910, Professor N. F. Drozdov formulated his tables for determining the maximum pressure p_m and the initial velocity $v_Д$, involving the coincidental determination of the position of the end of burning of the powder (ℓ_K/ℓ_0), this simplified considerably the solution of the direct problem of internal ballistics and permitted the rapid and simple solution of a number of inverse problems relating to the ballistic design of the barrel, such as determination of the length of path of the projectile necessary to assure attainment of the required initial (muzzle) velocity at a given density of loading and under the conditions of complete combustion of the powder in the bore ($\ell_K < \ell_Д$), determination of the thickness of powder necessary to assure the attainment of a predetermined maximum pressure, solution of diverse variants involving changes in the weight of the charge and in the thickness of the powder at the same maximum pressure to determine the most advantageous conditions of loading, etc.

The tables formulated by Professor Drozdov played an important part in perfecting and accelerating the solution of the problem of ballistic design and received widespread recognition. For convenience in use, they were later interpolated for smaller variations in the arguments entering into them. In 1933, they were perfected and expanded by the author himself. They also served as a model for the compilation in 1933 of the "Tables of the Chair of Internal Ballistics" for powders with a constant burning area.

A further development and continuation of the tables of Professor N. F. Drozdov is represented by the "ANII Tables," published in 1933, which make it possible, under given conditions of loading, to determine not only p_m, ℓ_K, ℓ_m, and $v_Д$, but also all curves for the gas pressure, the velocity of the projectile, and the time of motion as

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functions of the path of the projectile. These tables made it possible still further to accelerate the solution of a number of problems connected with the field of ballistic design.

Following the introduction of the tables of Professor N. F. Drozdov, and then of the ANII Tables, into artillery practice, the empirical tables of Heidenreich lost all of their importance.

In 1942, with the ANII Tables as a model, there were compiled under the editorship of V. E. Slukhotsky and S. I. Ermolaev detailed "GAU Tables," which were published in three parts, with the subsequent addition of a special Part 4 for the ballistic computation of guns.

They are more convenient and do not contain the errors present in the ANII Tables.

Part 4 of the tables (TBC) is especially convenient for ballistic computations.

In the present chapter, we shall discuss tables which represent numerical values of the principal elements p, v, ℓ, and t, obtained on the basis of formulas for the analytical solution of the direct problem for a large number of variants of loading conditions. Such tables make it possible, almost without computations, to find all elements of a shot, such as the gas pressure and the velocity of the projectile as functions of the path of the projectile and as functions of time, there being determined among others the elements for maximum pressure, for the end of burning of the powder, and for the muzzle face.

Some such (abbreviated) tables give directly only certain individual elements of the shot, including the maximum pressure, its position, and the position of the end of burning of the powder, making it necessary to conduct additional relatively simple computations for the calculation of the muzzle velocity.

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Contrary to the practice adopted by some authors, tabular methods cannot be interpreted to include those analytical methods for the solution of the direct problem comprising tables of various functions of internal ballistics, such as, for example, the D and ε functions of Professor Oppokov, the $\int_0^\beta z_x^{B/B_1} d\beta$ function of Sviridov, etc., which play an auxiliary part in the solution of the direct problem and in the computation of the elements of the shot.

PROCEDURE FOR FORMULATION OF TABLES(*)

In formulating tables on the basis of analytical methods, there is conducted a large number of computations leading to the solution of the direct problem of internal ballistics for various conditions of loading, which are chosen within definite limits.

To render the tables adaptable to guns of any desired caliber, the initial equations are reduced to such a form that they contain relative quantities wherever possible. For example, instead of weights of charges ω, which vary within very wide limits, use is made of densities of loading Δ, which vary but little for definite types of guns; instead of absolute paths of the projectile, there are determined the relative quantities $\Lambda = \ell/\ell_0$, where $\ell_0 = W_0/s$ is the adjusted length of the chamber. The quantity $\Lambda = \ell/\ell_0$ may be considered either as the relative path of the projectile or as the number of volumes of expansion of the gases, $\Lambda = s\ell/s\ell_0 = W/W_0$, which, in our artillery systems, varies only within circumscribed limits.

Instead of the absolute pressures p, there are sometimes determined the ratios of the pressure to the propellant force of the powder, p/f or p/p_1, where $p_1 = f\Delta/1 - \alpha\Delta$.

Moreover, the constants which characterize the conditions of

(*)Cf. Professor D. A. Ventsel, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics) /10/

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loading are grouped together in the form of dimensionless parameters independent of the caliber of the gun. Such parameters include, for example:

$$B = \frac{s^2 I_K}{f\omega\varphi m}$$

(in the method of Professor Drozdov)

or

$$H = \frac{2f\omega\varphi m}{s^2 I_K^2} = \frac{2}{B}$$

(in the method of Bianchi-Grave)

or

$$C = \frac{\theta}{H} = \frac{B\theta}{2}$$

(in the same method at $\chi = 1$)

To make it possible to construct the tables, it is necessary to establish the number of variables and parameters entering into the system of equations of internal ballistics. For this purpose, we shall consider the principal formulas for the elements of the shot (p, v, ℓ, and Ψ).

For the first period, in the case of a powder of degressive form, we have:

$$-\chi\lambda = \chi - 1;$$

$$\varphi = \varphi_0 + \chi\sigma_0 x + \chi\lambda x^2;$$

$$\chi\sigma_0 = k_1 = \chi + 2\chi\lambda z_0;$$

$$z_0 = \frac{2\Psi_0}{\chi(\sigma_0 + 1)} \approx \frac{\Psi_0}{\chi};$$

$$\Psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{\delta}{\Delta} - 1}{\frac{f\delta}{p_0} + \alpha\delta - 1};$$

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$$v = v_{np} \sqrt{\frac{B\theta}{2}}\, x = \sqrt{\frac{\omega}{\varphi q}} \sqrt{fgB}\, x;$$

or

$$v \sqrt{\frac{\varphi q}{\omega}} = \sqrt{fgB}\; x;$$

$$\Lambda = \frac{\ell}{\ell_0};$$

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi;$$

$$p = f\Delta \frac{\psi - \frac{B\theta}{2} x^2}{\Lambda_\psi + \Lambda};$$

$$\Lambda = \Lambda_{\psi_{av.}}\left(Z_x^{-\frac{B}{B_1}} - 1\right),$$

where

$$B_1 = \frac{B\theta}{2} - \chi\lambda,$$

B_1 entering into the quantities $\gamma = B_1\psi_0/k_1^2$ and $\xi = B_1\chi/k_1$, in accordance with which the function $\log Z_x^{-1}$ is determined.

The chamber volume W_0 and the cross-sectional area of the chamber s enter only into the expression $\Lambda = \ell/\ell_0$ by way of the quantity $\ell_0 = W_0/s$. The weight of the charge and the weight of the projectile enter in the form of the ratio ω/q into the formula for the velocity v; the coefficient $\varphi = a + b\,\omega/q$ depends upon the same ratio.

Consideration of the quantities p, $v\sqrt{\frac{\varphi q}{\omega}}$, and $\Lambda = \ell/\ell_0$ shows them to represent functions of the argument x and of the following eight parameters:

f, α, and δ, which characterize the nature of the powder;

$\theta = c_p/c_w - 1$, which characterizes the composition of the gases and the conditions of their expansion in the bore of the gun;

χ, which characterizes the form of the powder;

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p_0, which characterizes the arrangement of the belt of the projectile and of the grooves in the bore;

B and Δ, which characterize the conditions of loading.

The values of the same variables p, $v\sqrt{\varphi q/\omega}$, and Λ at the pressure maximum and at the end of burning of the powder depend upon the same eight parameters.

In the second period, p and $v\sqrt{\varphi q/\omega}$ are defined by the following expressions:

$$p = p_K\left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{1+\theta},$$

where

$$p_K = f\Delta\frac{1 - \frac{B\theta}{2}(1 - z_0)^2}{\Lambda_1 + \Lambda_K} \text{ and } \Lambda_1 = 1 - \alpha\Delta.$$

and

$$\left(v\sqrt{\frac{\varphi q}{\omega}}\right)^2 = \frac{2gf}{\theta}\left\{1 - \left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\},$$

where the argument is Λ; the remaining constants and the parameters B and Δ are the same as in the first period.

The large number of constants and parameters makes it necessary, in formulating the tables, to assume that some of the constants, which vary within definite limits, such as f, α, δ, χ, p_0, etc., are constant average values, which narrows down the field of applicability of the tables. Some authors choose the alternative of introducing more complex variables and parameters, which makes it possible to reduce the number of entries in the tables, but also complicates the use of the latter.

If f, α, δ, χ, θ, and p_0 are assumed to be constant, the quantities p, Λ, and $v\sqrt{\varphi q/\omega}$ will be functions of only the two parameters STAT

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Δ and B, and it becomes possible to formulate tables with only the two entries Δ and B.

Let us designate the quantity $v \sqrt{\varphi q/\omega}$ as $v_{tab.}$. After determining $v_{tab.}$ from the tables, the actual velocity of the projectile v is found by multiplying $v_{tab.}$ by the factor $n = \sqrt{\omega/\varphi q}$, which is known for the predetermined loading conditions:

$$v = v_{tab.}\, n,$$

where $\varphi = a + b\,\omega/q$.

The time of motion is expressed by the following integral:

$$t = \int_0^{\ell} \frac{d\ell}{v}.$$

If, in this integral, ℓ is expressed in terms of Λ and v in terms of $v_{tab.}$, we obtain for the time of motion the following expression:

$$t = \ell_0 \sqrt{\frac{\varphi q}{\omega}} \int_0^{\Lambda} \frac{d\Lambda}{v_{tab.}},$$

in which the integral $\int_0^{\Lambda} d\Lambda \cdot v_{tab.}$ is likewise a function of the same Δ, B, and Λ. The tables give the following quantity:

$$t_{tab.} = \frac{10^6}{\ell_0} \sqrt{\frac{\omega}{\varphi q}}\, t.$$

It should be pointed out that $v_{tab.}$ is given in the tables in m/sec^{-1}, while ℓ_0 in the formula for the time is expressed in dm.

The transition from tabular values for conditional time, $t_{tab.}$, to actual time values is carried out in accordance with the follow-

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ing formula:

$$t = l_{0\ dm} \sqrt{\frac{\varphi q}{\omega}}\, 10^{-6} t_{tab.}$$

In formulating the tables, there are first established the limits of variation of the parameters Δ and B which may be encountered in practice, together with the intervals of variation of these parameters that are convenient for interpolation of intermediate values. For example, Δ is taken between the limits of 0.20 and 0.80 or 0.10 and 0.90, and B is taken between 1 and 3 or 0 and 4.

The intervals for Δ are best chosen to be equal to 0.04, in order that later, by interpolating half-way and then half-way again, there may be obtained variations in the tabular data for the values of Δ at intervals of 0.01. The intervals for B may be taken to be 0.4, in order that two half-way interpolations may give tabular data for the values of B at intervals of 0.1.

We thus obtain two series of values for the principal parameters:

$$\Delta = 0.20;\ 0.24;\ 0.28;\ \ldots\ 0.72;\ 0.76;\ 0.80.$$

$$B = 1.0;\ 1.4;\ 1.8;\ 2.2;\ 2.6;\ 3.0.$$

Thereupon, with one of the values of Δ (for example 0.20) as a basis, there is carried out for all the values of B written out above a complete computation of the solution of the problem of internal ballistics, involving the determination of p, $v_{tab.}$, Λ, and ψ, both for the maximum-pressure values and for the end of burning of the powder, and in some tables also for a series of intermediate points in the first and second periods until a definite value of Λ is obtained.

This is then repeated for all the chosen values of Δ.

Upon completion of the computations, the values of the quantities which must be entered into the tables (for example p_m, ℓ_m/ℓ_0, ℓ_K/ℓ_0) are plotted on a large scale on cross-sectional paper, and curves showing the variations of these quantities, for example of p_m as a function of Δ at given values of B, are constructed. Thereupon, for the same values of Δ, there are constructed curves showing the variation of p_m as a function of B; these two systems of curves make it possible to carry out interpolations for intermediate values of Δ and B, and thus to formulate a full table of variations of p_m as a function of Δ at intervals of 0.01 and as a function of B at intervals of 0.1 or 0.05. Analogous curves are also constructed for the other quantities (ℓ_m/ℓ_0, ℓ_K/ℓ_0, etc.) as well, and interpolations are carried out in a similar manner.

The data obtained after interpolation are entered into tables, which make it possible to solve both direct problems on the determination of p_m, p_K, ℓ_m, ℓ_K, and $v_Д$ and inverse problems connected with the ballistic computation of guns.

CHAPTER 2 - TABLES FOR DETERMINING PRINCIPAL ELEMENTS OF SHOT (p_m, ℓ_m, ℓ_K, $v_Д$)

1. TABLES OF PROFESSOR N. F. DROZDOV

The tables were compiled for strip-type powders possessing the following form characteristics: $\chi = 1.06$ and $\chi\lambda = -0.06$.

In the tables, the following characteristics were assumed to be constant.

Propellant force of powder $f = 950{,}000$ kg-dm/kg.

Covolume $\alpha = 0.98$ dm^3/kg.

Density of powder $\delta = 1.6$ kg/dm^3.

Coefficient $\varphi = 1.05$.

Forcing pressure $p_0 = 300$ kg/cm^2.

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Adiabatic index $k = 1 + \theta = 1.2$ or $\theta = 0.2$.

Acceleration due to gravity $g = 98.1$ dm/sec^2.

$\alpha = 1/\delta = 1/\delta_1 = 0.355 = 1/2.82$.

The initially developed tables were brief and consisted of three tables: a basic table B and auxiliary tables A and C. They were subsequently modified and rendered more convenient and universal.

The basic data entered into the tables (cf. Tables I, II, III, and IV in the Appendix) are the density of loading Δ and the parameter of the loading conditions B:

$$B = \frac{s^2 e_1^2}{u_1^2} \frac{1}{f\omega\varphi m} = \frac{s^2 I^2_K}{f\omega\varphi m} .$$

The upper horizontal row contains quantities Δ from 0.07 to 0.80 at intervals of 0.01; the left-hand vertical column contains quantities B from 0.7 to 3.0 at intervals of 0.05.

The numbers in Table I give values for the maximum pressure p_m.

The numbers in Table II give the ratio ℓ_K/ℓ_0, where ℓ_K is the path of the projectile at the end of burning and ℓ_0 is the adjusted length of the chamber.

The numbers in Table III give the ratio ℓ_m/ℓ_0, where ℓ_m is the path of the projectile in the instant of attainment of maximum pressure.

The numbers in Table IV give values of the quantity $\log \left[1 - B\theta/2 \, (1 - z_0)^2\right]$, which enters into the expression for the velocity of the projectile in the second period.

The muzzle velocity is computed with the aid of the usual formula:

$$v_A = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q} \left\{ 1 - \eta_1^{\theta} \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right] \right\}}, \qquad (114)$$

where

$$\eta_1 = \frac{\frac{l_K}{l_0} + 1 - \alpha\Delta}{\frac{l_\text{д}}{l_0} + 1 - \alpha\Delta}$$

represents the ratio of free volumes of the initial air space in the instant of the end of burning and in the instant of emergence of the projectile.

Under given loading conditions, it is necessary for computing $v_\text{д}$ to find the value of l_K/l_0 from Table II and the quantity log [1 - $B\theta/2\ (1 - z_0)^2$] from Table IV, and then to substitute these into formula (114).

If there are first substituted into formula (114) the assumed values for the constants 2fg and $\varphi\theta$, that formula will be written as

$$v_\text{д} = 29,790 \sqrt{1 - \eta_1^{0.2} \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]} \quad \sqrt{\frac{\omega}{q}}. \qquad (115)$$

On the basis of the predetermined values for Δ and B (for example Δ = 0.60 and B = 2.0), there is found the maximum pressure (p_m = 2,255 kg/cm^2). At the same B = 2, Table II is used to find the value l_K/l_0 = 2.96, and Table IV is used to find log [1 - $B\theta/2\ (1 - z_0)^2$] = $\bar{1}$.9097.

Substitution of these values for l_K/l_0 and log [1 - $B\theta/2\ (1 - z_0)^2$] into formula (115) makes it possible to compute the muzzle velocity of the projectile.

Inspection of Table I indicates that, at a given density of loading Δ, p_m decreases and l_K/l_0 increases with increasing B. In a given gun, there may vary in the quantity $B = (s^2e_1^2/u_1^2)(1/f\omega\varphi m)$ the following principal items: the thickness of the powder $2e_1$, the weight of the charge ω, and the mass of the projectile m; f, φ, and s are constant and u_1 does not experience a strong enough variation.

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If the density of loading Δ remains constant in a given gun while B increases, this is due principally to an increase in the thickness of the powder $2e_1$. The table shows that, as the thickness $2e_1$ increases, the pressure p_m decreases, the end of burning moves closer to the muzzle, and incomplete combustion may result from a large increase in B:

$$\frac{\ell_K}{\ell_0} > \frac{\ell_{Д}}{\ell_0} \quad .$$

Analysis of Table I shows that, in a given gun, it is possible to obtain one and the same pressure p_m at different Δ by varying B at the same time. Identical pressures p_m are arranged in the table along slanting lines from the upper left to the lower right; for example, the pressure p_m = 2,400 kg/cm^2 is obtained at the following combinations of Δ and B, ℓ_K/ℓ_0 varying at the same time.

Table 12

p_m = 2400 kg/cm^2.

Δ	0.40	0.50	0.60	0.70
B	1.00	~1.39	1.87	2.40
$\frac{\ell_K}{\ell_0}$	0.85	1.43	2.54	4.55

If Δ increases as a result of an increase in the weight of the projectile (and not as a result of a decrease in the chamber volume W_0), an increase in ω should reduce B; for this reason, to maintain the same p_m, it is necessary to increase the thickness of the powder

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$2e_1$ in such a manner as to have its change not only compensate for the influence of the increase in ω, but also augment B to the values indicated in Table 12. Since the thickness of the powder increases, while p_m remains the same, the end of burning moves closer and closer to the muzzle (ℓ_K/ℓ_0 increases from 0.85 to 4.55). As the charge in- creases up to a certain limit, the initial velocity of the projectile v_ℓ will likewise increase; as the charge increases further, the velocity will cease increasing because of the incomplete combustion of the powder.

If Δ increases as a result of a decrease in the volume of the chamber while the weight of the charge remains unchanged (large base of projectile), the same p_m can be maintained by changing B as is indi- cated in the table, but the thickness of the powder will change less than in the first case, since, in this connection, it is not necessary to compensate for the increase in the weight of the charge ω.

If Δ is changed only by changing the weight of the charge ω, without any change in the thickness of the powder and in the other conditions of loading, the parameter B will simultaneously change in the reverse ratio ($B_2:B_1 = \omega_1:\omega_2$). For this reason, under otherwise identical conditions, the pressures corresponding to the change in the charge will be arranged along lines running from the lower left toward the upper right.

For example, if, at B = 2 and Δ = 0.50, p_m = 1750, then, at Δ = = 0.40, B = 2.4 and p_m = 1180; at Δ = 0.60, B = 1.67 and p_m = 2670.

Comparison of the results indicates that, as Δ changes from 0.40 to 0.50, i.e., by 25%, the pressure changes by $(1750 - 1180)/(1180) \cdot 100 =$ = 48% (almost twice as much); and as Δ changes from 0.50 to 0.60, i.e., by 20%, the pressure changes by $(2670 - 1750)/(1750) \cdot 100 = 52.5\%$ (more than 2.5 times as much).

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Consequently, as the density of loading increases, the same relative increase in the charge is associated with a larger and larger increase in pressure. For this reason, in selecting a charge in practice, its weight must be increased very cautiously if the density of loading is high.

A. Application of Tables to Solution of Various Problems

With the aid of the tables, it is possible to solve very rapidly a number of problems possessing great practical importance.

a) Determination of thickness of powder to assure attainment of predetermined maximum pressure p_m. If the data for the gun W_0, s, and $\ell_{\text{Д}}$, and for the weights of the charge ω and of the projectile q are known, and if p_m is predetermined, then, to determine the thickness of powder to assure attainment of the predetermined pressure, there is first computed $\Delta = \omega/W_0$; this Δ is used to enter the corresponding column in Table I; and the predetermined pressure is found in this column. In accordance with the value of p_m, the quantity B is found in the same row of the left-hand column, and the thickness $2e_1$ is found with the aid of the following formula:

$$2e_1 = \frac{2u_1}{s} \sqrt{Bf\omega\varphi m} \text{ dm.}$$

Using the same value of Δ and the value of B found from Table I, Table II is used to find ℓ_K/ℓ_0 and ℓ_K, and the latter is compared with $\ell_{\text{Д}}$ to determine whether all of the powder burns ($\ell_K < \ell_{\text{Д}}$) or does not burn ($\ell_K > \ell_{\text{Д}}$) in the bore. The procedure adopted for the solution may be represented by the following scheme.

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Example

Table I Table II

$$B \leftarrow p_m B \rightarrow \frac{\ell_K}{\ell_0} \rightarrow \ell_K \lesseqgtr \ell_\pi$$

$$2e_1 = \frac{2u_1}{s}\sqrt{Bf\omega\varphi m}$$

Table I

$$\Delta = 0.60$$

$$B = 1.90 \leftarrow p_m = 2365$$

$$2e_1 = \frac{2u_1}{s}\sqrt{1.90 \cdot 95 \cdot 10^4 \cdot \omega 1.05 \cdot m}$$

Table II

$$\Delta = 0.60$$

$$B = 1.90 \rightarrow \frac{\ell_K}{\ell_0} = 2.60$$

The quantity u_1 may be determined either with the aid of the following pyrostatic formula:

$$u_1 = \frac{0.175(N - 6.36)10^{-8}}{0.04(220^\circ - t_n^\circ) + 3h + h'}$$

or with the aid of the tabulation presented below, which gives an approximate dependence upon the thickness of the powder (which is connected with the varying volatile content) for pyroxylin powders.

Table 13

	Ordinary pyroxylin powders						Phlegmatized powders	
$2e_1$, mm	0.1	0.5	1.0	2.0	3.0	4.0	BT/0.3	0.7
$u_1 \cdot 10^7 \frac{dm}{sec} : \frac{kg}{dm^2}$	90	80	75	70	65	62	72	70

It is also possible to use the following approximate empirical formula, which gives the dependence of u_1 upon the total volatile content:

$$u_1 = 0.0000120 - 0.0000010(H\%).$$

b) Determination of length of path of projectile to assure attainment of required initial velocity under predetermined loading

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conditions. Given are the gun caliber d, the bore cross section s (including the grooves), the volume of the chamber W_0, the weight of the projectile q, the weight of the charge ω, and the required initial velocity of the projectile v_{Π}; let the magnitude of the maximum pressure p_m likewise be predetermined. It is necessary to find the length of the path of the projectile ℓ_{Π}.

To start with, $\Delta = \omega/W_0$ is computed; in the column of Table I corresponding to this Δ, we find the predetermined p_m, whereupon we use the latter to determine B. At these values for Δ and B, we find ℓ_K/ℓ_0 from Table II, $\log \left[1 + B\theta/2\,(1 - z_0)^2\right]$ from Table IV.

From formula (115) for the velocity in the second period, we find ℓ_{Π}:

$$\ell_{\Pi} = \ell_0 \left\{ \left(\frac{\ell_K}{\ell_0} + 1 - \alpha\Delta \right) \frac{\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{\frac{1}{\theta}}}{\left(1 - \frac{v_{\Pi}^2}{v_{\Pi p}^2}\right)^{\frac{1}{\theta}}} - (1 - \alpha\Delta) \right\}. \quad (116)$$

All quantities entering into the right-hand side of formula (116) are known.

Formula (116) makes it possible to determine that path of the projectile along the bore which will assure the attainment of the predetermined initial velocity under the given pressure p_m; the thickness of the powder will be determined in accordance with the scheme of the first problem.

Analysis of the tables of Professor N. F. Drozdov and the above exemplary problems that can be solved with their aid show their importance for the practice of artillery and their convenience and flexibility for ballistic design and for the choice of powder, whereas

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empirical tables express merely the general character of the variation of pressure and velocity as functions of the path and time without making it possible to draw any conclusions about the powder.

A certain disadvantage of the tables resides in the fact that they were formulated for strip-type powder possessing the definite characteristics $\varkappa = 1.06$ and $\varkappa\lambda = 0.06$, and for a constant propellant force of the powder $f = 950{,}000$ kg · dm/kg. Practice has shown, however, that, under conditions of equal charges, a powder with seven perforations gives results in firing that are practically identical with the results obtained with strip-type powder if the thicknesses of the powders are related as follows:

$$2e_1 \text{ strip-type} = \frac{10}{7}\, 2e_1 \text{ with 7 perforations or}$$

$$2e_1 \text{ with 7 perforations} = \frac{7}{10}\, 2e_1 \text{ strip-type}$$

In using the tables of Professor Drozdov for computing the action of a powder with seven perforations, its wall thickness must be multiplied by 10/7, whereupon the entire problem is solved as in the case of strip-type powder.

It is true, of course, that the end of burning of this powder and the ℓ_K/ℓ_0 obtained from the tables will not correspond to the actual values for a powder with seven perforations having decomposition products of greater thickness ($\rho = 0.532\ e_1$); but, as has been shown by firing tests from a gun equipped with lateral crusher gages, the pressure curves of standard powders - strip-type and with seven perforations - coincide almost completely.

If the full thickness $e_1 + \rho$ of the grain with seven perforations is computed in relation to an equivalent strip-type powder:

$$\left(e_1 \text{ with 7 perforations} = \frac{7}{10}\, e_1 \text{ strip-type}\right),$$

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then:

$$(e_1 + \rho)_{\text{with 7 perforations}} = 1.532\, e_{1\ \text{with 7 perforations}} =$$

$$= 1.532 \frac{7}{10} e_{1\ \text{strip-type}} = 1.07\, e_{1\ \text{strip-type}} .$$

Consequently, the full thickness of the powder with seven perforations together with the thickness of the decomposition products is somewhat greater than the thickness of the equivalent strip-type powder, and for this reason its end of burning will be found to be somewhat farther than in the case of strip-type powder, and the ℓ_K/ℓ_0 determined from the tables will be somewhat smaller than the true value. As concerns the utilization of the tables at a propellant force of the powder $f \neq 950{,}000$, to determine the pressure p_m it is possible, taking from the tables p_m at a given B, to multiply it by the ratio $f_1/950{,}000$, where f_1 is the new propellant force of the powder. On the other hand, to determine the velocity v_A, the value obtained by computation must be multiplied by $\sqrt{\frac{f_1}{950{,}000}}$.

These values will be approximate, since the paths ℓ_m and ℓ_K will also change simultaneously with p_m and v_A; but, for purposes of taking into account the order of the corrections of the values of p_m and v_A, they may be employed as a first approximation and in the presence of a force not too different from the normal value of 950,000 kg · dm/kg = = 95 t · m/kg.

At $\varphi \neq 1.05$, it is possible also to apply a correction to the value of v_A in accordance with the following formula:

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$$v_A = v_{A\,1.05} \sqrt{\frac{1.05}{\varphi}} .$$

At a given B, the quantity p_m is clearly independent of φ.

2. TABLES OF CHAIR OF INTERNAL BALLISTICS

In 1933, on the initiative of Professor I. P. Grave, the Chair of Internal Ballistics of the Dzerzhinskii Artillery Academy compiled tables for any values of f and φ and for a powder with a constant burning area, which is closely approached by long tubular powder.

The tables were computed on the basis of the analytical formulas of Bianchi, as modified by Professor Grave.

The following constants were assumed in the tables:

$\alpha = 0.98$, $\delta = 1.6$, $\varkappa = 1$, $\lambda = 0$, $\theta = 0.2$, $p_0/f = 0.035$.

The basic quantities used were the density of loading Δ and the parameter of the loading conditions $H = 2f\omega\varphi m/s^2 I_K^2 = 2/B$ or the reciprocal quantity $C = \theta/H$. It is not difficult to see that, at $\theta = 0.2$, $C = 0.1$ B.

In the tables, the left-hand column contains the quantity C, which varies uniformly from 0.10 to 0.40 at intervals of 0.01, and the next column contains the corresponding quantity $H = 0.2/C$, which varies nonuniformly from 2.0 to 0.5. The loading densities in the upper horizontal row vary from 0.10 to 0.90 at intervals of 0.01.

On each page of the tables, for six loading densities and for all values of C from 0.10 to 0.40, there are written the corresponding values of the ratios p_m/f, p_K/f, ℓ_m/ℓ_0, and ℓ_K/ℓ_0 and of the two auxiliary quantities D and B, which enter into the formula for the muzzle velocity of the projectile:

$$D = 1 - \varphi r_K = 1 - C(1 - \psi_0)^2, \quad B = \left(1 - \alpha\Delta + \frac{\ell_K}{\ell_0}\right)^{\theta},$$

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where

$$v_{Д} = \sqrt{\frac{2f\omega}{\varphi\theta m}\left[1 - \frac{BD}{\left(1 - \alpha\Delta + \frac{\ell_{Д}}{\ell_0}\right)^{\theta}}\right]} . \qquad (117)$$

It is not difficult to see that this formula coincides with the previously derived formula for the velocity in the second period and with the formula presented in the initial table of Professor Drozdov, since, for a powder with a constant burning area, $1 - C(1 - \psi_0)^2 = 1 - B\theta/2(1 - z_0)^2$, and:

$$\frac{B}{\left(1 - \alpha\Delta + \frac{\ell_{Д}}{\ell_0}\right)^{\theta}} = \eta_1^{\theta} .$$

The quantity φ enters into the parameter B or C.

Having found for a given set of Δ and C the values of p_m/f and p_K/f and knowing the propellant force of the powder f, we obtain the values for p_m and p_K; and having found ℓ_m/ℓ_0 and ℓ_K/ℓ_0 and knowing $\ell_0 = W_0/s$, we find ℓ_m and ℓ_K.

We thus find the nodal points of the pressure curve: the maximum pressure p_m and its position in the bore, i.e., the path of the projectile ℓ_m, as well as the pressure p_K at the instant of complete combustion of the powder and the corresponding path of the projectile ℓ_K; thereupon, having found B and D from the tables by means of formula (117), we determine with the aid of a simple computation the value of the muzzle velocity $v_{Д}$.

If, in the presence of the identical constants, values for p_m and $v_{Д}$ are computed from the tables of Professor Drozdov and from the tables of the Chair, the tables of the Chair are found to give lower

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values for p_m and $v_Д$.

For example, for $f = 950{,}000$, $\varphi = 1.05$, $\Delta = 0.60$, $B = 2.0$, $C = 0.2$, and $\ell_Д/\ell_0 = 5$, we obtain on the basis of the tables of Professor Drozdov:

$$p_m = 2255 \text{ kg/cm}^2,\ \frac{\ell_m}{\ell_0} = 0.630;\ \frac{\ell_K}{\ell_0} = 2.96;$$

$$\sqrt{1 - \eta_1^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]} = 0.510;$$

whereas the tables of the Chair, in the presence of the same constants and loading conditions, give:

$$\frac{p_m}{f} = 0.2155;\ p_m = 2045;\ \frac{\ell_m}{\ell_0} = 0.6942;\ \frac{\ell_K}{\ell_0} = 3.156;\ B = 1.29;\ D = 0.8142,$$

$$\sqrt{1 - \frac{BD}{\left(1 - \alpha\Delta + \frac{\ell_Д}{\ell_0}\right)^{\theta}}} = 0.500\ .$$

Since $v_{np} = \sqrt{2f\omega/\varphi\theta m}$ is identical under identical conditions, it follows that the numbers 0.510 and 0.500 are proportional to the muzzle velocities computed in accordance with the tables of Professor Drozdov and of the Chair, respectively.

Using the tables of Professor Drozdov, p_m was found to be 10% (2255/2045 = 1.10) higher, and the velocity $v_Д$, 2% (510/500 = 1.02) higher, than using the tables of the Chair.

The formula for x_m:

$$x_m = \frac{\varkappa\sigma_0}{\dfrac{B(1+\theta)}{\left(1 + \dfrac{p_m}{f\delta_1}\right)} - 2\varkappa\lambda}$$

shows that, as $\varkappa$ increases, x_m, ψ_m, and consequently also p_m all

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increase, it being shown by the computations that, under otherwise identical conditions, the maximum pressure is almost proportional to the magnitude of the form characteristic χ, which, according to Professor Drozdov, equals 1.06, while, in the tables of the Chair, $\chi = 1$ for a powder with a constant burning area.

For this reason, 6% of the 10% difference in pressure must have been obtained at the expense of χ; as concerns the remaining 4% difference, it is explained by the modification introduced by Professor I. P. Grave to integrate the differential equation connecting the path ℓ and x. This approximate supplementary modification leads to an increased value for the path ℓ and to a lower pressure as compared with ℓ and p_m obtained by the exact method of Professor Drozdov.

But if a comparison is made of the results of computations of the velocity v_A under the identical maximum pressure p_m and under the identical loading conditions Δ, W_0, and ℓ_A, then the tables of the Chair give values for v_A that are somewhat higher than the v_A obtained in accordance with the tables of Professor Drozdov; in this connection, the quantity ℓ_K - the path of the projectile at the end of burning of the powder - is smaller than is obtained with the aid of the tables of Professor Drozdov.

The fundamental disadvantage of the tables of the Chair resides in the fact that, if the propellant force of the powder changes, the forcing pressure p_0 changes simultaneously and proportionally, since, in the tables, p_0/f = const. = 0.035.

In any case, it should be pointed out that even the most exact method, and especially any tables formulated on the basis of such a method at definite values for the constants, cannot yield perfect agreement with experimental data obtained by firing different guns.

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This is due to the fact that the relations employed to account for the phenomena accompanying the shot, as well as all methods of solution, are to one or another degree approximate with respect to the actual phenomenon of the shot.

This circumstance demands preliminary computations for the selection of some constants, with the aid of which the computed data are obtained close to or coincident with the experimentally observed results of firing tests (p_m, $v_д$, and ℓ_m); and each method demands the selection of its own constants, which must give the best agreement with the results of firing tests.

For example, to obtain the data for p_m and $v_д$, the geometric law of burning demands certain constants, while the physical law of burning demands other constants.

As concerns the application of the tables of the Chair of Internal Ballistics to the investigation of the influence of various loading conditions, as well as to the solution of a series of direct and inverse problems, all the statements made above relating to the use of the tables of Professor Drozdov remains in force for the tables of the Chair as well.

For example, to determine the thickness of the powder to assure attainment of a predetermined pressure p_m provided the propellant force of the powder f is known, it is necessary first to find p_m/f, whereupon, using the table, the predetermined value of p_m/f is found at the corresponding Δ, C or H are taken accordingly in this row, and finally the thickness of the powder $2e_1$ is determined in accordance with the following formulas:

$$2e_1 = \frac{2u_1}{s}\sqrt{10Cf\omega\varphi m} = \frac{2u_1}{s}\sqrt{\frac{2f\omega\varphi m}{H}} .$$

The following scheme is employed in using the tables:

$$
\begin{array}{ccccc}
 & & & & \Delta \\
 & & & & \downarrow \\
(H) & - & C & - & \dfrac{p_m}{f} \\
 & & \downarrow & & \\
 & & 2e_1 & = & \dfrac{2u_1}{s}\sqrt{10Cf\omega\varphi m} \; .
\end{array}
$$

Once the thickness of the powder with a constant burning area (tubular powder) has been found, the wall thickness of a grain with seven perforations can be determined by multiplying $2e_1$ by the coefficient 0.75, the reverse transition involving multiplication by 4/3.

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CHAPTER 3 - DETAILED TABLES FOR CONSTRUCTION OF PRESSURE AND VELOCITY CURVES

1. ANII Tables (1933).

The tables of Professor N. F. Drozdov and of the Chair of Internal Ballistics make it possible to find the maximum pressure p_m, its location l_m, the pressure p_K at the end of burning, and its location l_K. But in order to determine the initial (muzzle) velocity of the projectile, it is necessary to perform additional computations, since this velocity depends upon the length of the bore. The tables also do not give intermediate values for p, v, and l, and they do not take into account the time of motion of the projectile through the bore of the gun.

The ANII Tables (1933) represented a further step forward and considerably facilitated the conduct of the ballistic computation of guns.

They were formulated on the basis of the same constants as those used by Professor Drozdov, and the computations leading to them were based on his formulas; they were computed by the method of numerical integration. For a given density of loading Δ and a given parameter B, using a strip-type powder with the characteristics $\chi = 1.06$ and $\chi\lambda = -0.06$, they make it possible to find curves for the gas pressure, the velocity of the projectile, and the time of motion as functions of the path of the projectile through the bore, thus permitting the complete solution of the fundamental problems of internal ballistics, both direct and inverse.

The arrangement of the tables is apparent from the scheme presented on page 748. They are formulated for loading densities ranging from 0.05 to 0.95; one page is reserved for each Δ at interva-

Scheme of ANII Tables

Pressures

$\Lambda = \frac{t}{\tau_0}$ $\Delta = 0.61$

B \ Λ	0	0.1	0.2	. . .	14	15	Λ_K	p_K	Λ_m	p_m
0.7	300	2824	4516		212	195	0.448	6439	0.448	6439
0.8	300	2591	4000		214	197	0.525	5724	0.525	5724
0.9	300	2379	3592	Second Period	216	199	0.593	5100	0.539	5100
.	.	.	.		.	.	.	.	.	.
2.0	300	1317	1782	First Period	242	222	2.944	1442	0.628	2315
.	.	.	.		.	.	.	.	.	.
3.0	300	1094	1333	* **	267	247	10.12	390	0.575	1630

Velocities

B \ Λ	0.1	0.2	0.3	. . .	14	15	Λ_K	v_K	Λ_m	v_m
0.7	234	427	590	Second Period	2044	2057	0.448	791	0.448	793
0.8	225	391	542		2032	2046	0.525	813	0.525	821
0.9	218	364	504		2019	2035	0.593	834	0.593	834
.	.	.	.	Tabular Velocities	.	.	.	.	.	.
2.0	162	268	259		1868	1888	2.944	1296	0.628	596
.	.	.	.	First Period	.	.	.	.	.	.
3.0	149	235	302	* **	1716	1741	10.12	1584	0.575	485

Time

.	.	.	.	Second Period	.	.	.	.	.	.
2.0	300	1317	1782	First Period	242	222	2.944	1442	0.628	2315
.	.	.	.		.	.	.	.	.	.
3.0	300	1094	1333	* **	267	247	10.12	390	0.575	1630

Velocities

B \ Λ	0.1	0.2	0.3	. . .	14	15	Λ_K	v_K	Λ_m	v_m
0.7	234	427	590	Second Period	2044	2057	0.448	791	0.448	793
0.8	225	391	542		2032	2046	0.525	813	0.525	821
0.9	218	364	504		2019	2035	0.593	834	0.593	834
.	.	.	.	Tabular Velocities	.	.	.	.	.	.
2.0	162	268	259		1868	1888	2.944	1296	0.628	596
.	.	.	.	First Period	.	.	.	.	.	.
3.0	149	235	302	* **	1716	1741	10.12	1584	0.575	485

Time

B \ Λ	0.1	0.2	0.3	. . .	14	15	Λ_K	t_K	Λ_m	t_m
0.7	59	92	110	Second Period	901	950	0.448	131	0.448	131
0.8	60	95	117		923	973	0.525	150	0.525	150
0.9	62	98	120		944	993	0.593	175	0.593	166
.	.	.	.	Tabular Times	.	.	.	.	.	.
2.0	74	121	153		1125	1179	2.944	457	0.628	222
.	.	.	.	First Period	.	.	.	.	.	.
3.0	80	134	173	* **	1285	1343	10.12	1027	0.575	242

$\psi_0 = 0.03173$
$z_0 = 0.03009$

*Position of Pressure Maximum
**Position of End of Burning of Powder

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of 0.01; the basic numbers are the parameter B and the relative length of path of the projectile $\Lambda = l/l_0$, where l_0 is the corrected length of the chamber (in these tables, as in the tables of Professor Drozdov, it is designated as l_1). The parameter B varies from 0.7 to 3, and at Δ greater than 0.80 it varies from 1.2 to 4; Λ varies from 0 to 15 at unequal intervals, which at first are smaller (0.1) and subsequently increase (to 1).

The velocities and times are given in the tables in arbitrary units, and in order to obtain the actual velocity of the projectile the tabular velocity values v_{tab} must be multiplied by $\sqrt{\omega/q}$. To obtain the actual time of motion of the projectile, the tabular t must be multiplied by $l_0 \sqrt{q/\omega} \cdot 10^{-6}$ if l_0 is expressed in decimeters:

$$t = t_{tab} l_0 \sqrt{\frac{q}{\omega}} \cdot 10^{-6}; \quad v = v_{tab} \sqrt{\frac{\omega}{q}}.$$

In each of the three tables, there are recorded on the right-hand side the exact values of the quantities corresponding to the maximum pressure and to the end of burning; i.e., p_K, p_m, v_K, v_m, t_K, and t_m are given for $\Lambda_m = l_m/l_0$ and for $\Lambda_K = l_K/l_0$.

The heavy broken line marks those intervals between neighboring values of Λ between which the end of burning of the powder is located. To the left and downward from this broken line are located the values of p, v, and t corresponding to the first period of the shot; those corresponding to the second period are located to the right and upward from the broken line.

As B increases, and consequently as the thickness of the powder at a given Δ increases, the end of burning shifts closer and closer to the muzzle face.

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The exact value of $l_K/l_0 = \Lambda_K$ is contained in the fourth column from the right. If $\Lambda_K < \Lambda_Д = l_Д/l_0$, this signifies that the burning of the powder is complete; if $\Lambda_K > \Lambda_Д$, it means that the powder does not burn completely in the bore.

The thin vertical lines in the range of $\Lambda = 0.1-0.7$ show that the maximum pressure p_m is located in this region (in the particular interval marked by the thin vertical line).

At the bottom of each page, there are presented the values of ψ_0 and z_0 corresponding to the instant of initial pressure at the given Δ.

A detailed description of the use of the tables is presented in the tables themselves.

Example and procedure for computation. Given a 76-mm 1902 model gun:

$$W_0 = 1.654 \text{ dm}^3;\ s = 0.4693 \text{ dm}^2;\ l_Д = 18.44;\ q = 6.5 \text{ kg};$$

$$\omega = 0.900 \text{ kg};\ 2e_1 = 1 \text{ mm} = 0.04 \text{ dm};\ u_1 = 0.0000075 \frac{\text{dm}}{\text{sec}} : \frac{\text{kg}}{\text{dm}^2}$$

Computation of constants:

$$\Delta = \frac{\omega}{W_0} = \frac{0.900}{1.654} = 0.543;\ l_0 = \frac{W_0}{s} = \frac{1.654}{0.4693} = 3.53 \text{ dm};$$

$$\Lambda_Д = \frac{l_Д}{l_0} = \frac{18.44}{3.55} = 5.20;\ I_K = \frac{e_1}{u_1} = \frac{0.005}{0.0000075} = 667 \frac{\text{kg} \cdot \text{sec}}{\text{dm}^2}$$

$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{0.4693^2 \cdot 667^2 \cdot 98.1}{95 \cdot 10^4 \cdot 0.90 \cdot 1.05 \cdot 6.5} = 1.645 \approx 1.65;\ \varphi = 1.03 + \frac{1}{3}\frac{0.900}{6.5} = 1.076;$$

$$\sqrt{\frac{1.05\omega}{1.076q}} = \sqrt{\frac{0.900 \cdot 1.05}{6.5 \cdot 1.076}} = 0.368;$$

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[redacted]

Table 14

$\Delta = 0.54$

B \ Λ		0.2	0.4	0.682 Λ_m = 0.679 0.676	1.0	1.808 Λ_K = 1.929 2.050	5.0	$\Lambda_Д$ = 5.20	5.5
p kg/cm^2	$B = 1.60$	1774	2228	2368	2296	1878	672		606
	$B = 1.65$	1735	2176	p_m = 2310	2240	1770	675	$p_Д$ = 650	609
	$B = 1.70$	1696	2124	2253	2184	1661	679		612
$v_{tab.}$	$B = 1.6$	288	475	676	849	1115	1589		1624
	$B = 1.65$	285	469	664	838	1147	1579	1593	1615
	$B = 1.70$	282	463	653	828	1179	1569		1605
$v_{tab.}\sqrt{\frac{\omega}{q}\frac{1.05}{\varphi}}$		105	172	243	307	422	577	$v_Д$ = 587 m/sec	
$t_{tab.}$	$B = 1.6$	112	166	213	255	336	565		596
	$B = 1.65$	113	168	215	258	350	571	583	601
	$B = 1.70$	114	169	216	260	365	577		607
$t = t_{tab.}\sqrt{\frac{1.076\ q}{1.05\ \omega}}\ l_0 \times 10^{-6}$ sec.		0.00108	0.00162	0.00206	0.00248	0.00335	0.00549	0.00560 sec. = $t_Д$	
$l = l_0 \Lambda d m$		0.705	1.41	2.39	3.53	6.80	17.65	18.44 = $l_Д$	

$$l_0 \sqrt{\frac{\varphi q}{1.05\omega}}\, 10^{-6} = 3.53\, \frac{10^{-6}}{0.368} = 0.0000096.$$

All computations are summarized in Table 14, with special consideration of the necessity of interpolating for B between 1.6 and 1.7.

The curves for p and v may be constructed either as functions of the path l or as functions of the time t.

All problems which were solved with aid of the tables of Professor N. F. Drozdov are solved in exactly the same manner with the aid of the ANII Tables. By using these tables, it is possible to determine considerably more rapidly the muzzle velocity $v_Д$ and the length of the path necessary in planning to obtain the required muzzle velocity.

To solve this last problem at a given Δ, the quantity p_m is used to find B and then the thickness of the powder $2e_1$. Knowing the predetermined value of $v_Д$, $v_{Д\,tab.} = v_Д \sqrt{\omega/q\ 1.05/\varphi}$ is found, and the table of velocities is used at the value of B found to seek the column containing the value of $v_{Д\,tab.}$. By ascending the column, $\Lambda_Д$ and then $l_Д$ are determined.

Schematically, this procedure for the solution will be represented as follows:

Pressures

$$\Delta \downarrow$$

$$2e_1 = \frac{2u_1}{s}\sqrt{Bf\omega\varphi m} \longleftarrow B \longleftarrow p_m$$

Velocities

$$\Lambda_Д \rightarrow l_Д = \Lambda_Д \cdot l_0$$

$$B \longrightarrow v_{tab.} \ (\uparrow \Lambda_Д)$$

The ANII Tables are likewise subject to the rule for conversion from the thickness of the strip-type powder, for which they are computed, to the thickness of a grain with seven perforations:

$$2e_1 \text{ with 7 perforations} = 0.7 \cdot 2e_1 \text{ strip-type}$$

Defects of ANII Tables. In the ANII Tables, the time of passage by the projectile of the first segment from 0 to $\Lambda = 0.1$ is computed incorrectly; these values are nearly twice as small as the values computed in accordance with the more exact formulas proposed by Professor E. L. Bravin [11], who noticed this error. This error distorts the first segment of the curves for the pressure, velocity, and path as functions of time and shifts all curves toward the origin by an amount equal to the magnitude of the error.

Professor Bravin proposed a formula to permit computation to a great degree of exactness the first element of time during the passage of the path $\Lambda = 0.1$, provided that there are given curves for the pressure and velocity as functions of the path, which is exactly what is available in the ANII Tables.

Having an initial pressure $p_0 = 300$ kg/cm^2, the pressure p', and the velocity v' (the latter two corresponding to the path $\Lambda' = 0.1$), it is possible to compute the time interval t' in accordance with the following formula:

$$t' = \frac{3l'}{v'} \frac{p_0 + p'}{2p_0 + p'} .$$

By subtracting from t' the quantity t'_Λ in the ANII Tables corresponding to the same path $\Lambda' = 0.1$, there is found the constant correction $\Delta t' = t' - t'_\Lambda$, which must be added to each time value found

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in the given row of the ANII Tables. Professor Bravin derived tables of corrections $\Delta t'$ to be applied to the ANII Tables for various Δ and B.

Aside from this error inherent in the computations, the ANII Tables suffer from poorly performed interpolation and contain many misprints. For this reason, in using them, it is recommended either to construct curves, which will make it possible, by the departure of points, immediately to detect errors and misprints, or else to pay close attention to the consistent character of the variation of the quantity being determined with the aid of the tables (p, $v_{tab.}$, $t_{tab.}$).

In spite of these defects, the ANII Tables represent a good aid in the solution of the most diverse, both direct and inverse, problems in internal ballistics and in the ballistic design of guns.

Some additional applications of tables of the type of the ANII Tables are cited in the chapter on the ballistic design of guns.

2. GAU Tables (1942)

There have now been published the more convenient and exact GAU Tables of 1942, which are formulated on the same general principle as the ANII Tables, but with a different arrangement of the fundamental parameters and elements of the shot. Moreover, the range of variation of the parameter B has been considerably expanded in the GAU Tables (from zero to 4.0).

The GAU Tables were formulated under the direction of Professor V. E. Slukhotsky and S. I. Ermolaev/ 12_7.

They consist of four parts. The first part comprises the tables of pressures, the second the tables of nominal velocities $v_{tab.}$ = $= v\sqrt{\varphi q/\omega}$, and the third the tables of nominal times t STAT. = STAT $1/I_0$

$\sqrt{\omega/\varphi q}$. With the aid of these tables, it is possible to conduct all ballistic computations of a gun in designing an artillery system. However, for convenience in computation, the three parts mentioned above are supplemented by a fourth, which comprises special tables for ballistic computation (TBR).

The tables of pressures, velocities, and times are characterized by the density of loading, whose values are given from 0.05 to 0.95 kg/dm^3 at 0.01 kg/dm^3 intervals.

The basic numbers in the tables of pressures, velocities, and times are the quantities:

$$B = \frac{s^2 I_K^2}{f\omega\varphi m}$$

and the relative path of the projectile:

$$\Lambda = \frac{l}{l_0}.$$

The values of Λ in each table are varied in the range of 0-20 at varying intervals. In each of the three tables, there are also contained exact values for the quantities Λ_m, Λ_K, p_m, p_K, v_{Tm}, v_{TK}, t_{Tm}, and t_{TK}, which correspond to the instant of attainment of maximum powder-gas pressure in the barrel and to the instant of the end of burning of the powder. The pressures are given in kg/cm^2.

The tables of pressures have the following form (cf. scheme).

In the tables of the first part, there are presented the true values for the pressures corresponding to the predetermined values of Δ, B, and Λ. By taking in the tables Λ from 0.1 to $\Lambda_Д = l_Д/l_0$ for the given gun, we shall obtain the corresponding values for the pressure in kg/cm^2 and shall be able to plot by points the p-Λ or p-l curve,

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since $l = t_0 \Lambda$.

Pressures (kg/cm^2), $\Delta = 0, \ldots$

Λ \ B	0.1	0.2	0.3	. . . .	. . .	4.0
0.1						
0.2						
.						
.						
.						
1.0						
1.5						
.						
.						
.						
.						
19						
20						
Λ_K						
p_K						
Λ_m						
p_m						

The one or two thin horizontal lines in the tables show that the maximum pressure is located in the given interval of Λ; the heavy "stairway" indicates the boundary between the first and second periods.

The tables of nominal velocities $v_{tab.}$ in the second part and of nominal times $t_{tab.}$ in the third part are arranged in exactly the same

manner as the tables in the first part, except that, in the last line and in the third line from the bottom, there are presented, respectively, v_m and v_K in the second part and t_m and t_K in the third part.

The actual velocities of the projectile are defined by the following expression:

$$v = v_{tab.} \sqrt{\frac{\omega}{\varphi q}} .$$

The actual times are defined by the following formula:

$$t = t_{tab.} \, l_0 \sqrt{\frac{\varphi q}{\omega}} \cdot 10^{-6} ,$$

where l_0 is in decimeters.

The tables are formulated on the basis of the following data:

Propellant force of powder	$f = 950{,}000$ kg · dm/kg
Covolume	$\alpha = 1.00$ dm^3/kg
Specific gravity of powder	$\delta = 1.6$ kg/dm^3
Initial pressure	$p_0 = 300$ kg/cm^2

In addition, the following assumptions were made in formulating the tables: $\theta = 0.2$, $\varkappa = 1.06$, $\varkappa\lambda = -0.06$. The velocities and times of motion of the projectile through the bore were computed on the condition that $\varphi = 1$.

The results obtained by the computations should be summarized in the form of a "Table of Principal Elements of Shot from Gun" at the following data:

$d = 107$ mm; $W_0 = 4{,}600$ dm^3; $s = 0.9165$ dm^2; $l_Д = 34.20$ dm; $q = 17.0$ kg; $\omega = 3.0$ kg; $p_m = 2500$ kg/cm^2; $\frac{\omega}{q} = 0.1765$; $\varphi = 1.05 + \frac{1}{3}\frac{\omega}{q} = 1.109$;

$l_0 = \frac{W_0}{s} = 5.019$ dm; $\Lambda_\text{Д} = \frac{l_\text{Д}}{l_0} = 6.814$; $\Delta = 0.65$ (rounded off to 0.01);

$$n_v = \sqrt{\frac{\omega}{\varphi q}} = 0.399; \; n_t = l_0 \sqrt{\frac{\varphi q}{\omega}} \, 10^{-6} = 12.56 \cdot 10^{-6}.$$

Summary of Results Obtained

Λ	0	0.2	0.4	$\Lambda_m = 0.623$	1.0	2.0	$\Lambda_K = 3.19$	5.0	$\Lambda_\text{Д} = 6.814$
l dm	0	1.00	2.01	3.13	5.02	10.04	16.01	25.10	34.20
p kg/cm^2	300	2008	2415	2500	2361	1827	1392	850	602
$v_{tab.}$	0	294	472	623	816	1136	1568	1565	1686
$v = n_v v_{tab.}$	0	117	188	249	325	453	546	624	673
$t_{tab.}$	0	204	258	299	351	452	546	667	779
$t_{sec} \cdot 10^3$	0	2.57	3.24	3.76	4.41	5.68	6.86	8.38	9.79

CHAPTER 4 - TABLES BASED ON GENERALIZED FORMULAS WITH REDUCED NUMBER OF PARAMETERS AND WITH RELATIVE VARIABLES

1. FORMULAS AND TABLES OF PROFESSOR B.N. OKUNEV.

Toward the end of the thirties, there were published several investigations in which groupings of parameters and relative variables were introduced for the purpose of reducing the large number of parameters and characteristic constants, as well as for the purpose of avoiding absolute values for the principal elements of the shot.

Such investigations include those by the Soviet workers Professor N.F. Drozdov/_16_7, Professor B.N. Okunev/_13_7, M.S. Gorokhov and A.I.Sviridov/_14_7, and Professor G.V. Oppokov/_15_7.

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As an example, we shall consider the method of Professor Okunev,

in which there introduced the relative variables $p_T = p/p_1$, where $p_1 = \frac{f\Delta}{1 - \alpha\Delta}$; $\nu = \frac{v}{v_{np}}$; $\tau = t/T$, where $T = \frac{\varphi q}{gs}\frac{v_{np}}{p_1}$; $x = \frac{\Lambda}{1 - \alpha\Delta} = l/l_1$; and the generalized parameters $R = \sqrt{\frac{2}{B\Theta}} = \sqrt{\frac{H}{\Theta}}$ and $\Lambda_\Delta = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} = l_\Delta / l_1$.

Professor Okunev' quantity Λ_Δ is the reciprocal of the quantity ∂, which was introduced by us in pyrostatics for the computation of ψ: $\Lambda_\Delta = 1/\partial$.

$$x = \frac{\Lambda}{1 - \alpha\Delta} = \frac{l}{l_1} .$$

Let us divide by $1 - \alpha\Delta$ the numerator and denominator of the formula for pressure:

$$p = f\Delta \frac{\psi - \frac{B\Theta}{2} x^2}{\Lambda_\psi + \Lambda} ,$$

where

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta \left(\alpha - \frac{1}{\delta}\right)\psi .$$

By transferring p_1 to the left, and keeping in mind that

$$x_\psi = \frac{\Lambda_\psi}{1 - \alpha\Delta} = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} - \left(\frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} - 1\right)\psi = \Lambda_\Delta - (\Lambda_\Delta - 1)\psi ,$$

where

$$\Lambda_\Delta = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} .$$

we obtain the tabular pressure:

$$p_{tab.} = \frac{\psi - \frac{B\Theta}{2} x^2}{x_\psi + x} ,$$

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where

$$p_{tab.} = \frac{p}{p_1} .$$

After dividing both sides of the differential equation:

$$\frac{dL}{dx} + \frac{B}{B_1}\frac{x}{\xi(x)} L = -\frac{\Delta}{\delta}(\alpha\delta - 1)(k_1 - 2\varkappa\lambda x)$$

by $1 - \alpha\Delta$, we obtain the following equation:

$$\frac{d(X_\psi + X)}{dx} + \frac{B}{B_1}\frac{x}{\xi(x)}(X_\psi + X) = -(\Lambda_\Delta - 1)(k_1 - 2\varkappa\lambda x).$$

Consideration of the expressions derived above shows that the quantity p_T and the quantity X are in the first period functions of the argument x and not of eight parameters, as previously, but of only five:

$$\theta, \varkappa, z_0, \Lambda_\Delta, B.$$

In the second period:

$$\frac{p}{p_K} = \left(\frac{1 - \alpha\Delta + \Lambda_K}{1 - \alpha\Delta + \Lambda}\right)^{1+\theta} = \left(\frac{1 + X_K}{1 + X}\right)^{1+\theta} .$$

The values of $p_{tab.}$ and X at the pressure maximum and at the end of burning depend upon the same five parameters. It should, however, be noted that, instead of p_0, the parameters include z_0. At a predetermined z_0, different p_0 will be obtained at different values of $\varkappa, \Lambda_\Delta$, and B, so that, in formulating the tables, it is not possible to base them on definite z_0, it being instead necessary to accept z_0 as one of the variables, whose variation, it is true, is encompassed within a narrow range. If, furthermore, the form of

the powder, i.e., $\varkappa$, and the ratio of heat capacities $1 + \Theta$ are predetermined, the values of $p_{tab.}$ and X at the pressure maximum and at the end of burning can be summarized in the form of tables with three entries: Λ_{Δ}, B, and z_0. In this connection, the remaining quantities which have not received definite values are the propellant force of the powder f, the covolume of the powder α, and the density of the powder δ. Unfortunately, it is impossible to vary the propellant force of the powder within wide limits, since the quantity $1 + \Theta$, which is connected with it, is predetermined.

This principle was used by Professor B.N. Okunev in the formulation of his tables [13]. In the first table, the values for $p_{tab.}$, X, $\nu = v/v_{np}$, and $\tau = t/T$ are given as functions of Λ_{Δ}, $R = \sqrt{2/B\Theta}$, and z_0 in the supporting points of the pressure curve. In the expressions for ν and τ, v_{np} is the limiting velocity, and $T = \varphi q/gs \; v_{np}/p_1$. In the second table, $p_{tab.}$, X, and τ are given as functions of the parameters Λ_{Δ}, R, and z_0 and of the argument ν. This table makes it possible to construct curves for the pressures and velocities as functions of paths and times.

In respect to these tables, there remains in force what was said above. In varying f, it is necessary to vary $1 + \Theta$, but this quantity, in the tables of Professor B.N. Okunev, has the definite value $1 + \Theta = 1.20$; for this reason, the tables relate to a definite value of the propellant force of powder f corresponding to this value.

It should also be pointed out that the necessity of placing tabular pressures in the tables creates great complications in all those cases when it is necessary to solve problems under the condition of maintaining p_m constant, as is usually the case in ballistic design.

2. METHOD OF PROFESSOR N.F. DROZDOV [16]

In his work, Professor N.F. Drozdov chose as the relative variable the ratio of the current pressure to the initial pressure: $\Pi = p/p_0$. Likewise, taking the parameter of Professor Okunev, $\Lambda_\Delta = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta}$ Professor Drozdov replaces it by $\zeta = 1 - 1/\Lambda_\Delta = 1 - \partial$ and introduces two additional parameters:

$$R = \frac{p_0\left(\alpha - \frac{1}{\delta}\right)}{f} = \frac{p_0}{f\delta_1} \quad \text{and} \quad R_1 = \frac{R}{1 + R} = \frac{1}{\frac{f\delta_1}{p_0} + 1}$$

which are functions of f, α, δ, and p_0.

For the normal tabular values of these constants:

$$R = 0.01121; \; R_1 = 0.01108.$$

Transforming his equations for a powder with a constant burning area, Professor Drozdov reduces the relative maximum pressure and the pressure at the end of burning, as well as the velocity of the projectile in the first period, to the following expressions, where:

$$\gamma = B_1\psi_0/k_1^2, \; \beta = B_1 x/k_1;$$

$$B_1 = \frac{B\theta}{2} - \chi\lambda; \text{ at } \chi = 1 \; \frac{B}{B_1} = \frac{2}{\theta};$$

$$\beta_m = \frac{1}{\frac{B}{B_1} + 2}(1 + \Pi_m R) = \frac{\theta}{2(1 + \theta)}(1 + \Pi_m R);$$

$$\Pi_m = \frac{p_m}{p_0} = \frac{\frac{\gamma + \beta_m - \beta_m^2}{\gamma} Z_m^{\frac{B}{B_1}}}{1 - R\frac{s_m}{\gamma}},$$

where $s' = \int_0^\beta Z^{\frac{B}{B_1}} d\beta$ is found from the tables in the Appendix (Appendix IV) for B/B_1 from 6 to 10.

These expressions are convenient for investigating the question of the influence of variations in powder characteristics and of various parameters generally upon the value of the maximum pressure, as well as for solving the problem of the transition from one set of powder characteristics to another.

For the end of burning:

$$\Pi_K = \frac{p_K}{p_0} = \frac{\frac{\chi + \beta_K - \beta_K^2}{\chi} Z_K^{\frac{B}{B_1}}}{1 - R\frac{s_K'}{\chi}} = \frac{(\chi + \beta_K - \beta_K^2) Z_K^{\frac{B}{B_1}}}{\chi - Rs_K'} .$$

The velocity of the projectile in the first period:

$$v = \beta\sqrt{\frac{B}{B_1}\frac{R_1}{R}\frac{p_0 s \ell_0\left(1 - \frac{\Delta}{\delta}\right)}{\chi\varphi m}} ,$$

where

$$R_1/R = 1/1 + R = 1/1 + p_0/f\delta_1 .$$

In the second period:

$$\Pi_Д = \frac{p_Д}{p_0} = \Pi_K\left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda_Д + 1 - \alpha\Delta}\right)^{1 + \theta} \Pi = \Pi_K \eta_1^{1 + \theta} ;$$

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$$v_{A}^{2} = v_{np}^{2} \left[1 - \eta_{1}^{\Theta} K \right],$$

where

$$K = \left[1 - \frac{B\Theta}{2} (1 - z_0)^2 \right].$$

On the basis of the relations obtained, Professor N.F. Drozdov solved a number of problems relating to the influence of the parameters entering into the equation for the maximum pressure upon the magnitude of this pressure, utilizing for this purpose a large number of new tables formulated by him and appended to his above-mentioned work.

Thus, he determined the influence of the following factors upon the variation of the maximum pressure p_m: variation of the initial pressure p_0, variation of the propellant force of the powder f, the quantity Θ, the powder density δ, the covolume α, and the transition from a powder with one set of form characteristics to a powder with another set of characteristics.

For certain partial loading conditions, his computations led to the following results.

a) Influence of Initial Pressure p_0 upon Value of p_m.

p_0	Δp_0	p_m	Δp_m
200		2364	
	100		158
300		2522	
	150		240
450		2762	

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From this, it is possible to derive the relation:

$$\Delta p_m \approx 1.6\Delta p \ .$$

i.e., the difference between maximum pressures is greater than the difference between initial pressures.

The difference increases with diminishing p_0.

p_0	Δp_0	p_m	Δp_m	k_p
450		2762		
	150		240	1.60
300		2522		
	100		158	1.58
200		2364		
	92		180	1.96
108		2184		
	54		146	2.70
54		2038		

b) Influence of Variation of α.

α	$\Delta\alpha$	p_m	Δp_m
0.98		2767	
	0.08		-104
0.90		2663	

As the covolume α diminishes by 0.01, the pressure p_m decreases by 13 kg/cm^2.

c) Influence of Variation of δ.

δ	$\Delta\delta$	p_m	Δp_m	$\frac{1}{\delta}$	$\Delta\left(\frac{1}{\delta}\right)$
1.60		2522		0.625	
	-0.04		+24		+0.015
1.56		2546		0.640	

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As the specific volume of the powder increases by 0.01, the pressure p_m increases by 16 kg/cm^2.

Consequently, similar variations in α and $1/\delta$ result in nearly the same change in the quantity p_m.

d) Influence of Variation of $\varkappa$.

$\varkappa$	1.00	1.02	1.045	1.06
p_m	2377	2425	2481	2500

p_m increases almost proportionally to the quantity :

$$\frac{\Delta p_m}{p_m} \approx \frac{\Delta \varkappa}{\varkappa} .$$

e) Influence of θ.

θ	p_m	Δp_m	$k=1+\theta$
0.25	3366		1.25
		121	
0.20	3487		1.20
		114	
0.16	3601		1.16

$$\frac{\Delta p_m}{p_m} \approx -1.1\frac{\Delta k}{k} .$$

f) Influence of f.

B = const.

$f. \frac{T\cdot m}{kg}$	Δf	p_m	Δp_m
104.5		2724	
	-9.5		-202
95		2522	
	-15.8		-328
79.2		2194	

$$\frac{\ell_K^2}{f} = \text{const.}$$

$$\frac{\Delta p_m}{p_m} \approx 0.8\,\frac{\Delta f}{f}$$

l_K = const.

$f\ \frac{T \cdot m}{kg}$	Δf	p_m	Δp_m
104.5		2997	
	-9.5		-475
95		2522	
	-8.7		-374
86.3		2148	

$$\frac{\Delta p_m}{p_m} \approx 1.75\,\frac{\Delta f}{f}$$

Subsequently, these new tables of Professor N.F. Drozdov were utilized in certain investigations connected with the consideration of variations in the initial pressure.

3. FORMULAS AND TABLES OF M.S. GOROKHOV AND L.I. SVIRIDOV /14/

By utilizing the parameters and variables γ and β of the method of N.F. Drozdov, and by introducing a new parameter D_1, it becomes possible to transform the formula for the path of the projectile to the following form:

$$\Theta = \frac{l}{l_\Delta} + 1 = N(\gamma,\beta)\,[1 - D_1 L(\gamma,\beta,n)] + \rho,$$

where

$$l_Д = l_0\left(1 - \frac{\Delta}{\delta}\right);\quad D_1 = D\,\frac{n(1+\Theta)}{2+n};\quad D = \frac{\alpha - \frac{1}{\delta}}{\frac{1}{\Delta} - \frac{1}{\delta}}\,\frac{k_1^2}{B_1};$$

$$k_1^2 = x^2 - 4x\lambda\psi_0;\quad B_1 = -x\lambda + B\,\frac{\Theta}{2};\quad n = \frac{B}{B_1};$$

$$N(\gamma,\beta,n) = Z^{-n}(\gamma,\beta);$$

$$L(\gamma,\beta,n) = \gamma + \int_0^{\beta} \frac{d\beta}{N(\gamma,\beta,n)} - \frac{\gamma+\beta-\beta^2}{N(\gamma,\beta,n)};\quad \rho = \frac{\Theta}{2}\,Dn\beta^2.$$

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For the function $\log Z^{-1}(\gamma, \beta)$, detailed four-place tables relating it to γ and to β have been set up. For the function $L(\gamma, \beta, n)$, tables relating it to β, γ, and n have been set up; in this connection, the parameters and variables vary within the following ranges:

$$0.00 \leqslant \gamma \leqslant 0.13;\ 0.00 \leqslant \beta \leqslant 0.70;\ 3 \leqslant n \leqslant 14.$$

The velocity of the projectile in the period of burning is determined with the aid of the usual formula.

The formula for the pressure can be transformed to the following form:

$$\Pi = \frac{\alpha - \frac{1}{\delta}}{f} p = \frac{D\xi(\gamma, B)}{\Theta - \rho - D\xi(\gamma, \beta)},$$

where

$$\xi(\gamma, \beta) = \gamma + \beta - \beta^2.$$

To determine the maximum pressure, it is necessary to determine the β_m corresponding to it with the aid of the following formula:

$$\beta_m - \frac{1}{2+n} = D_1 \left\{ \left(\gamma + \int_0^{\beta_m} \frac{\alpha\beta}{N(\gamma, \beta, n)} \right) \left(\beta_m - \frac{1}{2+n} \right) + \frac{\gamma + \beta_m - \beta_m^2}{(2+n) N(\gamma, \beta, n)} \right\}.$$

Detailed three-place tables have been set up for $\beta_m - 1/2 + n$. With the aid of the tables mentioned above, the fundamental problem of internal ballistics is solved for any values of the constants α, δ, f, Θ, p_0, and x present as definite values in the tables of N.F. Drozdov and of the GAU Artillery Committee.

The above-mentioned formulas and tables were first published in the work by M. Gorokhov on "Internal Ballistics" in 1943 [17].

Analogous formulas and tables (at the predetermined value $\Theta = 0.2$) were obtained by Gorokhov and Sviridov in 1939 [14, 17].

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Generalized formulas, accompanied by the use of tables, make it possible to solve problems in cases involving deviations from the usually accepted values for the constants (α, δ, f, p_0, θ, and $\varkappa$), as well as in the case of a combined charge consisting of any desired number of grades of powders and in the case of its being necessary to take into account the afterburning of decomposition residues from the burning of powders of the progressive form.

In this connection, in the course of the first phase, when all the powders burn simultaneously, the formulas remain the same as those presented above; but in the course of the second and subsequent phases, the formulas become somewhat more complex.

CHAPTER 5 - FUNDAMENTALS OF THEORY OF SIMILITUDE

1. THEORETICAL PRINCIPLES.

Ballistically similar guns are those in which the gas-pressure curves (p-l) and the projectile-velocity curves (v-l) are geometrically similar, i.e., can be made to coincide merely by changing the scale alone.

Algebraically, the condition of similitude may be expressed by the equations $F_1(p, l) = F_2(\alpha_1 p, \alpha_2 l)$ for pressure curves and $\Phi_1(v, l) = \Phi_2(\beta_1 v, \beta_2 l)$ for velocity curves, where α and β are coefficients of the change in scale leading to coincidence of the p-l and the v-l curves.

The theory of similitude in internal ballistics was developed in the Soviet Union by Professor I. P. Grave [18]; the field was further developed by Professor B.N. Okunev [19], who contributed generalized relations.

Conclusions based on the theory of similitude can find applica-

tion in the transition from guns of one caliber to those of another; they can also be applied to the ballistic design of new systems by making it possible to utilize data for already existing guns on the assumption that the conditions of the shot remain unchanged regardless of the size of the caliber of the gun, an assumption scarcely justified by actual facts.

Investigation shows that the p-l and v-l curves at different densities of loading can be similar only if $\alpha = 1/\delta$, which is not actually the case. For this reason, the case of different Δ is not considered, and only the case of Δ = const. is utilized.

The fundamental equations in the theory of similitude are the following equations of internal ballistics reduced in terms of relative variables for the formulation of ballistic tables:

$$\frac{p}{f} = \Delta \frac{\psi - \frac{B\theta}{2} x}{\Lambda_\psi + \Lambda} ;$$

$$\Lambda = \Lambda_{\psi_{av.}} (Z_x^{\frac{B}{B_1}} - 1);$$

$$v\sqrt{\frac{\varphi q}{\omega}} = v_{tab.} = \sqrt{fgB}\, x;$$

$$\psi = \psi_0 + \chi\sigma_0 x + \chi\lambda x^2;$$

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} ;$$

$$z_0 = \frac{2\psi_0}{\chi(\sigma_0 + 1)} \approx \frac{\psi_0}{\chi} ;$$

$$\Lambda_{\psi} = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi;$$

$$B_1 = \frac{B\theta}{2} - \varkappa\lambda;$$

$$B = \frac{s^2 I_K^2 g}{f\omega\varphi q}.$$

In order that the p-l and v-l curves in two guns be similar at the same Δ, it is necessary that the values of p, v, and Λ for one and the same value of x be the same. For this condition to be fulfilled, the following conditions must in turn be satisfied.

1) The nature of the powder (f, α, δ, θ) must be the same.

2) The form of the powder ($\varkappa$, λ) must be the same.

3) p_0/f or ψ_0 must be the same.

4) The parameter of the loading conditions B must be the same.

Only under these conditions will B_1, ψ_0, ψ, Λ_ψ, $\Lambda_{\psi av.}$, $v_{tab.}$, v $\left(\sqrt{\varphi q/\omega}\text{ is a scale factor}\right)$, $\gamma = B_1\psi_0/k_1^2$, $\xi = B_1 x/k_1$, $\log Z_x^{-1}$, Λ, and p also be the same, and consequently the curves for the pressure, p-Λ, and for the velocity, v-Λ, will coincide in all points, i.e., p-l and v-l will be similar for the entire first period.

For the end of burning ($\psi = 1$), we shall have the same values for v_K, p_K, Λ_K, and Λ_1.

For the second period:

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$$p = p_K\left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{1+\theta};$$

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$$\left(v\sqrt{\frac{\varphi q}{\omega}}\right)^2 = v_{tab.}^2 = \frac{2gf}{\theta}\left\{1 - \left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\}.$$

Here, the independent variable is $\Lambda > \Lambda_K$, and, since all remaining parameters in the two guns are the same, the p-Λ and $v_{tab.}$-Λ curves will also coincide, i.e., will be similar, in the second period as well.

In this connection, it is possible to draw the conclusion that, as a matter of fact, all our ballistic tables, beginning with the tables of Professor Drozdov and ending with the most complete GAU tables, are an expression and practical application of the theory of similitude. Indeed, at a given loading density Δ and at a given value of B, the quantity p_m and the p-Λ curve, as well as the $v_{tab.}$-Λ curve, depend neither upon the caliber of the gun nor upon its absolute dimensions, but only upon the ratio of Λ_m to Λ, while $v_{tab.\Delta}$ depends upon Λ_Δ. The factors $\sqrt{\omega/\varphi q}$ take into account the influence of the ratio ω/q and are scale factors used to reduce different v-Λ curves to one and the same common $v_{tab.}$-Λ curve.

The parameter of the loading conditions $B = s^2 I_K^2 g/f\omega\varphi q$ is dimensionless. For the analysis of the influence of various conditions of loading upon its magnitude, and, through it, upon the pressure and velocity curves, it is more convenient to write it differently:

$$B = \frac{n_s^2 g}{f} \frac{\left(\frac{I_K}{d}\right)^2}{\frac{\omega}{q}\varphi c_q^2} .$$

From an equality of the parameter B for two guns of different caliber, it follows that, in making the transition from one caliber to the other, with the weight of the projectile and ω/q remaining unchanged, the ratio $I_K/d:c_q$ or $I_K:q/d^2$ must also remain constant.

Consequently, it is possible to draw the following conclusion.

For similar guns of different calibers firing projectiles of the same weight, and with p_m = const., the pressure impulse I_K must be inversely proportional to the square of the caliber of the guns.

2. SOME THEOREMS OF THEORY OF SIMILITUDE.

Definitions. 1. Geometrically similar barrels are barrels in which the linear dimensions of the parts of the bore are proportional to the calibers, the cross sections of the bore are proportional to the squares of the calibers, and the chamber and bore volumes are proportional to the cubes of the calibers.

2. Similarly charged guns are guns in which the weights of the projectiles and of the charges are proportional to the cubes of the calibers.

Theorem 1. In geometrically similar and similarly charged guns, similar p-l and v-l curves can be obtained only if the powder thicknesses or the impulses I_K are proportional to the calibers of the bores.

From the condition of equality of the parameters B' and B", we have:

$$\frac{\left(\frac{I'_K}{d'}\right)^2}{\frac{\omega'}{q'}\varphi' c'^2_q} = \frac{\left(\frac{I''_K}{d''}\right)^2}{\frac{\omega''}{q''}\varphi'' c''^2_q}$$

or

$$\left(\frac{I''_K}{d''}\right)^2 : \left(\frac{I'_K}{d'}\right)^2 = \frac{\omega''}{\omega'}\frac{q'}{q''}\frac{\varphi''}{\varphi'}\frac{c''^2_q}{c'^2_q},$$

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but the condition of similarly charged guns gives us:

$$c''_q = c'_q; \quad \frac{\omega''}{\omega'} = \frac{d''^3}{d'^3}; \quad \frac{q'}{q''} = \frac{d'^3}{d''^3} \quad \frac{\omega''}{\omega'}\,\frac{q'}{q''} = 1.$$

Since

$$\omega'' = \omega' \left(\frac{d''}{d'}\right)^3; \quad q'' = q' \left(\frac{d''}{d'}\right)^3,$$

it follows that

$$\frac{\omega''}{q''} = \frac{\omega'}{q'} \text{ and } \frac{\varphi''}{\varphi'} = 1.$$

Consequently

$$\frac{I''_K}{d''} = \frac{I'_K}{d'},$$

and the theorem is proved.

<u>Theorem 2</u>. In similar and similarly charged guns, equal relative paths Λ will be associated with equal pressure and projectile velocities.

The equality of pressures follows from the ballistic similarity of curves at the same Δ and equal B, and since, for similarly charged guns:

$$\frac{\omega'}{\varphi' q'} = \frac{\omega''}{\varphi'' q''} \text{ and } v'_{tab.} = v''_{tab.},$$

it is also true that

$$v' = v''.$$

<u>Theorem 3</u>. In using one and the same gun for firing projectiles of different weights while maintaining the charge and the maxim pressure constant, the velocities of the projectiles are inversely

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proportional to the square root of the ratio of the products of the projectile weights multiplied by the corresponding coefficient φ. As a matter of fact, under the predetermined conditions, at given Δ and B and $\Lambda_{\text{Д}}$, $v'_{tab.} = v''_{tab.}$, or:

$$v'_{Д} : \sqrt{\frac{\omega}{\varphi' q'}} = v''_{Д} : \sqrt{\frac{\omega}{\varphi'' q''}} ,$$

from which

$$\frac{v''_{Д}}{v'_{Д}} = \sqrt{\frac{\varphi'}{\varphi''} \frac{q'}{q''}} .$$

In order to maintain p_m = const., we obtain from the condition B' = B"

$$\frac{I''^2_K}{\varphi' q'} = \frac{I''^2_K}{\varphi'' q''}$$

or

$$\frac{I''_K}{I'_K} = \sqrt{\frac{\varphi'' q''}{\varphi' q'}} ; \; I''_K v''_{Д} = I'_K v'_{Д} ,$$

i.e., the impulses I_K must be directly proportional to the square roo[t] of the product of the projectile weights multiplied by the correspond[-]ing coefficients φ.

In limiting consideration to these most important among the theorems of similitude and refraining from a discussion of the other numerous theorems, it is possible merely to point out that, as a rul[e] when a transition is made from a gun of one caliber to one of anothe[r] a change in the thickness of the powder is accompanied by a certain change in its nature, as well as in the initial pressure and in the

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heat loss.

This makes it necessary to consider the theorems of the theory of similitude in internal ballistics as being capable of giving merely approximate conclusions and relations.

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SECTION X - BALLISTIC DESIGN OF GUNS

General Remarks

The ballistic design of guns represents one of the most important departments concluding the theoretical course in applied interior ballistics, in which there is solved the principal problem of the latter, namely to determine the design data of the bore and the loading conditions at which a projectile of given caliber and weight, while being fired from a gun, acquires a definite predetermined initial velocity. In this connection, the maximum pressure of the gases evolved during the burning of the powder must not exceed a definite value p_m, which is usually stated in advance.

The design data of the bore comprise the following: the chamber volume W_0; the cross section s of the bore, including the rifling grooves; the length of the path of the projectile along the bore l_D; the length of the chamber l_{KM}, with proper allowance for its enlargement χ relative to the section of the barrel; the number of volumes of expansion of the gases $\Lambda_D = \frac{l_D}{l_0} = \frac{W_D}{W_0}$ or the relative path of the projectile through the bore; the length of the bore L_{KH}; the length of the barrel together with the breechblock L_{CT}; and the volume of the bore $W_{KH} = W_0 + s l_D = s(l_0 + l_D)$.

The loading conditions comprise the following: the relative weight of the charge ω/q at a predetermined definite nature of the powder (f, α, δ, θ); the loading density Δ; the shape and principal dimensions of the powder grains (type of powder); and $I_K = e_1/u_1$.

For the variant selected in the design, there is conducted a computation of the variation of the pressure of the powder gases

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and of the increase in the velocity of the projectile during the shot as functions of the path and of time. These data, plotted on diagrams in the form of p-l and v-l curves, as well as in the form of p-t and v-t curves, constitute the basic material for further computations by designers of the artillery system and ammunition.

The ballistic design of a gun provides the basic starting data for designing the extremely complex assembly represented by the modern artillery system together with the assortment of ammunition pertaining thereto.

On the basis of the data obtained in the ballistic design, the designer of the artillery system computes the barrel of the gun, the thickness of its walls, the fretting of the layers, the breechblock and the rifling grooves; these data also aid him in developing the design of the gun mount and of the recoil mechanism, which accumulates the energy of the recoil and returns the barrel of the gun to its original position after the shot.

Using the same ballistic-design data, the designer of the ammunition computes the body of the projectile and the rotating band, determines the stress in the explosive within the projectile, and computes the cartridge body and the percussion-cap mechanism, as well as the mechanisms of the firing devices and time fuzes.

On the basis of the shape and dimensions of the powder established in the design, the powder engineer computes the dies through which it is necessary to compact the powder mass of a given nature and determines the technological process required to produce the necessary dimensions and shape of the powder when the latter is finished in its final form.

Consequently, the design of the principal assemblies of an

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artillery system and of the ammunition pertaining thereto depends in considerable measure upon how rationally the ballistic design of the bore has been developed. The rational design of the bore, however, depends upon the thoroughness of the study and knowledge of the general relations which connect the elements of the shot (gas pressure, velocity and path of the projectile) and its characteristic features with the design data of the barrel and the loading conditions.

In contrast with the "direct" problem of interior ballistics (computation of the gas-pressure and projectile-velocity curves), which yields only one single solution for a predetermined barrel design and given loading conditions, the problem of ballistic design, even for a predetermined pressure p_m, admits of a multiplicity of solutions for the barrel design and for the loading conditions, which assure attainment of the predetermined initial velocity by a projectile of given weight and caliber.

In connection with such an indeterminately large number of variants of the solution of the problem, there arises the question of introducing a definite procedure for the computation of variants satisfying a given assignment and selecting a criterion for their evaluation.

A rational computing procedure must yield a solution of the problem within the shortest possible time and with a minimum number of variants.

For the theoretical justification of such a procedure, use must be made of the general relations of interior ballistics, which interconnect the design data of the bore, and of the loading conditions at definite values of the maximum gas pressure p_m and of the initial velocity of the projectile.

Once these relations have been established, it becomes possible

to outline the course of computation of the variants and to select for their evaluation one or another ballistic criterion, which characterizes the gun from the point of view of the rational nature of the solution.

But the ballistic criteria alone are not sufficient; it is necessary to take into account additional criteria, which are given in the tactical and technical requirements imposed upon the given gun when the assignment for the development of the design is issued.

On the basis of an analysis and a tactical evaluation of one or another tactical employment of artillery (destruction of live targets, attack upon tanks or aircraft, demolition of fortifications and obstacles, attack upon staffs and concentrations of enemy troops at great distances, etc.), there is issued an assignment to develop one or another system, for example an antiaircraft gun with a definite height of destruction of the target, or a heavy howitzer for the demolition of concrete fortifications, or else an antitank gun capable of piercing armor of definite thickness at a predetermined distance.

Knowing the action of the projectile at the target, and taking account of the ratio of the weight of the explosive to the weight of the projectile as a whole, the weight and caliber of the projectile are designated (ω_{BB} q = 0.10-0.20 for demolition shells, 0.02-0.05 for armor-piercing projectiles). Thereupon, using the formulas and tables of interior ballistics, there is computed the initial velocity of the projectile required in order that a projectile possessing definite caliber, weight, and shape give the necessary range, or else, for a predetermined range, have the impact velocity needed

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to pierce armor of predetermined thickness*).

In this connection, there are sometimes imposed additional requirements, for example that the gun have as small a weight as possible, or even a weight predetermined in advance, both in the traveling position and in the firing position, or else that the length of the gun be less than so many calibers, or else that an already existing shell case or gun mount be utilized.

The totality of all the above-mentioned requirements constitutes the so-called tactical and technical specifications imposed upon the gun being designed during the issuance of the assignment.

The additional conditions included in the tactical and technical specifications exert an influence upon the choice of the ballistic solution and sometimes makes it necessary to arrive at a design that is not completely rational from the ballistic point of view. For example, the 1927 model 76 mm regiment artillery gun has an excessively large chamber volume, combined with too small a number of expansion volumes and a cartridge of large weight and over-all dimensions. The 1909 model 76 mm mountain gun gives the same velocity of the projectile at the same gas pressure with a considerably smaller chamber volume, weight of the charge, and weight of the entire cartridge, and with the same bore volume. There is no doubt that the last-mentioned gun is much better designed from the ballistic point of view than the regiment artillery gun.

But the introduction of the regiment artillery gun of the above-mentioned design was dictated by considerations that were no longer

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*)To compute the velocity needed to pierce armor, use is made of the formula of Jacob and de Marre, $v_c = k[(d^{0.75}B^{0.7})/(q^{0.5}\cos\alpha)]$, where k is the coefficient of strength of the armor (k = 2200-2400).

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ballistic in character.

The art of the designer called upon to develop a ballistic design consists in considerable measure in arriving at a design that is rational from the point of view of interior ballistics while taking into account all the tactical and technical specifications.

A rational procedure for ballistic design must provide the shortest route to finding a solution satisfying all the requirements imposed by making a deliberate choice of each of the designated variants. In this connection, it is necessary to know in advance the direction and character of variation of the principal parameters and criteria which determine the expediency of the given variant; the computation must merely clarify the quantitative relations.

The principal relations which determine the connection between the design data of the bore of the gun and the loading conditions are obtained by solving the inverse problem for a predetermined caliber of the gun, weight of the projectile, and initial velocity of the projectile, and for a chosen maximum pressure.

The establishment of these general relations constitutes the subject matter of the chapter entitled "Theoretical Principles of Ballistic Design of Guns."

In establishing the general relations needed for ballistic design, it becomes necessary to resort to certain auxiliary functions and tables, which are obtained by additional treatment of the basic tables of Professor N.F. Drozdov, as well as of the ANII and GAU tables with "normal" values of the constants assumed therein.

These tables can also be utilized (and this is widely done in practice) for powders possessing a shape different from that of

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strip-type powders, with a propellant force of the powder that is not equal to 950,000 kg·dm/kg, and even for combination charges consisting of two types of powder.

Since the method of Professor Drozdov is based on the usual assumptions accepted in solving the principal problem of pyrodynamics, the same assumptions are also accepted in their entirety in the theoretical solution associated with the ballistic design (geometric law of burning, law of rate of burning $u = u_1 p$, average gas pressure in the initial air space, etc.).

There are available at the present time many investigations relating to the theory of ballistic design. Mention should be made of the work of the following authors in our country.

From 1910 through 1948, Professor N.F. Drozdov has illuminated a series of questions connected with ballistic design and has initiated the fundamental direction for the work of the Russian school with respect to the gun of maximum power as a gun capable of ensuring the maximum velocity of the projectile $\left(\frac{mv_D^2}{2}\right)$ max. in a gun of given length at a predetermined maximum pressure. The French school had defined the gun of maximum power as a gun with the maximum total work $\left(\frac{\varphi m v_D^2}{2}\right)$ max. The tables of Professor N.F. Drozdov have received widespread acceptance and application in design offices.

Professor I.P. Grave, in his course of pyrodynamics (1934-1937), gave the most complete investigation of the fundamental relations, stated the theory of ballistic design as developed by various authors, and presented a series of his own studies in this field.

Professor V.E. Slukhotsky, who has devoted his attention to

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problems of ballistic design since 1934, was the first to apply to ballistic design a consideration of the accuracy life of the barrel. Under his direction, there were compiled both the general 1942 GAU tables in three issues and the special issue of tables for ballistic computation (TBR), which are very convenient for practical use.

In 1939, Professor B.N. Okunev presented an analysis of the influence of certain parameters upon the "productivity" of an artillery system, compiled a number of tables, and outlined the general principles governing the choice of a ballistic solution.

In work performed in 1939-1946, Professor D.A. Ventsel presented the theory of ballistic design as applicable principally to small arms, in addition to which he also compiled special tables for various constants.

From 1940 through 1945, M.S. Gorokhov published a series of investigations supplemented by a large number of auxiliary tables and diagrams, which make it possible to establish general relations.

The author of the present book established in 1940 the concept of economic loading conditions, developed the theory of "the gun of minimum volume," which possesses considerably more advantageous characteristics than the earlier "gun of maximum power" advocated by the French school, and, on the basis of general relations, worked out a procedure of ballistic design with the use of a "directive diagram" for the choice of variants.

As a result of all these investigations by our scientists, the principles of ballistic design have in our country attained a high theoretical level and have been coordinated with tactical and technical requirements.

CHAPTER 1 - BASIC DATA

1. BALLISTIC CHARACTERISTICS OF GUNS.

In connection with the indefinitely large number of possible solutions in ballistic design, there arises the question of the choice of criteria for the evaluation of design variants obtained by computation.

Every gun is characterized by a definite system of ballistic characteristics, which can be broken up into three groups.

a) Design characteristics of the bore of the gun.

b) Characteristics of the loading conditions.

c) Energy characteristics of the shot.

Some of the characteristics, which have the most essential importance, may be selected as criteria for the evaluation of variants, in which connection it is also necessary to take into account the tactical and technical specifications imposed during the issuance of the assignment.

There is presented below an enumeration of the principal and most important ballistic characteristics of a gun.

A. Design Characteristics of Gun.

1. The chamber volume is characterized by its ratio to the weight of the projectile, W_0/q, which determines the magnitude of the initial velocity of the projectile. Depending upon the velocity v_D, the quantity W_0/q is varied within wide limits - from 0.1 to 2.0.

The chamber volume is sometimes characterized by the ratio W_0/d^3, which also varies within wide limits (1.6-33.0 according to V.E. Slukhotsky).

2. The length of the barrel and length of the bore in terms of calibers, L_{CT}/d and L_{KH}/d. These quantities increase as the velocity

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of the projectile and the coefficient of the weight of the projectile $c_q = q/d^3$ increase and may reach 150 and more calibers for $v_D = 1500$ m/sec.

3. The number of volumes of expansion of the gases in the bore $\Lambda_D = l_D/l_0 = W_D/W_0$, or the relative path of the projectile expressed in terms of nominal chamber lengths, is a most important design characteristic, which determines the type of gun. The larger the relative chamber volume the smaller is Λ_D. In modern guns, Λ_D varies in the range of 3.0-10.0; in guns of great power, $\Lambda_D = 3$-4; in automatic guns with small chambers, $\Lambda_D = 8$-10. Under otherwise identical conditions, a barrel of minimum weight is obtained with $\Lambda_D = 5$-6.

4. The characteristic of the depth of the rifling grooves n_s is determined from the formula $s = n_s d^2$, where n_s is about 0.80 at $t_n = 0.01d$ and $n_s = 0.83$ at $t_n = 0.02d$.

5. The coefficient of widening of the chamber $\chi = l_0/l_{KM} > 1$ (sometimes called the bottle-shape coefficient) influences the total length of the bore.

In artillery systems, χ varies from 1.05 to 3 (according to V.E.Slukhotsky); in small arms and antitank rifles, it reaches 4 and more.

B. Characteristics of Loading Conditions.

6. The loading density $\Delta = \omega/W_0$ varies within very wide limits, as follows:

In small arms	0.80-0.95
In powerful artillery systems	0.65-0.78
In ordinary guns	0.55-0.70
In howitzers with full charges	0.45-0.60

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In howitzers with reduced charges	0.10-0.35
In mortars	0.03-0.12

The loading density usually increases with increasing p_m and v_D.

7. The relative weight of the projectile ω q, which is what principally determines the velocity of the projectile and the work of displacement of the gases of the charge itself, varies within very wide limits - from 0.01 to 1.5.

8. The coefficient of the weight of the projectile $c_q = q\ d^3$ is one of the important characteristics determining the velocity of the projectile in a given gun.

For a predetermined velocity of the projectile, the quantities l_0 d, l_D d, L_{KH} d, and I_K d are directly proportional to c_q: the smaller c_q the smaller are the overall dimensions of the gun, and the finer is the powder required to maintain the predetermined p_{max}.

For armor-piercing shells of ordinary type	c_q = 16-18 kg/dm^3
For demolition shells	c_q = 12-16 kg/dm^3
For subcaliber armor-piercing projectiles with cores	c_q = 6-10 kg/dm^3
For coil projectiles with special armor-piercing cores	c_q = about 6-7 kg/dm^3
For light bullets	c_q = about 20-22 kg/dm^3
For heavy and armor-piercing bullets with special cores	c_q = about 25-30 kg/dm^3

9. The relative pressure impulse of the powder (expressed in calibers) I_K/d serves as a characteristic of the correspondence of the thickness of the powder to the caliber, as well as of the power of the gun.

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For a velocity $v_D = 350$-700 m/sec, $I_K/d = 500$-1000.

If the length of the barrel is increased with the caliber unchanged, I_K/d increases; for example, in a rifle at $v_D = 870$ m/sec, I_K/d = about 3000; in very powerful modern antitank rifles, I_K/d reaches the magnitude of 5000, where I_K is expressed in kg/dm^2·sec and d in dm.

10. The loading parameter $B = s^2 I_K^2 g / f\omega\varphi q$ combines all three preceding parameters (ω/q, c_q, and I_K/d) and represents the principal characteristic determining the magnitude of the maximum pressure p_m and the position of the projectile at the end of burning of the powder Λ_K.

The smaller B the higher is p_m and the smaller is Λ_K; and, on the contrary, as B increases, the magnitude of p_m decreases and Λ_K increases.

For normal loading conditions, independently of the magnitude of Δ and p_m, the average value of the quantity B is 1.9-2.0.

The parameter B is dimensionless, and for an analysis of the influence of the individual factors composing it, it is conveniently represented in the following form, which is based not on the absolute values, but on the relative values of the individual characteristics of the loading conditions:

$$B = \frac{n_s^2 d^4 I_K^2 g}{f \frac{\omega}{q} \varphi q^2} = \frac{g n_s^2 \left(\frac{I_K}{d}\right)^2}{f \frac{\omega}{q} \varphi \left(\frac{q}{d^3}\right)^2} = \frac{g n_s^2}{f} \; \frac{\left(\frac{I_K}{d}\right)^2}{\frac{\omega}{q} \varphi c_q^2},$$

where $\varphi = a + b\,\omega/q$ is a function of ω/q.

Consequently, the parameter B, with the propellant force of the powder constant, depends upon ω/q, $c_q = q/d^3$, and I_K/d, the ratio I_K/d being a means for changing the maximum pressure p_m and the position of the projectile at the end of burning of the powder without

considerably changing the velocity of the projectile; as for the quantities ω/q and c_q, they influence principally the initial velocity of the projectile, and their influence upon p_m may be compensated for by a corresponding change in I_K/d.

C. Energy Characteristics.

11. The coefficient of power $C_E = E_D/d^3 = c_q\, v_D^2/2g$ is the determining quantity for choosing in the design the initial elements p_m, χ, Δ, and W_0/d^3 (from the table of V.E. Slukhotsky or from the diagram of Schneider). As a rule, C_E varies in the range of 100-1700 tm/dm^3.

12. The coefficient of utilization of unit charge weight:

$$\eta_\omega = \frac{E_D}{\omega} = \frac{q v_D^2}{2g\omega} = \frac{v_D^2}{2g} : \frac{\omega}{q}\ \frac{\text{tm}}{\text{kg}}.$$

is as follows (in tm/kg).

For medium-power guns	$\eta_\omega = 120\text{-}140$
For very high projectile speeds	$\eta_\omega = 80\text{-}90$
For rifles and antitank rifles	$\eta_\omega = 100\text{-}110$
For howitzers with full charges	$\eta_\omega = 140\text{-}160$

13. The efficiency of the powder charge:

$$r_D = \frac{E_D\theta}{f\omega} = \eta_\omega \frac{\theta}{f}$$

is proportional to the coefficient η_ω and varies in the range of 0.20-0.30.

14. The characteristic of the position of the projectile at the end of burning of the powder:

$$\eta_K = \frac{l_K}{l_D} = \frac{\Lambda_K}{\Lambda_D}.$$

is as follows:

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For guns $\eta_K = 0.50\text{-}0.70$

For howitzers with full charges $\eta_K = 0.25\text{-}0.30$

15. The characteristic of utilization of the working volume of the chamber, or the characteristic of filling of the indicator diagram, is:

$$\eta_D = \frac{p_{CH}}{p_m} = \frac{\varphi E_D}{W_D p_m} = \frac{\varphi q v_D^2}{2g W_D p_m},$$

where $W_D = s l_D = W_0 \Lambda_D$.

As Λ_D increases from 3 to 10, η_D usually decreases from 0.70 to 0.40.

16. The characteristic of utilization of the total volume of the bore is:

$$R_D = \frac{\varphi E_D}{W_{KH} p_m} = \frac{\varphi q v_D^2}{2g W_{KH} p_m}$$

where:

$$W_{KH} = W_0(1 + \Lambda_D) = s(l_0 + l_D);$$

$$R_D = \eta_D \frac{\Lambda_D}{\Lambda_D + 1}$$

17. The characteristic of the accuracy life of the barrel N. In our country, one of the most widespread formulas for taking into account the accuracy life of the bore of guns is the formula of Professor V.E. Slukhotsky, which includes in itself a series of ballistic characteristics of the gun.

In firing single shots, the expression for N has the following form:

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$$N = k_1 k_2 k_3 \rho \frac{D_0^2 - d^2}{e^{0.0022 p_0 \frac{d}{\varepsilon} 10^{-3} + 0.002 t_1}} \cdot \frac{\Lambda_D + 1}{\omega v_D^2 \left[\Lambda_D \left(\frac{v_1}{v_D} \right)^2 + \left(\frac{v_2}{v_D} \right)^2 \right]} \qquad (118)$$

where k_1 is a coefficient depending upon the caliber of the gun;

k_2 is a coefficient varying with the rifling twist;

k_3 is a coefficient varying with the depth of rifling;

D_0 is the outer maximum diameter of the rotating band of the projectile;

ε is the thickness of the surface layer of the bore;

t_1 is the temperature of burning of the powder in °C:

$$t_1^o = T_1^o - 273;$$

ρ is the resilience of the metal of the tube;

v_1 is the average gas velocity in the throat of the chamber during the motion of the projectile through the bore;

v_2 is the average gas velocity in the throat of the chamber during the period of the aftereffect of the gases;

p_0 is the initial pressure.

In computing the magnitude of N in accordance with Formula (118), the quantities d and D_0 must be taken in mm, p_0 in kg/cm², v_D in m/sec.

The coefficient k_1, expressed as a function of caliber, is given in the following tabulation:

d, mm	50	100	150	200	250	300
$k_1 \cdot 10^{-6}$	15.80	7.10	3.40	2.00	1.96	1.93

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Professor Slukhotsky recommends that the coefficients k_2 and k_3 be taken as:

$$k_2 = 1 \text{ and } k_3 = 1$$

The quantity $d\,\varepsilon\,10^{-3}$ is taken, on the average, as 1.28 for artillery guns and as 1.40 for small arms.

The ratio v_2/v_D is usually small in comparison with $\Lambda_D(v_1/v_D)^2$ and may be neglected.

The ratio v_1/v_D is determined with the aid of a special table as a function of Λ_D and χ_H, where $\chi_H = l_0/\lambda_H$ and $\lambda_H = l_{KM} + 0.75d$ is the distance from the bottom to the throat of the chamber:

$$\chi_H = \frac{1}{\frac{1}{\chi} + \frac{0.75d}{l_0}}$$

Table 15 - Table of Values of v_1/v_D.

χ_H \ Λ_D	3	4	5	6	7	8	9	10
0.6	0.270	0.256	0.243	0.233	0.223	0.215	0.211	0.208
0.8	0.237	0.222	0.209	0.199	0.189	0.180	0.174	0.168
1.0	0.213	0.197	0.184	0.174	0.165	0.157	0.150	0.143
1.2	0.196	0.180	0.167	0.157	0.149	0.141	0.134	0.128
1.4	0.183	0.166	0.153	0.143	0.135	0.128	0.121	0.115
1.6	0.171	0.154	0.141	0.131	0.124	0.116	0.111	0.105
1.8	0.161	0.144	0.131	0.122	0.115	0.109	0.103	0.098
2.0	0.152	0.135	0.123	0.114	0.107	0.101	0.096	0.092
2.2	0.144	0.128	0.116	0.107	0.100	0.095	0.090	0.086
2.6	0.131	0.116	0.105	0.097	0.091	0.086	0.081	0.077
3.0	0.120	0.105	0.095	0.088	0.083	0.078	0.074	0.070

In accordance with Formula (118), it is possible to determine the actual number N of shots, which characterizes the accuracy life.

In a comparative evaluation of variants, it is possible to use

the following abbreviated conditional expression:

$$N_{ycn}{}^{*} = K \frac{\Lambda_D + 1}{\omega \Lambda_D \left(\frac{v_1}{v_D}\right)^2} .$$

(ycn* = conditional)

The product $\Lambda_D (v_1\ v_D)^2$ varies within narrow limits, for which reason N_{ycn} in most cases increases with increasing Λ_D and with diminishing weight of the charge ω.

2. COLLECTION AND TREATMENT OF PRELIMINARY DATA.

On the basis of the tactical and technical specifications imposed in the assignment for already chosen values of the caliber d, the weight of the projectile q, and the initial velocity v_D, it is necessary to collect preliminary data relating to the characteristics of guns approaching the gun being designed in type and in their loading and firing conditions.

For these "related," already existing artillery systems, it is necessary to find the following characteristics, which are in part available as such and must in part be determined by supplementary computations: d, W_0, s, l_{KM}, l_D, L_{HP}, L_{KH}/d, L_{CT} d, q, the type of projectile, $c_q = q\ d^3$, the nature of the powder (f, α, δ, θ), its shape and dimensions, the weight of the charge ω, $I_K = e_1 / u_1$, p_m, and v_D.

All these characteristics may be obtained in various handbooks, firing tables, service manuals, descriptions and drawings of charges and projectiles, and drawings of the chamber and barrel.

In firing tables and other sources, there is usually given not the length of path of the projectile along the bore, but the length of the rifled part L_{HP}, which is smaller than l_D. The difference

depends upon the arrangement of the base of the projectile, which is to be found precisely in the drawings of the projectiles. As an approximation, it may be considered that $l_D = L_{HP} + (0.5 - 1.0)d$, 0.5d relating to old flat-bottomed shells, and 1.0d relating to contemporary modernized projectiles.

On the basis of the data obtained, the following characteristics must be computed:

$$\Delta, \ \frac{\omega}{q}, \ \frac{I_K}{d}, \ \eta_\omega = \frac{E_D}{\omega} = \frac{v_D^2}{2g} : \frac{\omega}{q} \ ; \ C_\varepsilon = \frac{E_D}{d^3} = c_q \frac{v_D^2}{2g},$$

$$\eta_D = \frac{p_{av.}}{p_m} = \frac{\varphi m v_D^2}{2 s l_D p_m} = \frac{\varphi \eta_\omega \Delta}{\Lambda_D p_m} \ ;$$

$$r_D = \frac{E_D \theta}{f \omega} = \eta_\omega \frac{\theta}{f} \ ; \ r' = \varphi r_D \ ; \ R_D = \frac{\varphi m v_D^2}{2s(l_0 + l_D) p_m} = \eta_D \frac{\Lambda_D}{\Lambda_D + 1} .$$

After collecting all these data, they must be summarized and treated in such a manner as to utilize the data obtained by experimental means to coordinate the results of computation with experiment (determination in accordance with selected tables of coefficients) and to designate the basic data for use in the computations associated with the particular assignment in hand.

3. CHOICE OF BALLISTIC CRITERIA FOR EVALUATION OF VARIANTS.

Among the characteristics mentioned above, c_q and C_ε are known from the conditions of the assignment; the remaining characteristics are determined during the computation of the variants. The following characteristics are most essential:

$$\Lambda_D = \frac{W_D}{W_0} = \frac{l_D}{l_0}, \ \frac{L_{KH}}{d} \text{ or } \frac{W_{KH}}{q} = \frac{W_0}{q}(\Lambda_D + 1), \ \frac{\omega}{q}, \ \eta_K = \frac{\Lambda_K}{\Lambda_D},$$

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$$\eta_D = \frac{p_{av.}}{p_m}, \quad \eta_\omega = \frac{E_D}{\omega} \text{ and } N_{ycA} \text{ or } N.$$

In evaluating variants, it is usually attempted to select those with the largest possible η_D, R_D, and η_ω; with η_K in the range of 0.60-0.70, which corresponds to the economical utilization of the charge; with the smallest possible value of ωq; and with a large Λ_D, which increases N_{ycA}.

But, as η_D and R_D increase, the coefficient η_ω decreases; and, in order to reconcile these oppositely varying characteristics, it becomes necessary to select some intermediate relative solution.

In connection with this, mention should be made of attempts to provide a criterion combining the influence of these two conflicting criteria.

For example, Professor B.N. Okunev (1939) proposed to use as a characteristic of advantage of a variant the following quantity:

$$H = \sqrt{r'\eta_D},$$

where:

$$r' = \frac{\varphi q v_D^2 \theta}{2gf\omega} \quad \frac{\varphi\theta}{f}\eta_\omega.$$

This quantity, independently of the chosen pressure p_m, has a maximum at the ratio $\nu = v_D/v_{np} = 0.52$. For small arms, Professor Okunev recommends taking $\nu > 0.52$ (0.55-0.56), for guns of great power $\nu < 0.52$ (0.48-0.50).

Professor I.P. Grave, in developing the idea of Professor Okunev, gave the following expression:

$$H' = \sqrt[m+n]{r'^{m}\eta_D^{n}},$$

without, however, indicating a method and criterion for the choice of the exponents m and n. Professor M.E. Serebryakov, in developing this proposal, accepted as a measure of advantage the following quantity:

$$H'' = \sqrt[1+\gamma]{R_D r_D^{\gamma}},$$

in which connection he did give a procedure for computing the quantity γ and showed that, depending upon the type of system, the exponent γ varies from 0 to 1.5.

The exponent γ or the quantity H" depend in some measure upon the type of guns and in a certain measure reflect the influence of the tactical and technical specifications.

Professor V.E. Slukhotsky, while investigating a series of systems which had given good results in service, accepted for evaluating individual variants of a ballistic solution the criterion Z, as defined by the following expression:

$$Z = \eta_\omega \eta_L^{4} \sqrt{N_{yc\pi}}\ .$$

This criterion takes into account not only ballistic, but also design and economic requirements, such as are necessarily imposed upon the system being designed. The design factors include the length of the barrel; the economic factors include the accuracy life $N_{yc\pi}$ and the utilization of the charge η_ω.

As a characteristic of utilization of the length of the body of the gun, use is made of a quantity that is analogous to the

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quantity R_D:

$$\eta_L = \frac{qv_D^2}{L_{CT}} .$$

Professor Slukhotsky believes the most advantageous variant to be that in which the quantity Z is largest.

As is seen, the problem of establishing such combined criteria is in the stage of development and accumulation of experimental data.

In the selection of basic variants in ballistic design, use was made until quite recently (1940) of special diagrams or tables of relative gun characteristics. Among the most widely used diagrams, there was, for example, that shown in Fig. 133, which represents the result of treatment by Engineer N.A. Upornikov of the system of artillery equipment designed by the Schneider works.

Fig. 157 - Diagram of Basic Ballistic Characteristics.

1) Coefficient of power; 2) maximum pressure; 3) power; 4) pressure; 5) of chamber; 6) charge; 7) coefficient of volume of charge chamber and weight of charge;

Fig. 157 (cont'd.)

8) mortars; 9) howitzers; 10) field guns; 11) naval guns; 12) super guns.

The basic quantity in the diagram is the coefficient of power $C_\varepsilon = \frac{E_D}{d^3} \frac{tm}{dm^3}$. In accordance with this quantity, there are found the total length of the barrel (expressed in calibers), L_{CT}/d, as well as the relative chamber volume $c_{W_0} = W_0/d^3$, the relative weight of the charge $c_\omega = \omega/d^3$, and the maximum pressure p_m. Knowing c_{W_0} and c_ω, it is easy to find the loading density $\Delta = c_\omega/c_{W_0} = \omega/W_0$. To effect the transition from the relative chamber volume and the relative weight of the charge to the absolute values, it is necessary to multiply c_{W_0} and c_ω by the cube of the caliber in decimeters. The coefficient of utilization of the charge can be found from the ratio $C_\varepsilon/c_\omega = \eta_\omega$.

An advantage of the diagram is the independence of its data from the caliber of the system.

As guiding material, use was also made of the table compiled in 1934 by V.E. Slukhotsky [20] on the basis of a treatment of data relating to the most successful among our own and foreign systems. At the present time, this table has been revised by its author on a more modern basis.

Both in the table of V.E. Slukhotsky and in the Schneider diagram, the basic quantity is the coefficient of power of the system; values for the quantities η_ω, p_m, Δ, χ, and L_{CT}/d are given as functions of the quantity $C_\varepsilon = E_D/d^3$.

The table of V.E. Slukhotsky, as revised in accordance with the most recent data, is presented below:

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C_ε tm/dm^3	η_ω tm/kg	p_m crusher gage kg/cm^2	Δ kg/dm^3	$\chi = \frac{l_0}{l_{KM}}$	$\frac{L_{CT}}{d}$
100	124	1700	0.50	1.02	14
200	120	1950	0.55	1.09	23
300	117	2200	0.59	1.18	31
400	114	2400	0.62	1.28	38
500	112	2600	0.64	1.39	44
600	110	2800	0.66	1.50	51
700	108	2950	0.67	1.61	57
800	107	3100	0.68	1.73	64
900	106	3250	0.69	1.85	71
1000	105	3350	0.69	1.98	78
1100	104	3450	0.70	2.11	85
1200	104	3550	0.71	2.25	91
1300	103	3650	0.71	2.40	98
1400	103	3750	0.72	2.57	105
1500	102	3900	0.73	2.75	112
1600	102	4000	0.74	2.95	119

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CHAPTER 2 - THEORETICAL PRINCIPLES OF BALLISTIC DESIGN

1. MOST ADVANTAGEOUS AND MOST ECONOMICAL LOADING DENSITIES IN GUN AT GIVEN MAXIMUM PRESSURE [21]

In a given gun with a definite volumetric expansion ratio $\Lambda_D = l_D/l_0$, let the maximum gas pressure p_m be predetermined. It is necessary to follow the variation in the initial velocity of the projectile v_D required to maintain the pressure p_m constant during the simultaneous variation in the weight of the charge and in the thickness of the powder.

At the same time, it is also necessary to follow the variations in the coefficients of utilization of the unit weight of the charge $\eta_\omega = E_D/\omega$ and of utilization of the working space of the bore:

$$\eta_D = \frac{p_{av.}}{p_m} = \frac{\varphi m v_D^2}{2 s l_D p_m}$$

The minimum loading density at which a predetermined maximum pressure p_m is obtained in a given gun will be present in the case of instantaneous burning of the powder in the chamber space before the projectile begins moving.

In this case, we apply the following formula:

$$p_m = \frac{f\Delta_1}{1 - \alpha\Delta_1},$$

from which:

$$\Delta_1 = \frac{p_m}{f + \alpha p_m} = \frac{1}{\frac{f}{p_m} + \alpha} \text{ and } \omega_1 = W_0\Delta_1 = \frac{W_0}{\frac{f}{p_m} + \alpha}.$$

Now, in order to meet the condition p_m = const., we shall increase the charge while simultaneously increasing the parameter

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$B = s^2 I_K^2/f\omega\varphi m$ in conformity with the following previously established relation:

$$B = \frac{a_m \Delta}{1 - \alpha\Delta},$$

where:

$$a_m = \frac{fF_2(\Theta)}{p_m} \left(= \frac{0.32f}{p_m} \quad \text{at } \Theta = 0.2 \right).$$

In this case, the pressure impulse of the powder $I_K = e_1 u_1$ will be expressed by the following formula:

$$I_K = \frac{\sqrt{K_m}\,\omega}{\sqrt{W_0 - \alpha\omega}},$$

where:

$$K_m = \frac{f^2 F_2(\Theta)\varphi m}{p_m s^2} \quad \frac{a_m f\varphi m}{s^2}.$$

Note: The impulse increases with increasing ω; consequently, the thickness of the charge e_1 increases.

In the presence of such a simultaneous variation in the weight of the charge (or loading density) and in the thickness of the powder, we shall obtain curves for the pressure as a function of the path of the projectile on which the pressure maximum, while remaining unchanged in magnitude, will, in conformity with the expression $l_m = l_0(1 - \alpha\Delta)\,[F_1(\Theta) - 1]$, shift with increasing Δ toward the starting point of the motion, whereas the end of burning of the powder will shift toward the muzzle face. In this connection, we shall have a loading density Δ_i at which the end of burning will occur precisely at the muzzle face.

As Δ and I_K grow further, there will be obtained incomplete

burning of the powder and a decrease in v_D.

Experiments and computations show that the initial velocity of the projectile v_D accompanying such an increase in Δ from Δ_1 to Δ_i will first grow, then pass through a maximum at a certain $\Delta_m < \Delta_i$, and then slightly decrease; at $\Delta = \Delta_i$:

$$v_{Di} < v_{Dm}.$$

Consequently, for a powder of a given shape and nature, there exists a $\Delta = \Delta_m$ at which, with the pressure p_m predetermined, the initial velocity of the projectile will have a maximum value. This loading density $\Delta = \Delta_m$ will be designated by us as the most advantageous loading density.

The difference between v_{Di} and v_{Dm} is generally small (0.5-2.0%), and, in the previous investigations conducted by the French school, it was assumed as an approximation that the maximum velocity is obtained when the burning of the powder is completed precisely at the muzzle face.

As a matter of actual fact, this is not so; the relation between v_D and Δ in a given gun at a predetermined p_m is apparent from the curve in Fig. 158.

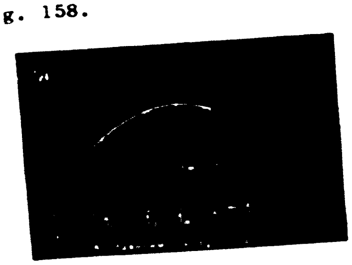

Fig. 158 - Relation between v_D and Δ in Given Gun at p_m = const.

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The v_D-Δ curve has its maximum at:

$$\Delta = \Delta_m;$$
$$v_{Di} \quad v_{Dm} \;,$$

and, consequently, there exists to the left from Δ_m a loading density $\Delta_E < \Delta_m < \Delta_i$ at which $v_{DE} = v_{Di}$.

Since the loading density Δ_E is considerably smaller than Δ_i (by 5-15%), we shall designate this loading density as the economical loading density.

At this loading density, the burning of the powder is completed sooner than at Δ_m or at Δ_i.

There is presented below a tabulation of some ballistic elements at various Δ for $\Lambda_D = 6.0$ and $p_m = 2500$ kg/cm^2.

Table 16

	Δ_1	Δ_2	Δ_E	Δ_m	Δ_i
Δ	0.21	0.55	0.65	0.70	0.75
$\eta_K = \frac{l_K}{l_D}$	0	0.30	0.55	0.72	1.00
v_D	425	613	644	648	644
$\eta_\omega = \frac{\varepsilon_D}{\omega}$	178	130	121	114	105
$\eta_D = \frac{p_{av.}}{p_m}$	0.277	0.582	0.643	0.650	0.643

The results of computations carried out with the aid of the ANII tables have confirmed the above theoretical conclusion relating to the existence of Δ_m, Δ_E, and Δ_i.

In this connection, the coefficient of utilization of the unit

weight of the charge η_ω is found to attain its maximum (178) for instantaneous burning, in spite of the low velocity of the projectile, and to decrease continuously to 105 at the maximum $\Delta = \Delta_i$.

The coefficient $\eta_D = p_{av.}/p_m$, on the other hand, increases rapidly at first, reaches its maximum at $\Delta = \Delta_m$, and thereupon slowly decreases. This decrease also continues to occur as Δ and the thickness of the powder further increase in the presence of incomplete burning of the powder.

The optimum utilization of the bore of the gun is obtained either at $\Delta = \Delta_m$ or, neglecting the small difference in velocities, at $\Delta = \Delta_E$, which gives a higher coefficient of utilization η_ω (121 instead of 114 at $\Delta = \Delta_m$).

We thus obtain the following relations:

$$\begin{array}{c|l}
\Delta & \Delta_1 < \Delta_2 < \Delta_E < \Delta_m < \Delta_i \\
v_D & v_1 < v_2 < v_E < v_m > v_i \quad (v = v_D) \\
\eta_K = \dfrac{l_K}{l_D} & 0 < \eta_K < \eta_{KE} < \eta_{K\,m} < \eta_K = 1 \\
\eta_\omega & \text{decreases} \\
\eta_D & \text{increases to } \Delta = \Delta_m
\end{array}$$

$$v_E = v_i;\ \Delta_i - \Delta_E = 0.07\ .\ .\ .\ 0.10 = (10 - 15\%)\Delta_i$$

$$v_m = (1.005 - 1.02)v_i.$$

<u>In practice, the economical loading density may be considered to be most advantageous</u>.

These relations are generally applicable to various p_m and Δ, as well as to guns with various volumetric expansion ratios Λ_D. The values of Δ_m, Δ_E, and Δ_i are functions of the gun character-

istic Λ_D and of the magnitude of p_m at a given shape of the powder.

There are presented below tabulations of values of Δ_i, Δ_E, and B_E as functions of p_m and Λ_D; these have been obtained by treatment of the ANII tables at the following powder characteristics: $\chi = 1.06$; $\chi\lambda = -0.06$; $\varphi = 1.05$; $f = 950{,}000$ kg·dm/kg; $\alpha = 0.98$ dm^3/kg; $\delta = 1.6$ kg/dm^3; $\theta = 0.20$; and for $p_0 = 300$ kg/cm^2 (standard constants adopted by Professor Drozdov in his tables).

Tables 17 and 18 show that Δ_i and Λ_E increase as p_m and Λ_D increase. In conformity with theoretical conclusions, the increase in Δ_E is accompanied by an increase in the parameter B_E, as is apparent from Table 19.

Table 20 shows that the quantity $p_{av.}$ p_m decreases with increasing Λ_D and with increasing p_m.

For economical Δ, $\eta_K = l_K/l_D$ at first decreases with increasing Λ_D, but then approaches a constant quantity.

Table 17 - Values of Δ_i (Burning of Powder at Muzzle Face). /20/

λ_D \ p_m	1800	2000	2200	2400	2600	2800	3000	3200	3400	3600
2	0.437	0.475	0.511	0.547	0.581	0.615	0.649	0.681	0.712	0.740
3	0.501	0.554	0.588	0.619	0.655	0.689	0.721	0.751	0.777	0.802
4	0.541	0.588	0.631	0.672	0.709	0.741	0.771	0.798	0.823	0.846
5	0.577	0.625	0.668	0.709	0.745	0.776	0.805	0.832	0.851	0.879
6	0.603	0.652	0.695	0.733	0.767	0.798	0.827	0.854	0.878	0.900
7	0.622	0.672	0.715	0.753	0.788	0.819	0.848	0.874	0.898	0.900
8	0.634	0.686	0.730	0.768	0.803	0.836	0.864	0.889	0.900	0.900
9	0.644	0.696	0.741	0.780	0.815	0.847	0.875	0.900	0.900	0.900
10	0.653	0.705	0.749	0.788	0.822	0.854	0.883	0.900	0.900	0.900

Table 18 - Economical Loading Densities Δ_E. [21]

λ_D \ p_m	1800	2000	2200	2400	2600	2800	3000	3200	3400	3600
2	0.41	0.44	0.48	0.52	0.55	0.58	0.62	0.65	0.68	0.71
3	0.46	0.51	0.55	0.58	0.62	0.65	0.68	0.71	0.74	0.76
4	0.50	0.54	0.59	0.63	0.66	0.70	0.73	0.75	0.78	0.80
5	0.52	0.57	0.61	0.65	0.69	0.72	0.75	0.78	0.80	0.82
6	0.54	0.59	0.63	0.67	0.70	0.73	0.76	0.79	0.81	0.84
7	0.55	0.60	0.64	0.68	0.72	0.75	0.78	0.80	0.83	0.86
8	0.56	0.61	0.66	0.70	0.73	0.76	0.79	0.82	0.84	0.87
9	0.58	0.63	0.68	0.72	0.75	0.79	0.81	0.84	0.86	0.88
10	0.60	0.65	0.70	0.74	0.77	0.80	0.83	0.86	0.88	0.90
Δ_1	0.160	0.1745	0.1885	0.2025	0.216	0.2285	0.241	0.253	0.265	0.276
Δ_H	0.492	0.532	0.568	0.600	0.631	0.660	0.687	0.711	0.736	0.762

Table 19 - Values of Parameter B_E for Economical Loading Conditions. [21]

Λ_D \ p_m	1800	2000	2200	2400	2600	2800	3000	3200	3400	3600
2	1.45	1.47	1.48	1.49	1.50	1.51	1.56	1.60	1.64	1.68
3	1.74	1.76	1.78	1.80	1.83	1.84	1.85	1.87	1.89	1.90
4	1.94	1.96	2.00	2.03	2.05	2.06	2.07	2.08	2.10	2.10
5	2.07	2.10	2.12	2.19	2.15	2.17	2.19	2.21	2.24	2.27
6	2.20	2.22	2.24	2.24	2.24	2.25	2.26	2.30	2.32	2.41
7	2.28	2.30	2.30	2.31	2.33	2.35	2.37	2.39	2.48	2.56
8	2.34	2.36	2.40	2.42	2.44	2.46	2.48	2.52	2.55	2.83
9	2.47	2.50	2.54	2.57	2.60	2.62	2.64	2.66	2.69	2.71
10	2.58	2.61	2.68	2.70	2.72	2.74	2.80	2.82	2.83	2.85

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Table 20 - Characteristics η_K and η_D for Economical Loading Conditions [21]

Λ_D		2	3	4	5	6	7	8	9	10
For all pressures $\eta_K = \frac{l_K}{l_D}$		0.82	0.75	0.70	0.66	0.63	0.60	0.57	0.59	0.63
$\eta_D = \frac{p_{av.}}{p_m}$	At p_m=1800	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.52	0.50
	At p_m=3600	0.79	0.73	0.66	0.60	0.55	0.50	0.45	0.42	0.40

2. FUNDAMENTAL RELATIONS AND DIAGRAMS FOR BORE DESIGN DATA.

It has been shown in the investigation of the most advantageous and economical loading density that the optimum utilization of the volume of the bore and of the powder charge is obtained under the condition of complete burning of the powder in the bore (l_K/l_D = 0.5-0.7).

For this reason, the basic formula for deriving the fundamental relations interconnecting the design data for the bore and the loading conditions is the formula for the velocity of the projectile in the second period, in the instant of emergence of the projectile from the bore, where v_D is predetermined, and the volume and length of the bore and the length of the path of the projectile are to be found:

$$v_D^2 = v_{np}^2\left\{1 - \left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda_D + 1 - \alpha\Delta}\right)^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\}, \qquad (119)$$

where:

$$v_{np}^2 = \frac{2gf\omega}{\varphi\theta q}\ ;\ \Lambda_K = \frac{l_K}{l_0}\ ;\ \Lambda_D = \frac{l_D}{l_0}\ ;\ \Lambda_D + 1 = \frac{l_D + l_0}{l_0} = \frac{W_{KH}}{W_0}$$

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It will be desirable to represent equation (119) in the following form:

$$(\Lambda_D + 1 - \alpha\Delta)(1 - r')^{\frac{1}{\theta}} = (\Lambda_K + 1 - \alpha\Delta)\ (1 - r'_K)^{\frac{1}{\theta}},$$

where:

$$\frac{v_D^2}{v_{np}^2} = \varphi r_D = r'; \quad \frac{v_K^2}{v_{np}} = \frac{B\theta}{2}(1 - z_0)^2 = r'_K .$$

Let us adopt the following designation:

$$(\Lambda_K + 1 - \alpha\Delta)\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{\frac{1}{\theta}} = K.$$

By solving the basic equation with respect to $\Lambda_D = 1 = l_D/l_0 + 1 = W_{KH}/W_0$ and transferring W_0 to the right-hand side, we obtain:

$$W_{KH} = W_0\left[\frac{K}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta\right] \tag{120}$$

for the entire volume of the bore, and:

$$l_D = l_0\left[\frac{K}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta - 1\right] \tag{121}$$

for the total length of the path of the projectile through the bore.

As is known from the tables of Professor Drozdov, at a predetermined value for p_m and at a chosen loading density Δ, the quantities Λ_K and B entering into the expression for K, and consequently also K itself, depend only upon p_m and Δ:

$$K = f(p_m, \Delta).$$

At predetermined values for Δ and v_D, the quantity r' is a function of ω/q only, since:

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$$r' = \frac{\varphi q v_D^2 \theta}{2g\omega f} = \frac{a + b\frac{\omega}{q}}{\frac{\omega}{q}} \frac{\theta}{f} \frac{v_D^2}{2g} .$$

At a predetermined value for v_D, the product $\frac{\theta}{f} \frac{v_D}{2g}$ = const. = k_v.

At the tabular values f = 950,000, θ = 0.2, g = 98.1 dm/sec^2:

$$k_v = \frac{\theta}{f} \frac{v_D^2}{2g} = \frac{v_D^2 \left(\frac{dm}{sec}\right)}{932 \cdot 10^6} ;$$

It is possible to compute this quantity in advance, and therefore r' is a function of ω/q only:

$$r' = k_v \frac{a + b\frac{\omega}{q}}{\frac{\omega}{q}} = f_2 \left(\frac{\omega}{q}\right).$$

The volume of the chamber is $W_0 = \omega/\Delta$.

Subsequently, for convenience in graphical representations, we shall introduce the relative quantities W_{KH}/q and W_0/q; then:

$$\frac{W_0}{q} = \frac{1}{\Delta} \frac{\omega}{q} = f_3 \left(\frac{\omega}{q} , \Delta\right) ,$$

i.e., the chamber volume is a function of only two variables, namely Δ and ω/q.

In that case, the ratio of the entire volume of the bore W_{KH} to the weight of the projectile q, which is expressed by the formula:

$$\frac{W_{KH}}{q} = \frac{W_0}{q} \left[\frac{K(p_m \cdot \Delta)}{(1 - r')^{\frac{1}{\theta}}} + \alpha \Delta\right], \qquad (122)$$

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will, at predetermined d, q, v_D, and p_m, also be a function of only two variables, namely Δ and ω/q.

In place of the quantities W_{KH}/q and W_0/q, it is possible to introduce into equation (122) the relative lengths of the entire bore and chamber in calibers.

If the corrected length of the volume of the bore is designated as $l_0 + l_D = L'_{KH}$, where $sL'_{KH} = W_{KH}$, then:

$$\frac{L'_{KH}}{d} = \frac{l_0}{d}\left[\frac{K(p_m,\Delta)}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta\right], \qquad (123)$$

$\frac{l_0}{d} = \frac{W_0}{sd} = \frac{W_0}{n_s d^3} = \frac{1}{n_s}\frac{W_0}{q}\frac{q}{d^3} = \frac{c_q}{n_s}\frac{1}{\Delta}\frac{\omega}{q}$ being a function of Δ and ω/q.

The actual length of the bore is $L_{KH} = l_{KM} + l_D = l_0/\chi + l_D$.

Upon designating $n' = 1 - 1/\chi = 1 - l_{KM}/l_0 = \delta_0/l_0$, we obtain:

$$L_{KH} = \frac{l_0}{\chi} + l_D = l_0 + l_D - \left(l_0 - \frac{l_0}{\chi}\right) = L'_{KH} - l_0 n'.$$

By substituting for L'_{KH} its expression in (123), we obtain:

$$\frac{L_{KH}}{d} = \frac{l_0}{d}\left[\frac{K(p_m,\Delta)}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta - n'\right] \quad ; \qquad (124)$$

and furthermore:

$$\frac{l_D}{d} = \frac{l_0}{d}\left[\frac{K(p_m,\Delta)}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta - 1\right]. \qquad (125)$$

Intercomparison among all the formulas presented above leads to the conclusion that, at predetermined d, q, v_D, and p_m (which is in fact, what is usually predetermined in ballistic design), <u>the design data for</u>

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the bore, i.e., the volume of the entire bore and of its working part, the chamber volume, the lengths corresponding to these volumes, and the actual length of the bore with proper allowance for the widening of the chamber, are functions of only the two variable loading conditions Δ and ω/q.

Consequently, each of the above-mentioned design quantities can be expressed in a three-dimensional coordinate system by a surface in the following coordinates:

$$Z, \Delta, \frac{\omega}{q},$$

where Z may be $\frac{W_{KH}}{q}$, $\frac{W_D}{q}$, $\frac{W_0}{q}$, $\frac{sL_{KH}}{q}$, or $\frac{L'_{KH}}{d}$, $\frac{l_D}{d}$, $\frac{l_0}{d}$, and $\frac{L_{KH}}{d}$.

Investigation shows that, at predetermined v_D and p_m, these surfaces (except for the surface representing chamber volumes) have the form of unsymmetrical "hammocks" with their convex sides turned downward. The lowest point of the "hammock" corresponds to the minimum of the quantity under consideration, and, for each of them - W_{KH}, W_D, and L_{KH} - there exists its own pair of values of Δ and ω/q at which these quantities have their minimum value (Fig. 159).

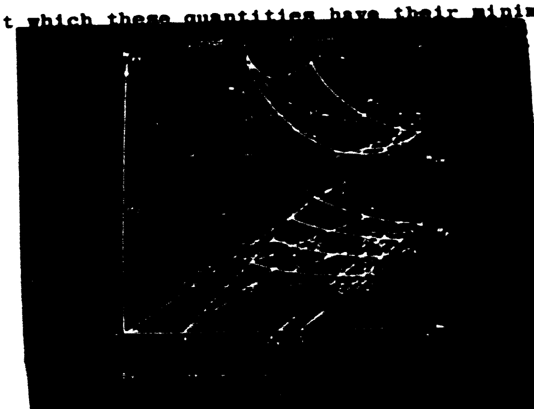

Fig. 159 - W_{KH} and W_0 as Functions of ω/q and Δ ("Hammock" and "Slope").

This existence of a minimum for all the fundamental design elements of the bore, such as the volume of the entire bore W_{KH}, its total length L_{KH}, the path of the projectile through the bore l_D, or the working volume of the bore $W_D = sl_D$, **has substantial significance in the development of a rational procedure for ballistic design.**

The form of the surface $W_0/q = (1/\Delta)(\omega/q)$ will be determined with the greatest ease; upon being intersected by Δ = const. planes, it gives as functions of ω/q straight lines, which intersect the Δ axis; upon being intersected by ω/q = const. planes, it gives equilateral hyperbolas $W_0/q\Delta$ = const., which are located the higher the greater is ω/q.

Consequently, this surface has the form of an asymmetric hyperbolic slope passing through the Δ axis, the slope having a greater gradient at small Δ than at large Δ.

The form of the W_{KH} and W_0 surfaces is represented in Fig. 159, where Δ and ω/q are plotted along the coordinate axes, and W_{KH} and W_0 are plotted along the Z axis.

The loading density varies from Δ_1, which corresponds to the given p_m as the charge burns instantaneously, to Δ_i, which corresponds to the burning of the powder at the muzzle face.

The points A and B on the $\Delta - \omega/q$ plane correspond to loading density Δ_1 and the charges ω/q = 0.6 and 1.2; the points D and C correspond to the loading density Δ_i and the same charges.

The points a, b, c, d define the surface corresponding to the chamber volumes whose magnitudes are expressed by the ordinates Aa, Bb, Cc, and Dd. The lines bc and ad are equilateral hyperbolas; the lines ab and dc are straight lines passing through the Δ axis.

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The ordinates Aa', Bb', Cc', and Dd' express the magnitudes of the volumes of the bore of the gun at combinations of Δ and ω/q corresponding to the points A, B, C, and D. The figure makes it apparent that Aa' > Bb' > Dd' > Cc' > Mm'. The ordinate Mm' gives the minimum volume of the bore, and the point M on the Δ - - ω/q plane defines the values of Δ and ω/q at which this "gun with the minimum bore volume," or briefly "minimum-volume gun," is obtained.

The ordinate Mm defines the chamber volume of the gun with the minimum bore volume.

a) Case W_0 = const.

If an intersecting plane ZOAH is passed through the point A and the Z axis, its intersection with the chamber-volume surface will give the straight line ah, where Aa = Hh. On the W_{KH} surface, there corresponds to this straight line of equal chamber volumes the line a'h' of decreasing bore volumes.

By proceeding along the line AH on the ω q- Δ plane, we maintain the tangent of its angle of slope $(\omega/q)(1/\Delta) = W_0/q$ constant; consequently, the straight lines OAH and OIK issuing from the origin of the coordinate system represent lines of equal chamber volume on the ω/q - Δ plane; in this connection, the greater the angle of slope of the straight line the larger is the volume of the chamber (Aa = Hh > Ii).

The diagram shows that, if the design is subject to the condition that the chamber volume have a definite magnitude (W_0 = = const.), this condition will be satisfied by different bore volumes (lines a'h' and i'k') and different ω/q and Δ, from among which it is possible to select those that are most suitable. In

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this connection, ω/q varies in direct proportion to the loading density Δ .

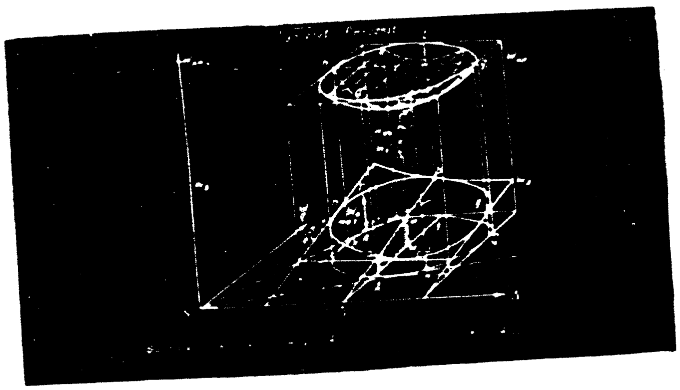

Fig. 160 - W_{KH} and W_0 as Functions of ω/q and Δ for W_{KH} = const.

b) Case W_{KH} = const. (Fig. 160).

Upon intersecting the surface of the "hammock" by a plane parallel to the $\omega/q - \Delta$ plane, i.e., W_{KH} = const., we obtain a line of intersection in the form of the oval a'h'b'g'. Consequently, the condition W_{KH} = const. can be satisfied by various combinations of Δ and ω/q; in this connection, except for the maximum and minimum values of ω/q and Δ, which define the extreme values of the boundaries of the oval (points a'b' and h'g'), two values for Δ correspond to every value of ω/q, and two values for ω/q correspond to every value of Δ. In conformity with this, the chamber volume also has two values - a greater and a smaller value - for every case.

The projection AHBG of the oval a'h'b'g' upon the $\omega/q - \Delta$ plane will also be an oval. On the hyperbolic-slope surface

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expressing the chamber volumes, its intersection with the cylinder Aa'Hh'Bb'Gg' gives the line ahbg, which possesses a complex curvature in space. If tangents are drawn from the origin of the coordinate system to the projection of the oval on the $\omega/q - \Delta$ plane, the line OR will give the maximum value for the chamber volume Rr at the given W_{KH}, and the line OS will give the minimum value for the chamber volume Ss at the same bore volume W_{KH}.

Thus, equal bore volumes are represented on the ω q $- \Delta$ plane by concentric ovals, whose center is the point M_0, which corresponds to the minimum bore volume at predetermined q, v_D, and p_m. The greater the bore volume the farther from the center M_0 does the corresponding oval lie.

3. DETERMINATION OF LOADING CONDITIONS Δ AND ω, q TO ATTAIN "MINIMUM-VOLUME GUN"

A. Determination of Loading Density Δ_{min} to Attain Minimum Bore Volume (at Constant Value of ω/q or r').

In the general case, the expression for W_{KH}/q has the following form:

$$\frac{W_{KH}}{q} = \frac{\omega}{q}\frac{1}{\Delta}\left[\frac{(\Lambda_K + 1 - \alpha\Delta)\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{\frac{1}{\theta}}}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta\right]. \quad (126)$$

The quantities Λ_K and B are not expressible analytically as functions of Δ, for which reason the partial differentiation of expression (126) to determine the minimum W_{KH}/q is not possible.

To investigate the form of the surface, it is necessary, after first assigning a constant value to ω/q, to vary Δ, i.e., to intersect our surface by planes parallel to the $Z-\Delta$ plane,

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and to determine the form of the curves obtained in each section as a function of Δ. Thereupon, after successively assigning definite values to Δ, it is necessary to vary ω/q and also to determine the form of the curves obtained in each given section as a function of ω/q.

An analytical solution at ω/q = const. is obtained only for the simplest case $\psi_0 = 0$, with a powder possessing a constant burning area.

In this case (from the formula for the velocity in the second period):

$$\frac{W_{KH}}{q} = \frac{\omega}{q}\frac{1}{\Delta}\left[\frac{1 - \alpha\Delta}{\left(1 - \frac{B\theta}{2}\right)^{\frac{1}{\theta}}}\frac{1}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta\right]; \tag{127}$$

Under the condition of constancy of the pressure p_m, the quantity B is connected with Δ by the following equation:

$$B = \frac{a_m\Delta}{1 - \alpha\Delta} = \frac{a_m}{\frac{1}{\Delta} - \alpha}, \tag{128}$$

where:

$$a_m = \frac{F_2(\theta)f}{p_m}$$

and, at $\theta = 0.2$, $F_2(\theta) = 0.320$.

By introducing $1/\Delta$ into the expression within the brackets, by replacing B by its expression (128), by introducing a new variable $y = 1/\Delta - \alpha$, and by designating $a_m\theta/2 = a''$, we obtain:

$$\frac{W_{KH}}{q} = \frac{\omega}{q}\left[\frac{y}{(1 - r')^{\frac{1}{\theta}}\left(1 - \frac{a''}{y}\right)^{\frac{1}{\theta}}} + \alpha\right] =$$

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$$= \frac{\omega}{q} \frac{1}{(1 - r')^{\frac{1}{\Theta}}} \left[\frac{y}{\left(1 - \frac{a''}{y}\right)^{\frac{1}{\Theta}}} + \alpha(1 - r')^{\frac{1}{\Theta}} \right].$$

Keeping in mind that r' is independent of Δ and y, differentiating the expressions within the brackets with respect to y, and equating the derivative to zero, we have:

$$1 - \frac{1}{\Theta} \frac{a''}{y_H} \frac{1}{\left(1 - \frac{a''}{y_H}\right)} = 0,$$

from which:

$$y_H = \frac{1}{\Delta_H} - \alpha = \frac{1 + \Theta}{\Theta} a'' = \frac{1 + \Theta}{2} a_m = \frac{1 + \Theta}{2} F_2(\Theta) \frac{f}{p_m} \quad (129)$$

and:

$$\Delta_H = \frac{1}{\alpha + \frac{1 + \Theta}{2} F_2(\Theta) \frac{f}{p_m}}$$

Let us designate:

$$\frac{1 + \Theta}{2} F_2(\Theta) = \frac{1}{2} \left(\frac{2 + \Theta}{2 + 2\Theta}\right)^{\frac{2 + \Theta}{\Theta}} = F_3(\Theta);$$

at $\Theta = 0.2$, $F_3(\Theta) = 0.192$, and:

$$\Delta_H = \frac{1}{\alpha + F_3(\Theta) \frac{f}{p_m}} \quad \frac{1}{\alpha + 0.192 \frac{f}{p_m}}.$$

The formula shows that, as p_m increases, an increase also

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occurs in the loading density Δ_H at which the minimum bore volume is obtained. It does not depend upon the quantity ω/q; consequently, at any desired value of ω/q = const., there exists a minimum bore volume, and it is always obtained at one and the same loading density Δ_H, which depends only upon the maximum gas pressure p_m.

We shall designate this loading density Δ_H as the most advantageous loading density.

The values of Δ_H applicable to our tables for the case $\psi_0 \neq 0$ were given approximately by Professor N.F. Drozdov [16] in 1940 and were then rendered more exact by the work of M.S. Gorokhov, [17] who gave a detailed table of values of Δ_H for various p_m in the range of 2000-4000 kg/cm^2 for χ = 1.06 and 1.00.

Excerpts from this table are presented below:

p_m	2000	2400	2800	3200	3600	4000	
Δ_H	0.53	0.60	0.66	0.71	0.76	0.81	For χ = 1.06

In the case of χ = 1.0, Δ_H increases by approximately 0.02 in comparison with the values shown in the above tabulation.

For the numbers in the first table at χ = 1.06, we have succeeded in deriving the following approximate empirical relation:

$$\Delta_H = \sqrt{\frac{p_m - 300}{5700}} ,$$

where p_m is given in kg/cm^2.

If expression (129) is substituted in the following formula for B:

$$B = \frac{a_m}{\frac{1}{\Delta} - \alpha}$$

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we shall obtain:

$$B_H = \frac{2}{1+\theta} = \text{const.}$$

Consequently, the minimum bore volume at any value of p_m and at the value of Δ_H corresponding thereto is always obtained at one and the same constant value of the parameter of loading B.

For the case of $\psi_0 = 0$, at $\theta = 0.2$, $B_H = 1.667$.

In accordance with the data of M.S. Gorokhov, for the tables of Professor Drozdov, at $p_0 = 300$ kg/cm^2, B_H also has a constant value:

$$B_H \approx 1.91 - 1.93$$

B. Determination of Most Advantageous Weight of Charge to Attain Minimum Bore Volume (at Δ = const.).

If, in equation (126), Δ is assigned different constant values, (i.e., if the $W_{KH}/q - \Delta - \omega/q$ surface is intersected by planes parallel to the $W_{KH}/q - \omega/q$ plane), it becomes possible to determine the conditions and values of ω/q at which, in these cases the minimum bore volume is obtained. In this case, in equation (126), not only Δ will be constant, but, for a selected value of p_m, the following function:

$$K = (\Lambda_K + 1 - \alpha\Delta) \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{\frac{1}{\theta}} = f(p_m, \Delta).$$

will likewise be constant.

Let us introduce in the place of ω/q a new variable r' (cf. p. 808):

$$r' = k_v \frac{a + b\frac{\omega}{q}}{\frac{\omega}{q}},$$

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where $k_v = v_D^2\Theta/2gf$; from this:

$$\frac{\omega}{q} = \frac{ak_v}{r' - bk_v} .$$

Expression (126) will be transformed to the following form:

$$\frac{W_{KH}}{q} = \frac{1}{\Delta} \frac{ak_v}{(r' - bk_v)} \left[\frac{K}{(1 - r')^{\frac{1}{\Theta}}} + \alpha\Delta \right] . \qquad (126')$$

By differentiating this expression with respect to r', we obtain the condition for minimum W_{KH}/q in the following form:

$$\frac{1}{(1 - r')^{\frac{1}{\Theta}}} - \frac{1}{\Theta} \frac{r' - bk_v}{(1 - r')^{\frac{1}{\Theta} + 1}} + \frac{\alpha\Delta}{K} = 0$$

or:

$$\frac{1}{\Theta} \frac{r' - bk_v}{(1 - r')^{\frac{1}{\Theta} + 1}} - \frac{1}{(1 - r')^{\frac{1}{\Theta}}} = \frac{\alpha\Delta}{K} = f(p_m, \Delta). \qquad (127)$$

At Θ = const., the left-hand side of the above equation is a function of r' and of the predetermined velocity v_D, which enters into $k_v = v_D^2\Theta/2gf$; the right-hand side $\alpha\Delta/K$ is a function of p_m and Δ ; it is known in advance. By selecting values of r', it is possible to satisfy equation (127) and to find the value r'_0 at which W_{KH}/q will have its minimum.

If the minimum bore length L_{KH} or the minimum total path of the projectile through the bore l_D are sought, then, by differentiating expressions (124) and (125), which have the same structure as expression (122), we shall obtain the following conditions

for the minima.

For the minimum bore length L_{KH}:

$$\frac{1}{\theta}\,\frac{r' - bk_v}{(1 - r')^{\frac{1}{\theta} + 1}} - \frac{1}{(1 - r')^{\frac{1}{\theta}}} = \frac{\alpha\Delta - n'}{K}\,; \tag{128}$$

and for the minimum length of path:

$$\frac{1}{\theta}\,\frac{r' - bk_v}{(1 - r')^{\frac{1}{\theta} + 1}} - \frac{1}{(1 - r')^{\frac{1}{\theta}}} = \frac{\alpha\Delta - 1}{K}\,. \tag{129}$$

Comparing all these three conditions, we see that their left-hand sides are the same, the differences residing only in their right-hand sides; the right-hand side of equation (127) has the greatest value, in (128) it is smaller, and in (129) it is negative (since $\alpha\Delta < 1$). In the first case, r'_0 will be greatest, in the last case least; as to the quantities $\omega/q = ak_v(r'_0 - bk_v)$, they vary in the opposite direction. The significance of these relations will be clarified later.

For determining r'_0 to satisfy this equation, N.A. Krinitsky designed a nomogram which makes it possible to use the quantities $\alpha\Delta/K$ and $x' = 1 - bk_v$ to find r'_0 (Fig. 161).

The nomogram consists of two uniform scales - a left-hand scale of values of $x' = 1 - bk_v = 1 - b\,\frac{\theta}{f}\,\frac{v_D^2}{2g}$ from 0.90 to 1.00 and a right-hand scale of values of $\alpha\Delta/K$ (or $\alpha\Delta - n'/K$ or $\alpha\Delta - 1/K$) - and a nonuniform curvilinear scale of values of r' in the middle.

Upon connecting with a straightedge the value of x' corresponding to the given velocity of the projectile and the value of

$\alpha\Delta/K$ corresponding to the chosen values of Δ and p_m (found in the table of the function $\alpha\Delta/K$ from the basic numbers for Δ and p_m, cf., p. 824), we read off at the point of intersection of the straight-edge with the r' scale the value r'_0 which satisfies the condition of minimum volume or length of the bore.

To determine the minimum bore volume, the value of $\alpha\Delta/K$ is taken on the right-hand scale. To determine the minimum length of the bore in the presence of an expansion (widening) of the chamber,

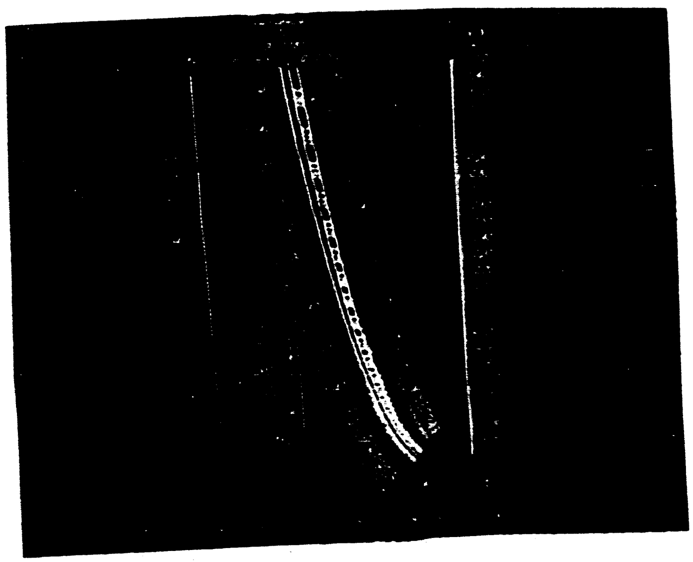

Fig. 161 - Nomogram for Determining Optimum r'_0 (Efficiency).

1) Values of r' at Θ = 0.181; 2) key.

there is taken on the right-hand scale the quantity $(\alpha\Delta - n')/K$, where $n' = 1 - (1/\chi)$. To determine the minimum length of the path of the projectile l_D, the quantity $(\alpha\Delta - 1)/K < 0$ is taken on the

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right-hand scale.

The nomogram shows that, in conformity with these three cases of determining the minima at a predetermined v_D, the point on the right-hand scale moves downward, the value of r' decreases accordingly (the straight line rotates around a predetermined point x'), and consequently the values of ω/q and W_0/q increase, since $\omega/q = a/\frac{r'}{k_v} - b$ and $W_0/q = (\omega/q)(1/\Delta)$, where Δ = const.

Thus, the gun with the minimum bore length L_{KH}, and to an even greater degree the gun with the minimum length of path l_D, are obtained with a larger chamber volume and a larger weight of the charge in comparison with the minimum-volume gun.

Therefore, the gun with the minimum bore volume which ensures the attainment of a predetermined initial velocity of the projectile v_D at a chosen pressure p_m has a smaller chamber volume and a smaller weight of the charge than the gun with the minimum bore length or with the minimum length of the path of the projectile, while having nearly the same actual bore length; for this reason, it may be designated as "optimum."

With the aid of the nomogram designed by Engineer Krinitsky, it is possible to follow the influence of other factors as well as upon the design data and the loading conditions.

For example, as the maximum pressure p_m for a predetermined value of v_D is increased, the quantity $\alpha\Delta/K$ increases, the point on the right-hand scale moves upward, and the quantity r' increases, but this reduces the weight of the charge and the chamber volume.

If Δ and p_m are maintained constant (the point $\alpha\Delta/K$ is fixed), then, as the velocity of the projectile changes (increases), x'

[redacted]

Table of $\alpha\Delta/K$ (from GAU Tables) ($\alpha = 1$).

Δ \ P_m	1600	1800	2000	2200	2400	2600	2800	3000	3200	3600	4000	4400	4800	5200	5600	6000	P_m / Δ
0.40	0.342	0.376	0.404	0.424	0.442	0.458	0.473	0.486	0.500	0.525	0.549	0.574	0.596	0.617	0.635	0.653	0.40
0.42	0.345	0.384	0.415	0.440	0.460	0.478	0.493	0.507	0.521	0.547	0.570	0.597	0.622	0.645	0.666	0.682	0.42
0.44	0.348	0.389	0.424	0.453	0.477	0.497	0.516	0.530	0.546	0.569	0.596	0.622	0.648	0.674	0.698	0.720	0.44
0.46	0.350	0.392	0.430	0.465	0.492	0.514	0.534	0.552	0.568	0.595	0.622	0.649	0.676	0.702	0.727	0.750	0.46
0.48	0.348	0.394	0.436	0.474	0.505	0.530	0.553	0.572	0.592	0.622	0.650	0.676	0.704	0.729	0.754	0.778	0.48
0.50	0.345	0.396	0.440	0.481	0.515	0.546	0.570	0.594	0.614	0.648	0.678	0.705	0.732	0.760	0.786	0.811	0.50
0.52	0.339	0.395	0.441	0.486	0.524	0.558	0.589	0.612	0.634	0.673	0.706	0.734	0.761	0.789	0.817	0.839	0.52
0.54	0.331	0.394	0.445	0.489	0.530	0.568	0.600	0.628	0.652	0.697	0.733	0.763	0.791	0.819	0.845	0.875	0.54
0.56	0.320	0.388	0.444	0.490	0.535	0.575	0.610	0.641	0.669	0.719	0.760	0.793	0.822	0.851	0.881	0.910	0.56
0.58	0.304	0.378	0.437	0.491	0.538	0.581	0.619	0.653	0.685	0.740	0.785	0.823	0.855	0.885	0.915	0.946	0.58
0.60	0.290	0.367	0.433	0.490	0.540	0.585	0.627	0.665	0.700	0.761	0.812	0.854	0.890	0.920	0.952	0.985	0.60
0.62	0.269	0.349	0.421	0.486	0.539	0.588	0.633	0.674	0.713	0.780	0.836	0.885	0.924	0.959	0.991	1.023	0.62
0.64	0.256	0.335	0.422	0.480	0.537	0.589	0.637	0.682	0.724	0.797	0.858	0.910	0.955	0.995	1.029	1.060	0.64
0.66	-	0.319	0.398	0.469	0.533	0.588	0.638	0.687	0.731	0.812	0.878	0.934	0.986	1.030	1.069	1.105	0.66
0.68	-	0.297	0.381	0.456	0.526	0.585	0.638	0.691	0.736	0.823	0.896	0.958	1.016	1.063	1.106	1.149	0.68
0.70	-	0.282	0.362	0.440	0.513	0.578	0.637	0.691	0.739	0.832	0.912	0.980	1.043	1.095	1.145	1.186	0.70
0.72	-	-	0.339	0.420	0.496	0.566	0.632	0.689	0.738	0.838	0.926	1.001	1.068	1.127	1.180	1.228	0.72
0.74	-	-	0.317	0.402	0.475	0.550	0.622	0.684	0.735	0.842	0.938	1.021	1.091	1.156	1.212	1.269	0.74
0.76	-	-	-	0.377	0.459	0.531	0.605	0.674	0.730	0.844	0.946	1.036	1.113	1.184	1.245	1.300	0.76
0.78	-	-	-	0.351	0.433	0.512	0.585	0.657	0.722	0.842	0.951	1.047	1.130	1.208	1.280	1.344	0.78
0.80	-	-	-	-	0.410	0.490	0.567	0.641	0.711	0.839	0.953	1.054	1.144	1.231	1.308	1.380	0.80
0.82	-	-	-	-	0.381	0.464	0.542	0.619	0.696	0.834	0.952	1.057	1.153	1.250	1.332	1.409	0.82
0.84	-	-	-	-	-	0.433	0.345	0.592	0.673	0.823	0.949	1.058	1.161	1.262	1.354	1.441	0.84
0.86	-	-	-	-	-	0.402	0.486	0.565	0.644	0.803	0.942	1.057	1.166	1.269	1.374	1.464	0.86
0.88	-	-	-	-	-	-	0.450	0.533	0.612	0.773	0.930	1.054	1.169	1.276	1.382	1.482	0.88
0.90	-	-	-	-	-	-	0.414	0.498	0.580	0.739	0.908	1.046	1.172	1.281	1.390	1.495	0.90
0.92	-	-	-	-	-	-	-	0.463	0.545	0.702	0.871	1.032	1.171	1.290	1.395	1.503	0.92
0.94	-	-	-	-	-	-	-	-	0.501	0.665	0.831	1.003	1.169	1.298	1.408	1.511	0.94
0.95	-	-	-	-	-	-	-	-	-	0.644	0.807	0.986	1.162	1.306	1.421	1.526	0.95

Table of Values of Function $a_2 = \frac{1}{(1 - r')^{\frac{1}{\theta}}}$; $\theta = 0.2$.

r' Hundredths	r' Thousandths 0	1	2	3	4	5	6	7	8	9
0.17	2.538	2.553	2.569	2.584	2.600	2.615	2.631	2.648	2.665	2.681
0.18	2.698	2.715	2.731	2.748	2.764	2.780	2.797	2.814	2.832	2.850
0.19	2.867	2.885	2.903	2.921	2.939	2.958	2.976	2.995	3.014	3.033
0.20	3.052	3.071	3.090	3.109	3.128	3.148	3.169	3.188	3.209	3.230
0.21	3.251	3.271	3.291	3.312	3.333	3.354	3.375	3.397	3.419	3.441
0.22	3.463	3.485	3.508	3.531	3.554	3.577	3.600	3.623	3.646	3.670
0.23	3.694	3.718	3.742	3.766	3.790	3.815	3.841	3.867	3.893	3.919
0.24	3.945	3.971	3.998	4.025	4.052	4.079	4.105	4.131	4.158	4.185
0.25	4.212	4.240	4.268	4.297	4.326	4.355	4.385	4.415	4.446	4.477
0.26	4.508	4.538	4.568	4.598	4.630	4.661	4.693	4.746	4.759	4.792
0.27	4.825	4.859	4.893	4.927	4.961	4.995	5.030	5.065	5.100	5.135
0.28	5.170	5.206	5.242	5.278	5.315	5.352	5.389	5.426	5.464	5.502
0.29	5.540	5.580	5.620	5.660	5.700	5.741	5.784	5.827	5.870	5.913
0.30	5.957	5.998	6.040	6.082	6.124	6.166	6.212	6.258	6.304	6.350
0.31	6.397	6.443	6.489	6.536	6.583	6.630	6.679	6.729	6.779	6.829
0.32	6.879	6.930	6.981	7.033	7.085	7.137	7.190	7.243	7.296	7.351
0.33	7.404	7.461	7.518	7.575	7.633	7.691	7.750	7.809	7.869	7.929
0.34	7.989	8.051	8.113	8.175	8.237	8.299	8.363	8.427	8.491	8.555
0.35	8.620	8.686	8.753	8.820	8.887	8.954	9.025	9.096	9.167	9.239

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decreases, the point on the left-hand scale moves upward, the quantity r' increases and the quantities ω/q and W_0/q decrease, the volumetric expansion ratio Λ_D increases, and the gun is set up with a smaller relative chamber volume.

Knowing r_0', we determine the relative charge ω_0/q at which, in the presence of a given Δ, a bore of minimum volume is obtained, with the aid of the following formula:

$$\frac{\omega_0}{q} = \frac{ak_v}{r_0' - bk_v} = \frac{a}{\dfrac{r_0'}{k_v} - b}.$$

Thus, the values of r_0' and ω_0/q <u>at which the minimum bore volume is obtained depend upon the quantity</u>:

$$\Delta\left[\frac{\alpha\Delta}{K} - f(p_m, \Delta)\right]$$

and, for each value of Δ, r_0' and ω_0/q will have their own values.

The values of $\alpha\Delta/K$ as a function of Δ and p_m are presented in a separate table, which shows that, as Δ increases at a given p_m, $\alpha\Delta/K$ at first increases until it reaches a maximum value, and then begins decreasing again. The maximum $\alpha\Delta/K$ defines that most advantageous loading density Δ_H at which the minimum bore volume is obtained.

Fig. 162. - Relation between Function $\alpha\Delta/K$ and Loading Density.

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Curves for $\alpha\Delta/K$ as functions of Δ at various p_m are presented in Fig. 162.

Thus, on the basis of investigations of the general relations of interior ballistics, there have been established other general relations connecting the design elements of the bore of the gun with the loading conditions at a predetermined caliber, weight of the projectile, its initial velocity, and maximum gas pressure, and there has been derived a procedure for determining the loading conditions at which a gun with the minimum bore volume, or the optimum gun, is obtained.

The diagrams presented for purposes of illustration show that a given velocity v_D at a chosen maximum pressure p_m can be obtained in the presence of the most diverse combinations of design data and loading conditions - with shorter or longer barrels, large or small chamber volumes, and large or small loading densities and charge weights.

CHAPTER 3 - APPLICATION OF DERIVED RELATIONS TO PRACTICAL DESIGN

1. DIRECTIVE DIAGRAM, ITS CONSTRUCTION AND INVESTIGATION.

To guide the expedient choice of variants in ballistic design, there is presented below a combined diagram of the design characteristics and some ballistic characteristics of a gun, which makes it possible to take into account some of the tactical and technical specifications imposed upon the gun as well. This diagram is designated as a "directive diagram," since it gives directives and instructions concerning the direction that must be followed in choosing variants satisfying the specifications imposed.

In constructing the diagram, use has been made of the diagrams of the variations of W_{KH}/q and W_0/q as functions of Δ and ω/q ("hammock" and "slope") presented above, which give the fundamental design characteristics of the gun. At predetermined q, v_D, and p_m, the directive diagram represents a projection upon the $\Delta - \omega/q$ plane of the sections of the W_{KH}/q and W_0/q surfaces formed by planes parallel to the $\Delta - \omega/q$ plane, which give in the section lines of equal W_{KH}/q and W_0/q. There is obtained, in a manner of speaking, a topographic map of the W_{KH}/q and W_0/q surfaces on the $\Delta - \omega/q$ plane (Fig. 163).

1. The fundamental point of the diagram, its "center," is the point M_0 with the coordinates Δ_H (most advantageous loading density) and ω_0/q (optimum relative charge), which represents the <u>minimum-volume gun</u> at predetermined q, v_D, and p_m, and which corresponds to the lowest point of the "hammock" $W_{KH}/q = f(\omega/q, \Delta)$.

2. There are circumscribed around the point M_0 oval-shaped

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curves of equal bore volumes W'_{KH}/q, W''_{KH}/q, W'''_{KH}/q, known as the bore isochores, which are obtained as projections of intersections of the "hammock" by planes W_{KH}/q = const. running parallel to the $\omega/q - \Delta$ plane.

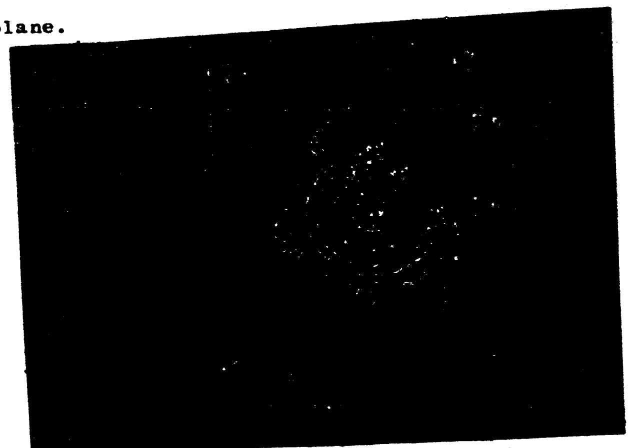

Fig. 163 - Directive Diagram for Choice of Variants.

The larger the bore volume the farther is the corresponding bore isochore from the point M_0:

$$W_{KH_0} < W'_{KH} < W''_{KH} < W'''_{KH} .$$

The point M_0 may be imagined to lie in the center of a hollow, whose sides rise from the center M_0 toward the periphery.

3. The straight lines drawn from the origin of the coordinate system and continued until they intersect the ordinate at $\Delta = 1$, on which is written the scale of values of W_0/q, represent lines of equal chamber volumes W_0/q, since $W_0/q = (\omega/q)(1/\Delta) = \tan \alpha$. The larger the angle of slope α the larger is the chamber volume.

These lines represent projections of lines of intersection

of the surface of chamber volumes $W_0/q = f_1(\omega/q, \Delta)$ in the form of a hyperbolic slope by planes parallel to the $\omega/q - \Delta$ plane at various distances from the latter (cf. Fig. 159 above).

The straight line passing through the point M_0 represents the chamber volume for the minimum-volume gun W_{OH}/q.

The straight lines of equal W_0/q tangent at the left and right to the oval with a given bore volume W_{KH}/q give the maximum and minimum values for the chamber volume at the given bore volume. Definite pairs of values for ω/q and Δ correspond to them.

4. Knowing the values of W_{KH}/q and W_0/q for every point of the $\omega/q - \Delta$ plane, it is possible to determine the corresponding values for the volumetric expansion ratio Λ_D ($\Lambda_D = (W_{KH} - W_0)/W_0 = W_D/W_0$), the most important design characteristic of the bore of the gun. By plotting lines of equal Λ_D on the same diagram, we obtain a family of "iso-Λ_D" curves in the form of dotted lines with values of Λ_D marked on them (from 2.5 to 8.0). The greater Λ_D the farther to the right and the lower is the curve of equal Λ_D located.

5. In addition to these purely design characteristics, there has also been plotted on the diagram a family of curves of equal quantities $\eta_K = l_K/l_D$, which characterize the position of the end of burning of the powder as a function of ω/q and Δ. On each of them is marked the corresponding value of η_K from 0.3 to 1.0.*)

The line $\eta_K = 1.0$ corresponds to the position of the projectile precisely at the muzzle face at the end of burning of the powder.

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*)The data have been obtained by treatment of tables contained in the work by M.S. Gorokhov, "BALLISTICHESKI RASCHET ORUDIYA" "Ballistic Computation of Guns," 1941.

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6. The loading parameter B at a given value of p_m is an increasing univalent monotonic function of Δ; at $\Delta = \Delta_1$ . . ., $B = 0$; at $\Delta = \Delta_H$, $B_H = 1.91$-1.93.

The B-Δ curve is also contained in the diagram.

7. Since a very important factor in ballistic computations is the accuracy-life characteristic of a gun of design under given loading conditions, the formula of V.E. Slukhotsky for the characteristic of the number of shots $N_{\text{усл}}$ was used to compute lines of equal values of N. These equal accuracy-life lines have also been plotted on the directive diagram; they are arranged in the form of straight lines nearly parallel to the Δ axis. The larger the quantities $N_{\text{усл}}$ - the number of shots which the system is capable of withstanding - the lower is the corresponding straight line located: $N_3 > N_2 > N_1$.

Consequently, in order to improve the accuracy life, it is more advantageous from the point of view of the design to take, as far as possible, small ω/q, large Δ, and small chambers.

8. The heavy dashed-dotted line E-E in the diagram corresponds to the economical loading conditions; it passes from the upper left toward the lower right part of the diagram and intersects the $\Delta = \Delta_H$ ordinate somewhat below the point M_0.

At pressures $p_m < 3200$ kg/cm^2, the economical loading conditions (Δ_E and ω_E/q) give good results in ballistic design.

9. The characteristic $\eta_\omega = \frac{qv_D^2}{2g\omega} = \frac{v_D^2}{2g} : \frac{\omega}{q}$ is reciprocal to the quantity ω/q plotted along the ordinate axis. As ω/q decreases at a predetermined value of v_0, η_ω increases. The lines of equal η_ω are straight lines running parallel to the Δ axis. At a predetermined v_D, the η_ω scale can be plotted parallel to the ω/q

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scale in the opposite direction.

10. An understanding of the variation of one of the fundamental ballistic characteristics, $\eta_D = p_{av.}/p_m = \frac{\varphi q v_D^2}{2gW_D p_m}$, or better $\eta_D' = \frac{q v_D^2}{2gW_D p_m}$ (without φ) can be obtained by proceeding along one of the straight lines of equal W_0/q.

At the points of its intersection with the oval of equal volumes W_{KH}/q, we have equal W_D/q and equal values of η_D'. The point on this line located on the perpendicular line dropped from the point M_0 will be closest to the point M_0 and will correspond to the minimum bore volume W_{KH}/q; at equal W_0/q, it will also correspond to the minimum working volume W_D/q and consequently to the maximum value of η_D'.

It can be stated that, in the zone below and to the right of the line OM_0, the closer the point under consideration to the point M_0 the smaller is W_{KH}/q, the larger W_0/q, and the larger the ratio of the muzzle energy to p_m and to the working volume of the bore.

2. APPLICATION OF DIRECTIVE DIAGRAM.

After clarifying the values and character of variations of all design characteristics of the bore of the gun, of the loading conditions, and of the conditions of the shot, it is possible to limit the zone of practical design on the directive diagram and to outline the general procedure for the selection of variants in order to obtain a solution with a minimum number of variants.

In the first place, there is eliminated from the region of

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practical design the area to the right and upward from the $\eta_K = 0.80$ line, since so large a magnitude of η_K does not guarantee the actual combustion of the powder.

In the second place, there is eliminated the zone to the left and upward from the straight line OM_0, since this is a zone of excessively large chamber volumes and small Λ_D.

There remains for practical design a zone in the form of a sector downward from the point M_0, it being preferable, if conditions permit, to use the right-hand part of this sector at $\Delta > \Delta_H$. Such loading densities are in practice attainable at pressures of 2500-3200 kg/cm^2, to which correspond the most advantageous $\Delta_H = 0.62$-0.71. It is very difficult to attain $\Delta > 0.75$ with existing tubular powders, so that $\Delta = 0.75$ is as yet the limiting possible loading density. Grained powders with seven channels and fine powders for small arms make it possible to raise Δ to 0.80 and even to 0.90.

At very high pressures p_m (> 3500 kg/cm^2), the most advantageous Δ_H increases to above 0.75, but it is in practice unattainable for powders possessing a tubular shape, and, in selecting variants, it becomes necessary to move to the portion of the sector on the left of Δ_H, to small loading densities for the given p_m, which leads to increased chamber volumes, reduced parameters B, and an earlier burning of the powder; η_K may be smaller than 0.40. It is in this same zone that the solution for howitzers should be sought, in order to obtain $\eta_K \approx 0.25$-0.30 with a full charge; this will make it possible to obtain complete burning of the powder with reduced charges as well.

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3. SEQUENCE OF COMPUTATIONS

A. Preliminary Choice of Basic Quantities.

On the basis of the relations established above among the design data for the bore of the gun, the loading conditions, and the energy characteristics of the shot, it is possible to outline the following sequence of ballistic computations.

1. From the predetermined magnitudes of the caliber d, the weight of the projectile q, and the initial velocity v_D, there are determined:

a) the coefficient of the weight of the projectile $c_q = q/d^3$ kg/dm^3;

b) the coefficient of the power of the projectile $C_\varepsilon = E_D/d^3 = \frac{qv_D^2}{2gd^3} = c_q \frac{v_D^2}{2g} \frac{\text{tm}}{\text{dm}^3}$.

2. From the quantity C_ε, there is chosen in Table 21 the maximum gas pressure p_m, rounded off to the nearest 100 kg/cm^2 in the upward direction, as well as the coefficient of widening of the chamber χ.

Table 21 - Table for Selection of p_m and χ.

C_ε	100	200	300	400	500	600	700	800	1000	1200	1400	1600
p_m	1840	2120	2300	2450	2600	2750	2875	3000	3200	3350	3500	3600
χ	1.04	1.09	1.14	1.20	1.26	1.33	1.40	1.49	1.70	1.97	2.35	2.91

The experience of the Great Patriotic War has shown the existence of a tendency toward increasing the pressure p_m at a given coefficient C_ε.

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For example, at $C_\varepsilon = 1600$, $p_m = 3900\text{-}4000$. It is true that such high pressures cause difficulties with the extraction of shell cases and with obturation.

3. From the magnitude of the maximum pressure p_m, there is selected with the aid of the formula $\Delta_H = \sqrt{\dfrac{p_m - 300}{5700}}$ the most advantageous loading density Δ_H, at which, for the given charge ω/q and pressure p_m, the required velocity of the projectile v_D is obtained with the minimum volume of the bore of the gun.

B. Determination of Data for Minimum-Volume Gun.

4. For every pair of values of v_D and p_m, there exists at the loading density Δ_H an optimum value ω_0/q at which the bore volume has its minimum value (minimum minimorum).

The optimum weight of the charge ω_0/q is a function of p_m, v_D, Δ_H, and the coefficient b in the formula $\varphi = a + b(\omega/q)$. To find it in a preliminary way on the basis of the nomogram of N. A. Krinitsky from the basic quantities $\alpha\Delta/K = f(p_m, \Delta)$ (given in the table under the two headings p_m and Δ) and $x' = 1 - bk_v$, where:

$$k_v = \frac{\theta}{f}\frac{v_D^2}{2g} = \frac{v_D^2}{932\cdot 10^6}\left[v_D - \frac{dm}{sec}\right],$$

there is found the complete coefficient of efficiency $r' = \varphi r_D$.

Thereupon, there is found:

$$\frac{\omega_0}{q} = \frac{a}{\dfrac{r_0'}{k_v} - b},$$

and then:

$$\frac{W_0}{q} = \frac{\omega_0}{q}\frac{1}{\Delta};\ W_0 = \frac{W_0}{q}q \text{ and } l_0 = \frac{W_0}{s}.$$

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After determining the auxiliary quantities:

$$K = \alpha\Delta : \frac{\alpha\Delta}{K} \text{ and } a_2 \frac{1}{(1 - r')^{\frac{1}{\theta}}}$$

(taken from the table in the preceding Chapter), there is found the value of:

$$\Lambda_D + 1 = Ka_2 + \alpha\Delta$$

followed by:

$$W_D = W_0\Lambda_D;\ l_D = l_0\Lambda_D;\ W_{KH} = W_0(\Lambda_D + 1).$$

From p_m and Δ, using the tables of Professor N.F. Drozdov or the GAU Tables, Issue No. IV, B_H and Λ_K are found; thereupon it is possible to determine:

$$\eta_K = \frac{\Lambda_K}{\Lambda_D};\ \frac{I_K}{d} = \sqrt{\frac{f}{g}}\,\frac{c_q}{n_s}\sqrt{B\varphi\frac{\omega}{q}} \text{ and } I_K = \frac{I_K}{d}\,d;$$

$$\left[\text{at } f = 950{,}000\ \frac{\text{kg}\cdot\text{dm}}{\text{kg}};\ g = 98.1;\ \sqrt{\frac{f}{g}} = 98.4\right].$$

In this manner, there are found the data for the minimum-volume gun with the aid of analytical formulas and tables set up for the case of standard constants (constants of GAU Tables of 1943).

This gun is represented by the point M_0 on the directive diagram.

4a. If the special tables and nomogram for the determination of r_0' do not happen to be accessible, but the GAU Tables, Part IV (TBR) are at hand, then, for computing the characteristics of the

minimum-volume gun, it is possible to make use of another method of approximation, since it is known that, as has been shown by computations, the quantity η_ω for the optimum gun is a function of C_ε, as defined by the following tabulation:

C_ε	100-1000	1200	1400	1600
$\eta_{\omega 0}$	85	84	83	82

After $\eta_{\omega 0}$ is derived from the above, there is found:

$$\frac{\omega_0}{q} = \frac{v_D^2}{2g}\,\frac{1}{\eta_{\omega 0}}\,.$$

Making use of the special form for ballistic computation, there is performed in the first column of the second page of the form a computation of the data for the gun having the minimum bore volume with predetermined d, q, v_D, and p_m. This is done with the aid of the 1943 GAU Tables, Part IV.

5. On the basis of the predetermined Δ_H, p_m, and $\eta_{\omega 0}$, the following quantities are found in the sequence indicated in the form.

Ballistic Computation of Barrel

(Determination of Loading Conditions and of Fundamental Dimensions of Bore from GAU Tables)

Type of system being designed . . .

Supplementary conditions . . .

Caliber d = . . . mm

Weight of projectile q = . . . kg

Muzzle velocity v_D = . . . m/sec

Cross-sectional area of bore $s = n_s d^2$ = . . . dm^2

Coefficient of weight of projectile $c_q = q/d^3 = \ldots$ kg/dm^3

Muzzle energy $E_D = q\,\dfrac{v_D^2}{2g} = \ldots$ tm

Coefficient of power $C_\varepsilon = c_q\,\dfrac{v_D^2}{2g} = \ldots$ tm/dm^3

Coefficient of chamber widening $\chi = \dfrac{l_0}{l_{KM}} = \ldots$

Coefficient of allowance for secondary work $\varphi = a + b(\omega/q)\begin{cases} a = \ldots \\ b = \ldots \end{cases}$

$$n_s \approx \begin{cases} 0.80\text{-}0.82 \text{ for artillery guns} \\ 0.82 \text{ for small arms; } \dfrac{v_D^2}{2g} = \ldots \end{cases}$$

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Form for Ballistic Design

$\frac{98 \cdot 4 c_q}{n_s} = \frac{v_D^2}{2g}$ - Variants		Minimum-Volume Gun	I	II
	p_m			
	Δ			
	η_ω			
1	$\frac{\omega}{q} = \frac{v_D^2}{2g} \frac{1}{\eta_\omega}$			
2	$b \frac{\omega}{q}$			
3	a			
4	$\varphi = a + b \frac{\omega}{q}$			
5	$n_v = \sqrt{\frac{\omega}{q\varphi}}$			
6	$v_{mD} = \frac{v_D}{n_v}$			
7	B			
8	λ_K			
9	λ_D			
10	$\eta_K = \frac{\lambda_K}{\lambda_D}$			

Variants		Minimum-Volume Gun	I	II
11	$\omega = \frac{\omega}{q} \cdot q$			
12	$W_0 = \frac{\omega}{\Delta}$			
13	$l_0 = \frac{W_0}{s}$			
14	$l_D = l_0 \cdot \lambda_D$			
15	$l_{KM} = \frac{l_0}{\chi}$			
16	$L_{KH} = l_{KM} + l_D$			
17	$\frac{L_{KH}}{d}$			
18	$\frac{L_{CT}}{d}$			
19	$\sqrt{B\varphi \frac{\omega}{q}}$			
20	$\frac{I_K}{d}$			
21	I_K			
22	$2e_1 = I_K \cdot 2u_1$			
23	$7/10 \cdot 2e_1$			
24	$\frac{p_{av.}}{p_m} = \frac{\eta_\omega \varphi \Delta}{\lambda_D\ p_m}$			

Data for Construction of Pressure and Velocity Curves from GAU Tables

λ	$l = l_0\lambda$	p	$v_{tab.}$	v	$t_{tab.}$	t

1) $\frac{\omega_0}{q} = \frac{v_D^2}{2g}$: η_{ω_0} ; 2) $b\frac{\omega}{q}$; 3) $a = 1.03$;

4) $\varphi = a + b\frac{\omega}{q}$; 5) $n_v = \sqrt{\frac{\omega}{\varphi q}}$; 6) $v_{tab.\,D} = \frac{v_D}{n}$.

Thereupon, with the aid of the GAU Tables, Part IV (or of the ANII Tables, for which $n_v = \sqrt{\frac{\omega}{q}\,\frac{1.05}{\varphi}}$), there are found:

7) B; 8) Λ_K; 9) Λ_D;

and these data are used to determine:

10) $\eta_K = \frac{\Lambda_K}{\Lambda_D}$; 11) $\omega = \frac{\omega}{q} q$; 12) $W_0 = \frac{\omega}{\Delta}$; 13) $l_0 = \frac{W_0}{s}$;

14) $l_D = l_0\Lambda_D$; 15) $l_{KM} = \frac{l_0}{\chi}$; 16) $L_{KH} = l_{KM} + l$; 17) $\frac{L_{KH}}{d}$;

18) $\frac{L_{CT}}{d} = \frac{L_{KH}}{d} + 1.5 - 2.0$; 19) $\sqrt{B\varphi\frac{\omega}{q}}$; 20) $\frac{I_K}{d} = \frac{98.4}{n_s} c_q \sqrt{B\varphi\frac{\omega}{q}}$;

21) $I_K = \frac{I_K}{d} d$; 22) $2e_1 = I_K \cdot 2u_1$; 23) $\eta_D = \frac{p_{av.}}{p_m} = \frac{\varphi\eta_\omega\Delta}{\Lambda_D \cdot p_m}$

24) $L'_{KH} = l_0 + l_D$.

This computation is performed in the first column of the form, and its fundamental data are entered in the first row of the summary of results on the third page of the same form.

It is in this manner that there are determined the design data, loading conditions, and fundamental characteristics of a gun with the minimum bore volume at predetermined d, q, v_D, and p_m.

6. The minimum-volume gun is actually the optimum gun for velocities of the order of 1500 m/sec and higher, when, in consequence of the very large dimensions of the gun, the minimum volume and length of the bore constitute decisive and fundamental criteria in the choice of variants.

In this case, the solution is found at once, without supplementary variants, and is the only acceptable solution, even though the gun obtained has a relatively large volume chamber (Λ_D = 3.0-3.5), a relatively large charge weight ω q, and a small coefficient of utilization of the charge η_ω(82-85 tm kg).

For projectile velocities lower than 1500 m/sec, of the order of 600-1200 m/sec, the minimum-volume gun is not to be recommended and represents merely a point of departure for other variants by setting a lower limit upon the bore volume.

C. Design at Usual Projectile Velocities.

To judge in what direction and how the loading conditions and design data of the bore must be changed in order to obtain a solution satisfying the requirements with a minimum number of variants, it is necessary to make use of the "directive diagram."

To obtain practically convenient solutions at v_D = 600-1200

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m/sec, it is necessary to depart from the minimum-volume gun in the direction of reducing the weight of the charge and the chamber volume, the entire volume and length of the bore being somewhat increased, and the utilization of the unit weight of the charge η_ω increasing at the same time.

If, in this connection, there is retained for the first variant the same loading density $\Delta = \Delta_H$, this corresponds on the directive diagram to a descent from the center of the diagram (the point M_0). For the purpose of reducing the number of variants while reducing ω/q, the following formulas can be recommended:

$$\frac{\omega'}{q} = \frac{\omega_0}{q}\left(\frac{1}{4} + \frac{3}{4}\,\frac{v_D}{1500}\right) \text{or } \frac{\omega}{q}\left(0.4 + 0.6\,\frac{v_D}{1500}\right),$$

where v_D is stated in m/sec.

From this:

$$\frac{W_0'}{q} = \frac{\omega'}{q}\,\frac{1}{\Delta_H}.$$

If p_m and Δ_H are maintained constant, B and Λ_K likewise remain constant; and since Λ_D increases as ω/q decreases, $\eta_K = \Lambda_K/\Lambda_D$ declines, and the characteristics of utilization of the bore volume of the gun $p_{av.}/p_m$ and R_D decline at the same time.

Let this value ω'/q be represented in the diagram (Fig. 163) by the point N, through which passes the isochore W''_{KH}/q.

For the chosen value ω'/q = const., the resulting gun has a minimum volume, since, as they move along the horizontal while ω/q is maintained constant, the points will move farther from M_0, and the bore volume will increase.

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Consequently, in this case, Δ_H is most advantageous not generally, but only as long as the weight of the charge is maintained constant, and this Δ_H is in this case advantageous only in a conditional manner.

For this reason, in order to improve the utilization of the bore volume and to transfer the end of burning to the muzzle face, which will also somewhat increase the muzzle pressure, the loading density should be increased if the gravimetric density of the powder permits this (in practice, if $\Delta_H \leqslant 0.70$).

Depending upon the tactical and technical specifications imposed, and in their absence on the basis of the requirement to give a rational ballistic solution, it is possible, in choosing further variants, to proceed from the point N in the following three directions (Fig. 163a).

Fig. 163a.

1. While maintaining the new chamber volume constant at W_0''/q, by moving from the point N to the right and upward along the line ON as far as the point n located on the perpendicular dropped from the point M_0 upon the straight line ON, and consequently by approaching the center M_0, to obtain <u>the minimum-volume gun at the given chamber volume</u>.

In this connection, it follows from the condition $W_0/q = (\omega/q)(1/\Delta) = \text{const.}$ that the weights of the charges vary proportionately to the loading densities:

$$\omega_2 : \omega' = \Delta_2 : \Delta_H;$$

To reduce the number of variants, it is permissible to take:

$$\Delta_2 = \Delta_H + 0.01\,\Lambda_D'$$

(Λ_D' corresponding to the charge ω' q at $\Delta = \Delta_H$ in the point N).

In the point n, the maximum value is attained for $p_{av.}/p_m$ at the given volume chamber, η_K increases to 0.65-0.75, and Λ_D decreases slightly, since W_{KH} decreases at W_0 = const.

The resulting variant will undoubtedly be more rational than the variant corresponding to the point N, since, at the same chamber volume and at a smaller bore volume, it exhibits better energy characteristics η_D and η_K.

2. While maintaining the bore volume constant at W_{KH}, it is possible, by moving from the point N to the right along the line W''_{KH}/q = const., by increasing Δ, and by increasing ω/q somewhat less, to obtain a gun with the given bore volume and with a minimum chamber volume (point n"), since, at the point n", the angle of slope of the straight line On" will be minimal (On" is tangent to the curve W''_{KH}/q = const.). In this connection, $\Delta''_{n''}$ will be somewhat greater than Δ_n.

In this case (in comparison with the point N), Λ_D will increase because of the diminution of the chamber volume, η_K and B will increase, and the value of η_D will increase somewhat (but less than in the first case).

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3. While maintaining the weight of the charge constant at ω'/q, it is possible to proceed along the horizontal to the right from the point N, thus increasing Δ, moving away from the center M_0, and thereby increasing W_{KH} and reducing the chamber volume, which increases Λ_D and raises the accuracy life.

In this connection, B and η_K also increase.

At the maximum practically permissible loading density $\Delta = \Delta_{n'}$ (point n'), there will be obtained a gun with the minimum chamber volume at the given weight of the charge.

For each of these three cases, the most advantageous loading density will exceed Δ_H, and, generally speaking, it will depend not only upon p_m, but also upon Λ_D. At the same time, the parameter B will also exceed $B_H \approx 1.91$-1.95.

All loading densities and all charge weights corresponding to the three cases discussed above will be located very close to the line E-E, which characterizes the combinations of Δ and ω/q capable of satisfying economical loading conditions.

For this reason, by choosing in the above-presented table of Δ_E for a predetermined p_m increasing magnitudes of Λ_D and the corresponding Δ_E, it is possible to obtain all three cases just discussed by a shorter route, without resorting to the preliminary transition to the point N at the same loading density Δ_H.

All these computations are performed in the subsequent columns of the same form for ballistic design, using the GAU Tables, Part IV (TBR), 1943, with a certain change in the order of operations.

D. Particular Design Cases.

I. Given d, q, v_D, and the chamber volume W_0' is assigned.

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1. After computing C_ε, Table 21 on p. 834 is used to choose p_m and χ; Δ_H is found, and one of the methods indicated above is used to compute the gun with the minimum bore volume (point M_0); there is obtained a definite value W_{0H} at the (optimum) charge ω_0/q (the first column of the form is filled in).

2. Since, at Δ = const. = Δ_H, the weight of the charge is proportional to the chamber volume, it is changed in such a manner as to obtain at once the predetermined chamber volume W_0':

$$\frac{\omega'}{q} = \frac{\omega_0}{q} \frac{W_0'}{W_{0H}} ;$$

whereupon the ballistic computation of the assigned variant is carried through to the end, and the second column of the form is filled in.

On the basis of the resulting values of Λ_D' and η_K', it is determined whether it is possible to stop at once at this variant, or whether it is necessary to proceed to Δ_E at the same W_0' (to the point n).

In the latter case, with the aid of the formula $\Delta_2 = \Delta_H + 0.01$ Λ_D' or of Table 18, Δ_E is designated on the basis of p_m and Δ_D' (Δ_E and Δ_2 will be close to each other), and the computation of the second variant is performed at this Δ. There will be obtained a gun having a shorter length than in the first variant.

If this barrel length is for some reasons unsatisfactory, the pressure p_m should be changed, and the computation of all three variants should be repeated for the new pressure.

An increase in pressure will increase Δ_H and, with the same chamber volume W_0', will reduce Λ_D and shorten the barrel.

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II. Given d, q, v_D, and the length of the bore in calibers $L_{KH}/d = (l_{KH} + l_D)/d$ is assigned.

Preliminarily, on the basis of the coefficient $C_\varepsilon = c_q \frac{v_D^2}{2g}$, with the aid of Table 21, p_m and χ are determined, there is assigned for the minimum-volume gun:

$$\Lambda_{DH} = 3.0 + 0.04 \frac{C_\varepsilon - 100}{100}$$

(empirical formula), and the adjusted bore length $L'_{KH} = l_0 + l_D$, the total bore volume $W_{KH} = sL'_{KH}$, and W_{KH}/q are found for the assigned bore length L_{KH}:

$$L'_{KH} = L_{KH} \frac{\Lambda_{DH} + 1}{\Lambda_{DH} + \frac{1}{\chi}} .$$

For the chosen value of p_m, there is found $\Delta_H = \sqrt{\frac{p_m - 300}{5700}}$, whereupon the minimum-volume gun and its volume W_{KH} are computed.

If the resulting volume W_{KH} exceeds the assigned W'_{KH}, then it is impossible at the chosen pressure p_m to obtain a gun of the assigned volume and length, and it will be necessary to increase the pressure p_m (by 200-300 kg/cm^2) and again compute for the new pressure its own minimum-volume gun.

In increasing the pressure for the computation of W_{KH}, it is possible to use as a guide the following approximate formula:

$$p_m \cdot W_{KH} \approx \text{const.}$$

If this new volume of the minimum-volume gun is smaller than the assigned volume, it is possible to proceed with the computation

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of variants ensuring the attainment of a gun of the assigned length.

For this purpose, it is simplest of all to bracket the assigned bore length by choosing for the pressure p_m two values Λ_D' and Λ_D'', and to take the two corresponding values Δ_E' and Δ_E'' from the table of Δ_E.

From the GAU Table IV (or from the ANII Tables), on the basis of the assigned Δ, p_m, and Λ_D, v_{TD}' and v_{TD}'' are found.

With the aid of the formulas:

$$\frac{\omega}{q} = \frac{a}{\frac{v_{TD}^2}{v_D^2} - b}$$

(for the GAU Tables, where $\varphi_{tab.} = 1$) and:

$$\frac{\omega}{q} = \frac{a}{1.05\ \frac{v_{TD}^2}{v_D^2} - b}$$

(for the ANII Tables, where $\varphi_{tab.} = 1.05$), ω'/q and ω''/q are determined; there are found:

$$\frac{W_0'}{q} = \frac{\omega'}{q}\,\frac{1}{\Delta'} \text{ and } \frac{W_0''}{q} = \frac{\omega''}{q}\,\frac{1}{\Delta''},$$

followed by:

$$W_{KH}' = W_0(\Lambda_D' + 1);\ W_{KH}'' = W_0(\Lambda_D'' + 1);$$

$$(L_{KH}')' = \frac{W_{KH}'}{s};\ (L_{KH}')'' = \frac{W_{KH}''}{s};$$

and finally by:

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$$(L_{KH})' = (L'_{KH})' \frac{\Lambda'_D + \frac{1}{\chi}}{\Lambda'_D + 1};\ (L_{KH})'' = (L'_{KH})'' \frac{\Lambda''_D + \frac{1}{\chi}}{\Lambda''_D + 1}.$$

Following this, the resulting values of L_{KH} are compared with the assigned length. If the latter is comprised between those found, the required values of Δ_E and Λ_D will be found by interpolation, and one more supplementary computation will be needed for verification.

If the assigned value of L_{KH} is either larger or smaller than the two values obtained, the required values of Δ_E and Λ_D will be found by extrapolation, and one supplementary variant will be needed for a check.

The variant obtained in either case ensures the assigned bore length at the minimum chamber volume.

If, together with the assignment of the bore length, there is imposed the supplementary requirement to obtain the smallest possible charge even if the chamber volume is somewhat increased, then, after establishing the bracket, both variants should be taken at $\Delta = \Delta_H$ and at different ω_1/q and ω_2/q (descending downward from the point M_0), and the results obtained should be used to apply corrections to ω/q for the final variant.

Example. To design an 85 mm antiaircraft gun having a bore length of about 60 calibers; $q = 9.2$ kg; $v_D = 900$ m/sec; $c_q = 15.0$; $C_\varepsilon = 618$ tm/dm^3; $\chi = 1.35$; $p_m = 2800$ kg/cm^2; $\Delta_H = 0.66$. By the simplified method:

$$\eta_H = 85 \text{ tm/kg } \frac{v_D^2}{2g} = 41.2 \cdot 10^4 \text{ dm}.$$

The computation is conducted with the aid of a 25 cm or 50 cm slide rule.

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Two variants are computed in parallel: a minimum-volume gun at $\Delta_H = 0.66$, and a gun at the same Δ and with a charge diminished by means of the following formula:

$$\frac{\omega'}{q} = \frac{\omega_H}{q}\left(\frac{1}{4} + \frac{3}{4}\,\frac{v_D}{1500}\right) = \frac{\omega_H}{q}\left(\frac{1}{4} + \frac{3}{4}\,\frac{900}{1500}\right) = 0.70\,\frac{\omega_H}{q}$$

For the first variant, there are taken for $C_\varepsilon = 618$ by the simplified method:

$$\eta_{\omega_H} = 85 \text{ tm/kg};$$

$$\eta_\omega = 85;$$

$$\frac{\omega_H}{q} = \frac{v_D^2}{2g} : \eta_{\omega_H} = \frac{41.2}{85} = 0.485;$$

$$\varphi = 1.03 + \frac{1}{3}\,\frac{\omega}{q} = 1.03 + \frac{1}{3}\,0.485 = 1.192;$$

$$n = \sqrt{\frac{\omega}{q}\,\frac{1}{\varphi}} = \sqrt{\frac{0.485}{1.192}} = 0.638;$$

$$v_{tab.D} = \frac{v_D}{n} = \frac{900}{0.638} = 1411;$$

$$\eta_{\omega'} = 121;$$

$$\frac{\omega'}{q} = 0.70 \cdot 0.485 = 0.3395;$$

$$\varphi' = 1.03 + \frac{1}{3}\,0.3395 = 1.143;$$

$$n' = 0.545;$$

$$v'_{tab.D} = \frac{900}{0.545} = 1652.$$

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From the GAU Tables, Part IV, at $\Delta = 0.66$ and $p_m = 2800$, there are found:

$$B_H = 1.926;$$

$$\Lambda_K = 2.498;$$

$$\Lambda_D = 3.104;$$

$$\eta_K = \frac{\Lambda_K}{\Lambda_D} = 0.805;$$

$$\omega = \frac{\omega}{q}\, q = 0.485 \cdot 9.2 = 4.46;$$

$$W_0 = \frac{\omega}{\Delta} = \frac{4.46}{0.66} = 6.76;$$

$$l_0 = \frac{W_0}{s} = \frac{6.76}{0.59} = 11.46 \text{ dm};$$

$$l_D = l_0 \Lambda_D = 11.46 \cdot 3.104 = 35.55;$$

$$l_{KM} = \frac{l_0}{\chi} = \frac{11.46}{1.35} = 8.49;$$

$$L_{KH} = l_{KM} + l_D = 44.04;$$

$$\frac{L_{KH}}{d} = \frac{44.04}{0.85} = 51.8;$$

$$\frac{L_{CT}}{d} = 51.8 + 1.7 = 53.5;$$

$$\frac{p_{av.}}{p_m} = \frac{\varphi \eta_\omega \Delta}{\Lambda_D p_m} = 0.77;$$

$$p_D = 1440;$$

$$B_H = 1.926;$$

$$\Lambda_K = 2.498;$$

$$\Lambda_D = 5.49;$$

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$$\eta_K' = 0.455;$$

$$\omega' = 0.3395 \cdot 9.2 = 3.123;$$

$$W_0' = \frac{3.123}{0.66} = 4.732;$$

$$l_0' = \frac{4.732}{0.59} = 8.02 \text{ dm};$$

$$l_D' = 8.02 \cdot 5.49 = 44.0;$$

$$l_{KM}' = \frac{8.02}{1.35} = 5.94;$$

$$L_{KH}' = 49.94;$$

$$\frac{L_{KH}'}{d} = 58.75;$$

$$\frac{L_{CT}'}{d} = 58.75 + 1.75 = 60.5;$$

$$\frac{p_{av.}}{p_m} = 0.595;$$

$$p_D = 760.$$

Comparison between the first and second variants shows that the minimum-volume gun has a very large chamber volume and charge weight, a small Λ_D, and a delayed burning of the powder ($\eta_K = 0.805$), but that the resulting barrel length is considerably shorter than that assigned; $p_{av.} / p_m = 0.77$ is very high.

In the second variant, the chamber volume and charge are 30% smaller, Λ_D has increased to 5.5, the end of burning of the powder occurs early ($\eta_K = 0.455$), but the barrel length has increased by 7 calibers (in consequence of a strong increase in the length of the path of the projectile l_D and of a certain decrease in the chamber length) and has come close to that required (60.5 instead

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of 60).

$p_{av.}/p_m = 0.595$ is considerably lower than in the first variant, and, in conformity with this, the muzzle pressure is $p_m = 760$ instead of 1440 in the first variant, i.e., is smaller by a factor of nearly two.

Now, while maintaining the chamber volume constant (proceeding along the line ON in Fig. 163), Δ and ω/q are proportionally increased, there being taken to reduce the number of variants:

$$\Delta_E = \Delta_H + \frac{\Lambda_D}{100} = 0.66 + \frac{5.5}{100} = 0.715 \text{ and } \frac{\omega_E}{\omega_H} = \frac{\Delta_E}{\Delta_H}.$$

Taking $\Delta_E = 0.71$ and $\Delta_E' = 0.72$, and performing the computation as in the first two variants, there are obtained:

N	Δ	$\frac{\omega}{q}$	B	Λ_D	η_K	W_0	l_D	L_{KH}	$\frac{L_{CT}}{d}$	$\frac{p_{av.}}{p_m}$	p_D	η_ω
3	0.71	0.365	2.216	5.37	0.625	4.732	43.05	49.0	59.4	0.615	860	113
3'	0.72	0.370	2.277	5.33	0.675	4.732	42.74	48.7	59.0	0.620	870	111.5

The data for the two variants nearly coincide; in comparison with the second variant, the length of the barrel has become shortened by 1.0-1.5 calibers, the muzzle pressure has increased by 100 kg/cm^2, the end of burning has shifted toward the muzzle face, and the value of η_K is good, being about 0.65.

Both these variants may be considered as being ballistically acceptable and as satisfying the imposed requirement to obtain a barrel length equivalent to about 60 calibers.

For the chosen variant (No. 3), the thickness of the powder is computed as follows:

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$$\frac{I_K}{d} = \sqrt{\frac{f}{g}}\,\frac{c_q}{n_s}\sqrt{B\varphi\frac{\omega}{q}} = 98.4\,\frac{c_q}{n_s}\sqrt{B\varphi\frac{\omega}{q}};$$

$$\frac{98.4 \cdot c_q}{n_s} = \frac{98.4 \cdot 15}{0.815} = 1810;$$

$$\sqrt{B\varphi\frac{\omega}{q}} = \sqrt{2.216 \cdot 1.152 \cdot 0.365} = 0.964;$$

$$\frac{I_K}{d} = 1810 \cdot 0.964 = 1745; \quad I_K = 1745 \cdot 0.85 = 1483;$$

$$2e_{1strip} = 2u_1 \cdot I_K = 2 \cdot 0.0_5 75 \cdot 1483 = 0.0222 \text{ dm} = 2.22 \text{ mm}.$$

From the table correlating u_1 with the thickness of pyroxylin powder, a powder thickness of about 2 mm is associated with $u_1 = 0.0000073$.

Upon introducing the correction, there is obtained:

$$2e_{1strip} = 2.22\,\frac{73}{75} = 2.16 \text{ mm; type } \frac{22}{1}$$

For a powder with seven channels:

$$2e_1 = 0.7 \cdot 2e_{1strip} = 0.7 \cdot 2.16 = 1.51 \text{ mm; type } \frac{15}{7}$$

Ballistic computation with the aid of the above procedure has required the computation of four variants.

Instead of first proceeding from the point M_0 downward to the point N while reducing ω/q and W_0 by 30% at the same Δ_H, and then ascending upward and to the right along the line ON while maintaining W_0 = const. and increasing Δ and ω/q, it is possible, immediately following the computation of the data for the minimum-volume gun, to change over to the economic loading densities Δ_E, taking them from the table of Δ_E and selecting Λ_D for them.

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In the table of Δ_E, for a pressure $p_m = 2800$ at $\Delta_H = 0.66$, the value $\Delta_E = 0.72$ is associated with $\Lambda_D = 5.0$, while $\Delta = 0.73$ is associated with $\Lambda_D = 6.0$.

The course of the computation in this case differs somewhat from that presented above.

The following values are assigned:

$$p_m = 2800 \text{ and}$$

$$\Delta = 0.72,\ \Lambda_D = 5.0;$$

$$\Delta = 0.73,\ \Lambda_D = 6.0.$$

From the GAU Tables, Part IV (TBR), the following values are found:

Variants	Δ	Λ_D	B	Λ_K	η_K	$v_{tab.D}$
II'	0.72	5.0	2.277	3.596	0.719	1565
III'	0.73	6.0	2.341	3.86	0.643	1625

ω/q is found in accordance with the following formula:

$$\frac{\omega}{q} = \frac{a}{\left(\dfrac{v_{tab.D}}{v_D}\right)^2 - b} = \frac{1.03}{\left(\dfrac{v_{tab.}}{900}\right)^2 - \dfrac{1}{3}}.$$

The remaining data are found as in the first computation.

Variants	$\frac{\omega}{q}$	w_0	l_D	L_{KH}	$\frac{L_{CT}}{d}$	η_ω	$\frac{p_{av.}}{p_m}$	p_D	Λ_D
II'	0.3835	4.900	41.5	47.65	57.8	107.5	0.640	950	5.0
III'	0.352	4.435	45.1	50.66	61.3	117	0.584	795	6.0

In Variant II', the resulting barrel length is smaller than that required (57.8 d); in III', it is somewhat greater (61.3 d).

It is possible to interpolate these two variants to their mean and to obtain:

$$\Delta_E = 0.725; \quad \Lambda_D = 5.5; \quad \eta_K = 0.68; \quad B = 2.309;$$

$$\frac{\omega}{q} = 0.368; \quad W_0 = 4.67; \quad \ell_D = 43.3; \quad L_{KH} = 49.15;$$

$$\frac{L_{CT}}{d} = 59.5; \quad \eta_\omega = 112.3; \quad \frac{p_{av.}}{p_m} = 0.612; \quad p_D = 870.$$

The results of this computation coincide almost completely with the results of Computation No. 3', in which a different route was adopted, but almost the same point of the directive diagram was reached. The computation fully satisfies both the ballistic criteria (η_K, η_D, η_ω) and the requirement for a definite barrel length. The type of powder is obviously the same as in the preceding case.

The entire computation has been performed in three variants.

All computations are performed and recorded in a form for the ballistic computation of the barrel, which contains a series of columns and headings for the operations and quantities.

For the finally selected variant, the GAU Tables (Parts I, II, and III) are used to solve the direct problem; i.e., the values of p, v, and t are found as functions of Λ or ℓ, and these data are used to construct p-ℓ, v-ℓ, p-t, and v-t curves for utilization in computing and designing the barrel, gun mount, tubes, and fuzes.

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CHAPTER 4 - SUPPLEMENTARY INFORMATION

1. INFLUENCE OF VARIATION OF PRESSURE p_m UPON BORE DESIGN DATA AND LOADING CONDITIONS.

It is known that, in a given gun, in the presence of the same charge, the gas pressure p_m, the area under the curve $\int_0^{l_D} p\,dl$, and at the same time the velocity of the projectile increases as the thickness of the powder decreases, since:

$$v_D = \sqrt{\frac{2s}{\varphi m} \int_0^{l_D} p\,dl} .$$

Consequently, in inverting the problem, the same area $\int_0^{l_D} p\,dl$, which ensures the attainment of the predetermined velocity v_D, can be obtained with a shorter length of path of the projectile l_D by increasing the pressure p_m while maintaining the chamber volume and weight of the charge unchanged.

Consequently, an increase in pressure with a given chamber volume and a given weight of the charge must reduce the length of path of the projectile l_D and the length of the entire bore L_{KH}.

Together with an increase in the pressure p_m, the following loading densities grow correspondingly:

Δ_1 for the instantaneous burning of the charge;

Δ_H - the most advantageous loading density;

Δ_E - the economical loading density;

Δ_i for the burning of the powder at the muzzle face.

The diagram expressing the dependence of W_{KH} and W_0 upon Δ at ω = const, at a predetermined v_D, and at $p_m'' > p_m'$ will have the form

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represented in fig. 164.

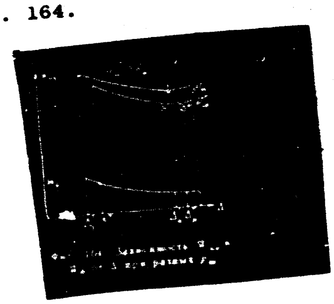

Fig. 164. Dependence of W_{KH} and W_0 Upon Δ at Various p_m.

1) An increase in the pressure p_m while Δ, ω, and W_0 are maintained constant (loading density Δ_H', transition from the point c' to the point b') reduces the length of the bore to a lesser degree than if Δ_H were increased simultaneously in conformity with the increase in pressure (transition from the point c' to the point b").

This is explained by the fact that the increased Δ_H'' is the most advantageous for the new pressure p_m'', ensuring a bore of minimum length; on the other hand, the previous Δ_H' is no longer the most advantageous for the new pressure, and the length of the bore is greater.

2) As the pressure increases, apart from the total decrease in the length of the bore, the quantity Λ_D also decreases, i.e., the relative chamber volume increases.

3) The characteristic $\eta_K = l_K / l_D$ decreases considerably at Δ = const as p_m is varied; but, as Δ increases in conformity with the increased pressure p_m'', η_K undergoes almost no change.

4) The product $p_m \cdot W_{KH,H}$ is close to being a constant.

Consequently, it may be considered that, as p_m increases in the presence of the same weight of the charge ω, the volume and length of the bore are inversely proportional to the pressure p_m.

This formula can be utilized for exploratory computations in ballistic design:

$$W''_{KH} = W'_{KH}\frac{p'_m}{p''_m} \quad \text{or} \quad L''_{KH} \approx L'_{KH}\frac{p'_m}{p''_m},$$

In this connection:

$$l''_0 = l'_0\frac{\Delta'}{\Delta''}.$$

If, in computing the minimum-volume gun, its length is obtained greater than the predetermined length, an increase in pressure constitutes the only means available to satisfy the imposed condition.

2. EFFECT OF DIFFERENT POWDERS ON THE LOADING CONDITIONS AND GUN DESIGN DATA BASED ON BALLISTICS.

Besides pyroxylin powders, use is also made of more powerful nitroglycerol powders, which contain 20-40% of nitroglycerol. By having a higher burning temperature, these powders considerably shorten the accuracy life of the barrel. The search for means to increase the accuracy life of guns has also led to the use of so-called "cold" powders, which have a lower burning temperature and a smaller propellant force; these include, for example, nitroguanidine powders.

Since our tables are set up for definite powder characteristics, which correspond to pyroxylin powders, there arises the question of how a variation in the nature of the powder will be reflected in the

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design data of the gun and in the loading conditions at given values of d, q, v_D, and p_m, a situation encountered in ballistic design.

The nature of a powder is characterized by the following factors: f - the propellant force of the powder; α - the covolume of the powder gases; u_1 - the rate of burning of the powder at p = = 1 kg/cm^2; the adiabatic index $k = 1 + \theta$, which depends upon the composition of the gases and upon their temperature in the bore.

It is known from courses in interior ballistics and from the study of powders that, for pyroxylin powders, f = 85-95 tm/kg, α = = about 1 dm^3/kg, and θ = 0.20; for nitroglycerol powders, f and u_1 increase, and the covolume α and the index θ decrease, as the nitroglycerol content increases; in nitroguanidine powders, on the contrary, f and u_1 decrease, and the quantities α and θ increase.

In this connection, although the rate of burning u_1 changes with variations in the nature of the powder, it enters into the fundamental equations and relations not as a separate entity, but as a component of the pressure impulse I_K, which depends both upon u_1 and upon the thickness of the powder $2e_1$ ($I_K = \frac{e_1}{u_1}$). For this reason, in the subsequent discussion, the quantity u_1 will not be considered separately, but the impulse will be included among the characteristics of the loading conditions which are determined in ballistic computations.

Since, in ballistic design, the variant which serves as the point of departure is the minimum-volume gun (the center of the directive diagram), it is convenient in investigating the influence of variations in the nature of the powder to compare the design data and loading conditions for minimum-volume guns at predetermined v_D and p_m.

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There are presented below some of the results of such investigations conducted by M.E. Serebryakov [22] and N.A. Krinitskii.

The computations were conducted for the following characteristics, which correspond to powders of different natures.

No.	Powder	f $\frac{tm}{kg}$	α $\frac{dm^3}{kg}$	θ	$\frac{f}{f_2}$ %
1	Nitroguanidine	86.5	1.10	0.220	91
2	Pyroxylin	95	1.00	0.200	100
3	Medium-power nitroglycerol	105	0.905	0.181	110.5
4	High-power nitroglycerol	115	0.825	0.165	121

Subsequently, each powder will be designated by its number; the data for pyroxylin are accepted as the reference unit = 100%.

The data for minimum-volume guns were determined by the general procedure involving the use of the nomogram of Krinitskii for the simplified case ($\psi_0 = 0$, $\varkappa = 1$, $\alpha = 1/\delta$).

In the summary table below, the fundamental ballistic characteristics of minimum-volume guns are given as percentages for powders of various natures.

Table 22

Characteristics	Powder No.	1	2	3	4
	No.	Percentage			
Propellant force of powder f	1	91	100	110.5	121
Burning temperature T_1	2	83	100	122	146
Loading parameter B_H	3	98.3	100	101.6	103
Most advantageous loading density Δ_H	4	97	100	102.2	103.7
Optimum efficiency r'_0	5	111.2	100	89.2	80.6
Optimum relative charge ω_0/q	6	110.7	100	90.1	81.3
Chamber volume W_0	7	114.1	100	88.2	78.4
Volumetric expansion ratio Λ_D	8	91.8	100	108.3	117.4
Length of path of projectile l_D	9	104.7	100	95.2	92.0
Bore volume W_{KH}	10	107	100	93.3	88.6
Path at end of burning l_K	11	99.5	100	101.5	101
Pressure impulse I_K	12	100.6	100	99.6	98.9
$\eta_K = \Lambda_K/\Lambda_D = l_K/l_D$	13	94.8	100	105.8	109.8
Muzzle pressure p_D	14	93.4	100	107.4	112.4
Coefficient of utilization of unit charge weight η_ω	15	90.8	100	110.7	122.7
Coefficient φ	16	102	100	98.1	96.4
Gas temperature at emergence of projectile T_D	17	79	100	126	155
Average pressure $\eta_p = p_{av}/p_m$	18	97.5	100	102.9	104.7
Accuracy [illegible]	19	[illegible]	100	37.8	12.6
[illegible]	20	[illegible]	100	90.5	82.5

[illegible] presented in the table shows the [illegible]

[illegible] loading density and the parameter B_H increase [illegible] propellant force of the powder increases (lines 3 [illegible]

[illegible] optimum efficiency r'_0 and the optimum charge ω_0/q vary

Table 22

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Length of path of projectile l_D	9	104.7	100	95.2	92.0
Bore volume W_{KH}	10	107	100	93.3	88.6
Path at end of burning l_K	11	99.5	100	101.5	101
Pressure impulse I_K	12	100.6	100	99.6	98.9
$\eta_K = \Lambda_K/\Lambda_D = l_K/l_D$	13	94.8	100	105.8	109.8
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Coefficient of utilization of unit charge weight η_ω	15	90.8	100	110.7	122.7
Coefficient φ	16	102	100	98.1	96.4
Gas temperature at emergence of projectile T_D	17	79	100	126	155
Average pressure $\eta_D = p_{av.}/p_m$	18	97.5	100	102.9	104.7
Accuracy life $N_{ycл}$	19	209	100	37.8	12.6
$1/f$	20	110	100	90.5	82.5

Investigation of the data presented in the table shows the following facts.

1) The optimum loading density and the parameter B_H increase slightly as the propellant force of the powder increases (lines 3 and 4).

2) The optimum efficiency r'_0 and the optimum charge ω_0/q vary

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inversely proportionally to the propellant force of the powder
5 and 6).

3) The coefficient of utilization of the unit weight of th
charge η_ω varies directly proportionally to the propellant force
the powder (line 15).

4) The volumetric expansion ratio Λ_D varies almost proport
ally to the propellant force of the powder (in somewhat lesser de
(line 8).

5) The chamber volume varies in the opposite direction to th
propellant force of the powder, but in somewhat greater degree (li
7).

6) The length of path of the projectile l_D also varies in th
opposite direction to the propellant force of the powder, but in
lesser degree (line 9).

7) The total volume of the bore varies in the opposite direct
and in somewhat greater degree than l_D, but in lesser degree than th
propellant force of the powder (line 10).

8) <u>The path of the projectile</u> at the end of burning l_K and <u>th</u>
<u>full pressure impulse</u> I_K <u>are practically independent of the nature of</u>
<u>the powder</u> (lines 11 and 12), but the thickness of powders with a
greater propellant force will grow in conformity with the increase in
u_1, since $e_1 = I_K \cdot u_1$.

9) The characteristic of the end of burning of the powder η_K =
$= l_K / l_D$ varies inversely proportionally to the variation in l_D, sinc
l_K is approximately constant (line 13).

10) The muzzle pressure increases somewhat more slowly than the
propellant force of the powder (line 14).

inversely proportionally to the propellant force of the powder (lines 5 and 6).

3) The coefficient of utilization of the unit weight of the charge η_ω varies directly proportionally to the propellant force of the powder (line 15).

4) The volumetric expansion ratio Λ_D varies almost proportionally to the propellant force of the powder (in somewhat lesser degree) (line 8).

5) The chamber volume varies in the opposite direction to the propellant force of the powder, but in somewhat greater degree (line 7).

6) The length of path of the projectile l_D also varies in the opposite direction to the propellant force of the powder, but in lesser degree (line 9).

7) The total volume of the bore varies in the opposite direction and in somewhat greater degree than l_D, but in lesser degree than the propellant force of the powder (line 10).

8) The path of the projectile at the end of burning l_K and the full pressure impulse I_K are practically independent of the nature of the powder (lines 11 and 12), but the thickness of powders with a greater propellant force will grow in conformity with the increase in u_1, since $e_1 = I_K \cdot u_1$.

9) The characteristic of the end of burning of the powder $\eta_K = l_K / l_D$ varies inversely proportionally to the variation in l_D, since l_K is approximately constant (line 13).

10) The muzzle pressure increases somewhat more slowly than the propellant force of the powder (line 14).

11) The muzzle gas temperature increases very sharply with an increase in the propellant force of the powder (line 17).

12) The average pressure increases very slowly with increasing propellant force of the powder (being nearly proportional to the loading density Δ_H) (line 18).

13) The characteristic of the accuracy life of the bore varies extremely sharply (line 19); the change from pyroxylin powder to No. 4 nitroglycerol powder is accompanied by an eightfold drop in $N_{ycл}$, in spite of the more advantageous design data and loading conditions; the change to nitroguanidine powders is accompanied by a greater than twofold rise in $N_{ycл}$.

All these conclusions present a perfectly clear picture of the variation in the fundamental design data, energy characteristics, and loading conditions for the minimum-volume gun accompanying a variation in the nature of the powder.

As has already been shown in the theoretical fundamentals of ballistic design, the minimum-volume gun can be recommended for practical realization only at very high projectile velocities (1500 m/sec).

At lower projectile velocities, a departure from the minimum-volume gun in the direction of a smaller charge and chamber volume (on the directive diagram, downward and to the right from the point M_0) is indicated.

There arises the question whether such a departure will not alter the relations established in Table 22, and whether the relative character of the resulting relations will be maintained for such guns possessing other than a minimum volume.

The investigation presented in the same chapter shows that the relations indicated in Table 22 are maintained with small changes even if the minimum-volume gun is not used as the starting variant.

Thus, if the nature of the powder differs from that assumed in the GAU Tables, it is possible, once a gun has been designed in accordance with these tables, to introduce changes into the results of the computation by making use of the data in Table 22 in the present chapter, whereupon the results may be checked by the use of analytical formulas, of the method of Professor Drozdov, or of the method of the average l_ψ.

3. RELATION BETWEEN WEIGHT OF BARREL AND ITS DESIGN ELEMENTS AT PREDETERMINED PROJECTILE VELOCITY AND AT GIVEN MAXIMUM PRESSURE p_m.

In some cases, the weight of the barrel may be one of the criteria in the selection of one or another variant.

For this reason, along with the knowledge of the variation in the design elements and loading conditions at predetermined v_D and p_m, it is also desirable to know the variation in the weight of the barrel Q_{CT}.

One and the same v_D at the same p_m can be obtained both from a short barrel with a relatively large chamber (minimum-volume gun) at Λ_D = about 3 and from a long barrel with a small chamber (Λ_D = about 10). In this connection, the muzzle pressure, which determines the barrel-wall thickness at the muzzle face, will in the first case equal about one-half the maximum pressure (p_D/p_m = about 0.5), while in the second case p_D/p_m = about 0.15-0.20 (fig. 165).

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Fig. 165 - Pressure Curves in Guns with Various Λ_D at Predetermined v_D and p_m.

In the first case, we have a long cylindrical part and a short conical part with thick walls at the muzzle face; in the second case, we have a considerably shorter cylindrical part with a long cone and a thin wall at the muzzle face.

It is necessary to clarify in an exploratory manner how the weight of the barrel will vary with variations in the characteristic Λ_D.

A longitudinal section through the barrel is represented schematically in fig. 166.

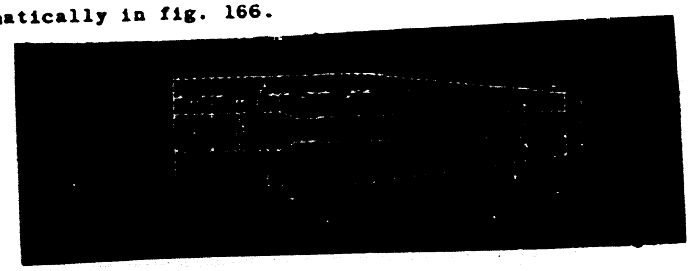

Fig. 166 - Scheme of Longitudinal Section of Barrel.

The gun has the following characteristics:

Outer diameter of cylindrical breech D.

Outer diameter of muzzle end of barrel d_2.

Caliber of gun d.

Average chamber diameter d_{KM}.

Length of breech ring l_{KA3}

Chamber length $l_{KM} = l_0/\chi$.

Path of projectile at instant of maximum pressure l_m.

Reserve in cylindrical part for possible shift of maximum pressure toward muzzle face l' = about $0.6\, l_0 \approx l_m$.

Length of conical part of barrel l''.

$$l'' = l_D - (l_m + l') \approx l_D - 1.2\, l_0.$$

If the density of steel is designated as δ' ($\delta' = 7.85$ kg/dm^3), the weight of the barrel will be expressed by the following formula:

$$Q_{CT} = \delta' \left\{ \frac{\pi}{4} D^2 (l_{KA3} + l_{KM} + l_m + l') + \frac{1}{3}\frac{\pi}{4}(D^2 + Dd_2 + d_2^2)\, l'' - \frac{\pi}{4} d^2 (l_0 + l_D) \right\},$$

where the first term in parentheses is the volume of the solid cylindrical part of diameter D, the second is the volume of the truncated cone of diameters D and d_2 and length l'', and the third is the volume of the entire bore.

We separate from the above expression the weight of the breech ring $Q_{KA3} = (\pi/4) D^2 l_{KA3} \delta'$, and, by designating $A = D/d$ and $a_2 = d_2/d$, taking l_0 out of the parentheses, and dividing both sides of the equation by d^3 in order to represent the weight of the

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barrel in relative units, we obtain:

$$\frac{Q_{CT}}{d^3} = \frac{Q_{KA3}}{d^3} + \delta' \frac{\pi}{4} \frac{l_0}{d} \left\{ \left(\frac{1}{\chi} + 1 \right) A^2 + \frac{1}{3} (A^2 + Aa_2 + a_2^2)(\Lambda_D - 1) - (1 + \Lambda_D) \right\}.$$

After dividing this by $c_q = q/d^3$, we have:

$$\frac{Q_{CT}}{q} \quad \frac{Q_{KA3}}{q} + \delta' \frac{W_0}{q} \left\{ \left(\frac{1}{\chi} + 1 \right) A^2 + \frac{1}{3} \left[1 + \frac{a_2}{A} + \left(\frac{a_2}{A} \right)^2 \right] A^2 (\Lambda_D - 1) - (1 + \Lambda_D) \right\}. \qquad (127)$$

The quantity $A = \frac{D}{d}$ is a function of the maximum pressure p_m; the quantity $a_2 = \frac{d_2}{d}$ is a function of the muzzle pressure p_D, which is itself a function of p_m and Λ_D; the values of PD/p_m, a_2/A, and A are tabulated below as functions of p_m and Λ_D.

Table of Values for $A = \frac{D}{d} = f(p_m)$

p_m	1800	2200	2600	3000	3600
A	1.68	2.00	2.56	3.79	5.10

Table of Values for $\frac{p_D}{p_m} = f(p_m, \Lambda_D)$
Under Economical Loading Conditions

p_m \ Λ_D	3	4	6	8	10
1800	0.543	0.451	0.323	0.244	0.208
2200	0.545	0.445	0.305	0.237	0.198
2600	0.521	0.430	0.390	0.220	0.185
3000	0.500	0.405	0.271	0.207	0.172
3600	0.465	0.373	0.250	0.189	0.154

Table of Values for $\frac{a_2}{A} = \frac{d_2}{D} = f(p_m, \Lambda_D)$

p_m \ Λ_D	3	4	6	8	10
1800	0.744	0.738	0.696	0.667	0.660
2200	0.690	0.650	0.600	0.570	0.560
2600	0.563	0.539	0.477	0.465	0.442
3000	0.396	0.368	0.328	0.312	0.301
3600	0.314	0.285	0.249	0.236	0.228

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The dependences of A and a_2 upon the pressure have been taken from Table 2 in the "Handbook on Design of Gun Barrels and Breech-blocks" by E.K. Larman, the mechanical-strength safety factor used being 1.33, and the elastic limit being assumed to be $\sigma_e = 6000$ kg/cm^2.

Consequently:

$$\frac{p_1}{\sigma_e} = \frac{1.33}{6000} p = \frac{p}{4500} \text{ kg/cm}^2.$$

Since the length of the breech ring $l_{KA3} \approx 1.5d$, it follows that:

$$Q_{KA3} = \frac{\pi}{4} D^2 \cdot 1.5 d \delta';$$

$$\frac{Q_{KA3}}{d^3} = 7.85 \frac{\pi}{4} A^2 1.5 \approx 10 A^2.$$

In Formula (127), the relative weight of the barrel is expressed in terms of the ballistic characteristics of the bore W_0/q, Λ_D, and χ, and of the quantities A and a_2/A, which also depend upon the ballistic characteristics p_m, Λ_D, and p_D. The formula is useful for exploratory computations.

Computations performed with the aid of this formula have shown that, at predetermined v_D and p_m, the weight of the barrel as a function of Λ_D varies along a curve possessing a minimum at $\Lambda_D \approx 5$-6 (Fig. 167).

Consequently, if there has been imposed the requirement to obtain a minimum-weight barrel, it is necessary to take $\Lambda_D \approx 5.6$.

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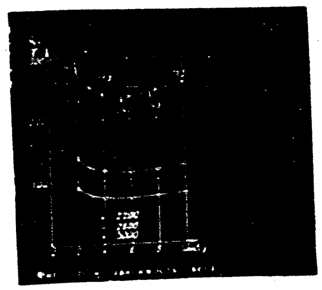

Fig. 167 - Weight of Barrel as a Function of Λ_D at Predetermined v_D and p_m.

4. APPLICATION OF VARIOUS BALLISTIC TABLES.

Ballistic computations may be carried out with the aid of any desired available tables, including the tables of Professor Drozdov for strip-type powder ($\varkappa = 1.06$) with "normal" constants, the 1933 ANII Tables with the same constants, the 1943 GAU Tables with somewhat modified constants ($\alpha = 1$, $\varphi = 1$), the 1933 tables of the Chair of Interior Ballistics for powders with a constant burning area ($\varkappa = 1$, $\lambda = 0$) and for any desired values of f and φ, and the tables of M.S. Gorokhov [17] for $\varkappa = 1.06$ and for $\varkappa = 1.00$ with the remaining constants "normal."

Maximum convenience for ballistic computations attaches to the GAU Tables, Part IV (TBR), and to the tables of M.S. Gorokhov.

Prior to the start of the computation, it is necessary to select for any table a coefficient of agreement between the computations and experiments, for which purpose it is, in turn, necessary to process the results of firing tests from artillery systems already accepted

for armament and approaching in type the system being designed.

Computations for ten of our systems firing with small relative charges ($\omega/q < 0.20$) have shown [21] that good agreement with experimental v_D and p_m in computations with the aid of the ANII Tables is obtained at $\varphi = 1.05$, whereas, when a correction is applied to make $\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q} > 1.05$ at a predetermined pressure p_m, the initial velocities obtained fall short of the experimental velocities by approximately 3%.

Consequently, in spite of the fact that the ANII Tables are compiled for strip-type powder ($\varkappa = 1.06$), whereas the systems accepted for armament fire either with tubular powders ($\varkappa = 1$) or with powders possessing seven perforations ($\varkappa \approx 0.7$), both of which burn more progressively than strip-type powder, nevertheless the adoption at $\omega/q < 0.20$ of the value 1.05 for φ - a value smaller than the actual value - compensates somewhat for the more degressive character of burning of strip-type powders and gives computed results for v_D that are very close to the results of firing tests conducted with tubular or strip-type powders.

<u>Note</u>: If, in conducting computations with the aid of the ANII Tables at the same $\omega/q < 0.20$, φ is taken in accordance with the theoretical formula $\varphi = a + \frac{1}{3}\frac{\omega}{q}$, where $a = 1.03$-1.06 depending upon the type of gun (according to Slukhotskii), but v_D is determined by means of the formula $v_D = v_{tab.}\sqrt{\frac{\omega}{q}\frac{1.05}{\varphi}}$, then, if the computed values for v_D coincide with the experimental values, the pressure p_m determined from the tables is obtained 10% higher than the experimental p_m.

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The application of a correction to φ should be adopted at high values of $\omega/q > 0.20$, when $\varphi = 1.05$ will differ too much from $\varphi = a + \frac{1}{3}\frac{\omega}{q}$. Nevertheless, the above-mentioned lack of consistency in the character of burning of powders, which requires some compensation by the reduction of φ, will manifest itself at large ω/q as well, and this will make it necessary to reduce the coefficient $b = 1/3$. For example, some designers take the formula $\varphi = 1.05(1 + \frac{1}{4}\frac{\omega}{q})$, which, at large ω/q, gives a smaller value than $\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q}$.

As for the quantity φ, it exerts a rather considerable effect upon the computed design data of the bore (as φ decreases, the length of path and the length of the bore also decrease).

Comparison among the tables of Professor Drozdov, the tables of Gorokhov, the ANII Tables, and the GAU Tables indicates that, at predetermined Δ and p_m, different values are obtained for B and Λ_K, as follows:

B	< B	< B	< B
Drozdov	ANII	Gorokhov	GAU
Λ_K	$< \Lambda_K$	$< \Lambda_K$	$< \Lambda_K$
Drozdov	Gorokhov	GAU	ANII

and this, at a predetermined v_D and at identical φ or b, leads to different values for the length of path l_D and the length of the bore L_{KH}, the difference increasing with increasing v_D; the smallest values for l_D and L_{KH} are obtained in working with the tables of Professor Drozdov, and the largest with the ANII tables, the difference being small (about 2%) for $v_D = 1000$ m/sec and as large as 7-8% for $v_D = 1500$ m/sec.

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At identical Δ and B, the GAU Tables give higher values for p_m than the tables of Professor Drozdov, the difference increasing with the increase in Δ. Thus, for $\Delta = 0.50$, at identical B, $p_{m_{GAU}} - p_{m_{Dr}}$ = about 2%; for $\Delta = 0.60$ the difference is 2.5-3.0%; for $\Delta = 0.70$ it is 4-5%; and for $\Delta = 0.80$ it is 5-6%. Such a discrepancy cannot be explained by the fact that the covolume in the GAU Tables is $\alpha = 1$ instead of 0.98 as in the tables of Professor Drozdov.

In the ANII tables the pressures p_m approach the values given by Drozdov; in Gorokhov's tables these values are 1-2% smaller than the p_m values given in the GAU tables.

From the tables of Gorokhov compiled for different $\varkappa$ (1.06 and 1.00), it is possible to draw the conclusion that, as $\varkappa$ varies from 1.06 to 1.00 at predetermined Δ and B, the pressure p_m decreases by 4-6%, the change in p_m being the greater the larger B and Δ.

In any case, tables compiled even on the basis of a mathematically exact method, for constants of definite values, cannot in all cases give complete agreement with experimental data, since the theoretical solution does not take into account all details of the phenomenon of the shot, and every mathematically exact method based on definite assumptions is merely an approximation with respect to the actual phenomenon, which is much more complex than the scheme adopted in the assumptions.

It is for this reason that, for every method of solution and for every table, it is necessary to select its own coefficient of agreement, which will give the best coincidence with experiment. In using one of the tables enumerated above for a ballistic computation, it is necessary, on the basis of experimental data for

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"related" guns under firing conditions close to those provided for by the design, to determine the coefficient of agreement of the given table with experiment and to utilize this coefficient in the design.

In the case of pyroxylin powders, agreement with experiment is attained best of all by the selection of the coefficient b in the formula $\varphi = a + b\frac{\omega}{q}$.

It has already been indicated above that, at identical φ and b, for predetermined p_m and $v_D = 1500$ m/sec, the ANII Tables give a length of path of the projectile l_D that is 7-8% greater than that obtained with the fundamental tables of Professor Drozdov.

Identical values of l_D can be obtained with the aid of either set of tables by selecting <u>different</u> values for the coefficient φ or b.

For example, if, on the basis of the tables of Professor Drozdov, the values obtained for b are 1/3 in one case and 1/5 in another, the corresponding values for b obtained from the ANII Tables will be 1/4 and 1/6, respectively.

This circumstance confirms the necessity of selecting the coefficient b for the purpose of ensuring agreement between each type of tables and experiment; it also indicates the errors in the procedure of compiling the ANII Tables at high velocities.

There is presented below a procedure for determining φ and b on the basis of firing tests with the aid of various tables.

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5. DETERMINATION OF COEFFICIENT b FROM TABLES

The quantity a in the coefficient $\varphi = a + b\frac{\omega}{q}$ is usually assumed to be 1.03 for high-power guns, 1.04-1.05 for guns of moderate power, 1.05-1.06 for howitzers, and 1.10 for smallarms; the quantity b is subject to determination on the basis of the results of firing tests.

For a given gun, let there be known(*):

$$W_0,\ s,\ l_D,\ q,\ \omega,\ p_m \text{ and } v_{D\ op};$$

In addition, there are determined:

$$\Delta = \frac{\omega}{W_0},\quad l_0 = \frac{W_0}{s} \text{ and } \Lambda_D = \frac{l_D}{l_0}.$$

The table for the given Δ having been selected, B is found from Δ and from p_m, and, in the table of velocities, at the same Δ, B and Λ_D are used to determine $v_{tab.\ D}$ and $v_{D\ calc.} = v_{tab.\ D}\sqrt{\frac{\omega}{q}}$. Since $v_{D\ op} = v_{tab\ D}\sqrt{\frac{\omega}{\varphi_{op}\ q}}$ (from the GAU Tables) and $v_{D\ op} = v_{tab\ D}\sqrt{\frac{\omega}{q}\frac{1.05}{\varphi}}$ (from the ANII Tables), it follows that, in working with the

GAU Tables ($\varphi = 1$):

$$\varphi_{op} = \left(\frac{v_{D\ calc.}}{v_{D\ op}}\right)^2;$$

$$b_{op} = \frac{\varphi_{op} - a}{\frac{\omega}{q}};$$

ANII Tables ($\varphi = 1.05$):

$$\varphi_{op} = 1.05\left(\frac{v_{D\ calc.}}{v_{D\ op}}\right)^2;$$

$$b_{op} = \frac{\varphi_{op} - a}{\frac{\omega}{q}}.$$

(*) The subscript op of the last value stands for "determined." translator.

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To determine the coefficient b in using the tables of Professor Drozdov and the tables of the Chair of Interior Ballistics, it is necessary, first of all, on the basis of the quantities Δ and p_m, to determine B (or C), and then to determine

$$\Lambda_K = \frac{l_K}{l_0} \text{ and } \eta_K = \frac{\Lambda_K}{\Lambda_D}.$$

If $\eta_K < 1$ (the burning of the powder is complete), $v^2_{D\ \text{calc.}}$ is computed as follows from the tables of Professor Drozdov ($\varphi = 1.05$);

$$v^2_{D\ \text{calc.}} = \frac{2g}{1.05}\frac{f}{\theta}\frac{\omega}{q}\left\{1 - \frac{(\Lambda_K + 1 - \alpha\Delta)^\theta}{(\Lambda_D + 1 - \alpha\Delta)^\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\} = 29{,}790^2 \cdot$$

$$\cdot \frac{\omega}{q}\left\{1 - \frac{K^\theta}{(\Lambda_D + 1 - \alpha\Delta)^\theta}\right\}; \quad \varphi_{op} = 1.05\left(\frac{v_{D\ \text{calc.}}}{v_{D\ op}}\right)^2; \quad b_{op} = \frac{\varphi - a}{\frac{\omega}{q}}.$$

According to the tables compiled by the Department of Interior Ballistics ($\varphi = 1$), for any f and $\theta = 0.2$:

$$v^2_{D\ \text{calc.}} = \frac{2gf}{\theta}\frac{\omega}{q}\left[1 - \frac{BD}{(\Lambda_D + 1 - \alpha\Lambda)^\theta}\right],$$

where B and D are found from the table on the basis of the same values for p_m, Δ, and C.

$$\varphi_{op} = \frac{v^2_{D\ \text{calc.}}}{v^2_{D\ op}}; \quad b_{op} = \frac{\varphi - a}{\omega/q}.$$

The tables of M.S. Gorokhov are in part constructed in the same manner as the tables of Professor Drozdov, in that the quantities B and Λ_K are given directly as functions of Δ and p_m (Appendix III). In addition, there exist special tables for determining the most

advantageous solutions at various p_m in the case of maximum φE_D (Δ_H, r', $\frac{W_{KH}}{\omega}$, $\Lambda_D + 1$, etc.).

If $\eta_K > 1$ (the burning of the powder is incomplete), it is impossible to employ the formula for $v^2_{D\,calc.}$ and to compute it with the aid of the tables of Professor Drozdov and of the Department of Interior Ballistics, and it becomes necessary to use only the ANII or GAU Tables in conjunction with the formulas presented above.

6. INFLUENCE OF VARIATION OF φ AND b UPON RESULTS OF COMPUTATIONS OF DESIGN DATA (Λ_D, l_D).

From the fundamental equation, we have:

$$\Lambda_D + 1 - \alpha\Delta = \frac{(\Lambda_K + 1 - \alpha\Delta)\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{1/\theta}}{(1 - r')^{1/\theta}} = \frac{K}{(1 - r')^{1/\theta}} =$$

$$= \frac{K}{(1 - \varphi r_D)^{1/\theta}}. \tag{128}$$

At predetermined Δ and p_m, the quantity $K = \text{const}$; at a given $\frac{\omega}{q}$, $r_D = \text{const}$. We differentiate equation (128) with respect to φ, whereupon we multiply and divide the right-hand side by φ:

$$d\Lambda_D = \frac{K}{\theta}\,\frac{r'}{(1 - r')^{\frac{1}{\theta}+1}}\,\frac{d\varphi}{\varphi}. \tag{129}$$

Upon dividing (129) by (128), we obtain:

$$\frac{d\Lambda_D}{\Lambda_D + 1 - \alpha\Delta} = \frac{1}{\theta}\,\frac{r'}{(1 - r')}\,\frac{d\varphi}{\varphi}, \tag{130}$$

where, as r' varies in the range of 0.200-0.333 and $\theta = 0.2$, the

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quantity $\frac{1}{\theta}\frac{r'}{1 - r'}$ (the coefficient of $\frac{d\varphi}{\varphi}$) varies in the range of 1.25-2.50.

It is seen from formula (130) that Λ_D increases and decreases with φ, this relative variation of Λ_D being greater than the relative variation of φ.

From the formula presented above:

$$b = \frac{\varphi - a}{\frac{\omega}{q}};$$

it follows that:

$$db = \frac{d\varphi}{\frac{\omega}{q}} \text{ and } \frac{db}{b} = \frac{d\varphi}{\varphi - a}. \qquad (131)$$

Since the difference $\varphi - a$ is usually small, the relative variation of b is considerably greater than the variation of φ, and the quantity $\varphi = \left(\frac{v_{D\ calc.}}{v_{D\ op}}\right)^2$. Consequently, the divergence in the velocities $v_{D\ calc.}$ obtained by computation with the aid of various tables necessitates a change in b_{op} required to give the same values of v_D.

It follows from Formula (131) that:

$$\frac{d\varphi}{\varphi} = \frac{\varphi - a}{\varphi}\frac{db}{b} \qquad (132)$$

Upon substituting (132) into (130), we obtain a direct connection

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between the variation of Λ_D and the variation of b:

$$\frac{d\Lambda_D}{\Lambda_D + 1 - \alpha\Delta} = \frac{1}{\theta}\,\frac{r'}{1 - r'}\,\frac{\varphi - a}{b}\,\frac{db}{\varphi} = \frac{1}{\theta}\,\frac{r'}{1 - r'}\,\frac{\omega}{q}\,\frac{db}{\varphi}.$$

Upon substituting $r' = \dfrac{\varphi k_v}{\frac{\omega}{q}}$, where $k_v = \dfrac{\theta}{f}\,\dfrac{v_D^2}{2g}$, we obtain:

$$\frac{d\Lambda_D}{\Lambda_D + 1 - \alpha\Delta} = \frac{k_v}{\theta}\,\frac{db}{1 - r'} = \frac{v_D^2}{2gf}\,\frac{db}{(1 - r')}.$$

Consequently, the influence of the difference in the quantity b increases with an increase in the initial velocity of the projectile v_D, confirming the results obtained in performing computations with the aid of the ANII Tables and the tables of Professor Drozdov.

The relations presented above confirm the necessity of exact selection of the value for the coefficient b_{op} on the basis of the results of firing tests from an existing "related" gun which is close in its data to the gun being designed. This is especially important in the case of high initial velocities.

Only in such a case is it possible to expect that the results of the design will agree well with practice.

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PART THREE - SOLUTION OF PROBLEMS OF INTERNAL BALLISTICS IN COMPLICATED CASES

SECTION ELEVEN - COMPLICATED CASES

CHAPTER 1 - SOLUTION FOR CASE OF COMBINED CHARGES

1. GENERAL INFORMATION.

In practice, use is made in many cases of charges consisting of a mixture of two samples of powders, one being usually thinner and the other thicker; in this connection, the powders may differ in the shape of their grain - being degressive or progressive - and in their nature - having different propellant forces f and rates of burning u_1.

Such composite or combined charges are employed principally in firing from howitzers to obtain different projectile velocities depending upon combat conditions, for the purpose of destroying targets at all ranges under a definite sufficiently large angle of fall.

Furthermore, combined charges are employed on the firing ground in testing artillery and ammunition equipment whenever it is necessary to select a combination of maximum gas pressure p_m and projectile velocity v_D which it is impossible to obtain with a charge composed of a single type of powder. For example, let it be assumed that "regulation" values for p_m and v_D have been obtained with a definite charge of a given type of powder, but that it is necessary to test the barrel or ammunition at a 10-15% higher pressure p_m and at the same velocity v_D, or else that it is necessary at the regulation pressure p_m to obtain a higher pro-

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jectile velocity v_D for the purpose of testing the action of the gun carriage at a higher recoil velocity. In this case, the problem can be solved by the use of a combined charge, by replacing a part of the regulation charge either with a thinner powder while reducing the total weight of the charge or with a thicker powder while increasing the total weight.

As a rule, on the basis of the tactical and technical requirements, there are predetermined a maximum initial velocity v_{Do} for the full charge (designated as No. 0) and a corresponding velocity v_{Dn} for the minimum charge (charge No. n).

The ballistic computation of the barrel for the full charge at predetermined d, q, and v_{Do} is performed in the usual manner, with certain modifications which take into account the burning characteristics of the powder under declining pressures as the charge weights are reduced. The number of velocities and the number of charges are designated on the basis of the firing conditions, depending upon the predetermined angles of fall of the projectile and the values set for the overlapping of ranges. The number n is set at 5-10 and even higher.

The maximum pressure for the full charge (No. 0) p_{mo} is designated in the usual manner on the basis of the quantity C_{ε}; the maximum pressure for the minimum charge p_{mn} is determined from the cocking conditions of the firing device ($p_{mn} > 500-700$ kg/cm^2).

The corresponding loading densities are found to be in the ranges of $\Delta_0 = 0.40-0.60$, $\Delta_n = 0.10-0.15$. After a scale of initial velocities v_{Do}, v_{D1}, v_{D2}, ... v_{Dn} has been arrived at

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on the basis of the solution of the problem of exterior ballistics with proper consideration of the overlapping ranges with adjacent charges, internal ballistics must give the magnitudes of the charges necessary to ensure attainment of the predetermined scale of velocities and the weight ratios of the thin and thick powders composing each of these charges, under the condition that the pressures p_{mi} do not exceed the limits imposed upon them.

In order to solve this problem, it is necessary first to give the procedure for solving the problem of pyrodynamics in the case of a combined charge.

This subject is left completely untouched in the treatises and textbooks of foreign authors, but has been elaborated in detail by many of our own authors [1, 2, 4-6].

2. CHARACTERISTICS OF COMBINED CHARGE.

Let a charge ω_{kg} consist of ω'_{kg} of thin powder and ω''_{kg} of thick powder: $\omega = \omega' + \omega''$. The relative weight of each powder will be designated as follows:

$$\frac{\omega'}{\omega} = \alpha', \quad \frac{\omega''}{\omega} = \alpha'';$$

$$\alpha' + \alpha'' = 1, \quad \alpha'' = 1 - \alpha'.$$

Let it be assumed that these powders possess the following characteristics:

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Thin	ω'	α'	$2e'_1$	u'_1	$\frac{e'_1}{u'_1} = I'_K$	$\varkappa'$, λ'	f'
Thick	ω''	α''	$2e''_1$	u''_1	$\frac{e''_1}{u''_1} = I''_K$	$\varkappa''$, λ''	f''

The propellant force of the powder in the composite charge is, in the first approximation, computed in accordance with the usual mixing formula:

$$f = \alpha'f' + \alpha''f''. \qquad (1)$$

In solving the problem in greater detail, it is necessary to take into consideration the magnitude of f for each instant as a function of the composition of the gases formed prior to that instant:

$$f = \alpha'f'\psi' + \alpha''f''\psi''. \qquad (1')$$

Since, as a rule, in the case of combined charges, use is made of pyroxylin powders, whose propellant forces f' and f" are close to each other, formula (1) can be employed with a sufficient degree of precision. During the burning of a mixture of powders under pressure conditions common to both of them, we shall have:

$$de' = u'_1 pdt; \quad de'' = u''_1 pdt.$$

Since, under the common pressure conditions $p = f(t)$, the quantity $\int_0^t pdt$ will have one and the same value for both powders,

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the integration of these expressions will give the following equations:

$$\frac{e'}{u'_1} = \frac{e''}{u''_1} = \int_0^t p\,dt.$$

where e' is one-half of the thickness of the layer of thin powder burnt prior to a given instant, and e" is the same for the thick powder.

Since the quantity $I = \int_0^t p\,dt$ is common to both powders, it is precisely this quantity that is most conveniently taken as the independent variable in solving problems of internal ballistics for a composite charge. This gives a general solution both for the geometric law of burning and for the physical law of burning.

Prior to a certain instant, let there be burned a fraction ψ' of the thin powder and a fraction ψ'' of the thick powder. In weight units, there will burn $\omega'\psi'_{kg}$ of the former type and $\omega''\psi''_{kg}$ of the latter type; the sum of these weights $\omega'\psi' + \omega''\psi''$ will constitute a certain fraction ψ of the total weight of the mixture ω:

$$\psi = \frac{\omega'\psi' + \omega''\psi''}{\omega} = \alpha'\psi' + \alpha''\psi''. \qquad (2)$$

The problem involved in determining the characteristics of the combined charge consists in establishing the form coefficients, Γ, and I_K for the mixture on the basis of the known form coefficients, Γ, and I_K for each of the two types of powders of which the mixture

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is composed.

Let us introduce into the general expression for ψ:

$$\psi = \varkappa z + \varkappa \lambda z^2$$

the new independent variable $I = \int_0^t p dt$ to replace z. Upon designating:

$$\frac{I}{I_K} = z; \quad \frac{\varkappa}{I_K} = K \quad \text{and} \quad \frac{\lambda}{I_K} = \Lambda,$$

there is obtained:

$$\psi = \frac{\varkappa}{I_K} I + \frac{\varkappa}{I_K} \frac{\lambda}{I_K} I^2 = KI + K\Lambda I^2. \qquad (3)$$

Application of this formula to each of the components of the charge gives:

$$\psi' = K'I + K'\Lambda' I^2; \qquad (3')$$

$$\psi'' = K''I + K''\Lambda'' I^2, \qquad (3'')$$

where:

$$K' = \frac{\varkappa'}{I'_K}; \quad \Lambda' = \frac{\lambda'}{I'_K}; \quad K'' = \frac{\varkappa''}{I''_K}; \quad \Lambda'' = \frac{\lambda''}{I''_K}.$$

Upon now substituting expressions (3), (3'), and (3") into (2), we have:

$$KI + K\Lambda I^2 = \alpha'(K'I + K'\Lambda' I^2) + \alpha''(K''I + K''\Lambda'' I^2)$$

By equating in this identity the coefficients of the same powers of I, we obtain the following expressions for the

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characteristics K, KЛ, and Л:

$$K = \alpha' K' + \alpha'' K''; \qquad (4)$$

$$K\text{Л} = \alpha' K'\text{Л}' + \alpha'' K''\text{Л}''; \qquad (5)$$

$$\text{Л} = \frac{K\text{Л}}{K}.$$

Consequently, the quantities f, ψ, K, and KЛ for the combined charge are obtained from the corresponding characteristics of the individual components in accordance with the ordinary rule of mixtures.

By differentiating equation (2) with respect to I, and keeping in mind that $dI = pdt$, we obtain:

$$\frac{d\psi}{dI} = \alpha' \frac{d\psi'}{dI} + \alpha'' \frac{d\psi''}{dI},$$

but:

$$\frac{d\psi}{dI} = \frac{d\psi}{pdt} = \Gamma.$$

Consequently:

$$\Gamma = \alpha'\Gamma' + \alpha''\Gamma''. \qquad (6)$$

In the coordinate axes ψ-I, the quantity Γ is the tangent of the slope of the ψ-I curve with respect to the I axis. Let us designate it as γ. Then:

$$\tan\gamma = \alpha' \tan\gamma' + \alpha'' \tan\alpha''. \qquad (6')$$

Formula (6) is applicable both to the geometric and to the physical law of burning. In the former case, we shall have:

$$\Gamma' = \frac{\varkappa'}{I'_K}\sigma'; \quad \Gamma'' = \frac{\varkappa''}{I''_K}\sigma''$$

and for the mixture:

$$\Gamma = \frac{\varkappa}{I_K}\sigma = \alpha'\frac{\varkappa'}{I'_K}\sigma' + \alpha''\frac{\varkappa''}{I''_K}\sigma''. \qquad (7)$$

For the start of burning, $\sigma' = \sigma'' = \sigma = 1$. For powders with the same grain shape, $\varkappa' = \varkappa''$. Thus, equation (7) will assume the following form:

$$\frac{\varkappa}{I_K} = \alpha'\frac{\varkappa'}{I'_K} + \alpha''\frac{\varkappa''}{I''_K}.$$

This equation connects two unknown quantities $\varkappa$ and I_K for the mixture with the corresponding quantities for the components.

One of these - $\varkappa$ - may be assigned arbitrarily: $\varkappa = \varkappa' = \varkappa''$; then, by cancelling out, we obtain a correlation expressing the nominal average impulse of the mixture of two powders in the following form:

$$\frac{1}{I_K} = \frac{\alpha'}{I'_K} + \frac{\alpha''}{I''_K},$$

from which:

$$I_K = \frac{I'_K I''_K}{\alpha' I''_K + \alpha'' I'_K}$$

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or:

$$I_K = \frac{I'_K}{\alpha' + \alpha'' z'_K} = \frac{I''_K}{\frac{\alpha'}{z'_K} + \alpha''}, \tag{8}$$

where $z'_K = \frac{I'_K}{I''_K} < 1$ is the relative impulse of the thin powder (*).

Since $\alpha' + \alpha'' = 1$, it follows that:

$$I'_K < I_K < I''_K .$$

Equation (8) shows that, in finding the nominal impulse of the mixture I_K by the rule of mixtures, what is added together are not I'_K and I''_K, but the reciprocal quantities $1/I'_K$ and $1/I''_K$.

As has been shown by investigations, the formula:

$$I_K = \alpha' I'_K + \alpha'' I''_K , \tag{9}$$

employed at one time gives values that are too high in comparison with I_K as computed in accordance with formula (8).

In using formula (9) for the computation, the quantity α' is obtained larger, which, in firing, may lead to too high a pressure in comparison with that required.

(*) $z'_K = y_1$ (according to Drozdov), and $z'_K = \beta_1$ (according to Grave).

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3. GRAPHICAL REPRESENTATION OF CORRELATION ψ-I.

The progressive burning characteristic Γ depicted in ψ-I coordinates represents a tangent of angle γ formed by the slope of curve ψ-I and the I-axis. For powders whose burning surface area is constant, $\tan \gamma = 1/I_K$ = const.

In the case of combined charges:

$$\psi = \alpha'\psi' + \alpha''\psi'';$$
$$\Gamma = \alpha'\Gamma' + \alpha''\Gamma''$$

or:

$$\tan \gamma = \alpha' \tan \gamma' + \alpha'' \tan \gamma'',$$

where ψ, Γ, and $\tan \gamma$ are expressed as functions of I.

Since:

$$\tan \gamma' = \frac{1}{I_K'}, \quad \tan \gamma'' = \frac{1}{I_K''} \quad \text{and} \quad \tan \gamma = \frac{1}{I_K},$$

we obtain equation (8):

$$\frac{1}{I_K} = \frac{\alpha'}{I_K'} + \frac{\alpha''}{I_K''}, \qquad (8)$$

where I_K is subject to graphical determination.

On the basis of the formulas presented above, there is obtained a simple graphical construction of the law of variation of ψ and Γ as functions of I, both during the burning of the mixture and during the completion of the burning of the remaining thicker powder after the burning of the thinner powder is complete.

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for simplicity, there is presented in the diagram the solution for powders with a constant burning area, it being assumed that $\alpha' = 0.4$ and $\alpha'' = 0.6$.

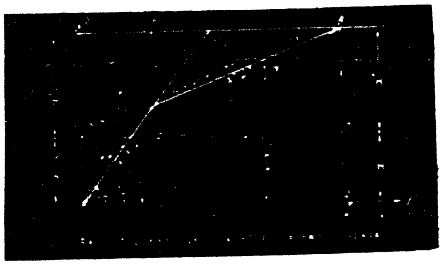

Fig. 168 - Scheme of Burning of Combined Charge.

1) completion of burning of thick powder; 2) mixture.

In fig. 168, the straight line 1 expresses the law of variation ψ'-1 for a thin powder with the impulse I_K', while the straight line 2 expresses the law of variation of ψ''-1 for a thick powder (I_K'').

In this connection:

$$\tan \gamma' = \frac{1}{I_K'}\,; \quad \tan \gamma'' = \frac{1}{I_K''}\,; \quad \tan \gamma = \frac{1}{I_K}\,.$$

To construct the law of variation of ψ for a mixture of powders, the diagram is divided along its height into two parts - a lower part α' (OO') and an upper part α'' (O'O"):

$$\alpha' + \alpha'' = 1.$$

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From the point O' at the height α', there is drawn the straight line O'B', which is parallel to the abscissa.

Along the ordinate aA, which corresponds to the impulse I'_K, the segment aA' = α'; the straight line OA' gives the values of $\alpha'\psi'$ (the values of the ordinates of the straight line OA multiplied by α'). In the upper part of the diagram, there is drawn from the point O' the straight line O'B; its ordinates, measured from the line O'B', give the values of the second component $\alpha''\psi''$.

In accordance with the formula $\psi = \alpha'\psi' + \alpha''\psi''$, there are added together the corresponding ordinates of the straight lines OA' and O'B; there is obtained the resultant line OC, which expresses the law of variation of ψ as a function of I for the combined charge as long as the two powders burn together.

From the similar triangles OCa and ODd on the one hand and O'CA' and OBB' on the other hand, we obtain:

$$\tan \gamma = \frac{Dd}{Od} = \frac{1}{I_K} = \frac{aC}{Oa},$$

but:

$$\frac{aC}{Oa} = \frac{aA' + A'C}{Oa} = \frac{\alpha'}{I'_K} + \frac{\alpha''}{I''_K},$$

since:

$$\frac{A'C}{Oa} = \frac{A'C}{O'A'} = \frac{B'B}{O'B'} = \frac{\alpha''}{I''_K}.$$

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Consequently, we obtain formula (8) by graphical means:

$$\tan \gamma = \frac{1}{I_K} = \frac{\alpha'}{I_K'} + \frac{\alpha''}{I_K''} . \qquad (8)$$

Toward the end of burning of the thin powder (I_K'), there will have burned the following part of the total charge:

$$\psi_K' = aA' + A'C = aA' + BB' \frac{I_K'}{I_K''} = \alpha' + \alpha'' \frac{I_K'}{I_K''} =$$

$$= \left(\frac{\alpha'}{I_K'} + \frac{\alpha''}{I_K''} \right) I_K' = \frac{I_K'}{I_K} ,$$

from which $\psi_K'/I_K' = 1/I_K$; in the diagram, this is represented by the ratio:

$$\frac{aC}{Oa} = \frac{Dd}{I_K} = \frac{1}{I_K} .$$

Consequently, the nominal impulse of the mixture I_K will be obtained by continuing the line OC until it intersects the straight line $\psi = 1$; the corresponding abscissa Od gives the impulse I_K for the mixture.

A break occurs at the point C along the line ψ-I, and thenceforth the law of completion of burning of the thick powder and of the variation of ψ is expressed by the line CB and by the equation:

$$\psi = \alpha' + \alpha''\psi'' .$$

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Fig. 169 - Variation of Intensive Gas Formation from Combined Charge.

1) Γ of mixture.

Figure 169 shows the construction of the Γ-I diagram for the mixture of the same powders with a constant burning area.

Γ' is characterized by the ordinates of the straight line O'A'; Γ'' is characterized by the straight line O"B"; in this connection $\int_0^{I_K} \Gamma dI = 1$ must be fulfilled as an identity, and since, in the case under consideration, Γ = const, there will prevail for each powder separately $\Gamma' I'_K = 1$ and $\Gamma'' I''_K = 1$.

In conformity with the formula:

$$\Gamma = \alpha'\Gamma' + \alpha''\Gamma'' \qquad (6)$$

we first multiply the ordinates Γ'' by α'', obtaining the straight line bb'b" ($\alpha''\Gamma''$); to its ordinates, for the abscissas from zero to I'_K, we add the quantities ba = b'a' ($\alpha'\Gamma'$). The ordinates of the line aa' give the characteristic Γ for the combined charge:

$$\Gamma = \alpha'\Gamma' + \alpha''\Gamma''.$$

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In the instant of complete burning of the thin powder with the impulse I_K', the characteristic Γ changes suddenly (from a' to b'), thereupon assuming the form of the straight line b'b" ($a''\Gamma''$), which expresses the intensity of gas formation in the process of completion of burning of the thicker powder with the impulse I_K'' alone.

It is not difficult to show that the shaded area, which expresses the intensity of burning of the combined charge, equals unity, just as in the case of single charges.

As a matter of fact:

(This formula is illegible on the original photostat. Editor.)

since $\Gamma' I_K' = \Gamma'' I_K'' = 1$.

Now, knowing the correlations Γ-I and ψ-I, it is possible to establish the diagram for the correlation Γ-ψ and to apply the resulting data on Γ, ψ, and I for the combined charge to the solution of the problem of internal ballistics.

In solving the problem for the case of the geometric law of burning, it is necessary to know the form characteristics K and KΛ in accordance with formulas (4) and (5) and to apply them in the same manner as in solving the problem for a charge consisting of a single type of powder until the thin powder and the corresponding part of the thick powder have burned.

Following this, the law of gas formation changes, the intensity of gas formation diminishes, and, in solving the problem of pyrodynamics, it becomes necessary to take into account the change in the initial conditions for this phase of burning of

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the composite charge.

A detailed theoretical solution of this problem is given in the theoretical part on pyrodynamics.

In the case of the geometric law of burning, the dependence of ψ for the mixture is usually expressed in terms of z" - the relative thickness of the thicker powder. For powders with the form characteristics of the grain $\varkappa'$, λ' and $\varkappa''$, λ'', we have:

$$\psi' = \varkappa' z' + \varkappa' \lambda' z'^2; \quad \psi'' = \varkappa'' z'' + \varkappa'' \lambda'' z''^2,$$

where:

$$z' = \frac{e'}{e_1'} = \frac{I'}{I_K'}; \quad z'' = \frac{e''}{e_1''} = \frac{I''}{I_K''}.$$

Inasmuch as, for both powders, in a given instant, $I' = I'' = I$ and $I_K' < I_K''$, it follows that $z' > z''$.

Substituting the quantities z' and z" into the formulas for ψ' and ψ'', we obtain:

$$\psi' = \frac{\varkappa'}{I_K'} I + \frac{\varkappa'}{I_K'} \frac{\lambda'}{I_K'} I^2; \quad \psi'' = \frac{\varkappa''}{I_K''} I + \frac{\varkappa''}{I_K''} \frac{\lambda''}{I_K''} I^2.$$

By expressing the dependence of ψ' in terms of z", multiplying and dividing by I_K'', and designating $I_K'/I_K'' = z_K'$, we obtain:

$$\psi' = \frac{\varkappa'}{z_K'} \frac{1}{I_K''} + \frac{\varkappa'}{z_K'} \frac{\lambda'}{z_K'} \frac{1}{I_K''} = \frac{\varkappa'}{z_K'} z'' + \frac{\varkappa'}{z_K'} \frac{\lambda'}{z_K'} z'',$$

but:

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$$\psi = \alpha'\psi' + \alpha''\psi''.$$

Representation of ψ as a function of z" gives:

$$\psi = \varkappa_{CM} z'' + \varkappa_{CM}\lambda_{CM} z''^2 = \alpha' \frac{\varkappa'}{z'_K} z'' + \alpha' \frac{\varkappa'}{z'_K}\frac{\lambda'}{z'_K} z''^2 + \alpha''\varkappa'' z'' + \alpha''\varkappa''\lambda'' z''^2.$$

We equate the coefficients of the same powers of z":

$$\varkappa_{CM} = \varkappa' \frac{\alpha'}{z'_K} + \alpha''\varkappa''; \quad \varkappa_{CM}\lambda_{CM} = \varkappa'\lambda' \frac{\alpha'}{z'^2_K} + \alpha''\varkappa''\lambda''.$$

Thus, there has been obtained an expression for ψ mixture in the usual form, as a function of z" - the relative thickness of the burnt layer of the thicker powder.

Since the quantity ψ_0 is computed with the aid of the usual formula:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f_1}{p_0} + \alpha - \frac{1}{\delta}},$$

where $f_1 = \alpha'f' + \alpha''f''$ is the propellant force of the powder in the combined charge, it follows that:

$$z''_0 = \frac{2\psi_0}{\varkappa_{CM}(1 + \sigma_0)} \approx \frac{\psi_0}{\varkappa_{CM}} = \frac{\psi_0}{\varkappa\left(\frac{\alpha'}{z'_K} + \alpha''\right)},$$

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since $\sigma \approx 1$.

Since $z_K' < 1$, it follows that:

$$\frac{\alpha'}{z_K'} + \alpha'' > \alpha' + \alpha'' = 1$$

and z_0'' is smaller than z_0 for a single charge at the same value of ψ_0.

The formulas derived above apply as long as both powders are burning, i.e., until the instant when:

$$z'' = z_K' = \frac{I_K'}{I_K''} ; \quad z' = 1 \quad \text{and} \quad \psi' = 1.$$

In that instant:

$$\psi_{K'} = \alpha' + \alpha''\psi_{K'}'' ,$$

where $\psi_{K'}'' = \varkappa z_K' + \varkappa\lambda z_{K'}'^2$ is the part of the charge of thick powder which has burned by the time the burning of the thin powder is complete.

4. ANALYTICAL SOLUTION OF PROBLEM

(Written by Professor G. V. Oppokov).

We shall here consider in detail only the simplest case, when the charge consists of powders of two types, both of which are degressive in form and possess the same physico-chemical nature. It is already known that the total interval of burning of the charge must in this case be divided into two phases; by the end of the first phase, all of the thin powder and a part of the thick powder have burned, and the burning of the thick powder is completed in

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the course of the second phase.

For the first phase, in the presence of a binomial relation for the law of gas formation, we have:

$$\psi = \varkappa z'' + \varkappa\lambda z''^2 .$$

Consequently, the formulas for the single charge retain their significance, as long as the following particulars are observed.

1) The form characteristics $\varkappa$ and $\varkappa\lambda$ are determined with the aid of the following formulas:

$$\varkappa = \frac{\varkappa'}{z'_K}\alpha' + \varkappa''\alpha'' \quad \text{and} \quad \varkappa\lambda = \frac{\varkappa'\lambda'}{z'^2_K}\alpha' + \varkappa''\lambda''\alpha'',$$

where:

$$z'_K = \frac{I'_K}{I''_K}, \quad \alpha' = \frac{\omega'}{\omega}, \quad \alpha'' = \frac{\omega''}{\omega} .$$

2) The quantities z_0 and I_K must be replaced by z''_0 and I''_K for the thick powder, where:

$$z''_0 = \frac{\psi_0}{\varkappa} = \frac{\psi_0}{\varkappa''\left(\frac{\alpha'}{z'_K} + \alpha''\right)} .$$

3) The argument:

$$x = z'' - z''_0; \quad x_{K'} = z''_{K'} - z''_0 = z'_K - z''_0 .$$

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4) At the end of the first phase, we shall have:

$$z''_{K'} = \frac{e'_1}{e''_1}; \quad x_{K'} = z''_{K'} - z''_0; \quad v'_K = v_{K,0} x_{K'},$$

where:

$$v_{K,0} = \frac{SI''_K}{\varphi m}$$

<u>In the second phase</u>, the differential equation:

$$\frac{dl}{dx} = \frac{Bx(l_\psi + l)}{\psi - \frac{B\theta}{2} x^2} \qquad (10)$$

is retained, as is the formula for the velocity of the projectile:

$$v = v_{K,0} x;$$

and the law of gas formation has the following form:

$$\psi = \frac{\omega'}{\omega} + \frac{\omega''}{\omega} \varkappa'' z''(1 + \lambda'' z'').$$

By substituting here the quantity:

$$z'' = z''_0 + x$$

and designating:

$$\psi_{0,2} = \frac{\omega'}{\omega} + \left(\frac{\omega''}{\omega}\varkappa''\right) z''_0 + \left(\frac{\omega''}{\omega}\varkappa''\lambda''\right) z''^2_0;$$

$$k_{1,2} = \frac{\omega''}{\omega}\varkappa'' + 2\left(\frac{\omega''}{\omega}\varkappa''\lambda''\right) z''_0 =$$

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$$= \frac{\omega''}{\omega} \varkappa''(1 + 2\lambda'' z''_0) = \frac{\omega''}{\omega} \varkappa'' \sigma''_0,$$

we shall obtain for the second phase:

$$\psi = \psi_{0,2} + k_{1,2}x + \left(\frac{\omega''}{\omega} \varkappa'' \lambda''\right) x^2.$$

Thus, the law of gas formation has a form analogous to the form of the law of gas formation for the first phase:

$$\psi = \psi_0 + k_1 x + \varkappa \lambda x^2.$$

It follows from this that, for the second phase, instead of B and C, use must be made of the quantities B_2 and C_2, where:

$$B_2 = \frac{B\theta}{2} - \left(\frac{\omega''}{\omega} \varkappa'' \lambda''\right); \quad C_2 = \frac{B_2}{k_{1,2}},$$

and then the tabular parameters will be:

$$\beta = C_2 x; \quad \gamma = \frac{C_2 \psi_{0,2}}{k_{1,2}}.$$

Furthermore, in integrating the equation:

$$\frac{dl}{dx} = \frac{Bx(l_\psi + l)}{\psi_{0,2} + k_{1,2}x - B_2 x^2} \tag{11}$$

it must be taken into consideration that, at the start of the second phase, the path l is equal to the path $l_{K,1}$ of the projectile at the end of the first phase, and the initial value of β equals:

$$\beta_{0,2} = C_2 x_{K,1}.$$

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Consequently, approximate integration of equation (11) gives:

$$\ln \frac{l_{\psi_{av.}} + l}{l_{\psi_{av.}} + l_{K,1}} = \int_{x_{K,1}}^{x} \frac{Bx\,dx}{\psi_{0,2} + k_{1,2}x - B_2x^2} = \frac{B}{B_2} \int_{\beta_{0,2}}^{\beta} \frac{\beta\,d\beta}{\gamma + \beta - \beta^2}, \qquad (12)$$

where, as in the first phase:

$$l_{\psi_{av.}} = l_{\Delta} - a\psi_{av.},$$

but, in contrast with that phase:

$$\psi_{av.} = \frac{\psi_{K,1} + \psi}{2},$$

in which connection $\psi_{K,1}$, the relative fraction of the burnt part of the total charge, is equal at the start of the second phase to:

$$\psi_{K,1} = \varkappa z''_{K,1} + \varkappa\lambda z''^{2}_{K,1} = \psi_0 + k_1 x_{K,1} + \varkappa\lambda x^2_{K,1}.$$

In determining the integral of the right-hand side of equation (11), it is possible to make use of the same table for $\log Z^{-1}$, while keeping in mind, however, that:

$$\log e \int_{\beta_{0,2}}^{\beta} \frac{\beta\,d\beta}{\gamma + \beta - \beta^2} = \log e \int_{0}^{\beta} \frac{\beta\,d\beta}{\gamma + \beta - \beta^2} - \log e \int_{0}^{\beta_{0,2}} \frac{\beta\,d\beta}{\gamma + \beta - \beta^2} =$$

$$= \log(Z^{-1}Z_{0,2}),$$

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where:

$$\log Z^{-1} = \log e \int_0^{\beta} \frac{\beta d\beta}{\gamma + \beta - \beta^2} ; \quad \log Z_{0,2}^{-1} = \log e \int_0^{\beta_{0,2}} \frac{\beta d\beta}{\gamma + \beta - \beta^2} .$$

Thus:

$$\ln = \frac{l_{\psi_{av.}} + l}{l_{\psi_{av.}} + l_{K,1}} = \ln (Z^{-B/B_2} Z_{0,2}^{B/B_2}),$$

from which there is finally obtained the desired general relation for the path of the projectile in the second phase:

$$l = (l_{\psi_{av.}} + l_{K,1}) Z_{0,2}^{B/B_2} Z^{-B/B_2} - l_{\psi_{av.}} .$$

Therefore, in comparison with the first phase, the coefficient $l_{\psi_{av.}}$ applied to Z^{-B/B_1} is replaced by the following product:

$$(l_{\psi_{av.}} + l_{K,1}) Z_{0,2}^{B/B_2} ,$$

where $Z_{0,2}^{B/B_2}$ is a constant quantity for all points in the second phase, which is determined from the table for $\log Z^{-1}$ on the basis of data for γ and $\beta_{0,2}$.

Note: At $\gamma > 0.2$, it is possible to make use of the following formula:

$$\log Z^{-1} = \frac{1}{2} \frac{1}{\sqrt{1 + 4\gamma}} \log \frac{1 + \frac{\beta}{2\gamma} (\sqrt{1 + 4\gamma} + 1)}{1 - \frac{\beta}{2\gamma} (\sqrt{1 + 4\gamma} - 1)}$$

$$- \frac{1}{2} \log \left[1 + \frac{\beta}{\gamma} (1 - \beta) \right].$$

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Example 1. To find the principal ballistic elements for the 1909 model, 152-mm field howitzer with a full charge composed of thin Γ_6 powder and thick Γ_6 powder under the following conditions:

$f = 925{,}000$; $\alpha = 0.98$; $\delta = 1.6$; $\theta = 0.18$; $u_1 = 7.1 \cdot 10^{-6}$;

$\omega' = 0.6015$; $0.675 \cdot 14 \cdot 100$ (mm);

$\omega'' = 1.228$; $1.055 \cdot 20 \cdot 100$ (mm);

$s = 1.868$; $W_0 = 4.04$; $l_D = 14.58$;

$q = 40.95$; $\varphi = 1.06 + \frac{1}{3}\frac{\omega}{q}$; $p_0 = 30{,}000$; $g = 98.1$.

Solution. The computations are broken up into separate stages.

I. Preliminary Period.

1) $\varkappa' = 1 + \alpha' + \beta' - \frac{1}{2}\alpha'\beta' = 1.0548$; $\varkappa'\lambda' = -0.0548$;

2) $\varkappa'' = 1 + \alpha'' + \beta'' - \frac{\lambda}{2}\alpha''\beta'' = 1.063$; $\varkappa''\lambda'' = -0.0630$;

3) $\frac{\omega'}{\omega} = \frac{\omega'}{\omega' + \omega''} = 0.3287$;

4) $\frac{\omega''}{\omega} = 0.6713$;

5) $z''_{K,1} = \frac{2e'_1}{2e''_1} = 0.6398$;

6) $\varkappa = \frac{\omega'}{\omega}\frac{\varkappa'}{z'_{K,1}} + \frac{\omega''}{\omega}\varkappa'' = 0.5419 + 0.7136 = 1.2555$;

7) $\varkappa\lambda = \frac{\omega'}{\omega}\frac{\varkappa'\lambda'}{z''^2_{K,1}} + \frac{\omega''}{\omega}\varkappa''\lambda'' = -0.04399 - 0.04229 = -0.08628$;

8) $\frac{1}{\delta_2} = \frac{W_0}{\omega} - \frac{1}{\delta} = 1.583$;

9) $\frac{1}{\delta_1} = \alpha - \frac{1}{\delta} = 0.355;$

10) $\psi_0 = \frac{1}{\delta_2} : \left(\frac{f}{p_0} + \frac{1}{\delta_1}\right) = 0.0508;$

11) $k_1 = \sqrt{x^2 + 4\varkappa\lambda\psi_0} = 1.248;$

12) $z_0'' = \frac{2\psi_0}{\varkappa + k_1} = 0.0405; \quad x_{K,1} = z_{K,1}'' - z_0'' = 0.5993;$

$x_{K,2} = 1 - z_0'' = 0.9595.$

II. Preliminary Computations for First Phase of First Period.

1) $\varphi = 1.06 + \frac{1}{3}\frac{\omega}{q} = 1.075;$

2) $\frac{\varphi m}{s} = 0.2402;$

3) $I_K'' = \frac{e_1''}{u_1} = 742.9;$

4) $v_{K,0} = I_K'' : \frac{\varphi m}{s} = 3092;$

5) $\frac{\omega}{s} = 0.9800;$

6) $f\,\frac{\omega}{s} = 906,300;$

7) $l_\Delta = \frac{\omega}{s}\,\frac{1}{\delta_2} = 1.552;$

8) $a = \frac{\omega}{s}\,\frac{1}{\delta_1} = 0.3478.$

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III. Tabular Constants for First Phase of First Period.

1) $B = I''^{2}_{K} : \left(f \frac{\omega}{s} \frac{\varphi m}{s} \right) = 2.534;$

2) $B_1 = \frac{B\theta}{2} - \varkappa\lambda = 0.3144$ (cf. No. 7 in Stage I);

3) $\frac{B}{B_1} = 8.061;$

4) $C = \frac{B_1}{k_1} = 0.2520;$

5) $\gamma = \frac{C\psi_0}{k_1} = 0.01024.$

IV. Ballistic Elements of Shot at End of First Phase of First Period.

1) $\beta_{K,1} = C\,(1 - z''_0) = Cx_{K,1} = 0.1510;$

2) $v_{K,1} = v_{K,0}x_{K,1} = 185.3$ m/sec;

3) $\psi_{K,1} = \psi_0 + k_1 x_{K,1} + \varkappa\lambda x^2_{K,1} = 0.7676;$

4) $\psi_{av.} = \frac{\psi_{K,1} + \psi_0}{2} = 0.4092;$

5) $l_{\psi_{av.}} = l_\Delta - a\psi_{av.} = 1.410;$

6) $\log Z^{-1}_{K,1} = 0.0577$ (from table of $\log Z^{-1}$);

7) $l_{K,1} = l_{\psi_{av.}} Z_{K,1}^{-B/B_1} - l_{\psi_{av.}} = 2.704$ dm;

8) $l_{\psi_{K,1}} = l_\Delta - a\psi_{K,1} = 1.285$ dm;

9) $p_{K,1} = f\,\frac{\omega}{s}\,\frac{\psi_{K,1} - \frac{B\theta}{2}x^2_{K,1}}{l_{\psi_{K,1}} + l_{K,1}} = 1558\ \text{kg/cm}^2$

V. Ballistic Elements of Shot at p_m.

We anticipate $p_m = 1700$ kg/cm^2. In the first approximation, we find:

$$x_{m,1} = \frac{k_1}{\frac{B(1+\theta)}{1 + \frac{p_m}{f\delta_1}} - 2\varkappa\lambda} = 0.4187.$$

1) $\beta_{m,1} = Cx_{m,1} = 0.1054$;

2) $v_{m,1} = v_{K,0}x_{m,1} = 129.5$ m/sec;

3) $\psi_{m,1} = \psi_0 + k_1x_{m,1} + \varkappa\lambda x^2_{m,1} = 0.5572$;

4) $\psi_{av.} = \frac{\psi_{m,1} + \psi_0}{2} = 0.3045$;

5) $l_{\psi_{av.}} = l_\Delta - a\psi_{av.} = 1.446$;

6) $\log Z^{-1}_{m,1} = 0.0369$;

7) $l_{m,1} = l_{\psi_{av.}}Z^{-B/B_1}_{m,1} - l_{\psi_{av.}} = 1.423$;

8) $l_{\psi_{m,1}} = l_\Delta - a\psi = 1.358$ dm;

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9) $p_{m,1} = f \frac{\omega}{s} \frac{\psi_{m,1} - \frac{B\theta}{2} x^2_{m,1}}{\ell_{\psi m,1} + \ell_{m,1}} = 1689$ kg/cm^2

There is no sense in making further approximations, since, generally speaking, there is allowed a ±50 kg/cm^2 discrepancy between the initial and computed p_m. In accordance with literature data, the maximum pressure obtained in firing tests is 1650-1700 kg/cm^2.

VI. Preliminary Computations for Second Phase of First Period.

1) $\psi_{0,2} = \frac{\omega'}{\omega} + \left(\frac{\omega''}{\omega}\varkappa''\right) z''_0 + \left(\frac{\omega''}{\omega}\varkappa''\lambda''\right) z''^2_0 = 0.3576$ (cf. Nos. 6 and 7 in Stage I);

2) $k_{1,2} = \frac{\omega''}{\omega}\varkappa'' + 2\left(\frac{\omega''}{\omega}\varkappa''\lambda''\right) z''_0 = 0.7102.$

VII. Tabular Constants for Second Phase of First Period.

1) $B_2 = \frac{B\theta}{2} - \frac{\omega''}{\omega}\varkappa''\lambda'' = 0.2704;$

2) $\frac{B}{B_2} = 9.374;$

3) $C_2 = \frac{B_2}{k_{1,2}} = 0.3807;$

4) $\gamma = \frac{C_2\psi_{0,2}}{k_{1,2}} = 0.1916;$

5) $\beta_{0,2} = C_2 x_{K,1} = 0.2281;$

6) $\log Z^{-1}_{0,2} = 0.0364$ (knowing γ and $\beta_{0,2}$)

7) $Z_{0,2}^{B_2} = 0.4558.$

VIII. Ballistic Elements at End of Burning of Powder.

1) $\beta_{K,2} = C_2 x_{K,2} = 0.3653;$

2) $v_{K,2} = v_{K,0} x_{K,0} = 296.7$ m/sec;

3) $\psi_{K,2} = \psi_2 + k_{1,2} x_{K,2} + \left(\frac{\omega''}{\omega} \varkappa'' \lambda''\right) x^2_{K,2} = 1;$

4) $\psi_{av.} = \dfrac{\psi_{K,2} + \psi_{K,1}}{2} = 0.8838;$

5) $l_{\psi_{av.}} = l_\Delta - a\psi_{av.} = 1.245$ dm;

6) $\log Z_{K,2}^{-1} = 0.0807;$

7) $l_{K,2} = (l_{\psi_{av.}} + l_{K,1}) Z_{0,2}^{B/B_2} Z_{K,2}^{-B/B_2} - l_{\psi_{av.}} = 9.04$ dm;

8) $l_{\psi K,2} = l_\Delta - a\psi_{K,2} = 1.203$ dm;

9) $p_{K,2} = \dfrac{f\omega}{s} \dfrac{\psi_{K,2} - \frac{B\theta}{2} x^2_{K,2}}{l_{\psi_{K,2}} + l_{K,2}} = 699$ kg/cm^2.

The resulting values:

$$\psi_{K,2} - \frac{B\theta}{2} x^2_{K,2} = 1 - \frac{v_K^2}{v_{np}^2} = 0.7901;$$

$l_{\psi_{K,2}} = l_1 = 1.203;$ $\log p_{K,2}$ and $-\log(l_1 + l_{K,2})$

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will be needed for the next stage.

IX. Ballistic Elements of Shot at Muzzle.

As assigned, $l_D = 14.58$.

1) $v^2_{np} = \frac{2}{\theta}\left(f\frac{\omega}{s}\right):\left(\frac{\varphi m}{s}\right) = 4.193 \cdot 10^7$;

2) $\eta_D = \frac{l_1 + l_D}{l_1 + l_{K,2}} = 1.539$;

3) $p_D = p_{K,2}\eta_D^{-1-\theta} = 420\ kg/cm^2$;

4) $v_D = v_{np}\sqrt{1 - \left(1 - \frac{v^2_K}{v^2_{np}}\right)\eta_D^{-0.2}} = 335.9\ m/sec.$

The tabular muzzle velocity is $v_D = 335.3$ m/sec.

5. Use of GAU Tables for the Case of Combined Charges.

The GAU Tables, ANII Tables, and tables of Professor Drozdov are set up for a charge consisting of a single type of powder, which is characterized by the strip-type grain shape ($\varkappa = 1.06$), the strip thickness $2e_1$, and the burning rate u_1 or full impulse $I_K = e_1/u_1$.

The magnitude of the impulse I_K enters into the loading parame[illegible] $B = \frac{s^2 I_K^2}{f\omega\varphi m}$, which is a basic quantity in the tables together with [illegible] quantity Δ. The relations found above for I_K of the mixture [illegible] the other characteristics make it possible approximately, b[illegible]

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will be needed for the next stage.

IX. Ballistic Elements of Shot at Muzzle.

As assigned, $l_D = 14.58$.

1) $v_{\text{пр}}^2 = \frac{2}{\theta}\left(f\frac{\omega}{s}\right):\left(\frac{\varphi m}{s}\right) = 4.193 \cdot 10^7$;

2) $\eta_D = \frac{l_1 + l_D}{l_1 + l_{K,2}} = 1.539$;

3) $p_D = p_{K,2}\eta_D^{-1-\theta} = 420\ \text{kg/cm}^2$;

4) $v_D = v_{\text{пр}}\sqrt{1 - \left(1 - \frac{v_K^2}{v_{\text{пр}}^2}\right)\eta_D^{-0.2}} = 335.9\ \text{m/sec}$.

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The magnitude of the impulse I_K enters into the loading parameter $B = \frac{s^2 I_K^2}{f\omega\varphi m}$, which is a basic quantity in the tables together with the quantity Δ. The relations found above for I_K of the mixture and for the other characteristics make it possible approximately, but with

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an accuracy sufficient for practical purposes, to utilize the GAU and other table for the solution of problems in interior ballistics in the case of combined charges.

If the powders composing the charge have the same strip shape, then, knowing I_K' and I_K'', as well as ω, ω', and ω'', the nominal pressure impulse of the mixture of powders I_K is found with the aid of formula (8), substituted into the expression for B, and used in solving the problem in the same manner as in the case of a single powder.

If the powders have different grain shapes, for example degressive (strip or grain of the 4/1 or 7/1 type) and progressive with seven perforations (7/7, 9/7, 12/7, etc.), then, for the degressive powders, $\varkappa = 1.06$ is assumed, and, instead of powders with seven perforations, there is taken the equivalent strip-type powder with the following strip thickness:

$$2e_{1_{\text{strip}}} = \frac{10}{7}\, 2e_{1_{7\ \text{perfor.}}}$$

and the impulse $I_K'' = e_1''/u_1''$ is determined for it.

For strip-type as well as for 4/1 and 7/1 powders, the thickness of the powder is not altered, and I_K' is determined in the usual manner: $I_K' = e_1'/u_1'$.

These values for the impulses I_K' and I_K'' are thereupon substituted into formula (8), I_K for the combined charge is found from the known values of α' and α'', and, this quantity having been used to compute the parameter B, the problem is thenceforth solved in the usual manner, as in the case of a single powder.

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Thus the solution applicable to a combined charge contains only one additional operation for determining the pulse I_K of the mixture according to formula (8).

The results of computations on the basis of the tables, performed parallel with computations by the analytical methods of Professor Drozdov and Professor Grave, show an almost perfect agreement in the quantities p_m and v_D. But the tabular method with the use of the impulse I_K of the mixture and the parameter B corresponding thereto does not make it possible to determine the actual position of the projectile at the end of burning of the entire charge, which corresponds to the end of burning of the thick powder with the impulse I_K''.

The quantity I_K for the mixture is merely a nominal quantity suitable to characterize the rate of gas formation as long as both powders are burning together (as far as the point C in fig. 168). The actual end of burning can be determined from the velocity curve $v-l$ or $v-\lambda$, if there is marked thereon the ordinate v_K'' corresponding to the end of burning of the thick powder, which is determined for the formula:

$$v_K'' = \frac{sI_K''}{\varphi m}(1 - z_0), \qquad (13)$$

where:

$$z_0 \approx \frac{\psi_0}{\varkappa}, \quad \text{and} \quad \psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}$$

From the $v-l$ diagram, on the basis of the quantity v_K'', we can determine the magnitude of the path l_K'', i.e., the position of the

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end of burning of the total charge, which is important for establishing the completeness of burning of the powder with a given charge.

In exactly the same manner, with the aid of the formula:

$$v''_K = \frac{sI''_K}{\varphi m}(1 - z_0),$$

it is possible to determine from the v-l diagram the position of the projectile at the end of burning of the thin powder l'_K.

To determine the percentage content of the thin and thick powders in the mixture for a gun with predetermined design characteristics (W_0, s, Λ_D) at a predetermined Δ, the quantity p_m is used to establish the parameter B from the GAU Tables, Issue No. I, whereupon there is found the impulse of the mixture:

$$I_K = \frac{1}{s}\sqrt{Bl\omega\varphi m}.$$

Knowing the impulses I'_K and I''_K of the powders composing the mixture and the total weight of the charge $\omega = W_0 \cdot \Delta$, it is possible to find the quantity α' from formula (8), assuming that $\alpha'' = 1 - \alpha'$.

We obtain:

$$\alpha' = \frac{\dfrac{I_K}{I'_K} - 1}{\dfrac{I''_K}{I_K} - 1} \qquad (14)$$

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and then:

$$\omega' = \omega\alpha'; \quad \omega'' = (1 - \alpha')\omega.$$

In the treatment of results of firing tests from guns, there is usually observed a diminution of the coefficient of utilization of the unit weight of the charge η_ω as the weight of the charge ω/q increases, since, at large ω/q, a larger fraction of the external work is consumed in moving the gases of the charge and a smaller part remains for the useful work of moving the projectile.

In the treatment of results of firing tests from howitzers with combined charges, there is usually observed a different law of variation of the coefficient η_ω. In some howitzers, in the presence of the minimum charge, $\eta_{\omega n}$ has its minimum value and increases as the weight of the charge increases; in others, as the weight of the charge increases with the increase in the relative weight of the thick powder, the coefficient η_ω at first diminishes, passes through a minimum in the presence of one of the intermediate charges, and then increases again, it usually being the case that, with the full charge, $\eta_{\omega 0}$ is larger than $\eta_{\omega n}$ of the basic minimum charge.

In this connection, as a rule, the velocity v_{Di} varies in accordance with a linear law as a function of the weight of the charge (fig. 170), instead of being convex upward.

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Fig. 170 - Variation in v_D and η_ω in Firing with Combined Charge.
1) charge no.

6. PARTICULARS OF BALLISTIC DESIGN OF HOWITZERS AND COMPUTATION OF CHARGES.

The ballistic computation of a howitzer is conducted for the maximum initial velocity v_D, which corresponds to the full charge at the maximum pressure p_{mo}. In this connection, the following data are found: the weight of the full charge No. 0 - ω_0; the fundamental design characteristics of the bore - W_0, Λ_D, $L_{KH} = l_{KM} + l_D$, $W_{KH} = W_0(\Lambda_D + 1)$; and the nominal impulse of the powder mixture for the charge No. 0 - I_{Ko} - which is determined from the parameter B_0. The propellant force of the powder in the charge No. 0 is assumed to be 90-93 tm/kg, unless more accurate data relating to f_0 for a "related" gun are available.

In this connection, contrary to the design of guns firing with one charge and with one initial projectile velocity, for which it is desirable to obtain η_K in the range of 0.60-0.65, η_K for a howitzer with a full charge must be considerably lower (0.25-0.30), in order that the same thick powder be given enough time to burn in the presence of diminished charges and reduced pressures, whic

involves the transfer of the end of burning toward the muzzle face.

For this reason, in utilizing the directive diagram, after determining the minimum-volume gun (point M_0) and reducing the weight of the charge (point N), it is necessary to proceed in the direction of reducing Δ and η_K and increasing Λ_D, i.e., to take variants in the lower left sector from the point M_0. In this connection, the characteristic $\eta_D = p_{av.}/p_m$ is obtained smaller than in the case of guns (0.40-0.50).

For the designed gun, using the minimum charge No. n, the predetermined velocity $v_{D,n}$ and the pressure $p_{m,n}$, which is predetermined by the cocking conditions of the firing device, are used to assign Δ_n = 0.10-0.15 and to determine ω_n, B_n, and I'_K - the pressure impulse for the thin powder alone. On the basis of treatment of data for existing howitzers, the propellant force of the powder f' is obtained equal to 80-82 tm/kg.

The computation will in this case follow the course outlines below. Δ_n having been assigned, there are found:

$$\omega_n = W_0 \cdot \Delta_n,\ \varphi_n = (1.05\text{-}1.06) + \frac{1}{3}\frac{\omega}{q};$$

$$n_f = \sqrt{\frac{\omega}{\varphi q}\frac{f}{95}};\quad v_{T,D} = \frac{v_{D,n}}{n_f}.$$

At the given Δ_n, from Λ_D and $v_{T,D}$, there are found B_n and:

$$I'_K = \frac{1}{s}\sqrt{B_n f_n \omega_n \varphi_n m};$$

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From Δ and B, there are determined the pressure p_m tab. and:

$$p_{mf} = p_m \text{ tab.} \frac{f}{95} .$$

If p_{mf} is smaller than the required p_m, Δ is changed and the computation is repeated until the necessary magnitude is selected for $p_{m,n}$.

The impulse I'_K determined in the final variant will characterize the thin powder composing the basic charge ω_n in the remaining combined charges.

Knowing the weights of the full and minimum charges ω_0 and ω_n, we can find α'_0 for the charge No. 0:

$$\alpha'_0 = \frac{\omega_n}{\omega_0} ; \quad \alpha''_0 = 1 - \alpha'_0 .$$

Knowing I_K for the full charge, which is composed of a mixture of a thin powder with a known impulse I'_K and a thick powder with an unknown impulse I''_K, we determine the latter on the basis of equation (8):

$$\frac{1}{I_{K,o}} = \frac{\alpha'_0}{I'_K} + \frac{\alpha''_0}{I''_K} ,$$

from which:

$$I''_K = \frac{\alpha''_0}{\frac{1}{I_{K,o}} - \frac{\alpha'_0}{I'_K}} = I_{K,o} \frac{\alpha''_0}{1 - \alpha'_0 \frac{I_{K,o}}{I'_K}} .$$

The values of I'_K and I''_K will subsequently be the same for all intermediate charges.

For the thick powder, f'' is found from the quantities f_0 and f' for the full minimum charges:

$$f_0 = f'\alpha_0' + f''\alpha_0'',$$

from which:

$$f'' = \frac{f_0 - f'\alpha_0'}{\alpha_0''}$$

Usually, f'' is close to 95 tm/kg.

Knowing $v_{D,o}$, $v_{D,n}$, ω_0, ω_n, and the scale of velocities, we designate the weights of the intermediate charges on the basis of the linear relation between v_D and ω:

$$\omega_i = \omega_n + \frac{\omega_0 - \omega_n}{v_{D,o} - v_{D,i}} (v_{D,i} - v_{D,n}).$$

Knowing ω_i, we determine:

$$\Delta_i = W_0\omega_i; \quad \alpha_i' = \frac{\omega_n}{\omega_i}; \quad \alpha_i'' = 1 - \alpha_i';$$

$$f_i = f'\alpha_i' + f''\alpha_i'',$$

The subsequent procedure is as above: $n_{fi} = \sqrt{\frac{\omega_i}{q} \frac{1}{\varphi_i} \frac{f_i}{95}}$ and $v_{tab.D} = v_{D,i}/n_{f,i}$ are computed, and Δ_i, Λ_D, and $v_{tab.D}$ are used to determine B_i, whereupon Δ_i and B_i are used to determine p_m tab. and $p_{mf} = p_m \text{ tab.} \frac{f}{95}$.

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CHAPTER 2 - SOLUTION WITH CONSIDERATION OF GRADUAL CUTTING OF ROTATING BAND INTO RIFLING GROOVES OF BARREL

(Professor G. V. Oppokov)

For a consideration of the gradual cutting of the rotating band into the rifling grooves of the barrel, the preliminary period must be divided into two phases: the initial phase and the phase of acceleration of the projectile, at the start of which the pressure of the powder gases attains a magnitude sufficient for the onset of the cutting-in process.

For the initial phase of the preliminary period, it is possible to employ the usual formulas applicable to the preliminary period after replacing therein the quantities:

$$p_0, \psi_0, k_1 \text{ and } z_0$$

by the quantities:

$$p_H, \psi_H, k_H \text{ and } z_H.$$

In the acceleration phase, it will be necessary to deal with an equation of the motion of the projectile of the following type:

$$\varphi m \frac{dv}{dt} = (p - \Pi)s,$$

where Π is the force of resistance of the rotating band to the cutting-in process, related to the unit cross-sectional area s:

$$p = \frac{f\omega\psi - \theta A}{s(l_\psi + l)},$$

it being necessary to represent the total work A of the powder gases in the following form:

$$A = \frac{\varphi m v^2}{2} + A_\Pi.$$

The law governing the force Π and the corresponding work A_Π should be established by experimental means.

Finally, the gas inflow ψ will be found in accordance with the

law of gas formation:

$$\frac{d\psi}{dt} = \frac{p}{I_K}\sqrt{\varkappa^2 + 4\varkappa\lambda\psi},$$

if use is made of the geometric law of burning.

In summary, numerical integration must be applied to a system of equations of the following type:

$$\left.\begin{aligned} \frac{d\psi}{dt} &= \frac{p}{I_K}\sqrt{\varkappa^2 + 4\varkappa\lambda\psi}; \\ \frac{dv}{dt} &= (p - \Pi) : \left(\frac{\varphi m}{s}\right); \\ \frac{dl}{dt} &= v, \end{aligned}\right\} \quad (15)$$

where:

$$p = f\frac{\omega}{s}\,\frac{\psi - \frac{B\theta}{2}x^2 - \frac{\theta}{f\omega}A_{\Pi}}{l_{\psi} + l}. \quad (16)$$

This integration may be carried out in accordance with the variant presented in the books of Professor Oppokov [7-8].

In order to arrive at an analytical method of solution, it is necessary to make three simplifying assumptions for the purpose of determining the velocity of the projectile in the given phase.

1) The variation of ψ in the first equation of the system (15) is neglected:

$$\frac{d\psi}{dt} = \frac{p}{I_K}\sqrt{\varkappa^2 + 4\varkappa\lambda\psi_{\Pi}} = \frac{\varkappa_H}{I_K}p; \quad (17)$$

2) The small terms in the numerator of the fraction on the right-hand side of formula (16) are neglected, and the following substitution is performed in the denominator:

$$l_{\psi} + l = l_{\Delta} + l_c, \quad (18)$$

where the average length may be replaced by $\frac{\lambda}{2}$, l being considered to be constant during the integrating operation.

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3) A law of the following type is accepted for the force Π :

$$\Pi = \Pi_0 + \Pi_{av.} \frac{l}{l_{Kav.}} = p_H + \Pi_{av.} \frac{l}{l_{Kav.}}, \qquad (19)$$

where $l_{Kav.}$ is the path of the projectile at the end of the given phase, the force Π_0 characterizes the "sensitivity of the band", and $\Pi_{av.}$ characterizes its "rigidity".

As a result of the second assumption, we shall have in place of formula (16):

$$p = f \frac{\omega}{s} \frac{\psi}{l_\Delta + l_c},$$

from which, after differentiation, we shall obtain:

$$\frac{dp}{dt} = \frac{f \frac{\omega}{s}}{l_\Delta + l_c} \cdot \frac{d\psi}{dt}$$

or, in accordance with formula (17):

$$\frac{dp}{dt} = \frac{f \frac{\omega}{s}}{l_\Delta + l_c} \cdot \frac{\varkappa_H}{I_K} p.$$

We separate the variables and designate:

$$\tau_{av.} = \frac{I_K}{\varkappa_H} (l_\Delta + l_c) : \left(f \frac{\omega}{s}\right). \qquad (20)$$

We find:

$$\frac{dp}{p} = \frac{dt}{\tau_{av.}},$$

from which:

$$p = p_H e^{\frac{t}{\tau_{av.}}} \qquad (21)$$

Through the intermediacy of formulas (19) and (21), the second equation of the system (15) assumes the following form:

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$$\frac{d^2 l}{dt^2} = \frac{s}{\varphi m}\left(p_H e^{\frac{t}{\tau_{av.}}} - p_H - \Pi_{av.}\,\frac{l}{l_{Kav.}}\right), \qquad (22)$$

since:

$$\frac{dv}{dt} = \frac{d^2 l}{dt^2}$$

if the phenomenon of recoil is not taken into account in its explicit form.

By integrating equation (22) in accordance with the known rules of mathematical analysis, and designating for convenience:

$$\tau^2 = \frac{\varphi m}{s}\cdot\frac{l_{Kav.}}{\Pi_{av.}}\;; \quad k^2 = \frac{\tau^2_{av.}}{\tau^2}\;; \quad L = \frac{s p_H}{\varphi m}\tau^2_{av.}, \qquad (23)$$

we shall obtain:

$$\left.\begin{aligned} \frac{l}{L} &= \frac{e^{\frac{t}{\tau_{av.}}}}{1+k^2} - \frac{1}{k^2} + \frac{1}{k^2\sqrt{1+k^2}}\cos\left(\tan^{-1}k + k\frac{t}{\tau_{av.}}\right); \\ \frac{\tau_{av.} v}{L} &= \frac{e^{\frac{t}{\tau_{av.}}}}{1+k^2} - \frac{1}{k\sqrt{1+k^2}}\cdot\sin\left(\tan^{-1}k + k\frac{t}{\tau_{av.}}\right), \end{aligned}\right\} \qquad (24)$$

the time t being counted from the start of the given phase.

On the basis of formulas (21) and (24), there have been set up brief tables containing values for the following quantities:

$$\frac{p}{p_H} = e^{\frac{t}{\tau_{av.}}} \text{ (Table 1) and } \frac{\tau_{av.} v}{L} \text{ (Table 2)}$$

as functions of the two parameters $\frac{l}{L}$ and k^2.

These tables are necessary for the solution of the direct problem of interior ballistics with consideration of the gradual cutting of the rotating band of the projectile into the rifling grooves of the barrel.

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All formulas for performing computations in connection with the acceleration phase are summarized in Table 3 (p. 926).

The quantities:

$$\varkappa, \quad \frac{1}{\delta_2}, \quad \frac{1}{\delta_1}, \quad \psi_H, \quad k_H, \text{ and } z_H$$

have already been found in the computations for the initial phase. The quantities p_H and $\Pi_{av.}$ must be known in advance. The argument in this phase is the path l of the projectile. The formula for the pressure in Table 3 has been obtained from formula (16), in which there are taken:

$$A_{\Pi} = 0; \quad \frac{B\theta}{2} x^2 = \frac{v^2}{V^2_{\Pi p}} = \frac{\theta\varphi m}{2f\omega} v^2.$$

The formula for t in Table 3 is found from expression (21).

The theory of solution of the problem in the first period is analogous to the theory of its solution for a simple charge, except that it is necessary to take into consideration that, at the start of the first period, the projectile has the velocity $v_{Kav.}$ and has already traversed the path $l_{Kav.}$. For this reason, in the first place, integration of the equation for velocity will give:

$$v - v_{Kav.} = \frac{s}{\varphi m}(I - I_{Kav.}),$$

where:

$$I_{Kav.} = \int_0^{t_H + t_{Kav.}} pdt = I_K z_{Kav.}; \quad I = I_K \cdot z.$$

The quantity:

$$v_{K,0} = \frac{sI_K}{\varphi m}$$

may be retained, but for z_0 it is necessary to take:

$$z_0 = z_{Kav.} - \frac{v_{Kav.}}{v_{K,0}} \qquad (25)$$

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Table 2 - Values of $\frac{\tau_{av.} v}{L}$.

$\frac{l}{L}$ \ k^2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0	0	0	0	0	0	0	0	0	0	0
0.01	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.02	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.04	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.21
0.06	0.29	0.29	0.29	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
0.08	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
0.10	0.41	0.41	0.41	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.39
0.15	0.56	0.56	0.56	0.55	0.55	0.55	0.55	0.54	0.53		
0.20	0.68	0.68	0.67	0.67	0.67	0.66	0.66	0.65			
0.25	0.79	0.79	0.79	0.79	0.78	0.77	0.77				
0.30	0.90	0.90	0.90	0.90	0.89	0.88	0.88				
0.35	1.01	1.01	1.01	1.01	1.00	0.99					
0.40	1.12	1.12	1.12	1.11	1.10	1.08					
0.45	1.22	1.22	1.22	1.21	1.20	0.18					
0.50	1.32	1.32	1.31	1.30	1.29	1.28					
0.6	1.52	1.51	1.50	1.49	1.47						
0.7	1.70	1.69	1.68	1.67	1.65						
0.8	1.87	1.88	1.85	1.84	1.82						
0.9	2.05	2.04	2.02	2.01	2.00						
1.0	2.23	2.21	2.19	2.18	2.17						
1.1	2.40	2.38	2.36	2.34							
1.2	2.57	2.55	2.53	2.50							
1.3	2.73	2.71	2.69	2.67							
1.4	2.89	2.87	2.85	2.83							
1.5	3.05	3.03	3.00	2.98							
1.6	3.21	3.18	3.15	3.12							
1.7	3.37	3.33	3.30	3.27							
1.8	3.52	3.48	3.45	3.42							
1.9	3.66	3.63	3.59	3.56							
2.0	3.81	3.77	3.73	3.70							
2.2	4.11	4.06	4.02								
2.4	4.40	4.35	4.30								
2.6	4.69	4.64	4.58								
2.8	4.98	4.92	4.86								
3.0	5.26	5.20	5.14								
3.2	5.53	5.47	5.41								
3.4	5.81	5.74	5.67								
3.6	6.09	6.01	5.94								
3.8	6.36	6.28	6.21								
4.0	6.63	6.55	6.47								
4.2	6.90	6.81	6.73								
4.4	7.16	7.07	6.99								
4.6	7.42	7.33	7.24								
4.8	7.69	7.59	7.49								
5.0	7.96	7.85	7.74								
5.2	8.22	8.11									
5.4	8.48	8.36									
5.6	8.74										

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Table 3 - Formulas for Computing Acceleration Phase.

$$\varphi = K + \frac{1}{3}\frac{\omega}{q}; \quad \frac{\varphi m}{s}; \quad I_K = \frac{e_1}{u_1};$$

$$v_{K,0} = I_K : \frac{\varphi m}{s}; \quad \frac{\omega}{s}; \quad f\frac{\omega}{s};$$

$$l_\Delta = \frac{\omega}{s} \cdot \frac{1}{\delta_2}; \quad l_\alpha = \frac{\omega}{s} \cdot \frac{1}{\delta_1};$$

$$\tau^2 = \frac{\varphi m}{s} \cdot \frac{I_{Kav.}}{\Pi_{av.}}; \quad \tau_1 = \frac{I_K l_\Delta}{f\frac{\omega}{s}k_H}; \quad \frac{\tau_1}{2l_\Delta}; \quad \frac{sp_H}{\varphi m}.$$

$$\tau_{av.} = \tau_1 + \frac{\tau_1}{2l_\Delta} l; \quad k^2 = \frac{\tau_{av.}^2}{\tau^2}; \quad L = \frac{sp_H}{\varphi m} \cdot \tau_{av.}^2;$$

$$\psi = \frac{l_\Delta + l + \frac{\theta}{2} \cdot \frac{\varphi m}{s} \cdot \frac{v^2}{p}}{f\frac{\omega}{s} \cdot \frac{1}{p} + l_\Delta};$$

$$z = \frac{2\psi}{\varkappa + k_{av.}}; \quad t = 2.303\tau_{av.} \log \frac{p}{p_H}.$$

Table 4 - Formulas for Computing First Period.

$$x_{Kav.} = \frac{v_{Kav.}}{v_{K,0}}; \quad z_0 = z_{Kav.} - x_{Kav.};$$

$$\psi_0 = \varkappa z_0 + \varkappa\lambda z_0^2; \quad k_1 = \sqrt{\varkappa^2 + 4\varkappa\lambda\psi_0}.$$

$$B = I_K^2 : \left(f\frac{\omega}{s} \cdot \frac{\varphi m}{s}\right); \quad B_1 = \frac{B\theta}{2} - \varkappa\lambda;$$

$$\frac{B}{B_1}; \quad C = \frac{B_1}{k_1}; \quad \gamma = \frac{C\psi_0}{k_1};$$

$$\beta_{Kav.} = Cx_{Kav.}; \quad \log z_{Kav.}^{-1}; \quad z_{Kav.}^{\frac{B}{B_1}}$$

$$\beta = Cx; \quad v = v_{K,0}x; \quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

$$\psi_{av.} = \frac{\psi + \psi_{Kav.}}{2}; \quad l_c = l_\Delta - l_\alpha\psi_{av.}; \quad l = (l_c + l_{Kav.}) z_{Kav.}^{\frac{B}{B_1}} z^{-\frac{B}{B_1}} - l_c.$$

$$l_\psi = l_\Delta - l_\alpha\psi; \quad p = f\frac{\omega}{s}\frac{\psi - \frac{B\theta}{2}x^2}{l_\psi + l}.$$

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<u>Example 1</u>. To find the ballistic elements of a shot at the end of the preliminary period with consideration of the gradual cutting of the rotating band of the projectile into the rifling grooves of the barrel for the 1942 model 76 mm light division gun (ZIS-3) under the following loading conditions:

$$f = 896{,}000;\quad \alpha = 1;\quad \delta = 1.6;\quad \theta = 0.2;$$

$$\omega = 1.08;\quad 2e_1 = 1.4 \text{ mm};\quad \varkappa = 1.06;$$

$$W_0 = 1.49;\quad s = 0.4692;\quad l_D = 26.88;\quad l_{Kav.} = 0.33;$$

$$q = 6.2;\quad \varphi = 1;\quad p_H = 15{,}000;\quad \Pi_{av.} = 10{,}000.$$

I. Initial Phase of Preliminary Period.

1) $\varkappa = 1.06$ (given); 2) $\dfrac{1}{\delta_2} = \dfrac{W_0}{\omega} - \dfrac{1}{\delta} = 0.755$; 3) $\dfrac{1}{\delta_1} = \alpha - \dfrac{1}{\delta} = 0.375$;

4) $\psi_H = \dfrac{1}{\delta_2} : \left(\dfrac{f}{p_0} + \dfrac{1}{\delta_1}\right)$; $\log\psi_H = \bar{2}.0987$; 5) $\varkappa_H = \sqrt{\varkappa^2 + 4\varkappa\lambda\psi_H}$; $\log k_H = 0.0248$.

II. Preliminary Computations for Acceleration Phase.

1) $\varphi = K + \dfrac{1}{3}\dfrac{\omega}{q} = 1$ (given); 2) $\log\dfrac{\varphi m}{s} = \bar{1}.1293$; 3) $I_K = 938.5$; $2e_1 = 1.4$;

4) $\log v_{K,0} = \log\left(I_K : \dfrac{\varphi m}{s}\right) = 3.8431$; 5) $\log\dfrac{\omega}{s} = 0.3620$; 6) $\log f\dfrac{\omega}{s} = 6.3143$;

7) $\log l_\Delta = \log\dfrac{\omega}{s}\cdot\dfrac{1}{\delta_2} = 0.2399$; 8) $\log l_\alpha = \log\dfrac{\omega}{s}\cdot\dfrac{1}{\delta_1} = \bar{1}.9360$;

9) $\log\tau^2 = \log\left(\dfrac{\varphi m}{s}\cdot\dfrac{l_{Kav.}}{\Pi_{av.}}\right) = 6.6478$; 10) $\log\tau_1 = \log\dfrac{I_K l_\Delta}{f\dfrac{\omega}{s}k_H} = 4.8732$;

11) $\log\dfrac{\tau_1}{2l_\Delta} = \bar{4}.3323$; 12) $\log\left(p_H : \dfrac{\varphi m}{s}\right) = 5.0468$.

III. Ballistic Elements at End of Preliminary Period.

Knowing from the assignment that $l = l_{Kav.} = 0.33$ dm, we obtain from the formulas in the middle section of Table 3:

1) $\tau_{av.} = \tau_1 + \frac{\tau_1}{2l_\Delta} l_{Kav.} = 0.709 \cdot 10^{-4}$; 2) $k^2 = \frac{\tau_{av.}^2}{\tau^2} = 0.150$;

3) $L = \tau_{av.}^2 \left(p_H : \frac{\varphi m}{s}\right)$; $\log L = \bar{2}.8720$; 4) $\frac{l}{L} \frac{l_{Kav.}}{L} = 4.43.$

Knowing that $\frac{l}{L} = 4.43$ and $k^2 = 0.150$, we use Tables 1 and 2 to obtain by double interpolation:

5) $\frac{p_{Kav.}}{p_H} = 10.78$ and 6) $\frac{\tau_{av.} v_{Kav.}}{L} = 7.07$, which makes it possible to find for the end of the preliminary period:

7) $p_{Kav.} = 10.82 p_H = 1617$ kg/cm^2; 8) $v_{Kav.} = \frac{7.13L}{\tau_{av.}} = 64.3$ m/sec.

We continue the computations in accordance with the formulas in the lower section of Table 3:

$$8)\ \psi_{Kav.} = \frac{l_\Delta + l_{Kav.} + \frac{\theta}{2}\frac{\varphi m}{s} \cdot \frac{v_{Kav.}^2}{p_{Kav.}}}{f\frac{\omega}{s} \cdot \frac{1}{p_{Kav.}} + l_a};\quad \log \psi_{Kav.} = \bar{1}.1887;$$

9) $k_{av.} = \sqrt{\varkappa^2 + 4\varkappa\lambda\psi_{Kav.}} = 0.0181$; 10) $z_{Kav.} = \frac{2\psi_{Kav.}}{\varkappa + k_{av.}} = 0.1469$;

$$11)\ t_{Kav.} = 2.303\tau_{av.} \log \frac{p_{Kav.}}{p_H} = 0.001944 \text{ sec}.$$

Without consideration of the gradual cutting-in process, there were at the end of the period:

$$l = 0;\quad v_0 = 0;\quad p_0 = 300 \text{ kg/cm}^2$$

With consideration of the cutting-in process, these changed to:

$$l_{Kav.} = 0.33;\quad v_{Kav.} = 64.3 \text{ m/sec};\quad p_{Kav.} = 1617 \text{ kg/cm}^2$$

__Example 2__. To find the principal ballistic elements of the shot under the conditions of Example 1 and from its results.

Solution. We perform the computations with the aid of the formulas in Table 4.

IV. Preliminary Computations for First Period.

1) $x_{Kav.} = \frac{v_{Kav.}}{v_{K,0}} = 0.0924$; 2) $z_0 = z_{Kav.} - x_{Kav.} = 0.0545$;

3) $\psi_0 = \varkappa z_0 + \varkappa\lambda z_0^2 = 0.0756$; 4) $k_1 = \sqrt{\varkappa^2 + 4\varkappa\lambda\psi_0} = 1.0226$.

V. Tabular Constants for First Period.

1) $B = I_K^2 : \left(f\frac{\omega}{s}\cdot\frac{\varphi m}{s}\right) = 3.171$; 2) $B_1 = \frac{B\theta}{2} - \varkappa\lambda = 0.3771$;

3) $\log\frac{B}{B_1} = 0.9248$; 4) $\log C = \log\frac{B}{k_1} = \bar{1}.5538$; 5) $\gamma = \frac{C\psi_0}{k_1} = 0.0196$;

6) $\beta_{Kav.} = Cx_{Kav.} = 0.0331$; 7) $\log Z^{-1}_{Kav.} = 0.0060$ (form table $\log Z^{-1}$, knowing γ and $\beta_{Kav.}$); 8) $\log Z_{Kav.}^{\frac{B}{B_1}} = \bar{1}.9495$.

For the stages VI and VII, we employ the formulas in the lower section of Table 4 ($\log x_{m,1} = \bar{1}.4676$; $\log x_K = \bar{1}.9756$):

$v_K = 658.8$ m/sec; $\psi_K = 1$; $l_K = 22.53$ dm; $p_K = 606$ kg/cm^2; $v_{m,1} = 204.5$; $\psi_{m,1} = 0.3615$; $l_{m,1} = 1.502$; $p_{m,1} = 2345$.

For the stage VIII, we employ the formulas of Section 4, Chapter 1, Part Two:

$$\log \eta_D = 0.0555;$$

$$p_D = 519 \text{ kg/cm}^2; \quad v_D = 679.5 \text{ m/sec}.$$

It is already known that, by experiment:

$$p_m = 2320 \text{ kg/cm}^2 \text{ and } v_D = 680 \text{ m/sec}.$$

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CHAPTER 3 - SOLUTION OF PROBLEM OF INTERNAL BALLISTICS FOR MORTARS

1. General Information.

In comparison with a shot from an ordinary artillery gun, a shot from a mortar has a number of specific features. For this reason, the burning of the powder and the other processes connected with the work of the powder gases during a shot from a mortar proceed under conditions which are more complex and less known in some respects, but simpler in other respects.

The basic charge of a mortar (fig. 171) is contained in a cardboard cartridge (shell case) inserted in the stabilizer tube 1 (tail of the mortar shell). The tube has four or six rows of circular openings 2, through which the powder gases formed within the shell case must flow out into the space behind the mortar shell once the cardboard has been pierced.

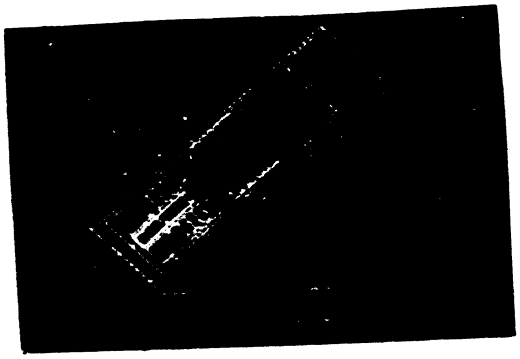

Fig. 171 - Sketch of Arrangement of Mortar.

During loading, the mortar shell is lowered to the bottom of

the bore, displacing the air through the clearance 3. The percussion cap of the cartridge with the basic charge strikes the firing pin 4, which is fixed in the bottom of the bore of the mortar, and ignition of the percussion cap and powder charge takes place; in this connection, the powder burns first in the closed space of the cartridge at a rather high loading density $\Delta_0 = 0.50-0.60$. In a certain instant, the gas pressure pierces the walls of the cardboard cartridge, and the gases of the basic charge flow through the openings 2 in the stabilizer tube into the chamber space W_0.

Under these conditions of very rapid burning of a very thin fine powder, the maximum gas pressure inside the stabilizer tube is, as has been shown by experiments, very sensitive both with respect to the size of the tube openings and the thickness of the cartridge walls, and with respect to the most insignificant delays or advances in the piercing of the cartridge walls.

As a result of a small difference in the pressures under which the cartridge walls are pierced, there may ensue a large scattering of the magnitudes of maximum pressure in the stabilizer tube.

For this reason, it is precisely in a mortar, in greater measure than in a gun, that importance attaches to the composition, the weight of the igniting percussion cap, and the rate of burning of the powder; the greater the impulse provided by the igniter the more uniformly is the powder ignited.

The next characteristic feature consists in the fact that the gases of the basic charge, which first burns inside the stabilizer chamber at $\Delta = 0.50-0.60$, undergo strong expansion and cooling as they flow out into the space behind the mortar shell. Since the surface of the stabilizer vanes 2 and of the bottom part of the mortar

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shell is large, while the loading density of the basic charge with respect to the total chamber volume W_0 is small (Δ = about 0.01), there occurs a large loss of heat to the walls of the bore and of the mortar shell, this loss being still more accentuated by the slow motion of the mortar shell and by the long interval of time during which the gases remain in contact with the walls of the mortar.

If additional charges ω are present, the powder contained therein is ignited by the action of the gases of the basic charge, and the motion of the mortar shell proceeds under the action of the total gas pressure produced by the basic and additional charges. Owing to the presence of the clearance 3 between the mortar shell and the walls of the bore, a portion of the gases will penetrate through this clearance from the very outset of the motion of the mortar shell, and consequently their energy will not be utilized.

In gas-regulator mortars, with the gas regulator open, a considerable part of the gases also escapes through the gas regulator. The existence of a penetration of gases through the clearance between the mortar shell and the bore and through the gas regulator constitutes the third characteristic feature of the shot from a mortar. The consumption of gases through the clearance and the gas regulator is accounted for on the basis of the general relations of gas dynamics.

As shown by slow-motion photographs, a considerable part of the gases is ejected from the bore of the mortar prior to the emergence of the mortar shell from the barrel, which is accompanied by the ejection of the principal mass of the gases. This part of the gases which is ejected through the clearance and does not participate in

communicating a velocity to the mortar shell constitutes as much as 10-15% of the total quantity of gases, whereas, in an ordinary gun, the fraction of gases ejected through the clearances between the rotating band and the grooves of the rifling is insignificant.

The fourth characteristic feature consists in the fact that the pressure to overcome the inertia of the projectile may in practice be considered as being equal to zero. In exactly the same manner, no energy has to be expended in a smooth barrel to overcome friction and to rotate the mortar shell.

Thus, the solution of the problem of internal ballistics is, on the one hand, simplified by the fact that the pressure to overcome the inertia of the projectile and a portion of the secondary work are assumed to be zero; on the other hand, the solution of the problem is complicated by the necessity of taking into account a greater heat loss and the escape of gases through the clearance and gas regulator, which requires the utilization of the fundamental relations of gas dynamics.

Since, in a shot from an ordinary mortar resting on a base plate, there occurs practically no recoil, and the relative weight of the charge ω/q is very small (of the order of 0.01-0.02 with a full charge), it is possible to assume in practice that the coefficient $\varphi = 1$.

To retain unity in the procedure and in the designations of the parameters and functions, the solution of the fundamental problem of internal ballistics for mortars, which is developed below, is presented in the designations of Professor Drozdov for ordinary artillery guns.

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2. ANALYTICAL SOLUTION OF FUNDAMENTAL PROBLEM FOR SMOOTH-BORE MORTARS.

(Simplified Method of Professor M.E. Serebryakov)

The analytical solution is based on the following assumptions:

1) There is no pressure to overcome the inertia of the projectile. The mortar has an annular clearance between the mortar shell and the bore.

2) The burning of the basic charge in the stabilizer tube is not considered.

The gases of the basic charge flowing from the stabilizer tube into the space behind the mortar shell produce in that space the pressure p_0, under which the powder of the additional charges is ignited. Thus, the basic charge acts as an igniter for the additional charges.

3) The ignition of the additional charges is assumed to be instantaneous and simultaneous for all grains and for all points on the surface of every grain.

4) The burning of the grains of the additional charges proceeds in parallel layers in conformity with the geometric law of burning and is expressed by the following known formulas:

$$\psi = \varkappa z + \varkappa \lambda z^2;$$

$$\sigma = 1 + 2\lambda z.$$

5) The rate of burning of the powder is proportional to the pressure (in the first power):

$$u = \frac{de}{dt} = u_1 p,$$

where u_1 is the rate of burning at $p = 1$.

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$$sp(l'_{\psi} + l) = f_0\omega_0 + f(\omega\psi - Y) - \frac{\theta}{2}\varphi m v^2, \qquad (26)$$

where:

$$l'_{\psi} = \frac{1}{\delta}\left[W_0 - \frac{\omega}{\delta}(1 - \psi) - \alpha(\omega\psi - Y) - \alpha_0\omega_0\right]$$

takes into account the loss of gases through the clearance $s - s'$.

2) The equation of motion of the mortar shell:

$$s' p dl = \varphi m v dv. \qquad (27)$$

$$\psi = \varkappa z + \varkappa\lambda z^2 \qquad (28)$$

3) The (geometric) law of burning for lamellar fine powders and $\psi = z$ for flat disk-shaped powders.

4) The formula for the velocity of the mortar shell:

$$v = \frac{s' I_K}{\varphi m} z. \qquad (29)$$

5) The relative consumption of gases:

$$\eta = \frac{Y}{\omega} = \frac{\zeta' A s_3 I_K}{\omega} z = \eta_K z, \qquad (30)$$

where:

$$\eta_K = \frac{\zeta' A s_3 I_K}{\omega} = \frac{\zeta' A s_3}{\omega}\frac{e_1}{u_1}. \qquad (31)$$

$\zeta' < 1$ is a coefficient which takes into account the shape of the opening through which the gases flow out; and η_K is the relative consumption of gases at the end of burning of the powder.

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The following designations are introduced:

$$B' = \frac{s'^2 I_K^2}{f\omega\varphi m} = \left(\frac{s'}{s}\right)^2 \frac{s^2 I_K^2}{f\omega\varphi m} = \left(\frac{s'}{s}\right)^2 B;$$

$\chi_0 = \frac{f_0\omega_0}{f\omega}$ is the relative energy of the basic charge.

By replacing in equation (26) the quantities ψ, v, and Y (or η) by their expressions (28), (29) and (30) in terms of z, we obtain the fundamental equation of pyrodynamics in the following form:

$$sp(l'_\psi + l) = f\omega\left[\chi_0 + \varkappa z + \varkappa\lambda z^2 - \eta_K z - \frac{B'\theta}{2} z^2\right] = f\omega\left[\chi_0 + (\varkappa - \eta_K) z - \left(\frac{B'\theta}{2} - \varkappa\lambda\right) z^2\right]. \quad (32)$$

From equation (27), we have:

$$s'p\, dl = \frac{s'^2 I_K^2}{\varphi m} z\, dz. \quad (33)$$

Upon dividing (33) by (32) term by term, we obtain:

$$\frac{s'}{s} \frac{dl}{l'_\psi + l} = B' \frac{zdz}{\chi_0 + k'_1 z - B'_1 z^2} = -\frac{B'}{B'_1} \frac{zdz}{z^2 - \frac{k'_1}{B'_1} z - \frac{\chi_0}{B'_1}} = -\frac{B'}{B'_1}\, d \ln Z,$$

where:

$$k'_1 = \varkappa - \eta_K; \quad B'_1 = \frac{B'\theta}{2} - \varkappa\lambda; \quad A_s = \frac{s}{s'} \frac{B'}{B'_1}.$$

Z is a known function developed by Professor N.F. Drozdov. We

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If $z_m \leqslant 1$, we have a real pressure maximum; if $z_m > 1$, the maximum is unreal, and in this case the highest actual pressure will be the pressure at the end of burning p_K:

$$p_K = \frac{f\omega}{s} \frac{1 + \chi_0 - \eta_K - \frac{B'\theta}{2}}{l_1' + l_K},$$

where:

$$l_1' = l_0 \left[1 - \alpha\Delta (1 - \eta_K) \right] \quad \text{and} \quad \Delta_K = \frac{\omega}{W_0} (1 - \eta_K) = \Delta (1 - \eta_K).$$

The remaining elements at the end of the first period will be:

$$v_K = \frac{s' I_K}{\varphi m}; \quad l_K = l' \psi_{av.} (Z_K^{-As} - 1),$$

in which connection:

$$\beta_K = \frac{B_1'}{k_1'}.$$

For solving the problem <u>in the second period</u>, we have the following system of equations:

$$sp(l_1' + l) = f_0\omega_0 + f\omega(1 - \eta_K z') - \frac{\theta}{2}\varphi m v^2, \qquad (37)$$

$$s'p\,dl = \varphi m v\,dv, \qquad (33)$$

where:

$$z' = \frac{I}{I_K} = \frac{\int_0^t p\,dt}{\int_0^{t_K} p\,dt} = \frac{v}{v_K},$$

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z' being here already greater than unity.

The quantity $\eta_K z'$ takes into account the escape of gases through the clearance, which continues in the second period as well. As in the first period, the complete escape is proportional to the pressure impulse of the gases, which, in turn, is proportional to the velocity of the projectile.

Equation (37) can be rewritten in the following manner:

$$sp(l_1' + l) = f\omega\left[\chi_0 + 1 - \frac{\eta_K}{v_K} v - \frac{v^2}{v^2_{\Pi p}}\right], \qquad (38)$$

where:

$$\eta_K = \frac{\zeta' A s_3 I_K}{\omega}; \quad v_K = \frac{s' I_K}{\varphi m};$$

$$v^2_{\Pi p} = \frac{2f\omega}{\varphi\theta m};$$

$$\frac{\eta_K}{v_K} = \frac{\zeta' A s_3 I_K}{\omega} \frac{\varphi m}{s' I_K} = \zeta' A \frac{\varphi}{g} \frac{q}{\omega} \frac{s_s}{s'} = \eta_K' .$$

We divide (33) by (38):

$$\frac{dl}{l_1' + l} = \frac{s}{s'} \frac{\varphi m}{f\omega} \frac{vdv}{1 + \chi_0 - \eta_K' v - \frac{v^2}{v^2_{\Pi p}}} = -\frac{s}{s'} \frac{2}{\theta} \frac{vdv}{v^2 + \eta_K' v^2_{\Pi p} v - (1 + \chi_0) v^2_{\Pi p}}$$

or:

$$\frac{dl}{l_1' + l} = -\frac{s}{s'} \frac{2}{\theta} \frac{vdv}{v^2 + \eta_2 v - \eta_3}, \qquad (39)$$

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where:

$$\eta_2 = \eta'_K v^2_{\text{пр}} - 2\zeta'_A \frac{s}{\theta}\frac{s_3}{s'} = \frac{\eta_K}{\dfrac{v_K}{v^2_{\text{пр}}}} = v_{\text{пр}} \frac{\eta_K}{\dfrac{v_K}{v_{\text{пр}}}} = \frac{\eta_K}{v_K} v^2_{\text{пр}};$$

$$\eta_3 = (1 + \chi'_0)\, v^2_{\text{пр}} = (1 + \chi_0) \frac{2f\omega}{\varphi\theta m} = v'^2_{\text{пр}} .$$

Integration of equation (39) gives:

$$\int_{l_K}^{l} \frac{dl}{l'_1 + l} = -\frac{s}{s'}\frac{2}{\theta}\int_{v_K}^{v} \frac{v\,dv}{v^2 + \eta_2 v - \eta_3}; \qquad (40)$$

$$\int_{l_K}^{l} \frac{dl}{l'_1 + l} = \ln \frac{l'_1 + l}{l'_1 + l_K}. \qquad (41)$$

We find the integral of the right-hand side by first resolving the function under the integral sign into the simplest fractions in accordance with the method proposed by Professor Drozdov.

We find the roots of the equation $v^2 + \eta_2 v - \eta_3 = \xi'(v) = 0$:

$$v = -\frac{\eta_3}{2}\left(1 \pm \sqrt{1 + 4\frac{\eta}{\eta_2^2}}\right) = -\frac{\eta_3}{2}(1 \pm b),$$

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where:

$$\frac{\eta_2}{2} = \frac{f}{\theta}\frac{s_3}{s'}\zeta' A; \quad b = \sqrt{1 + 4\frac{\eta_3}{\eta_2^2}} = \sqrt{1 + 4\gamma};$$

$$\gamma = \frac{\eta_3}{\eta_2^2} = \frac{(1 + \chi_0) v_{\Pi p}^2}{\eta'^2 (u_{\Pi p}^2)^2} = \frac{1 + \chi_0}{v_{\Pi p}^2 \cdot \eta_K'^2}, \text{ where } \eta_K' = \frac{\eta_K}{v_K};$$

$$\gamma = \frac{(1 + \chi_0)}{\eta_K}\left(\frac{v_K^2}{v_{\Pi p}^2}\right) = \frac{(1 + \chi_0)}{\eta_K} \cdot \frac{B'\theta}{2};$$

$$v_1 = -\frac{\eta_2}{2}(1 + b); \quad v_2 = -\frac{\eta_2}{2}(1 - b) = \frac{\eta_2}{2}(b - 1); \quad \eta_K = \frac{\zeta' A s_{clearance} I_K}{\omega};$$

$$v_2 - v_1 = \eta_2 b; \quad \frac{v}{\xi'(v)} = \frac{A_1}{v - v_1} + \frac{A_2}{v - v_2};$$

$$A_1 = \frac{b + 1}{2b}; \quad A_2 = \frac{b - 1}{2b};$$

$$\int_{v_K}^{v} \frac{v\,dv}{\xi'(v)} = \frac{b + 1}{2b}\int_{v_K}^{v} \frac{dv}{v - v_1} + \frac{b - 1}{2b}\int_{v_K}^{v} \frac{dv}{v - v_2} = \ln\left(\frac{v - v_1}{v_K - v_1}\right)^{\frac{b+1}{2b}} \cdot$$

$$\cdot \left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{2b}} = \ln\frac{Z'_v}{Z'_{v_K}}. \quad (42)$$

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Upon substituting the expressions (41) and (42) into (40), we obtain:

$$\left(\frac{l_1' - l}{l_1' - l_K}\right)^{-\frac{s'}{s}\frac{\theta}{2}} = \left(\frac{l_1' + l_K}{l_1' + l}\right)^{\frac{s'}{s}\frac{\theta}{2}} = \left(\frac{v - v_1}{v_K - v_1}\right)^{\frac{b+1}{2b}} \left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{2b}} =$$

$$= \frac{Z'_v}{Z'_{v_K}}$$

or, finally, we have:

$$\left(\frac{l_1' + l_K}{l_1' + l}\right)^{\frac{s'}{s}\theta} = \left(\frac{v - v_1}{v_K - v_1}\right)^{\frac{b+1}{b}} \left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{b}};$$

$$l = l_1' \left[\left(1 + \frac{l_K}{l_1'}\right)\left(\frac{Z'_v}{Z'_{v_K}}\right)^{-\frac{s}{s'}\frac{2}{\theta}} - 1\right].$$

In accordance with this equation, taking definite values for $v > v_K$, we find first the values for the left-hand side, and then the corresponding values for the path of the projectile l; upon constructing a diagram, we find by interpolation or graphically the value v_D corresponding to the value l_D, whereupon, for control purposes, we check the computation once more at $v = v_D$.

The pressure is determined with the aid of the following formula:

$$p = \frac{f\omega}{s} \frac{\left(1 + \chi_0 - \frac{\eta_K}{v_K} v - \frac{v^2}{v^2 \eta_D}\right)}{l_1' + l}.$$

Results of Computations

Basic data for 82-mm mortar; dimensions in the kg-dm-sec system:

$W_0 = 0.720$; $l_D = 10.20$; $s = 0.5277$; $s_{clearance} = 0.0082$; $f = 1120 \cdot 10^3$;
$\omega = 0.0366$; $\alpha = 0.85$;
$q = 3.4$; $l_0 = 1.363$; $\omega_0 = 0.0072$
$I_K = 55$; $A\xi_1' = 0.004$.
$f_0 = 679 \cdot 10^3$; $\Lambda_D = 7.48$; $\varkappa\lambda = -0.255$;
$\xi_1' = 0.666$;
$\theta = 0.15$; $c_q = 6.16$; $A = 0.006$;
$\varkappa = 1.255$; $\Delta = 0.0608$;
$\delta = 1.64$;

ξ_1' is a coefficient which takes into account the shape of the escape opening; its value 0.666 has been taken from the preliminary investigation of Greten; $\varphi = 1$.

Computation of Constants

$\chi_0 = 0.1192$; $\eta_K = 0.04923$; $B' = 0.5923$;
$B_1 = 0.2994$; $\frac{B'}{B_1'} = 1.979$; $\gamma = 0.02452$;
$\beta_K = 0.2483$.

The elements of the shot are $l_m = l_K = 0.700$ dm; $p_K = p_m = 398$ kg/cm²; $p_D = 48$ kg/cm²; $v_D = 205.5$ m/sec.

At the same constants and $\theta = 0.20$: $p_K = p_m = 392$, $v_D = 201.0$.

In the absence of an escape of gases through the clearance: $\theta = 0.20$, $l_m = l_K = 0.678$ dm, $p_K = p_m = 435$ kg/cm², $p_D = 50$ kg/cm², $v_D = 211.5$ m/sec.

The results of the computations are found to be close to the experimental data ($p_m = 380\text{-}390$ and $v_D = 202\text{-}205$).

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The best agreement between the results of the computations and the experimental data would be obtained at $\theta = 0.18$.

The results of computations presented above show that the above-derived analytical formulas make it possible to compute with good accuracy the ballistic elements of a shot from a mortar (p_K, p_m, l_K, l_m, v_K, v_m, p_D, v_D) and to construct curves for the pressures of the powder gases and for the velocity of the projectile as functions of its path.

In case only the basic charge ω_0 is present and the additional charges ω are absent, the problem is solved as for the case of the instantaneous burning of the charge, the heat loss being accounted for by considering the reduction in the propellant force of the powder f, as determined in a special apparatus.

3. EXAMPLE OF SOLUTION OF FUNDAMENTAL PROBLEM OF INTERNAL BALLISTICS FOR SMOOTH-BORE MORTARS (SIMPLIFIED METHOD).

Basic Data for 82-mm Mortar

Chamber volume W_0, dm^3	0.720
Cross-sectional area of mortar bore s, dm^2	0.5277
Cross-sectional area of clearance $s_{clearance}$, dm^2	0.0082
Coefficient characterizing shape and arrangement of clearance, ξ'	0.666
Escape coefficient A	0.006
Length of path of mortar shell through bore l_D, dm	10.20
Weight of mortar shell q, kg	3.4
Weight of charge ω, kg	0.0366
Weight of basic charge ω_0, kg	0.0072
Coefficient of consideration of secondary work φ	1.0
Propellant force of powder f, $\frac{kg \cdot dm}{kg}$	$1120 \cdot 10^3$

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Propellant force of powder of basic charge f_0, $\frac{kg.dm}{kg}$	679,000
Covolume α, dm^3/kg	0.85
Density of powder δ, kg/dm^3	1.64
Impulse of pressure increase at end of burning I_K	55
Form characteristics of powder: $\varkappa = 1.255$, $\varkappa\lambda = -0.255$	
Polytrope index k	1.15
$\theta = k - 1$	0.15

Basic Formulas for Computation

A. First Period.

$$z = \frac{I}{I_K} = \frac{e}{e_1}$$

$$\chi_0 = \frac{f_0\omega_0}{f\omega}$$ relative energy of basic charge;

$$s' = s - s_{clearance}$$ cross-sectional area of mortar shell at bourrelet;

$$v = \frac{s'I_K}{\varphi m} z$$ velocity of mortar shell;

$$\psi = \varkappa z + \varkappa\lambda z^2;$$

$$l = l_{\psi av.}'(Z^{-A_s} - 1)$$ expression for path of mortar shell;

$$p = \frac{f\omega}{s}\,\frac{\chi_0 + k_1' z - B_1' z^2}{l_\psi' + l}$$ expression for gas pressure;

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$\log Z^{-1}$ is determined from the table of Professor N.F. Drozdov on the basis of the basic quantities:

$$\gamma = \frac{B_1'\chi_0}{k_1'^2}; \quad \beta = \frac{B_1'}{k_1'} z,$$

where:

$$B' = \left(\frac{s'}{s}\right)^2 \cdot \frac{s^2 I_K^2}{f\omega\varphi m} = \left(\frac{s'}{s}\right)^2 \cdot B;$$

$$B_1' = \frac{B'\theta}{2} - \varkappa\lambda;$$

$$\Lambda_s = \frac{s}{s'} \cdot \frac{B'}{B_1'};$$

$$k_1' = \varkappa - \eta_K,$$

where $\eta_K = \frac{Y_K}{\omega}$, and $Y_K = \zeta' A s_{clearance}$. I_K is the quantity of gases escaping through the clearance;

$$l'_{\psi av.} = l'_{\psi};$$

$$l'_{\psi} = \frac{1}{s}\left[W_0 - \frac{\omega}{\delta}(1 - \psi) - \alpha(\omega\psi - Y) - \alpha_0\omega_0\right];$$

$$l_0 = \frac{W_0}{s}; \quad \Lambda_D = \frac{l_D}{l_0};$$

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$$z_m = \frac{\varkappa - \eta_K\left(\dfrac{1 + \dfrac{\alpha p_m}{f}}{1 + \dfrac{p_m}{f\delta_1}}\right)}{\dfrac{B'\left(\dfrac{s}{s'} + \theta\right)}{1 + \dfrac{p_m}{f\delta_1}} - 2\varkappa\lambda},$$

where:

$$\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}.$$

If $z_m < 1$, we have a real pressure maximum; if $z_m > 1$, the maximum is unreal, and in this case the highest actual pressure will be the pressure at the end of burning p_K.

B. Second Period.

$$v_{np}^2 = \frac{2f\omega}{\varphi\theta m};$$

$$\eta_K' = \frac{\eta_K}{v_K}, \text{ where } v_K = \frac{s' l_K}{\varphi m};$$

$$\eta_2 = \eta_K' v_{np}^2;$$

$$\eta_3 = (1 + \chi_0)\, v_{np}^2; \quad \gamma = \frac{\eta_3}{\eta_2^2}; \quad b = \sqrt{1 + 4\gamma};$$

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$$v_1 = -\frac{\eta_2}{2}(1 + b); \quad v_2 = \frac{\eta_2}{2}(b - 1);$$

$$\left(\frac{l_1' + l_K}{l_1' + l}\right)^{\frac{s'}{s}\theta} = \left(\frac{v - v_1}{v_K - v_1}\right)^{\frac{b+1}{b}} \left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{b}} . \qquad \text{(a)}$$

On the basis of this equation, taking definite values for $v > v_K$, we find first the values for the left-hand side, and then the corresponding values for the path of the projectile l. Upon constructing a diagram, we find by interpolation or graphically the value of v_D corresponding to the value of l_D, and, for control purposes, check the computations once more at $v = v_D$.

Or else, replacing in formula (a), v_1 and v_2 by their expressions, i.e., $v_1 = -\frac{\eta_2}{2}(1 + b)$ and $v_2 = \frac{\eta_2}{2}(b - 1)$, we find l with the aid of the following formula:

$$l = (l_1' + l_K)\left\{\left[\frac{v_K + \frac{\eta_2}{2}(b + 1)}{v + \frac{\eta_2}{2}(b + 1)}\right]^{\frac{b+1}{b}} \cdot \left[\frac{v_K - \frac{\eta_2}{2}(b - 1)}{v - \frac{\eta_2}{2}(b - 1)}\right]^{\frac{b-1}{b}}\right\}^{\frac{s}{s'}\cdot\frac{1}{\theta}} - l_1'.$$

Computation of constants (with a 50-cm slide rule):

$$\chi_0 = \frac{f_0\omega_0}{f\omega} = \frac{679{,}000 \cdot 0.0072}{1120 \cdot 10^3 \cdot 0.0366} = 0.1192;$$

$$s' = s - s_{clearance} = 0.5277 - 0.0082 = 0.5195;$$

$$\frac{s'I_K}{\varphi m} = \frac{0.5195 \cdot 55 \cdot 98.1}{3.4} = 824.5;$$

$$\frac{\omega f}{s} = \frac{0.0366 \cdot 1120 \cdot 10^3}{0.5277} = 77{,}680;$$

$$B' = \left(\frac{s'}{s}\right)^2 \cdot \frac{s^2 I_K^2}{f\omega\varphi m} = \left(\frac{0.5195}{0.5277}\right)^2 \frac{0.5277^2 \cdot 55^2 \cdot 98.1}{1120 \cdot 10^3 \cdot 0.0366 \cdot 3.4} = 0.5747;$$

$$B_1' = \frac{B'\theta}{2} - \varkappa\lambda = \frac{0.5747 \cdot 0.15}{2} - (-0.255) = 0.2952;$$

$$A_s = \frac{s}{s'} \cdot \frac{B'}{B_1'} = \frac{0.5277}{0.5195} \cdot \frac{0.5747}{0.2952} = 1.016 \cdot 1.947 = 1.978;$$

$$Y_K = \zeta' \cdot A \cdot s_{clearance} \cdot I_K = 0.666 \cdot 0.0060 \cdot 0.0082 \cdot 55 = 0.00003277 \cdot 55 =$$
$$= 0.001802;$$

$$\eta_K = \frac{Y_K}{\omega} = \frac{0.001802}{0.0366} = 0.04923;$$

$$k_1' = \varkappa - \eta_K = 1.255 - 0.04923 = 1.2058;$$

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$$\frac{B_1'}{k_1'} = \frac{0.2952}{1.2058} = 0.2448; \quad \frac{1}{\delta_1} = \alpha - \frac{1}{\delta} = 0.85 - \frac{1}{1\cdot 64} = 0.2402;$$

$$\gamma = \frac{B_1' \cdot \chi_0}{k_1'^2} = \frac{0.2952 \cdot 0.1192}{1.2058^2} = 0.02419; \quad \frac{\omega}{\delta} = \frac{0.0366}{1\cdot 64} = 0.02232;$$

$$l_0 = \frac{W_0}{s} = \frac{0.720}{0.5277} = 1.364; \quad \Lambda_D = \frac{l_D}{l_0} = 7.49; \quad \Delta = \frac{\omega}{W_0} = \frac{0.0366}{0.72} = 0.0508;$$

$$z_m = \frac{\varkappa - \gamma_K\left(\dfrac{1 + \dfrac{\alpha p_m}{f}}{1 + \dfrac{p_m}{f\delta_1}}\right)}{\dfrac{B'\left(\dfrac{s}{s'} + \theta\right)}{1 + \dfrac{p_m}{f\delta_1}} - 2\varkappa\lambda} = \frac{1.255 - 0.04923\left(\dfrac{1 + \dfrac{0.85 \cdot 40,000}{1,120,000}}{1 + \dfrac{40,000}{1,120,000} \cdot 0.2402}\right)}{\dfrac{0.5747\left(\dfrac{0.5277}{0.5195} + 0.15\right)}{1 + \dfrac{40,000}{1,120,000} \cdot 0.2402} + 0.510} = 1.0257$$

$z_m > 1$, the maximum is "unreal," and the maximum pressure will in this case be the pressure p_K at the end of burning of the powder.

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Form for Computation of Ballistic Elements (ψ, v, l, and p) for First Period (Computation with 50-cm Slide Rule).

Basic formulas	No.	Operations					End of burning
	1	z	0.3	0.5	0.7	0.9	1.0
$v = \frac{s'I_K}{\varphi m} z = 824.5z$	2	v, dm/sec	247.3	412.2	577.2	742.0	824.5
$\frac{B'_1}{k'_1} = 0.2448$	3	$\beta = \frac{B'_1}{k'_1} z$	0.07344	0.1224	0.1714	0.2203	0.2448
$\varkappa = 1.255$	4	(+) $\varkappa z$	0.3765	0.6275	0.8785	1.1295	1.255
$\varkappa\lambda = -0.255$	5	$\varkappa\lambda z^2$	-0.0229	-0.0637	-0.1249	-0.2066	-0.255
$\psi = \varkappa z + \varkappa\lambda z^2$	6	$\psi = \varkappa z + \varkappa\lambda z^2$	0.3536	0.5638	0.7536	0.9229	1.000
	7	$1 - \psi$	0.6464	0.4362	0.2464	0.0771	0
$z = \frac{I}{I_K}$	8	$I = I_K \cdot z$	16.5	27.5	38.5	49.5	55.0
$\zeta'\Lambda s_{clearance} = 0.0_{4}3277$	9	$\gamma = \zeta'\Lambda \cdot s_{clearance} \cdot I$	$0.0_{3}54$	$0.0_{3}90$	$0.0_{2}126$	$0.0_{2}162$	$0.0_{2}180$
	10	(-) $\omega\psi$	0.0129	0.0206	0.0276	0.0338	0.0366
	11	γ	0.0005	0.0009	0.0013	0.0016	0.0018
	12	$\omega\psi - \gamma$	0.0124	0.0197	0.0263	0.0322	0.0348
$\alpha_0 = \alpha = 0.85;$	13	(+) $\alpha(\omega\psi - \gamma)$	0.0105	0.0167	0.0223	0.0274	0.0296
$\frac{\omega}{\delta} = 0.02232;$	14	$\frac{\omega}{\delta}(1-\psi)$	0.0144	0.0097	0.0055	0.0017	0
$W_0 = 0.720;$	15	$\alpha_0\omega_0$	0.0061	0.0061	0.0061	0.0061	0.0061
	16	a	0.0310	0.0325	0.0339	0.0352	0.0357
$l'_\psi = \frac{1}{s}\left[W_0 - \frac{\omega}{\delta}\cdot(1-\psi) - \alpha(\omega\psi - \gamma) - \alpha\omega_0\right]$	17	$W_0 - a$	0.6890	0.6875	0.6861	0.6848	0.6843
$a = \alpha(\omega\psi - \gamma) + \frac{\omega}{\delta}\cdot(1-\psi) + \alpha_0\omega_0$	18	$l'_\psi = \frac{W_0 - a}{s}$	1.306	1.303	1.300	1.298	1.297

	12	$\omega\psi - \gamma$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\alpha_0 = \alpha = 0.85;$	13	(+) $\alpha(\omega\psi - \gamma)$	0.0105	0.0167	0.0223	0.0274	0.0296
$\frac{\omega}{\delta} = 0.02232;$	14	(+) $\frac{\omega}{\delta}(1-\psi)$	0.0144	0.0097	0.0055	0.0017	0
$W_0 = 0.720;$	15	(+) $\alpha_0\omega_0$	0.0061	0.0061	0.0061	0.0061	0.0061
	16	a	0.0310	0.0325	0.0339	0.0352	0.0357
$l'_\psi = \frac{1}{s}\left[W_0 - \frac{\omega}{\delta}\cdot(1-\psi) - \alpha(\omega\psi - \gamma) - \alpha\omega_0\right]$	17	$W_0 - a$	0.6890	0.6875	0.6861	0.6848	0.6843
$a = \alpha(\omega\psi - \gamma) + \frac{\omega}{\delta}\cdot(1-\psi) + \alpha_0\omega_0$	18	$l'_\psi = \frac{W_0 - a}{s}$	1.306	1.303	1.300	1.298	1.297
$\log Z^{-1}$ from table on page 290 (of the original - Editor)	19	$\log Z^{-1}$	0.01813	0.03663	0.05759	0.08047	0.0928
$\gamma = 0.02419$	20	$A_s \log Z^{-1}$	0.03586	0.07245	0.1139	0.1592	0.1835
$A_s = 1.978$	21	Z^{-A_s}	1.086	1.181	1.300	1.443	1.526
	22	(+) $l = l'_\psi\cdot(Z^{-A_s} - 1)$	0.1123	0.2358	0.390	0.5750	0.6822
	23	(+) l'_ψ	1.306	1.303	1.300	1.298	1.297
	24	$l + l'_\psi$	1.418	1.5388	1.690	1.873	1.979
$p = \frac{f\omega}{s}\,\frac{\chi_0 + k_1'z - B_1'z^2}{l'_\psi + l}$	25	(+) χ_0	0.1192	0.1192	0.1192	0.1192	0.1192
	26	(+) $k_1'z$	0.3617	0.6029	0.8440	1.0852	1.2058
	27	(−) $\chi_0 + k_1'z$	0.4809	0.7221	0.9632	1.2044	1.3250
	28	(−) $B_1'z^2$	0.0266	0.0738	0.1446	0.2391	0.2952
$\frac{f\omega}{s} = 77{,}680$	29	$\chi_0 + k_1'z - B_1'z^2$	0.4543	0.6483	0.8186	0.9653	1.0298
	30	p, kg/cm^2	249.0	327.0	376.0	400.0	404.2

Computation of Second Period

Computation of Constants for the Second Period

$$v^2_{\Pi p} = \frac{2f\omega}{\varphi\theta_m} = \frac{2\cdot 1,120,000\cdot 0.0366\cdot 98.1}{0.15\cdot 3.4} = 15,770,000;$$

$$\eta'_K = \frac{\eta_K}{v_K} = \frac{0.04923}{824\cdot 5} = 0.0_4 5971;$$

$$\eta_2 = \eta'_K \cdot v^2_{\Pi p} = 0.0_4 5971\cdot 1577\cdot 10^4 = 941.6;$$

$$\eta_3 = (1 + \chi_0)\, v^2_{\Pi p} = 1.1192\cdot 1577\cdot 10^4 = 17,650,000;$$

$$\gamma = \frac{\eta_3}{\eta_2^2} = \frac{17,650,000}{941.6^2} = 19.91;$$

$$b = \sqrt{1 + 4\gamma} = \sqrt{1 + 4\cdot 19.91} = 8.978;$$

$$v_1 = -\frac{\eta_2}{2}(1 + b) = \frac{-941.6}{2}(1 + 8.978) = -4698;$$

$$v_2 = \frac{\eta_2}{2}(b - 1) = \frac{941.6}{2}\cdot 7\cdot 978 = 3756;$$

$$\frac{b + 1}{b} = \frac{9.978}{8.978} = 1.111;$$

$$\frac{b - 1}{b} = \frac{7.978}{8.978} = 0.8886;$$

$$\frac{s}{s'}\frac{1}{\theta} = \frac{0.5277}{0.5195}\cdot\frac{1}{0.15} = 6.770;$$

$$\frac{\eta_K}{v_K} = \frac{0.04923}{824.5} = 0.00005971.$$

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Form for Computation of Elements for Second Period

Basic formulas	No.	Operations				Muzzle face
$\left(\frac{l_1' + l_K}{l_1' + l}\right)^{\frac{s'}{s}\theta} =$	1	(−) v	1200	1500	1800	2052
$= \left(\frac{v - v_1}{v_K - v_1}\right)^{\frac{b+1}{b}} \cdot$	2	v_1	-4698	-4698	-4698	-4698
$\cdot\left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{b}}$						
$v_1 = -\frac{\eta_2}{2}(1 + b) =$ $= -4698$	3	$v - v_1$	5898	6198	6498	6750
	4	(−) v_K	824.5	824.5	824.5	824.5
	5	v_1	-4698	-4698	-4698	-4698
$v_K = \frac{s'I_K}{\varphi m} = 824.5$	6	$v_K - v_1$	5522.5	5522.5	5522.5	5522.5
	7	$\frac{v - v_1}{v_K - v_1}\ldots$	1.068	1.122	1.176	1.222
	8	$\log \frac{v - v_1}{v_K - v_1}\ldots$	0.0285	0.0500	0.0704	0.0871
	9	$\frac{b + 1}{b}\log\frac{v - v_1}{v_K - v_1}\ldots$	0.03166	0.05555	0.0782	0.0968
$\frac{b + 1}{b} = 1.111$	10	$\left(\frac{v - v_1}{v_K - v_1}\right)^{\frac{b+1}{b}}$	1.076	1.137	1.198	1.249

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(cont'd)

Basic formulas	No.	Operations				Muzzle face
$v_2 = \frac{\eta_2}{2}(b-1) = 3756$	11 12	$(-)\begin{cases} v \\ v_2 \end{cases}$	1200 +3756	1500 3756	1800 3756	2052 3756
	13	$v - v_2$	-2556	-2256	-1956	-1704
	14 15	$(-)\begin{cases} v_K \\ v_2 \end{cases}$	824.5 3756	824.5 3756	824.5 3756	824.5 3756
	16	$v_K - v_2$	-2931.5	-2931.5	-2931.5	-2931.5
$\frac{b-1}{b} = 0.8886$	17	$\frac{v - v_2}{v_K - v_2}\ldots$	0.8719	0.7696	0.6672	0.5813
	18	$\left.\log \frac{v - v_2}{v_K - v_2}\right\}$	$\bar{1}.9404$ -0.0596	$\bar{1}.8862$ -0.1138	$\bar{1}.8242$ -0.1758	$\bar{1}.7644$ -0.2356
	19	$\frac{b-1}{b} \log \frac{v - v_2}{v_K - v_2}$	0.05296 $\bar{1}.9470$	-0.1011 $\bar{1}.8989$	-0.1526 $\bar{1}.8438$	-0.2094 $\bar{1}.7906$
$\frac{s}{s'}\frac{1}{\theta} = 6.77$	20	$\left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{b}}$	0.8851	0.7923	0.6979	0.6175
	21	$\Gamma = \left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b+1}{b}} \cdot \left(\frac{v - v_2}{v_K - v_2}\right)^{\frac{b-1}{b}}$	0.9524	0.9008	0.8361	0.7712
	22	$\log \Gamma$	$\bar{1}.9788$ -0.0212	$\bar{1}.9546$ -0.0454	$\bar{1}.9223$ -1.0777	$\bar{1}.8872$ -0.1128

(cont'd)

Basic formulas	No.	Operations				Muzzle face
$p = \frac{f\omega}{s} \frac{1 + \chi_0 - \frac{\eta_K}{v_K} v - \frac{v^2}{v^2_{\Pi p}}}{l'_1 + l}$	23	$\frac{s}{s'} \frac{1}{\theta} \log \Gamma \ldots$	-0.1435 $\bar{1}.8565$	-0.3073 $\bar{1}.6927$	-0.5260 $\bar{1}.4740$	-0.7636 $\bar{1}.2364$
$l'_1 = 1.297$	24	$\frac{s}{\Gamma s'} \frac{1}{\theta} \ldots$	0.7186	0.4928	0.2979	0.1724
$l'_1 + l_K = 1.297 + 0.682 =$ $= 1.979$	25	$l'_1 + l_K$	1.979	1.979	1.979	1.979
	26	$(-)\left\{\frac{l'_1 + l_K}{\Gamma \frac{s}{s'} \frac{1}{\theta}}\right.$	2.754	4.016	6.643	11.479
$\chi_0 = 0.1192$	27	$l'_1 \ldots$	1.297	1.297	1.297	1.297
	28	l	1.457	2.719	5.346	10.182
$\frac{\eta_K}{v_K} = 0.0_4 5971$	29	$l'_1 + l$	2.754	4.016	6.643	11.479
	30	$(-)\left\{1 + \chi_0\right.$	1.1192	1.1192	1.1192	1.1192
	31	$\frac{\eta_K}{v_K} \cdot v$	0.0716	0.0896	0.1075	0.1225
	32	$(-)\left\{1 + \chi_0 - \frac{\eta_K}{v_K} \cdot v\right.$	1.0476	1.0296	1.0117	0.9967
$\frac{f\omega}{s} = 77, 680$	33	$\frac{v^2}{v^2_{\Pi p}} \ldots$	0.0913	0.1427	0.2055	0.2671
	34	$1 + \chi_0 - \frac{\eta_K}{v_K} v - \frac{v^2}{v^2_{\Pi p}}$	0.9563	0.8869	0.8062	0.7296
	35	p, kg/cm^2	270	172	94.0	49.0

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The results of the computation are presented in fig. 172.

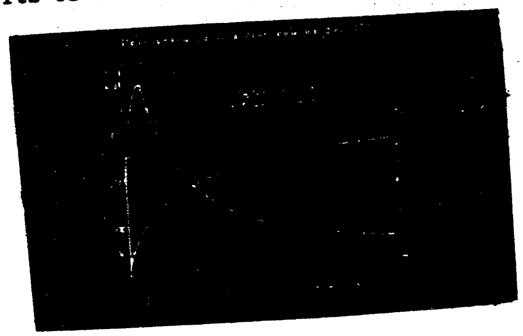

Fig. 172 - p-l and v-l Curves for Mortar.

1) kg/cm^2; 2) p-l and v-l curves for 82-mm mortar; 3) m/sec.

Fig. 172-a - p-l and v-l Curves for Mortar with Various Charges.

1) kg/cm^2; 2) m/sec; 3) designations; 4) charge nos.

Fundamental Ballistic Data for Mortars.

c_q	6.8	6.17	6.53	9.25	
C_ξ	3.13	12.6	34.1	33.1	
Δ	0.028	0.061	0.151	0.140	
p_m	300	450	850	1000	
ω/q	0.0053	0.0129	0.0387	0.0281	
w_0/d^3	1.29	1.31	1.67	1.87	
$\frac{\eta\omega}{\check{L}_{KH}\cdot d}$	10.3	11.2	11.45	11.05	$\sim$ const
$\eta_D = \frac{p_{av.}}{p_m}$	0.195	0.286	0.530	0.457	
w_0/q	0.190	0.212	0.256	0.202	
l_0/d	1.64	1.66	2.13	2.37	
Δ_0' (*)	0.67	0.56	0.64	0.64	
Λ_D	4.15	7.50	4.53	3.87	

(*) Δ_0' is the loading density of the basic charge in relation to the chamber of the tail cartridge.

SECTION TWELVE-GUNS WITH CONICAL BORE

INTRODUCTION

Guns with a conical bore had been proposed as experimental models as far back as the 1870's. After the First World War, the German engineer Herlich conducted firing tests from a rifle with a conical bore. In these tests, there was obtained an initial bullet velocity that was considerably higher than the usual velocity, as a result of which the armor-piercing effect was sharply increased.

In the course of the Second World War, use was made in the German army of conical guns of various calibers, which were employed principally as antitank artillery. The following guns were used: an antitank gun with a 28-mm entrance caliber and a 20-mm exit caliber (28/20), giving a projectile velocity of 1400 m/sec; a 42/28 antitank gun of the same type; and a 75/55 cylindrical-conical gun (v_D' = about 1250 m/sec), whose barrel consisted of a 75-mm rifled cylindrical tube of the usual type, followed by a smooth conical part with the diameters 75/55, and a smooth cylindrical end piece of 55-mm caliber. The projectile had two skirt bands: a thinner directing band in front and a thicker rotating band in the rear. A section through the armor-piercing projectile for the 75/55 gun is shown in fig. 173.

Firing from a conical barrel is in principle analogous to firing from an ordinary gun with a subcaliber projectile. For this reason, we shall discuss at the outset the possibility of increasing the velocity of the projectile by reducing its weight through a transition to a subcaliber model.

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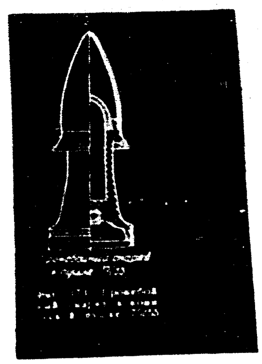

Fig. 173 - Armor-Piercing Projectile for 75/55 Conical Gun.

In recent years, and especially during the Great Patriotic War, attempts have been made to increase the armor-piercing ability of the projectile by increasing its velocity through a reduction in its weight.

Let us determine to what extent the velocity of a projectile in a given gun will change at a predetermined maximum pressure p_m if the weight of the projectile q is changed.

For an ordinary projectile, let us designate its weight as q_1', its initial velocity as v_D', the pressure impulse of the powder as I_K', and the thickness of the powder as $2e_1' = I_K' \cdot 2u_1'$. Let the new weight of the projectile be $q'' < q'$; the problem is to determine the changed velocity v_D''.

We have the simplest case by accepting the conditions that p_m and Δ (or ω) remain unchanged.

From the formula for the second period, we have:

$$\frac{\varphi q v_D^2 \theta}{2gf\omega} = 1 - \frac{(\Lambda_K + 1 - \alpha\Delta)^\theta}{(\Lambda_D + 1 - \alpha\Delta)^\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] = 1 - \frac{K^\theta}{(\Lambda_D + 1 - \alpha\Delta)^\theta}; \qquad (43)$$

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where:

$$K = (\Lambda_K + 1 - \alpha\Delta)\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{\frac{1}{\theta}}.$$

Under the condition of constancy of the values for p_m, Δ, and ω, we have B and Λ_K = const, K = const; for this reason, the left-hand side of the expression (43) also remains unchanged (r'_D = const).

Consequently, discarding the constant quantities, we obtain the following correlation:

$$\varphi q v_D^2 = \left(a + b\frac{\omega}{q}\right) q v_D^2 = \text{const}, \tag{44}$$

from which:

$$\left(\frac{v''_D}{v'_D}\right)^2 = \frac{\varphi' q'}{\varphi'' q''} = \frac{q'}{q''}\,\frac{a + b\frac{\omega}{q'}}{a + b\frac{\omega}{q''}}. \tag{45}$$

The condition (44) at ω = const is equivalent to the condition:

$$\frac{\varphi q v_D^2}{2g\omega} = \varphi\eta_\omega = \text{const},$$

and, since the quantity $\varphi = a + b\frac{\omega}{q}$ increases as the weight of the projectile decreases, it follows that, under the imposed conditions of maintaining p_m, Δ, and ω constant, the coefficient of utilization of the charge η_ω will be somewhat lower with the lighter projectile than with the ordinary projectile.

Thus:

$$v''_D = v'_D\sqrt{\frac{q'}{q''}\,\frac{\varphi'}{\varphi''}}; \tag{46}$$

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$$\eta_{\omega}'' = \eta_{\omega}' \frac{\varphi'}{\varphi''}.$$

From the condition B = const, we obtain the following additional condition:

$$\frac{I_K^2}{\varphi q} = \text{const} \tag{47}$$

or:

$$I_K'' = I_K' \sqrt{\frac{\varphi''}{\varphi'} \frac{q''}{q'}}.$$

It follows from (46) and (47) that:

$$I_K'' \, v_D'' = I_K' \, v_D' = \text{const}.$$

The formula makes it possible to select the weight of projectile q'' necessary to obtain in the given gun the required initial velocity v_D''.

How will V_{max} change as the light-weight projectile is adopted?

$$V_{max} = \frac{q}{Q_{CT}} \frac{1 + \beta \frac{\omega}{q}}{1.15} v_D,$$

where:

$$\beta \approx \frac{C_1}{v_D} = \frac{\text{const}}{v_D};$$

β being the coefficient of secondary action of the gases.

We change the weight of the projectile, retaining ω = const.

$$\frac{v''_D}{v'_D} = \sqrt{\frac{q'}{q''}\,\frac{\varphi'}{\varphi''}}; \quad \beta\,\frac{\omega}{q} = \frac{C_1\omega}{qv_D};$$

$$\frac{V''_{max}}{V'_{max}} = \frac{q''}{q'}\,\frac{1 + \dfrac{C_1\omega}{v''_D q''}}{1 + \dfrac{C_1\omega}{v'_D q'}}\,\frac{v''_D}{v'_D} = \frac{q''v''_D + C_1\omega}{q'v'_D + C_1\omega} =$$

$$= \frac{q'v'_D\sqrt{\dfrac{\varphi'}{\varphi''}\,\dfrac{q''}{q'}} + C_1\omega}{q'v'_D + C_1\omega} = \frac{\sqrt{\dfrac{\varphi'}{\varphi''}\,\dfrac{q''}{q'}} + \beta'\,\dfrac{\omega}{q'}}{1 + \beta'\,\dfrac{\omega}{q'}}.$$

Since:

$$\varphi' q'' < \varphi'' q',$$

it follows that:

$$V''_{max} : V'_{max} < 1.$$

Consequently, in adopting a light-weight type projectile while retaining the same weight of the charge, in spite of the increase in the velocity of the projectile, the maximum recoil velocity decreases, so that the load on the gun mount also decreases.

Thus, the obtaining of increased projectile velocities by reducing the weight of the projectile is a fully justified and practically realizable measure.

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Let a projectile with a weight coefficient $c_q' = 15.0$ have $v_D' = 1000$ m/sec at $\omega/q = 0.45$. If $\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q}$, $\varphi' = 1.18$.

We adopt a light-weight projectile with a weight coefficient $c_q'' = 7.5 = \frac{1}{2} c_q'$. Then $\omega/q'' = 0.90$, $\varphi'' = 1.03 + \frac{1}{3} \cdot 0.90 = 1.33$.

$$v_D'' = 1000 \sqrt{\frac{1.18}{1.33} \cdot \frac{15.0}{7.5}} = 1000 \cdot 1.33 = 1330 \text{ m/sec}.$$

If $\eta_\omega' = 130$, then:

$$\eta_\omega'' = 130 \, \frac{1.18}{1.33} = 115.3 \text{ tm/kg};$$

$$I_K'' = I_K'' \frac{1}{1.33} = 0.752.$$

Thus, if the weight of the projectile is halved, the velocity in the case under consideration increases by 33%, and the thickness of the powder decreases by 25% (the same p_m and Δ being retained).

Let us consider the condition on the basis of which it is possible to determine the weight of the projectile necessary to obtain a predetermined v_D'' in firing from a given gun.

From the condition (45), we have:

$$\left(\frac{v_D''}{v_D'}\right)^2 = \frac{a + b\frac{\omega}{q'}}{a + b\frac{\omega}{q''}} \frac{q'}{q''} = \frac{aq' + b\omega}{aq'' + b\omega}.$$

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From this, we obtain:

$$q'' = \frac{q'}{a}\left[\varphi'\left(\frac{v_D'}{v_D''}\right)^2 - b\,\frac{\omega}{q'}\right]. \qquad (48)$$

CHAPTER 1 - FUNDAMENTAL CHARACTERISTICS AND BALLISTIC PROPERTIES OF BARREL WITH CONICAL BORE

In order to slowly lose velocity in flight, a projectile must be "heavy," i.e., must have the greatest possible weight coefficient $c_q = q/d^3$ or transvorse load q/s. In order to attain a predetermined initial velocity in the bore after as short a path as possible, a projectile must be "light," i.e., must have the smallest possible weight coefficient c_q.

These two mutually contradictory requirements make it possible to reconcile barrels with a conical or cylindrical-conical bore.

Let d_0 be the entrance caliber of the conical bore and d_D its exit caliber, where $d_D < d_0$. In such a bore, the projectile, by having with respect to the entrance caliber a small coefficient $c_{q_0} = q/d_0^3$ and always retaining it smaller than $c_{q_D} = q/d_D^3$ until it is ejected from the bore, will acquire a predetermined velocity v_D after a considerably shorter path than a projectile of the same weight in a cylindrical bore with a caliber d_D equal to the exit caliber of the conical bore; but, as this projectile is ejected, it will already have a large weight coefficient $c_{q_D} = q/d_D^3$ with respect to the exit caliber d_D and will be advantageous for flight in the air.

The solution of the problem of the clarification of the fundamental ballistic properties of a conical bore must be sought in a comparison of cylindrical guns of different calibers firing a projectile possessing a given weight, and this problem is easily solved by theoretical means.

As a matter of fact, it is known from the general relations encountered in ballistic design that, at identical Δ, ω/q, p_m, and v_D, the lengths of the bore L_{KH} and of the chamber l_0, expressed in calibers, and the thicknesses of the powder in relation to the caliber are proportional to the projectile-weight coefficients c_q, and the absolute

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values of the same quantities are proportional to the values for q/s (transverse load).

The fundamental relations of internal ballistics for cylindrical bores give:

$$\frac{W_{KH}}{q} = \frac{W_0}{q}\left\{\frac{K(p_m,\Delta)}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta\right\}$$

or:

$$\frac{L'_{KH}}{d} = \frac{l_0}{d}\left\{\frac{K(p_m,\Delta)}{(1 - r')^{\frac{1}{\theta}}} + \alpha\Delta\right\}.$$

The expression enclosed in brackets equals $\Lambda_D + 1$;

$$\frac{l_0}{d} = \frac{W_0}{sd} = \frac{W_0}{n_s d^3} = \frac{\omega}{q}\frac{1}{n_s\Delta}\frac{q}{d^3} = \frac{\omega}{q}\frac{c_q}{n_s\Delta}.$$

At predetermined p_m, v_D, ω/q and Δ, the expression in the brackets is a constant quantity, and l_0/d is proportional to c_q; consequently, both $\frac{L'_{KH}}{d} = \frac{l_0}{d}\left\{\Lambda_D + 1\right\}$ and $\frac{l_D}{d} = \frac{l_0}{d}\left\{\Lambda_D\right\}$ are also proportional to the quantity c_q.

Since:

$$\frac{I_K}{d} = \sqrt{\frac{f}{g}}\frac{c_q}{n_s}\sqrt{B\varphi\frac{\omega}{q}},$$

it follows that, at predetermined p_m and Δ, the quantity B = const, and at a predetermined ω/q, the quantity φ is also constant. Consequently, the relative pressure impulse is likewise proportional to the weight

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coefficient c_q.

The larger the caliber of a gun at a given weight of the projectile, the smaller is I_K/d and the thinner is the powder necessary to ensure attainment of the same maximum pressure p_m in the presence of the same charge.

It further follows from this that the absolute values of l_0, l_D, L'_{KH}, and I_K are directly proportional to the transverse load q/s.

The relation between c_q and q/s for the entrance and exit calibers will be expressed as follows:

$$c_{q_0} = c_{q_D}\left(\frac{d_D}{d_0}\right)^3; \quad \frac{q}{s_0} = \frac{q}{s_D}\left(\frac{d_D}{d_0}\right)^2.$$

Since usually $d_0/d_D = 1.35\text{-}1.40$, it follows that:

$$\left(\frac{d_0}{d_D}\right)^3 = (1.40\ldots1.35)^3 \sim 2.75\ldots2.46; \left(\frac{d_D}{d_0}\right)^3 = 0.363\text{-}0.407;$$

$$\left(\frac{d_0}{d_D}\right)^2 = 1.96\ldots1.82; \quad \left(\frac{d_D}{d_0}\right)^2 = 0.51\text{-}0.55.$$

Consequently, at $d_0/d_D = 1.4$, a projectile of a given weight will attain a predetermined velocity v_D in a cylindrical gun of caliber d_0 after traversing a path nearly twice as short as in a similar cylindrical gun of caliber d_D.

At a given chamber volume, both the bore volumes and the quantities Λ_D will be identical. At a given p_m, the p-Λ and v-Λ curves will coincide.

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If now, while retaining the weight of the projectile, the chamber and bore volumes, and consequently also $\Lambda_D = W_D/W_0$ constant, a gradual transition is made in cylindrical guns from the larger to the smaller caliber, the lengths of the chambers and bores, as well as the weight coefficients c_q, will gradually increase. Since, in this connection, Δ, q, ω/q, p_m, and v_D remain the same, it follows that, for such cylindrical guns, the v-Λ or v-W curves coincide not only for c_{q_0} and c_{q_D}, but also for all intermediate calibers and c_q.

This is a fundamental property of ballistically similar guns of different calibers, which makes it possible to explain the properties of the v-l and p-l curves for a conical barrel.

As a matter of fact, in a conical barrel of the same volume, with the same chamber volume, and with the same value for $\Lambda_D = W_D/W_0$, there is accomplished a continuous transition from a cylindrical barrel with the initial entrance caliber d_0 and $c_{q_0} = \frac{q}{d_0^3}$ to a cylindrical barrel with the exit caliber d_D and $c_{q_D} = q/d_D^3$. Since there has already been established the identity of the curves for the velocity of the projectile as a function of $\Lambda = W/W_0$ for all cylindrical barrels with gradually diminishing calibers, it is possible to conclude <u>that, at a given weight of the projectile, the velocity curve v-Λ for the conical bore at the same Δ, ω/q, φ, and p_m will coincide with the same curves for all cylindrical barrels of various calibers, but having the same volume.</u>

This is one of the fundamental assumptions made by us with regard to the ballistic properties of a conical bore. At the same volume of the bore and of its working part, its length is smaller than the length of a cylindrical bore of caliber d_D and greater than the length of a

similar bore of caliber d_0. This is clear from the sketch presented in fig. 174.

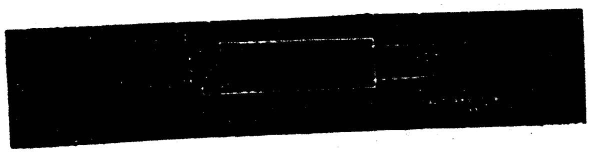

Fig. 174 – Sketch of Conical and Cylindrical Bores.

As will be shown subsequently, the pressure curve in the conical bore as a function of relative volume, will not coincide with the pressure curves for the same bores having a cylindrical shape; after reaching the maximum, this curve will be disposed higher than the p–Λ curve for cylindrical bores, and the end of burning at the same maximum pressure will occur earlier in the conical bore than in the cylindrical one:

$$\Lambda_{K_{con.}} < \Lambda_{K_{cyl.}}$$

1. DESIGNATIONS AND GEOMETRIC CHARACTERISTICS OF CONICAL BORE.

Entrance caliber.............	d_0
Exit caliber.................	d_D
Angle of conicity............	β
Length of path of projectile..	l_D
Total length of cone..........	$h_K = l_D \cdot \dfrac{d_0}{d_0 - d_D} = \dfrac{d_0}{2}\cot\beta$
Total volume of cone to apex..	$W_{con.}$
Volume of working part of cone.	W_D

Total bore volume including chamber.....$W_{KH} = W_0 + W_D$

Total relative volume of conical bore...$\Lambda_D = \frac{W_D}{W_0}$

Fig. 175 - Geometric Data of Conical Bore.

Let us introduce the relative diameter $y = d/d_0$. All remaining characteristics are expressed very simply as functions of this quantity:

$$y = \frac{d}{d_0} = \frac{h_K - l}{h_K} = 1 - \frac{l}{h_K}.$$

From this:

$$l = h_K(1 - y); \quad l_D = h_K(1 - y_D);$$

$$\frac{s}{s_0} = \left(\frac{d}{d_0}\right)^2 = y^2; \quad \frac{s_{av.}}{s_0} = \frac{1 + y + y^2}{3} \approx y;$$

$$W = s_0\left(\frac{1 + y + y^2}{3}\right) l = \frac{s_0 h_K}{3}(1 + y + y^2)(1 - y) =$$

$$= \frac{s_0 h_K}{3}(1 - y^3) = W_{con.}(1 - y^3);$$

$$\Lambda_D = \frac{W_D}{W_0} = \frac{s_0 h_K}{3 s_0 l_0}(1 - y_D^3) = \frac{h_K}{3 l_0}(1 - y_D^3).$$

Since we shall subsequently adopt $\Lambda = W/W_0$ as the independent variable, we find as a function of the latter:

$$y = \sqrt[3]{1 - \frac{W}{W_{con.}}} = \sqrt[3]{1 - \frac{\Lambda}{\Lambda_{con.}}},$$

where $\Lambda_{con.} = W_{con.}/W_0$.

The remaining quantities, as previously, are expressed in terms of y:

$$l = h_K(1 - y); \; s = s_0 y^2; \; s_{av.} = s_0 \frac{1 + y + y^2}{3}.$$

The system of equations for the conical bore differs in no way externally from the equations for the cylindrical bore[1]; only the value for the cross-sectional area s entering into the equations is variable, and this complicates the solution of the problem.

For an exact solution, it becomes necessary to resort to the method of numerical integration or resolution into a series.

However, the coincidence of the projectile-velocity curves as a function of Λ for the conical and cylindrical bores makes it possible approximately, but with a sufficient degree of precision, to solve the problem for the conical barrel in the final form by introducing an average value for s (which is possible in the presence of a slight conicity).

In this connection, we take for comparison a cylindrical barrel with a caliber equal to the entrance caliber d_0 and with the same system characteristics:

$$W_0, \; W_D, \; q, \; \frac{\omega}{q}, \; \Delta, \; \varphi, \; I_K, \; f, \; \alpha, \; \delta, \; \theta \text{ and } p_0.$$

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Let us write down the system of fundamental relations:

$spdt = \varphi m dv$ .... (1) Equation of motion;

$de = u_1 p dt$ .... (2) Law of burning rate;

$p(W_\psi + W) = f\omega\psi - \frac{\theta}{2}\varphi m v^2$ .. (3) Equation of transformation of energy.

Solving (3) with respect to p, and introducing Λ and Λ_ψ, we obtain:

$$p = f\omega \frac{\psi - \frac{v^2}{v_{\Pi p}^2}}{W_\psi + W} = f\Delta \frac{\psi - \frac{v^2}{v_{\Pi p}^2}}{\Lambda_\psi + \Lambda}, \tag{49}$$

where:

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\Delta\psi;$$

$$\frac{s}{s_0} = \left(1 - \frac{W}{W_{con.}}\right)^{\frac{2}{3}} = \left(1 - \frac{\Lambda}{\Lambda_{con.}}\right)^{\frac{2}{3}}$$

is the dependence of the cross-sectional area upon the relative bore volume;

$$W_{con.} = \frac{s_0 h_K}{3} = \frac{s_0}{3}\frac{d_0}{2}\cot\beta.$$

$$\psi = \psi_0 + \varkappa\sigma_0 x + \varkappa\lambda x^2 \ (*)$$

is the law of inflow of the gases.

From (1) and (2), as usual, we obtain:

$$sde = u_1\varphi m dv;$$

$$dv = \frac{s}{\varphi m}\frac{e_1}{u_1}dz = \frac{sI_K}{\varphi m}dx. \tag{50}$$

In contrast with the cylindrical barrel, in this case s is a variable which is not directly connected with either x or v; but the property, assumed above, of the coincidence of the $v = f(\Lambda)$ curves for the cylindrical and conical barrels at equal p_m, w_0, and Λ_D makes it possible to establish s as a function of v for the conical bore with the aid of the $v = f(\Lambda)$ curve obtained for the cylindrical barrel.

By introducing into equation (50) the quantity for the entrance cross section s_0, integrating, and taking s/s_0 on the right-hand side outside the integral sign in the form of an average value, we obtain:

$$v = \frac{s_0 I_K}{\varphi m} \frac{s_{av.}}{s_0} x. \qquad (51)$$

For the cylindrical barrel with the same cross section s_0, we have the usual relation (in which all elements are marked by '):

$$v' = \frac{s_0 I_K}{\varphi m} x',$$

φ and I_K being the same as in the preceding equation. Since for a given value of Λ:

$$v = v',$$

it follows that:

$$\frac{s_{av.}}{s_0} x = x'$$

or:

$$x = \frac{x'}{\frac{s_{av.}}{s_0}} > x'. \qquad (52)$$

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It is seen from this that, at one and the same value of v for the conical and cylindrical bores, and at the same values of I_K and φ, the relative thickness of the burnt powder x for the conical barrel is greater than the corresponding quantity x' for the cylindrical barrel: $x > x'$. Consequently, on the basis of formula (49), there follow the relations stated below.

(a) For a conical barrel with a variable cross section from s_0 to s_D:

$$p = f\Delta \frac{\psi - \frac{v^2}{v_{np}^2}}{\Lambda_\psi + \Lambda};$$

(b) For cylindrical bores with a cross section s_0 = const or s_D = const:

$$p' = f\Delta \frac{\psi' - \frac{v'^2}{v_{np}^2}}{\Lambda'_{\psi'} + \Lambda'}.$$

Since, from (52), $x > x'$; from (*), $\psi > \psi'$ and $\Lambda_\psi < \Lambda'_\psi$; and $v = v'$ and $\Lambda = \Lambda'$; it follows that $p > p'$.

We obtain the following fundamental conclusion, which characterizes the ballistic properties of the conical barrel: under identical loading conditions (W_0, ω/q, Δ, I_K), and in the presence of identical values for the working volume W and the projectile velocity v, the burnt part of the charge ψ and the gas pressure are greater in the conical barrel than in the cylindrical bore with the cross section s_0. The difference is the greater the greater the conicity of the bore.

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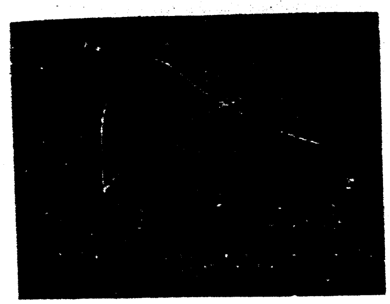

Fig. 176 - p-W Pressure Curves for Conical Bore.

Consequently, under loading conditions identical with those in a cylindrical barrel with the same W_0, W_D, and s_0, the gas-pressure curve in a conical barrel, expressed as a function of W, has a more progressive character than in the cylindrical barrel; and since the average pressure in the former is greater than in the latter, the end of burning of the powder will occur in the former sooner, at a smaller Λ_K, than in the latter.

2. RELATION OF POWDER THICKNESSES IN CONICAL AND CYLINDRICAL BORES UNDER EQUAL MAXIMUM PRESSURES.

It has been shown that, at identical entrance calibers d_0, under identical loading conditions (Δ, ω/q, W_0), and at identical magnitudes of the pressure impulse I_K, the gas pressure in a conical bore exceeds that in a cylindrical bore because of an increase in the burnt part of the charge $\psi = \psi_0 + x$. And since the ballistic properties of the barrels must be compared at the identical maximum pressure p_m, it follows that, without altering the other loading conditions, it is possible to obtain identical pressures in both bores as a result of a change in the impulse I_K.

Let us determine the relation between the pressure impulses (or powder thicknesses) for a conical barrel with the calibers d_0 and d_D and for cylindrical barrels with the same calibers.

For cylindrical barrels of different calibers, at a given weight of the projectile and at given p_m and Δ, $B_0 = B_D$, from which:

$$s_0 I_{K0} = s_D \cdot I_{KD}.$$

The greater the caliber the smaller I_K.

$$\frac{I_{K0}}{I_{KD}} = \frac{s_D}{s_0} = y^2; \quad I_{KD} = I_{K0} \frac{s_0}{s_D} = \frac{I_{K0}}{y_D^2}.$$

To start with, we shall find the relation between I_K for the conical barrel and I_{K0} for the cylindrical barrel of caliber d_0. Let us determine how the quantities x and ψ vary as a function of v in the conical and cylindrical barrels if the caliber is d_0.

For Cylindrical Barrel

$$x' = \frac{\varphi m}{s_0 I_{K0}} v$$

For Conical Barrel

$$x = \frac{\varphi m}{s_0 I_K} \frac{v}{\frac{s_{av.}}{s_0}} > x'$$

In this connection, as v increases and $s_{av.}/s_0$ decreases, the difference $x - x'$ grows uninterruptedly.

In fig. 177, x' and x are shown as functions of v. $x' - v$ is the straight line o'bx'; $x - v$ is the curve o'ax with the same tangent o'bx' at the start of motion of the projectile at $v = 0$.

Fig. 177 - Diagram of Variation of x in Conical and Ordinary Bores.

For a given value of v'_m at identical values of Λ'_m, $x_m > x'_m$, $\psi_m > \psi'_m$, $p_m > p'_m$. In order to obtain $p_m = p'_m$, it is necessary, by changing the pressure impulse, to lower the curve o'ax, equating the intensities of gas formation for the cylindrical and conical bores over the segment from 0 to v'_m.

Let us consider a powder with a constant burning area:

$$\psi = \psi_0 + x.$$

It is not difficult to see that the tangent of the angle of slope of the lines characterizes the intensity of gas formation. As a matter of fact:

$$\frac{dx}{dv} = \frac{d\psi}{dv} = \frac{d\psi}{dt}\frac{dt}{dv},$$

but:

$$\frac{dv}{dt} = \frac{sp}{\varphi m}.$$

Consequently:

$$\frac{dx}{dv} = \frac{\varphi m}{sp}\frac{d\psi}{dt} = \frac{\varphi m}{s}\Gamma,$$

where:

$$\Gamma = \frac{\varkappa}{I_K}\sigma.$$

For this reason, for a powder with a constant area of burning $\Gamma = 1/I_K$, $dx/dv = \varphi m/sI_K$; for a conical barrel, s decreases and dx/dv increases. We impose the requirement that the average value of dx/dv along the segment from zero to v'_m be equal to $\frac{dx'}{dv} = \frac{\varphi m}{s_0 I_{K0}}$ for the cylindrical barrel. Then:

$$\frac{\varphi m}{s^{(m)}_{av.} I_K} = \frac{\varphi m}{s_0 I_{K0}} \text{ and } I_K = I_{K0}\frac{s_0}{s^{(m)}_{av.}},$$

where $s^{(m)}_{av.}$ is the average value for the cross section of the conical bore from the start of motion to the attainment of v'_m, i.e., to the attainment of p_{max}. Since Λ'_m is known on the basis of the course of the $v' - \Lambda'$ curve for the cylindrical barrel, it is possible to determine:

$$\frac{s^{(m)}_{av.}}{s_0} \approx y^{(m)} = \sqrt[3]{1 - \frac{\Lambda_m}{\Lambda_{con.}}}.$$

Thus:

$$I_K = I_{K0}:y^{(m)} = I_{KD}\frac{s_D}{s_0}\frac{s_0}{s^{(m)}_{av.}}.$$

For $\Lambda_{con.}$ = about 6.0:

$$\Lambda'_m \approx 0.6,\ y^{(m)} = 0.97;$$

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For $\frac{d_0}{d_D} = 1.4$:

$$\frac{s_0}{s_D} = 1.96;\ I_K = I_{K0}\frac{1}{0.97} \approx 1.03 I_{K0};\ I_K = 1.03\frac{1}{1.96} I_{K0} = 0.527 I_{KD};$$

For $\Lambda_{con.} = 10$:

$$y_m = 0.983.$$

Consequently, in order to obtain an identical maximum pressure p_m, the thickness of the powder in the conical barrel ($d_0/d_D = 1.4$) must be a little greater (by 3%) than the thickness of the powder for the cylindrical barrel of caliber d_0, which is equal to the entrance caliber of the conical bore.

It must be considerably thinner (by nearly 50%) than the thickness of the powder for the cylindrical barrel of caliber d_D (at the same W_0, Δ, and ω/q):

$$I_{K0} < I_K \ll I_{KD}.$$

For this reason, it is erroneous to compare, as is sometimes done, a conical barrel and a cylindrical barrel of caliber d_D with the same powder thickness selected for this cylindrical barrel; in this case, the maximum pressure in the conical barrel will be obtained several times lower than in the cylindrical barrel, and the powder will not even burn to the end.

3. RELATION OF LENGTHS OF BARRELS WITH CONICAL AND CYLINDRICAL BORES.

Since the projectile is ejected from the conical bore while having a caliber d_D at a normal c_{qD}, which ensures its attaining the predetermined range and speed of encounter with an obstacle, the ballistic

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characteristics of the conical barrel should be compared with those for a cylindrical barrel having a caliber d_D, which is equal to the exit caliber of the conical bore.

At the same W_0, Δ, ω/q, p_m, and v_D, the working volumes of the bores W_D will also be equal.

Let us determine the relation between the lengths of the path l_D in the conical bore and $l_D^{(D)}$ in the cylindrical bore of caliber d_D.

From the condition of equality of working volumes W_D, we have:

$$W_D = s_D l_D^{(D)} = s_0 \frac{1 + y_D + y_D^2}{3} l_D$$

or:

$$l_D = l_D^{(D)} \frac{s_D}{s_0} \frac{3}{1 + y_D + y_D^2} = l_D^{(D)} \frac{3y_D^2}{1 + y_D + y_D^2}.$$

The relative diminution in the length of the conical bore is:

$$\frac{\delta l_D}{l_D^{(D)}} = \frac{l_D^{(D)} - l_D}{l_D^{(D)}} = \frac{1 + y_D - 2y_D^2}{1 + y_D + y_D^2}.$$

At the ratio $\frac{d_0}{d_D} = \frac{28}{20} = 1.4$:

$$y_D = 0.715; \; y_D^2 = 0.511;$$

$$\frac{\delta l_D}{l_D^{(D)}} = \frac{1.715 - 1.022}{1.715 + 0.511} = 0.311, \text{ or } 31.1\%.$$

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At $\frac{d_0}{d_D} = \frac{75}{55} = 1.362$:

$$y_D = 0.734; \quad y_D^2 = 0.539;$$

$$\frac{\delta l_D}{l_D^{(D)}} = \frac{1.734 - 1.078}{1.734 + 0.539} = \frac{0.656}{2.273} = 0.2885, \text{ or } 28.9\%.$$

With respect to the length of the conical barrel, the difference in length will give the following lengthening:

$$\frac{\delta l_D}{l_D^{(K)}} = \frac{l_D^{(D)} - l_D}{l_D^{(K)}} = \frac{l_D^{(D)}}{l_D^{(K)}} - 1 = \frac{1 + y_D - 2y_D^2}{3y_D^2}.$$

For 28/20: $\frac{\delta l_D}{l_D} = \frac{0.693}{3 \cdot 0.511} = 0.452$, or 45.2%;

for 75/55: $\frac{\delta l_D}{l_D} = \frac{0.656}{3 \cdot 0.539} = 0.405$, or 40.5%

<u>Conclusion</u>. At the same chamber and bore volumes W_0, W_D, and W_{KH}, and under the same loading conditions (q, ω, Δ, ω/q), a conical barrel d_0/d_D as compared with a cylindrical barrel of a caliber d_D equal to the exit caliber of the conical barrel, must give at existing ratios $d_0/d_D = 1.4$ the same initial velocity v_D and the same maximum pressure with a length of path of the projectile reduced by about 30% and with a powder

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thickness reduced in the following ratio:

$$\left(\frac{d_D}{d_0}\right)^2 = \frac{s_D}{s_0}.$$

The reduction in length constitutes the principal advantage of a conical barrel in comparison with a cylindrical barrel at equal exit calibers. This advantage possesses particular importance at high initial velocities of the projectile, when an excessively great length (about 150 d) is obtained for the cylindrical barrel, which makes the gun inconvenient for combat use and for transport. Moreover, great length combined with a small diameter results in sagging of the barrel and vibration during firing.

4. CONSIDERATION OF SECONDARY WORK IN CONICAL BORE.

The comparison between the ballistic characteristics of the conical and cylindrical bores presented above was conducted on the assumption that the coefficient φ, which takes into account the secondary work, is identical in the two cases. As a matter of actual fact, the gun with a conical bore has a number of features which reflect themselves in the magnitude and character of the secondary work.

The principal features include the following:

(a) The motion of the charge gases and of the as yet unburnt part of the charge takes place in a bore with a cross section which continuously decreases in the direction of motion of the projectile.

(b) There occurs a continuous deformation of the rotating bands of the projectile, which causes an equally continuous increase in the resistance forces until the projectile passes through the minimum cross

thickness reduced in the following ratio:

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section of the bore.

For the coefficient of secondary work φ, we shall adopt the following general expression: $\varphi = a_K + b_K \frac{\omega}{q}$. The first feature must be reflected in the coefficient b_K, which may be of considerable importance at high projectile velocities and at large $\frac{\omega}{q}$, at which, in fact, it is alone advantageous to employ conical barrels.

The second feature must be reflected in the magnitude of the term a_K, which takes into account the resistance forces developed during the deformation of the bands, these forces increasing continuously and retarding the motion of the projectile in considerably greater measure than is the case in a cylindrical bore. Simultaneously with this, there occurs an increase in the part of the work expended in the conical barrel to overcome friction over the ever-increasing area of contact between the bands of the projectile and the bore.

While in small-arms, where the entire lateral surface of the bullet cuts itself into the rifling grooves, a = 1.10 instead of 1.03 for artillery guns, a_K in the conical barrel must be still greater.

For a cylindrical barrel without chamber widening:

$$b = \frac{1}{3}.$$

For a cylindrical barrel with a chamber widening $\chi = l_0 / l_{KM}$:

$$b = \frac{1}{3} \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1},$$

where $\Lambda = \frac{l}{l_0}$ is the current value of the volumetric expansion ratio. As the projectile moves, b varies from $b_0 = \frac{1}{3}\frac{1}{\chi}$ at the start of the

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motion at $\Lambda = 0$ to $b_D = \frac{1}{3}\frac{\Lambda_D + \frac{1}{\chi}}{\Lambda_D + 1}$, where $b_D > b > b_0$, and approaches 1/3 as Λ_D increases.

In deriving the formula for b_K with consideration of chamber widening, the following assumptions have been made, which extend to the motion of the gases in the bore of the conical barrel.

The gas velocity in various cross sections varies in accordance with a linear law from the chamber bottom to the bottom of the projectile; the mass of the gases is uniformly distributed in the space, but what takes part in the motion is not the entire gas mass but only that which has a cross section s equal to the current cross section of the conical bore or to the cross section of the cylindrical bore. The internal friction of the gases and their friction against the walls of the bore are neglected.

Let the chamber of the conical bore have a widening characterized by the quantity $\chi = l_0 / l_{KM}$, and let the conical bore itself be characterized by the ratio of diameters $d_D / d_0 = y_D$ for the muzzle face and $d / d_0 = y$ for the current position of the projectile after the latter has traversed the path l.

The relative weight ω_{DB} of the gases taking part in the motion (upon the displacement of which the work $b_K \frac{\omega}{q}$ is expended), stated as a fraction of the total weight of the charge ω, is expressed by the following formula:

$$\frac{\omega_{DB}}{\omega} = \frac{s(l_{KM} + l)}{W_0 + W} = \frac{\frac{s l_{KM}}{s_0 l_0} + \frac{s_{av.}}{s_0}\frac{l}{l_0}\frac{s}{s_{av.}}}{1 + \Lambda} = \frac{\frac{s}{s_0}\frac{1}{\chi} + \frac{s}{s_{av.}}\frac{W}{W_0}}{1 + \Lambda} =$$

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motion at $\Lambda = 0$ to $b_D = \frac{1}{3}\frac{\Lambda_D + \frac{1}{\chi}}{\Lambda_D + 1}$, where $b_D > b > b_0$, and approaches 1/3 as Λ_D increases.

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$$= \frac{\frac{s}{s_0}\left(\frac{1}{\chi} + \frac{s_0}{s_{av.}}\Lambda\right)}{1 + \Lambda},$$

where $\Lambda = W/W_0$. Since:

$$\frac{s}{s_0} = y^2;\ \frac{s_{av.}}{s_0} = \frac{1 + y + y^2}{3} \approx y,$$

it follows that:

$$\omega_{DB} = \omega y^2 \frac{\frac{1}{\chi} + \frac{\Lambda}{y}}{1 + \Lambda} = \omega y \frac{\Lambda + \frac{y}{\chi}}{\Lambda + 1}.$$

The work expended upon the displacement of the cylindrical column of gas with the cross section s and weight ω_{DB} is represented in the over-all balance by the component:

$$\frac{1}{3}\frac{\omega_{DB}}{q} = \frac{1}{3}\frac{\omega}{q}\frac{\omega_{DB}}{\omega}.$$

Replacement of ω_{DB} by the expression for it gives:

$$b_K = \frac{1}{3}\, y \frac{\Lambda + \frac{y}{\chi}}{\Lambda + 1}.$$

For the cylindrical bore, $y = 1$, and we obtain the previously derived formula:

$$b = \frac{1}{3}\frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1};$$

Since for the conical bore $y < 1$, it follows that $b_K < b$.

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Thus, the work expended upon the displacement of the parts of the charge is smaller in a conical bore than in a cylindrical bore under conditions of identical values for χ, ω and Λ, the difference between the two continuously increasing as the projectile moves forward (since, as Λ increases, y in the expression for b_K decreases).

Examination of the expression for b_K shows that, at $\chi = 1$, $\Lambda = 0$, $y = 1$, $b_{K0} = 1/3$, and that, as Λ increases and y decreases, the quantity b_K decreases.

At $\chi > 1$, the quantity b_{K0} starts out with $b_{K0} = \frac{1}{3}\frac{1}{\chi}$, then, as Λ increases, it grows at first, passes through a maximum, and thereupon continusously decreases.

In this connection, the maximum b_K is obtained the later the larger χ, and the decrease in b_K proceeds the more rapidly the greater the conicity and the smaller $\Lambda_{con.} = W_{con.}/W_0$. There is presented below a table of values for the coefficients b_K and $b_{K_{av.}}$ in a conical barrel at $\Lambda_{con.} = 6.0$ and $\chi = 1.8$.

$\chi = 1.8$; $\Lambda_{con.} = 6.0$

Λ	0	0.2	0.4	0.6	0.8	1	2	3	4
b_K	0.185	0.205	0.219	0.228	0.235	0.238	0.241	0.227	0.203
$b_{K_{av.}}$	0.185	0.195	0.203	0.210	0.216	0.220	0.231	0.232	0.230

Diagrams of curves for $b_{K_{av.}}$ and $b_{av.}$ at various χ for conical and cylindrical bores are presented in fig. 178.

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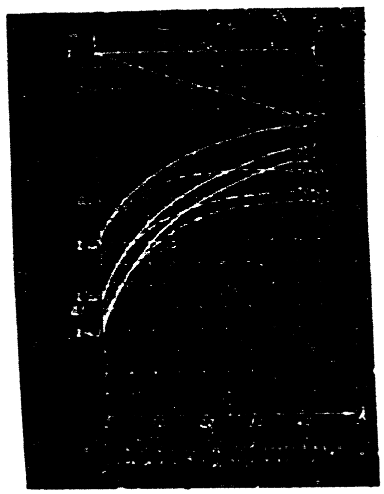

Fig. 178 - Variation of Coefficient b in Conical and Cylindrical Bores.

The guidance of a projectile with two skirt bands through a conical bore differs considerably from the guidance of an ordinary projectile with a copper rotating band through a cylindrical bore. In the latter, the resistance increases sharply as the band cuts itself into the rifling grooves. After the band has cut itself in to the full depth of the rifling grooves, the resistance drops sharply, and thereupon, to overcome the friction in the rifling grooves, there is consumed about 1% of the energy expended to communicate a forward motion to the projectile (k_3 = about 0.01). In this connection, the friction due to the radial force Φ is usually neglected.

During the motion of a projectile with two guiding bands through a conical bore, the cutting of the bands into the rifling grooves and the compression of progressively thicker parts of the bands must increase

the force required for their deformation.

Moreover, as a result of the diminution of the diameter and the bending of the bands toward the surface of the projectile, as well as because of the continuous increase in the surface of contact between the bands and the bore, the frictional force between the bands and the surface of the bore must progressively increase. This force must increase still more under the action of the gas pressure, which presses the rear band toward the surface of the bore, thus likewise increasing the frictional force.

There is presented below the procedure for taking into account the retarding forces in the conical bore.

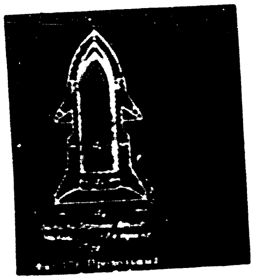

Fig. 179 - Longitudinal Section through 28/20 Armor-Piercing Projectile.

The arrangement of the rear obturating ring of the projectile for the German 28/20 gun is apparent from the sketch in fig. 179.

The part ab is the thinnest cylindrical part; the part bce is a conical part, which makes a smooth transition to the cylindrical part of the body; the inside part is turned to correspond to a certain curve.

The forward band has no appreciable cylindrical part. Both bands, in their longitudinal section, resemble bodies of equal bending strength, being wide at the base ef and narrowing down toward the end a. The

diameter d_0' is somewhat greater than the land diameter of the gun (d_0' = about 28.3, d_0 = 28.0). The diameter of the body of the projectile, d_1 = about 19 mm, is smaller than d_D = 20 mm, in order to give a clearance for the compression of the forward band:

$$b_0 = 3 \text{ mm}; \; b_1 \approx 9 \text{ mm}.$$

The angle of inclination of the generatrix of the cone ecb with respect to the axis of the projectile is α = about 30°. At the start of the motion, the projectile, while pressing apart the rolled-up part of the cartridge with its forward band, moves through the 35-mm long smooth cylindrical part of the bore, and only then cuts itself into the rifled conical part. The gases act upon the inner cavity of the rear band and press it toward the surface of the bore along the cylindrical part ab, whose surface at the start of the motion is:

$$S_{\Pi 0} = \pi d_0' b_0.$$

As they cut themselves into the conical part of the bore, both bands are compressed and elongated toward the rear. After ejection of the projectile, the rear band has the appearance represented in fig. 180; its outer surface shows traces of the rifling grooves.

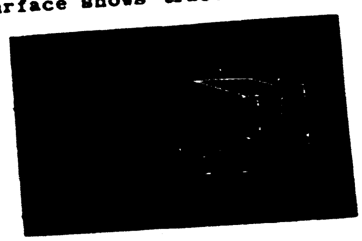

Fig. 180 - Rear Band after Compression.

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The deformation of the outer surface of the rear band and the law governing the change in the surface of contact can be determined with the aid of the simplified scheme represented in figs. 181 and 182.

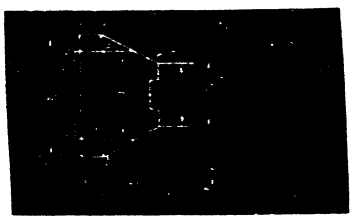

Fig. 181 - Scheme of Compression of Rear Band.

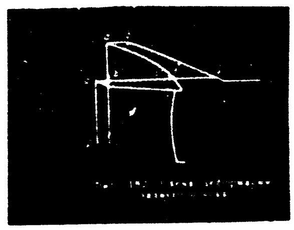

Fig. 182 - Scheme of Deformation of Rear Band.

The fundamental assumption made as a result of measuring the bands before and after the shot is that the length of the generatrix of the band always remains the same, i.e.:

$$l_n = ecba = ecb'a'.$$

The deformation of the surface of contact consists in the transformation of the initial cylindrical surface of diameter d_0' into a cylindrical surface of diameter d. The slight conicity is neglected, and the surface of contact is considered to be cylindrical:

$$S_n = \pi d(a'b' + b'c).$$

But:

$$b'c = bc = \frac{d_0' - d}{2 \sin \alpha} \approx \frac{d_0 - d}{2 \sin \alpha},$$

since the difference between the diameters d_0 and d_0' does not exceed 1%.

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Making use of the previously derived designation $y = d/d_0$, and taking into account that $s_0 = \frac{\pi}{4} d_0^2$, we have:

$$S_n = \pi d_0 y \left(b_0 + \frac{d_0 - d}{2 \sin \alpha} \right) = \frac{\pi d_0^2}{4} y \left[\frac{4b_0}{d_0} + \frac{2(1 - y)}{\sin \alpha} \right] =$$

$$= s_0 y^2 \left[\left(\frac{4b_0}{d_0} + \frac{2}{\sin \alpha} \right) \frac{1}{y} - \frac{2}{\sin \alpha} \right] =$$

$$= s \left[\left(\frac{4b_0}{d_0} + \frac{2}{\sin \alpha} \right) \frac{1}{y} - \frac{2}{\sin \alpha} \right],$$

where $s = s_0 y^2$ is the current value for the cross section of the conical bore.

Consequently, the bracketed expression represents the ratio of the surface of the band pressed against the surface of the conical bore to the cross section of this bore. Since y diminishes all the time, S_n / s continuously increases.

As the projectile moves through the conical bore, there will be developed the following frictional force:

$$R_T = \xi \nu_1 S_n p_{CH},$$

where $\xi < 1$, since, as a result of the rigidity of the metal, the pressure p is incompletely transmitted to the frictional surface. Moreover, the pressure p acts upon a surface which is smaller than S_n, especially at the end of compression of the band (cf. scheme in fig. 182, where the pressure is not transmitted to the segment ec').

The work required to overcome this force is:

$$\int_0^l R_T dl = \xi \nu_1 \int_0^l S_n p_{CH} dl .$$

Replacing S_n in accordance with the formula presented above, we obtain:

$$\int_0^l R_T dl = \xi \nu_1 \int_0^l \left[\left(4 \frac{b_0}{d_0} + \frac{2}{\sin \alpha} \right) \frac{1}{y} - \frac{2}{\sin \alpha} \right] p_{CH} \cdot s dl \ ;$$

but $s dl = dW$, and $\int p_{CH} dW$ = about $\frac{mv^2}{2}$.

Upon taking the bracketed expression outside the integral sign in the form of an average value, we obtain:

$$\int_0^l R_T dl = \xi \nu_1 \left[\left(4 \frac{b_0}{d_0} + \frac{2}{\sin \alpha} \right) \left(\frac{1}{y} \right)_{av.} - \frac{2}{\sin \alpha} \right] \frac{mv^2}{2} .$$

Consequently, the relative work expended to overcome friction is:

$$k_3'' = \xi \nu_1 \left[\left(4 \frac{b_0}{d_0} + \frac{2}{\sin \alpha} \right) \left(\frac{1}{y} \right)_{av.} - \frac{2}{\sin \alpha} \right] .$$

Since $y = \sqrt[3]{1 - \frac{\Lambda}{\Lambda_{con.}}}$, it follows that:

$$\frac{1}{y_{av.}} = \frac{1}{\Lambda / \Lambda_{con.}} \int_0^{\frac{\Lambda}{\Lambda_{con.}}} \left(1 - \frac{\Lambda}{\Lambda_{con.}} \right)^{-\frac{1}{3}} d \frac{\Lambda}{\Lambda_{con.}} .$$

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Upon introducing a new variable $t = 1 - \frac{\Lambda}{\Lambda_{con.}}$, we obtain after certain transformations:

$$\left(\frac{1}{y}\right)_{av.} = \frac{3}{2}\frac{1 - y^2}{\Lambda/\Lambda_{con.}}.$$

At $b_0/d_0 = 0.1$, $\alpha = 30^o$, $\xi = 1$, $\nu_1 = 0.10$, we obtain the following table of values of $(1/y)_{av.}$ and k_3'' (Table 5).

Table 5

$\frac{\Lambda}{\Lambda_{con.}}$	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70
$\left(\frac{1}{y}\right)_{av.}$	1	1.017	1.037	1.059	1.082	1.110	1.143	1.183
k_3''	0.040	0.9445	0.0563	0.0658	0.0762	0.0884	0.1028	0.1204

The numbers presented in the table show that the coefficient k_3'', in varying from 0.04 to 0.12, considerably exceeds in value the coefficient k_3 in the cylindrical barrel (k_3 = about 0.01).

But, in the expression for k_3'', no account is taken of the work expended upon the deformation of the bands and upon overcoming the resisting forces developing on the surface of contact between the bands and the bore.

To obtain more accurate data on the forces and energies expended during the drawing of a projectile through a conical bore, there were conducted in 1943-1945 tests on pressing projectiles for the 28/20 gun through dies of various conicities, the purpose being subsequently to compute the forces and energies required to press similar projectiles

through the barrel of the 28/20 gun.

In order to segregate the influence of each band, the forward band was reduced in diameter (on a lathe) on one set of projectiles, so that only the rear band remained in operation, while the rear band was reduced on another set of projectiles, so that only the forward band remained in operation; a third set of projectiles was pressed through with both bands intact.

The dies, 28 mm and 20 mm in diameter, differed in length and in their angles of conicity β (tan β = 0.040, 0.025, and 0.020).

The pressing through the dies was performed statically in an Amsler press with the aid of a rod which transmitted the pressure from the press to the bottom of the projectile.

The tests revealed the following relations:

1) In the presence of only one band - either the forward or the rear band - the force diagrams have the appearance represented in fig. 183, where (1) is the projectile with the forward band and (2) is the projectile with the rear band:

$$\Pi_{max\ 2} \approx 2\Pi_{max\ 1}.$$

Since the rear band is considerably thicker and more massive than the forward band, its entire force curve lies higher than the curve for the forward band.

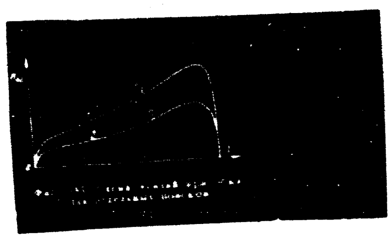

Fig. 183 - Scheme of Forces in Compressing Individual Bands. Ordinate: Π, kg.

2) In the presence of both bands, one of which is displaced with respect to the other by a certain distance, the force diagram has the appearance represented in fig. 184.

Fig. 184 - Summation of Forces in Compressing Both Bands.

The initial segment of the curve corresponds to the compression of only the forward band (while the rear band is still moving through the cylindrical part of the die); at the point a, the force Π_2 begins to be added to the force Π_1, and the Π_{1+2} curve is obtained by adding together the ordinates of the curves for Π_1 and Π_2, which are shifted with respect to each other by the distance a_1 between the bands.

The results of the pressing tests are summarized in Table 6.

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Table 6

Die no.	tan β	Length of conical part, mm	Π_{max} for both bands	$\Pi_{max\ 2}$ for rear band	$\Pi_{max\ 1}$ for forward band	A_2, kg·dm
			kg			
1	0.040	100	3500	2620	1350	1480/100
2	0.025	160	3250	2350	1180	1980/133
3	0.020	200	3100	2250	1100	2520/170

In all three cases, the conicity exerts a slight effect upon the magnitude of Π_{max}; as the conicity changes by a factor of two (from the first to the third case), Π_{max} changes by only 400 kg for both bands (equivalent to 11.5%) and by only 250 kg for the forward band (equivalent to 12.3%).

3) The area of the $\int_0^{l_M} \Pi dl$ diagram, where l_M is the length of the conical segment of the die, defines the magnitude of the work expended upon pressing the projectile through the die (fig. 185).

Fig. 185 - Forces as Functions of Angle of Slope of Cone.

The work is substantially dependent upon the path traversed by the projectile. It is least in the shortest die, so that:

$$\int_0^{l_M^{(1)}} \Pi dl < \int_0^{l_M^{(2)}} \Pi dl < \int_0^{l_M^{(3)}} \Pi dl .$$

To take into account the work necessary to press the projectile through the conical bore of the gun itself, the force diagram for the die, $\Pi_M = f_M(l)$, must be transformed into a force diagram applicable to the operation of pressing through the barrel, $\Pi_c = f_c(l)$.

5. DERIVATION OF FORMULA FOR RECOMPUTATION OF PRESSING FORCES FROM DIE TO BARREL (*)

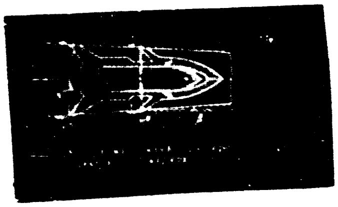

Fig. 186 - Scheme of Forces During Passage of Projectile.

As the projectile is pressed through a die or a barrel, it is acted upon by the following forces (Fig. 186):

1) The force of the press Π, which is directed along the axis of the projectile.

2) The reaction force N' perpendicular to the conical surface, which is uniformly distributed over the variable surface of contact of the forward band.

3) The analogous force N'' on the rear band.

4) The frictional force $\nu_1 N'$ on the forward band.

(*) Derivation performed by Engineer Shpigelburd.

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5) The frictional force $\nu_1 N''$ on the rear band.

Upon resolving the reaction forces N and νN into their components parallel and perpendicular to the axis of the bore and of the projectile, we find that the projectile is acted upon in the axial direction by the following three forces:

$$\Pi, \quad (N' + N'')\sin\beta, \quad (N' + N'')\nu_1 \cos\beta;$$

and that the following two forces act in the radial direction:

$$(N' + N'')\cos\beta, \ (N' + N'')\nu_1 \sin\beta.$$

For the static pressing of the projectile through the die, the equilibrium conditions for the forces in the axial direction will give the following equation (designating $N' + N'' = N$):

$$\Pi = N(\sin\beta + \nu_1 \cos\beta). \qquad (53)$$

The radial forces compressing each band:

$$\Phi' = N'(\cos\beta - \nu_1 \sin\beta) \text{ and } \Phi'' = N''(\cos\beta - \nu_1 \sin\beta)$$

cause the bands to undergo plastic deformation.

In transferring the pressing-force diagram from the die to the barrel, we make the assumption that identical radial forces act in similar cross sections of the die and of the barrel which correspond to one and the same diameter.

In such a case, for similar cross sections of the barrel and of the die, we can write the following equation of radial forces (the subscript "M" indicating the die, and the subscript "c" indicating the barrel):

$$N_M(\cos \beta_M - \nu_{1M} \sin \beta_M) = N_c(\cos \beta_c - \nu_{1c} \sin \beta_c). \qquad (54)$$

On the basis of formula (53), we have:

$$\Pi_M = N_M (\sin \beta_M + \nu_M \cos \beta_M); \ \Pi_c = N_c (\sin \beta_c + \nu_c \cos \beta_c),$$

from which:

$$\frac{\Pi_c}{\Pi_M} = \frac{N_c}{N_M} \frac{\sin \beta_c + \nu_c \cos \beta_c}{\sin \beta_M + \nu_M \cos \beta_M}.$$

Upon replacing the ratio N_c/N_M as indicated in expression (54), we obtain:

$$\Pi_c = \Pi_M \frac{\cos \beta_M - \nu_M \sin \beta_M}{\cos \beta_c - \nu_c \sin \beta_c} \frac{\sin \beta_c + \nu_c \cos \beta_c}{\sin \beta_M + \nu_M \cos \beta_M} =$$

$$= \Pi_M \frac{\tan \beta_M}{\tan \beta_c} \frac{\cot \beta_M - \nu_M}{\cot \beta_c - \nu_c} \frac{\tan \beta_c + \nu_c}{\tan \beta_M + \nu_M}. \qquad (55)$$

Since:

$$\cot \beta_M \gg \nu_M \text{ and } \cot \beta_c \gg \nu_c,$$

then, assuming:

$$\frac{\tan \beta}{\tan \beta_c} \frac{\cot \beta_M - \nu_M}{\cot \beta_c - \nu_c} = 1,$$

We can reduce formula (55) to the following simpler form:

$$\Pi_c = \Pi_M \frac{\tan \beta_c + \nu_c}{\tan \beta_M + \nu_M}. \qquad (56)$$

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Applying formula (56) to two dies of different conicities, assuming ν_M to be the same, and knowing Π_1 and Π_2 from the pressing diagram obtained with the aid of the press, there was obtained ν_M = = 0.16, which corresponds to the data accepted in technology. This demonstrates the correctness of the initial assumptions.

Formula (56) serves for recomputation of the forces required to press the projectile through the barrel.

Since the bore of the 28/20 gun consists of segments of different conicities, it follows that, in those places where the conicity changes, the rear band of the projectile will be on one segment, and its forward band will be on the other. For this reason, it is necessary to use formula (56) in such a manner as to transform from the die to the barrel the $\Pi - l$ diagrams for each band separately, whereupon the diagrams are added together.

During rapid motion of the projectile through the bore, the coefficient of friction decreases in conformity with the following formula:

$$\nu = \nu_0 \frac{1 + a_1 v}{1 + a_2 v},$$

where $a_1 < a_2$.

In accordance with the data of M.M. Shlyaposhnikov, $\nu_0 = 0.27$, $a_1 = 0.0213$, $a_2 = 0.133$. In accordance with the data of Robinson [3], ν is close to 0.05 at $v > 200$ m/sec.

Assuming ν_c to have an average value (0.1 or less), we can use formula (56) to obtain the forces involved in pressing the projectile through the barrel.

For the 28/20 gun, which comprises three segments of different conicities, there is obtained at $\nu_{av.} = 0.10$ the $\Pi = f(l)$ force

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diagram represented in fig. 187.

Fig. 187 - Resistance to Drawing through 28/20 Barrel.

1) cylindrical bore; 2) conical bore of 28/20 antitank gun; 3) profile of 28/20 antitank gun bore.

Computation of the work involved in static pressing through the bore at $\nu_c = 0.10$ gave the value of $A = 1320$ kg·m, which constitutes about 10% of the muzzle energy of the projectile. At existing conicities, $\int \Pi dl$ depends in considerably greater measure upon the coefficient of friction ν than upon $\tan \beta$.

As is seen from the results of the computation, this value is considerably greater than the work required to overcome friction in a cylindrical barrel, where k_3 = about 0.01, or approximately 1%.

The coefficient φ_K for the conical barrel can thus be represented in the following form:

$$\varphi_K = a_K + b_K \frac{\omega}{q},$$

where:

$$a_K = 1 + k_2 + k_3' + k_3'' + k_3^{(K)} + k_5.$$

Here, k_2 is the relative work consumed in rotating the projectile, k_2 is about 0.01;

diagram represented in fig. 187.

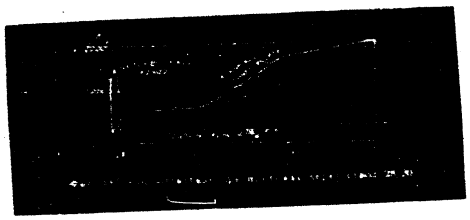

Fig. 187 - Resistance to Drawing through 28/20 Barrel.

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$$\varphi_K = a_K + b_K \frac{\omega}{q},$$

where:

$$a_K = 1 + k_2 + k_3' + k_3'' + k_3^{(K)} + k_5.$$

Here, k_2 is the relative work consumed in rotating the projectile, k_2 is about 0.01;

k_3' is the relative work consumed in overcoming friction on the driving edges of the two bands, k_3' = about 0.02;

$k_3^{(K)}$ is the relative work consumed in overcoming the resistance forces due to the friction of the bands against the surface of the bore and by the deformation

of the bands, $k_3^{(K)} = \dfrac{\int_0^l \Pi\, dl}{\dfrac{mv^2}{2}}$, $k_3^{(K)} \approx 0.10$;

k_3'' is the relative work consumed in overcoming the additional friction caused by the pressing of the rear band against the surface of the bore under the action of the gas pressure, $k_3'' = 0.04-0.08$;

k_5 is the relative work consumed by the recoil, $\dfrac{k_5 \approx 0.01}{\Sigma k_i = 0.18-0.22}$.

Thus:

$$a_{K_{av.}} = 1.20; \quad b_K = \frac{1}{3}\, y\, \frac{\Lambda + \dfrac{y}{\chi}}{\Lambda + 1} \approx 0.22.$$

For Cylindrical Bore: For Conical Bore:

For charge $\dfrac{\omega}{q} = 1$

$$\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q} = 1.363. \qquad \varphi_K = 1.20 + 0.222\,\frac{\omega}{q} = 1.422;$$

For charge $\dfrac{\omega}{q} = 1.5$

$$\varphi = 1.03 + \frac{1}{3}\,\frac{3}{2} = 1.53. \qquad \varphi_K = 1.20 + 0.333 = 1.533;$$

For charge $\frac{\omega}{q} = 2.0$

$$\varphi = 1.03 + \frac{2}{3} = 1.70. \qquad \varphi_K = 1.20 + 0.444 = 1.644.$$

Consequently, for the charge $\omega/q = 1.5$ (which corresponds to $v_D =$ = about 1500 m/sec), the identical coefficients φ and φ_K are obtained for the cylindrical and conical bores, even though the components a and $b\frac{\omega}{q}$ differ:

$$a_K > a, \; b_K < b.$$

At smaller relative charges ω/q, the coefficient φ_K for the conical barrel is greater than φ for the cylindrical barrel. At $\frac{\omega}{q} > 1.5$, at projectile velocities $v_D > 1500$ m/sec, $\varphi_K < \varphi$, and the conical bore is found to be more advantageous, since, at a large relative charge, the decrease in the coefficient $b_K\frac{\omega}{q}$ is more pronounced.

At very high initial projectile velocities, higher than 1500 m/sec, the barrel with the conical bore is more advantageous not only because it considerably reduces the length of the bore, but also because it reduces the quantity of work consumed in moving the gases in the narrowing bore $\left(\text{term } b_K\frac{\omega}{q}\right)$.

If the values for the coefficients k_3'' and $k_3^{(K)}$ are not averaged and the problem is solved for variable magnitudes of the resistance forces, the following system of equations is obtained:

1) Equation of motion:

$$sp_{CH} - \xi\nu_1 S_n p_{CH} - \Pi = \varphi_1 m\frac{dv}{dt}. \qquad \text{(a)}$$

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2) Equation of work or equation of transformation of energy:

$$p(W_{\psi} + W) = f\omega\psi - \frac{\theta}{2}\varphi_2 mv^2 - \theta\int_0^l \Pi dl - \xi\int_0^l \nu_1 S_{\Pi} p dl. \qquad \text{(b)}$$

3) Law of burning rate:

$$u = u_1 p. \qquad \text{(c)}$$

4) Law of gas formation:

$$\psi = \varkappa z + \varkappa\lambda z^2. \qquad \text{(d)}$$

5) Relation between p_{CH} and p (average):

$$p = p_{CH}\left[1 + \frac{1}{3}\frac{y}{\varphi_1}\frac{\Lambda + \frac{y}{\chi}}{\Lambda + 1}\right]. \qquad \text{(e)}$$

The coefficient φ_1 takes into account the work of the resistance forces on the driving edges of the rifling grooves and the work consumed in rotating the projectile. It is possible to assume that φ_1 = = about 1.02. The coefficient φ_2 takes into account all the usual forms of work, except for the work accounted for separately by the last two terms in equation (b):

$$\varphi_2 = 1 + k_2 + k_3' + k_4 + k_5 \approx 1.03 + b_K \frac{\omega}{q},$$

where:

$$b_K = \frac{1}{3} y \frac{\Lambda + \frac{y}{\chi}}{\Lambda + 1}, \quad y = \frac{d}{d_0}, \quad \chi = \frac{l_0}{l_{KM}}.$$

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The system represented by these equations is solved by the method of resolution into a Taylor's series, accompanied by the use of a series of diagrams expressing the dependence of Π, $\int \Pi dl$, S_{Π}, b_K and φ_2 upon W or Λ and the relation between ν_1 and the velocity of the projectile.

For verifying the correctness of the relations derived above by taking into account the various forms of secondary work, it is expedient to utilize the above system of equations (a-e) at very small charges, using a very rapid-burning powder.

In this case, which is close to the case of instantaneous burning of the powder, equations (c) and (d) are eliminated, the coefficient φ_2 is close to being constant because of the smallness of ω/q, and the retarding forces $R = \xi \nu_1 S_{\Pi} p_{CH}$ and Π acquire predominant importance and can more easily be taken into account.

If equation (a) is written in the following form:

$$sp_{CH}\left[1 - \xi \nu_1 \frac{S_{\Pi}}{s} - \frac{\Pi}{sp_{CH}}\right] = \varphi_1 m \frac{dv}{dt},$$

it becomes possible to select values of Δ and p_{CH} at which the bracketed expression will be very small, and may in a certain instant even become less than zero; the projectile will be retarded to such an extent that it will stop without emerging from the bore.

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CHAPTER 2 - METHOD OF SOLUTION OF PROBLEM OF INTERIOR BALLISTICS

The fundamental assumptions are the same as in solving the problem for the usual cylindrical barrel: instantaneous ignition, geometric law of burning of the powder, law of rate of burning $u = u_1 p$, instantaneous cutting of the band into the rifling grooves at the pressure p_0 to overcome the inertia of the projectile; unchanging gas composition during expansion.

The fundamental distinction is the variable cross section of the bore.

Fundamental Relations

Equation of motion:

$$\varphi m \frac{dv}{dt} = sp \tag{57}$$

Law of rate of burning:

$$u = \frac{de}{dt} = u_1 p \tag{58}$$

Equation of elementary work:

$$\varphi m v dv = ps dl = pdW \tag{59}$$

Equation of transformation of energy:

$$p(W_\psi + W) = f\omega\psi - \frac{\theta}{2}\varphi m v^2$$

where:

$$W_\psi = W_0 \left[1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi\right] \tag{60}$$

is the free volume of the chamber in the instant when the fraction of the charge ψ has burned.*)

From (57) and (58), we have, as usual: $sde = u_1\varphi m dv$, or:

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*) $\varphi = a_K + b_K \frac{\omega}{q}$ in accordance with the formulas in the preceding chapter.

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$$dv = \frac{s}{\varphi m}\,\frac{e_1}{u_1}dz - \frac{sI_K}{\varphi m}dx, \tag{61}$$

where:

$$x = z - z_0 = \frac{e}{e_1} - \frac{e_0}{e_1};$$

x is the relative thickness of the powder burnt since the start of motion of the projectile; e_0 is the thickness of the powder burnt in the instant of the start of motion; z_0 is the relative powder thickness; and ψ_0 is the fraction of the charge burnt prior to the same instant of time.

We introduce into equation (61) the relative quantity $\frac{s}{s_0}$:

$$dv = \frac{s_0 I_K}{\varphi m}\quad\frac{s}{s_0}dx. \tag{62}$$

The fundamental difficulty in solving the system of equations for the conical barrel consists in the fact that the connection between $\frac{s}{s_0}$ and v or x is not known in advance; it will be established later.

This makes it impossible to integrate equation (61).

We propose taking this connection from the solution of the problem for the cylindrical barrel, where we obtain a connection between v and W or:

$$\Lambda = \frac{W}{W_0}$$

From the fundamental assumption that, under identical loading conditions for the conical and cylindrical barrels, the v-Λ curves for both coincide, we obtain the relation between $\frac{s}{s_0}$ and W or Λ. Since the quantity Λ will subsequently be the argument, the ratio $\frac{s}{s_0}$ will be known in accordance with it, and the equation can be

integrated.

By integrating it, we obtain:

$$v = \frac{s_0 I_K}{\varphi m} \int_0^x \frac{s}{s_0} dx.$$

Taking $\frac{s}{s_0}$ outside the integral sign in the form of the average value of $\frac{s_{av.}}{s_0}$ from 1 to $\frac{s}{s_0}$, corresponding to the relative volume Λ and the velocity v in the cylindrical barrel, we have:

$$v = \frac{s_0 I_K}{\varphi m} \frac{s_{av.}}{s_0} x, \tag{63}$$

and, since we have accepted the condition that $v = v_u$, it follows that:

$$\frac{s_0 I_K}{\varphi m} \frac{s_{av.}}{s_0} x = \frac{s_0 I_K}{\varphi m} x_u,$$

from which:

$$x = \frac{x_u}{\frac{s_{av.}}{s_0}} > x_u. \tag{64}$$

From equations (59) and (60), we obtain the following relation between W and Λ and x:

$$\frac{dW}{W_\psi + W} = \frac{d\Lambda}{\Lambda_\psi + \Lambda} = \frac{1}{f\omega} \frac{\varphi m v dv}{\psi - \frac{v^2}{v_{np}^2}}, \tag{65}$$

where $\psi = f(z)$ is expressed by the following binomial formula:

$$\psi = \varkappa z + \varkappa\lambda z^2 = \psi_0 + \varkappa\sigma_0 x + \varkappa\lambda x^2.$$

Upon substituting into the right-hand side of equation (65) the values for dv in accordance with equation (61) and for v in accordance with equation (63), and upon introducing the designation:

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$$B_0 = \frac{s_0^2 I_K^2}{f\omega\varphi m},$$

we obtain:

$$\frac{d\Lambda}{\Lambda_\psi + \Lambda} = \frac{B_0 \frac{s_{av.}}{s_0} \frac{s}{s_0} x\,dx}{\psi_0 + \varkappa G_0 x - \left[B_0 \left(\frac{s_{av.}}{s_0}\right)^2 \frac{\theta}{2} - \varkappa\lambda\right] x^2} = \frac{B_0 \frac{s_{av.}}{s_0} \frac{s}{s_0} x\,dx}{\psi_0 + k_1 x - B_{1s} x^2}, \tag{66}$$

where:

$$B_{1s} = \frac{B_0 \theta}{2} \left(\frac{s_{av.}}{s_0}\right)^2 - \varkappa\lambda.$$

The analogous equation for a cylindrical barrel of caliber d_0 has the following form:

$$\frac{d\Lambda}{\Lambda_\psi + \Lambda} = \frac{B_0 x_u dx_u}{\psi_0 + k_1 x_u - B_1 x_u^2}, \tag{67}$$

where:

$$B_1 = \frac{B_0 \theta}{2} - \varkappa\lambda,$$

and B_0 is the same as above:

$$B_0 = \frac{s_0^2 I_K^2}{f\omega\varphi m}.$$

Comparison of expressions (66) and (67) shows that the parameters B_0 and B_1, which are constant for the cylindrical barrel, become in equation (66) for the conical barrel variable and dependent upon the variation in the cross section of the barrel.

But the product $\frac{s_{av.}}{s_0} \quad \frac{s}{s_0}$ in the numerator of formula (66) is a function of Λ and can be transferred to the left-hand side during integration, while the last term in the denominator is relatively small in comparison with the sum of the first two, and the variable

quantity $\left(\frac{s_{av.}}{s_0}\right)^2$ therein may be assumed to equal an average value, either one and the same for the entire interval of integration or different depending upon the quantity x.

In this case, equation (66) can be integrated, and the relation between Λ and x can be obtained in its final form.

On the basis of the above discussion:

$$\frac{s}{s_0} = y^2; \quad \frac{s_{av.}}{s_0} \approx y; \qquad \frac{s_{av.}}{s_0}\frac{s}{s_0} = y^3 = 1 - \frac{\Lambda}{\Lambda_{con.}} = f(\Lambda).$$

Separating the variables, we obtain:

$$\frac{d\Lambda}{(\Lambda_\psi + \Lambda)\left(1 - \frac{\Lambda}{\Lambda_{con.}}\right)} = \frac{B_0 x dx}{\psi_0 + k_1 x - B_{1s} x^2} = -\frac{B_0}{B_{1s}}\frac{xdx}{x^2 - \frac{k_1}{B_{1s}}x - \frac{\psi_0}{B_{1s}}}. \tag{68}$$

The right-hand side of equation (68) shows no external differences from the right-hand side of a similar equation for the cylindrical barrel and represents a differential of the function of Professor N. F. Drozdov (in which connection, during integration, B_{1s} must be taken as an average value $\bar{B}_{1s}$ over the given interval of integration):

$$-\frac{B_0}{\bar{B}_{1s}}\int_0^x \frac{xdx}{x^2 - \frac{k_1}{\bar{B}_{1s}}x - \frac{\psi_0}{\bar{B}_{1s}}} = \ln Z_x^{-\frac{B_0}{\bar{B}_{1s}}},$$

This function is found from the basic quantities:

$$\gamma = \frac{\bar{B}_{1s}\psi_0}{k_1^2}, \quad \beta = \frac{\bar{B}_{1s}}{k_1}x,$$

the magnitude of $\bar{B}_{1s}$ either varied in moving from one value of x to another or else being retained constant for the entire first period.

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The left-hand side of equation (68) is likewise capable of integration at $\Lambda_\psi = \Lambda_{\psi av.}$, in which connection $\Lambda_{\psi av.}$ may likewise be taken either constant for all values of ψ, or else, which is better, its own value is taken each time for each value of $\psi = \psi_0 + k_1 x + \varkappa\lambda x$ used (as is commonly done to obtain a more exact solution at an average l_ψ). We resolve the function under the integral on the left-hand side of equation (68) into the simplest fractions:

$$\frac{1}{(\Lambda_{\psi av.}+\Lambda)\left(1-\frac{\Lambda}{\Lambda_{con.}}\right)} = \frac{\Lambda_{con.}}{(\Lambda_{\psi av.}+\Lambda)\ (\Lambda_{con.}-\Lambda)} = \frac{a}{\Lambda_{\psi av.}+\Lambda} + \frac{b}{\Lambda_{con.}-\Lambda};$$

$$\Lambda_{con.} = a\Lambda_{con.} - a\Lambda + b\Lambda_{\psi av.} + b\Lambda.$$

To determine a and b, we have two equations:

$$a\Lambda_{con.} + b\Lambda_{\psi av.} = \Lambda_{con.} \quad \text{and} \quad a - b = 0,$$

from which:

$$a = b \quad \text{and} \quad a + b\frac{\Lambda_{\psi av.}}{\Lambda_{con.}} = 1;$$

$$a = b = \frac{\Lambda_{con.}}{\Lambda_{\psi av.}+\Lambda_{con.}} = \frac{1}{1+\frac{\Lambda_{\psi av.}}{\Lambda_{con.}}}.$$

Consequently:

$$\frac{\Lambda_{con.}d\Lambda}{(\Lambda_{\psi av.}+\Lambda)(\Lambda_{con.}-\Lambda)} = \frac{\Lambda_{con.}}{\Lambda_{con.}+\Lambda_{\psi av.}}\left[\frac{d\Lambda}{\Lambda_{\psi av.}+\Lambda} - \frac{-d\Lambda}{\Lambda_{con.}-\Lambda}\right] =$$

$$= \frac{\Lambda_{con.}}{\Lambda_{con.}+\Lambda_{\psi av.}}\, d\ln\frac{\Lambda_{\psi av.}+\Lambda}{\Lambda_{con.}-\Lambda}.$$

By integrating equation (68) between the limits from zero to Λ and from zero to x, we obtain:

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$$\left(\frac{1 + \dfrac{\Lambda}{\Lambda_{\psi\,av.}}}{1 - \dfrac{\Lambda}{\Lambda_{con.}}}\right)^{\dfrac{\Lambda_{con.}}{\Lambda_{con.} + \Lambda_{\psi\,av.}}} = Z_x^{-\dfrac{B_0}{B_{1s}}} \qquad (69)$$

or, introducing the designation:

$$\frac{B_0}{B_{1s}}\left(1 + \frac{\Lambda_{\psi\,av.}}{\Lambda_{con.}}\right) = A_K,$$

$$\frac{1 + \dfrac{\Lambda}{\Lambda_{\psi\,av.}}}{1 - \dfrac{\Lambda}{\Lambda_{con.}}} = Z_x^{-A_K}. \qquad (70)$$

By solving equation (69) with respect to Λ, we obtain:

$$\Lambda = \Lambda_{\psi\,av.}\,\frac{Z_x^{-A_K} - 1}{1 + \dfrac{\Lambda_{\psi\,av.}}{\Lambda_{con.}}\,Z_x^{-A_K}}. \qquad (71)$$

For the cylindrical barrel, we had:

$$\Lambda_{ц} = \Lambda_{\psi\,av.}\left(Z_x^{-\dfrac{B_0}{B_1}} - 1\right), \qquad (72)$$

where:

$$B_1 = \frac{B\theta}{2} - \varkappa\lambda.$$

The numerator of formula (71) for the conical barrel has the same form, with the sole difference that the exponent $\frac{B_0}{B_1}$ is replaced by the exponent:

$$A_K = \frac{B_0}{B_{1s}}\left(1 + \frac{\Lambda_{\psi\,av.}}{\Lambda_{con.}}\right),$$

where:

$$\overline{B}_{1s} = \frac{B\Theta}{2}\left(\frac{s_{av.}}{s_0}\right)^2 - \varkappa\lambda.$$

is not constant, but varies depending upon the variation in the cross section of the bore. Moreover, there is present a denominator greater than unity, which increases as the powder burns during the motion of the projectile through the conical barrel.

Thus, formula (70) reflects the influence of the variable cross section of the bore and of those specific features which make their appearance as the projectile moves through the conical barrel.

The formula for the pressure will have the following form:

$$p = f\Delta \frac{\psi - \frac{v^2}{v^2_{\Pi p}}}{\Lambda_\psi + \Lambda} = f\Delta \frac{\psi_0 + k_1 x - B_{1s} x^2}{\Lambda_\psi + \Lambda}, \tag{73}$$

i.e., its structure does not differ from that of the formula for the cylindrical barrel. Since, for definite values of x or ψ, the velocity $v < v_ц, \Lambda < \Lambda_ц$ for the conical barrel, the pressure in the conical bore exceeds that in the cylindrical barrel ($p > p_ц$).

Formula (73) can be written in the following form:

$$p = f\Delta \frac{\psi - \frac{B_0\Theta}{2}\left(\frac{s_{av.}}{s_0}\right)^2 x^2}{\Lambda_\psi + \Lambda} \tag{74}$$

By differentiating it with respect to x and equating the derivative to zero, we obtain a formula for x_m at which the gas pressure reaches its maximum.

Without presenting a detailed derivation, we shall write down the formula in its final form:

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$$x_m = \frac{k_1}{\dfrac{B_0\left(\dfrac{s_{av.}}{s_0}\right)_m^2 \left[\left(\dfrac{s_{av.}}{s_0}\right)_m + \Theta\right]}{1 + \dfrac{p_m}{f}\left(a - \dfrac{1}{\delta}\right)} - 2\varkappa\lambda};$$

At s = const., this formula becomes transformed into the usual formula for the cylindrical bore.

The appearance of the $p - \Lambda$ and $v - \Lambda$ curves is represented in Fig. 188.

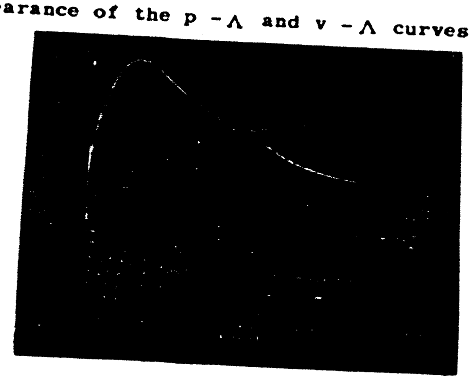

Fig. 188 - $p - \Lambda$ and $v - \Lambda$ Curves in Conical and Cylindrical Bores.

1) $p - \Lambda$ Curve } in gun with cylindrical bore
1') $v - \Lambda$ Curve }
2) $p - \Lambda$ Curve } in gun with conical bore
2') $v - \Lambda$ Curve }

In the method described above, there was accepted and utilized the proposition that the $v - \Lambda$ curves in the conical and cylindrical barrels coincide. In this connection, the quantity $x = z - z_0$ was taken as the argument. But if the quantity Λ is taken as the argument, the same formulas can be used as a basis to give a different order of computation of the elements of the curves, which has been proposed by I. M. Belenky. Knowing and assigning the quantity

and the corresponding values of $\frac{s}{s_0}$ and $\frac{s_{av.}}{s_0}$, and assuming in the first approximation for the computation of $\Lambda_{\psi av.}$ the value $\psi = 1$ and $\psi_{av.} = \frac{\psi_0 + 1}{2}$, it is possible to compute the function:

$$\log Z_x^{-1} = \frac{1}{\Lambda_K} \log \frac{1 + \frac{\Lambda}{\Lambda_{\psi av.}}}{1 - \frac{\Lambda}{\Lambda_{con.}}}, \tag{75}$$

where:

$$\Lambda_K = \frac{B_0}{B_{1s}} \left(1 + \frac{\Lambda_{\psi av.}}{\Lambda_{con.}}\right) \quad \text{and} \quad \bar{B}_{1s} = \frac{B_0 \Theta}{2} \left(\frac{s_{av.}}{s_0}\right)^2 - \varkappa\lambda.$$

Thereupon, from the table of the function $\log Z_x^{-1}$ and the quantity $\gamma = \frac{B_{1s}}{k_1^2} \psi_0$, there are found the values of $\beta = \frac{B_{1s}}{k_1} x$, from which $x = \frac{k_1}{B_{1s}} \beta$.

Since $\Lambda_{\psi av.}$ enters into the first part of formula (75), while ψ is not known in advance, it becomes necessary in the initial computations to accept an average value of $\psi = \frac{1}{2}$, which is correct only for the end of burning of the powder and introduces an error in determining the elements of the intermediate points of the first period. For these points, the pressure is obtained higher than is actually the case. It is necessary to proceed for these points by the method of successive approximations, the number of approximations being reducible to two if we assume $\psi = \left(\frac{\Lambda}{\Lambda_K}\right)^{2/3}$, where $\Lambda_K = \frac{W_K}{W_0}$ is the relative volume of the bore in the instant of the end of burning of the powder, and $\Lambda = \frac{W}{W_0}$ is the current value of the relative volume.

The character of the variation of Λ_ψ and $\Lambda_{\psi av.}$ and their influence upon the magnitude of the pressure are apparent from the diagram in Fig. 189.

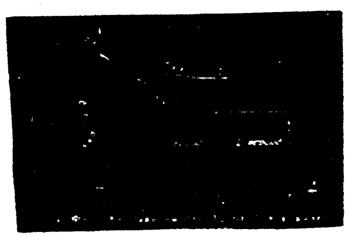

Fig. 189 - Dependence of Λ_ψ upon Λ in Conical Bore.
1) at; 2) period.

The actual values of $\Lambda_{\psi av.}$ vary as a function of Λ along a certain curve ab; acceptance of $\psi_0 = \frac{1}{2}$ for all points of the first period gives a straight line parallel to the abscissa.

The formulas for the second period have the usual form:

$$p = p_K \left(\frac{w_1 + w_K}{w_1 + w} \right)^{1+\theta} = p_K \left(\frac{1 - \alpha\Delta + \Lambda_K}{1 - \alpha\Delta + \Lambda} \right)^{1+\theta},$$

where:

$$w_1 = w_0(1 - \alpha\Delta) \quad \text{and} \quad \Lambda > \Lambda_K.$$

$$v = v_{np} \sqrt{1 - \left(\frac{1 - \alpha\Delta + \Lambda_K}{1 - \alpha\Delta + \Lambda_D} \right)^{\theta} \left(1 - \frac{v_K^2}{v_{np}^2} \right)},$$

where:

$$v_{np} = \sqrt{\frac{2gf\omega}{\varphi_K \theta q}} \quad \text{and} \quad v_K = \frac{s_0 I_K}{\varphi_K m} \frac{s_{K\,av.}}{s_0} (1 - z_0).$$

CHAPTER 3 - BALLISTIC DESIGN OF CONICAL BARREL

Since conical guns will be employed only for the purpose of attaining very high initial projectile velocities under conditions

when ordinary guns have an excessive length, we shall find in our design a conical barrel which corresponds to a cylindrical barrel with the minimum bore volume. For this reason, such a design will be based on the procedure and auxiliary tables presented in the division entitled "Ballistic Design of Guns".

On the basis of tactical and technical requirements, let there be predetermined the exit caliber d_D, the weight of the projectile q, and the initial projectile velocity v_D for the conical barrel.

Additionally, we determine $c_{qD} = \frac{q}{d_D^3}$; $\frac{v_D^2}{2g}$; and $C_\varepsilon = c_{qD} \frac{v_D^2}{2g}$. Since c_{qD} for armor-piercing projectiles may fluctuate in the range of 16-18, while the limit for the minimum c_{q0} based on the entrance caliber equals 6.0-7.0, the possible values for the ratios of the entrance to the exit caliber lie within rather narrow limits, namely:

$$\beta = \frac{d_0}{d_D} = \sqrt[3]{\frac{c_{qD}}{c_{q0}}} = \sqrt[3]{\frac{16}{7} \ldots \frac{18}{6}} = 1.32 \ldots 1.44.$$

In German guns, the accepted ratios are $\beta = 1.4$ for the 28/20 gun and $\beta = 1.363$ for the 75/55 gun.

The ratio:

$$\beta = \sqrt[3]{\frac{16}{6}} = \sqrt[3]{\frac{18 \cdot 67}{7}} = 1.385$$

may be recommended.

After selecting the entrance caliber d_0 in such a manner as to obtain c_{q0} = about 6.0, we find the fundamental characteristics of the minimum-volume cylindrical gun at $c_q = 6.0$ and at the chosen values of v_D and p_m:

$$\eta_{\omega_0} = 85 - 82 \text{ tm/kg}; \quad \frac{\omega_0}{q}; \quad \varphi_K = a_K + b_K \frac{\omega}{q}, \text{ where } a_K \approx 1.20;$$

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$$b = 0.222; \quad v_{tab.D} = \frac{v_D}{n}; \quad n = \sqrt{\frac{\omega_0}{\varphi_K q}};$$

From the GAU Tables, we find:

$$B, \Lambda_K, \Lambda_D, \eta_K = \frac{\Lambda_K}{\Lambda_D}; \frac{l_0}{d}; W_0, \frac{L'_{KH}}{d_0} = \frac{l_0}{d} \cdot (\Lambda_D + 1); \frac{l_D}{d_0} = \frac{l_0}{d} \cdot \Lambda_D.$$

Here, $d = d_0$ equals the entrance caliber of the conical gun.

On the basis of these data, we determine the chamber volume $W_0 = s_0 l_0$, where $s_0 = n_s d_0^2$. The working volume of the bore is $W_D = s_0 l_D$, and this volume will be equal to the volume of the cone. Knowing the volume of the truncated cone W_D and the ratio of its diameters $y_D = \frac{d_D}{d_0}$, we use the formula $W_D = W_{con.}(1 - y_D^3)$ to find the volume of the entire cone to its apex:

$$W_{con.} = \frac{W_D}{1 - y_D^3}.$$

Since for the truncated cone:

$$W_D = s_{av.} l_D = s_0 \frac{1 + y_D + y_D^2}{3} l_D,$$

it follows that the length of the path of the projectile through the conical bore is:

$$l_D = \frac{3W_D}{s_0(1 + y_D + y_D^2)} \quad \text{and} \quad \tan\beta = \frac{d_0 - d_D}{2 l_D}.$$

After designating:

$$\chi = \frac{l_0}{l_{KM}},$$

we find:

$$\frac{l_0}{\chi} = l_{KM} \quad \text{and} \quad L_{KH} = l_{KM} + l_D$$

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and finally the total length of the barrel:

$$L_{CT} = L_{KH} + 2d_0$$

($2d_0$ being reserved for the breechblock).

Having assigned $\Lambda_m = 0.6$ and found $W_m = 0.6W_0$, we determine:

$$y_m = \sqrt[3]{1 - \frac{W_T}{W_{con.}}} \quad \text{and} \quad \frac{\bar{s}_{av.m}}{s_0} = \frac{1 + y_m}{2} .$$

We find $\frac{I_K}{d_0} = \frac{I_{K,\text{ц}}}{d} \frac{s_0}{\bar{s}_{av.\ m}}$ or the pressure impulse $I_K = I_{K,\text{ц}0} \frac{s_0}{s_{av.\ m}}$ which ensures attainment of the predetermined pressure p_m. As is shown by theory, the end of burning will be transferred closer to the start of motion of the projectile, and $\eta_K = \frac{W_K}{W_D}$ in the conical barrel will be smaller than $\eta_{K\text{ц}} = \frac{W_K}{W_D} = \frac{l_{K\text{ц}}}{l_{D\text{ц}}}$ in the cylindrical barrel.

To compute the pressure and velocity curves for the conical barrel, we first compute and construct the $v - W$ or $v - \Lambda$ curve for the cylindrical barrel found. This is done most simply with the aid of the ANII or 1942 GAU Tables.

On the basis of the values for v obtained in this manner, we compute the values of $x_{\text{ц}} = \frac{v}{v_0}$, where $v_0 = \frac{s_0 I_K}{\varphi m}$ is the velocity of the projectile at the end of burning of the powder in the absence of any pressure to overcome the inertia of the projectile.

From the values of W corresponding to the values of v taken from the curve for the cylindrical barrel, we successively find the values of:

$$y = \sqrt[3]{1 - \frac{W}{W_{con.}}} ,$$

where:

$$W_{con.} = \frac{W_D}{1 - y_D^3} ; \quad \frac{s_{av.}}{s_0} = \frac{1 + y + y^2}{3} \approx y$$

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By dividing the values of x_u by $\frac{s_{av.}}{s_0}$, we find for the same v and W the values of $x = \frac{x_u}{s_{av.}/s_0}$ for the conical barrel.

The quantity $x_K = 1 - z_0$ will indicate the volume W_K and the velocity v_K in the conical barrel at the end of burning of the powder.

On the basis of the resulting values of x, we find:

$$\psi = \psi_0 + \varkappa \sigma_0 x + \varkappa \lambda x^2$$

and the pressure:

$$p = f\omega \frac{\psi - \frac{v^2}{v^2_{np}}}{W_\psi + W} = f\Delta \frac{\psi - \frac{v^2}{v^2_{np}}}{\Lambda_\psi + \Lambda},$$

where:

$$W_\psi = W_0\left(1 - \frac{\Delta}{\delta}\right) - \omega\left(\alpha - \frac{1}{\delta}\right)\psi; \qquad \Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi,$$

and v and W are the same as for the cylindrical barrel.

For the end of the first period, $\psi_K = 1$.

$$p_K = f\omega \frac{1 - \frac{v^2}{v^2_{np}}}{W_1 + W_K} = f\Delta \frac{1 - \frac{v_K^2}{v^2_{np}}}{1 - \alpha\Delta + \Lambda_K}.$$

We find the muzzle velocity and muzzle pressure with the aid of the following formulas:

$$v_D^2 = v_{np}^2\left[1 - \left(1 - \frac{v_K^2}{v_{np}^2}\right)\left(\frac{1 - \alpha\Delta + \Lambda_K}{1 - \alpha\Delta + \Lambda_D}\right)^\theta\right];$$

$$p_D = p_K\left(\frac{1 - \alpha\Delta + \Lambda_K}{1 - \alpha\Delta + \Lambda_D}\right)^{1+\theta}$$

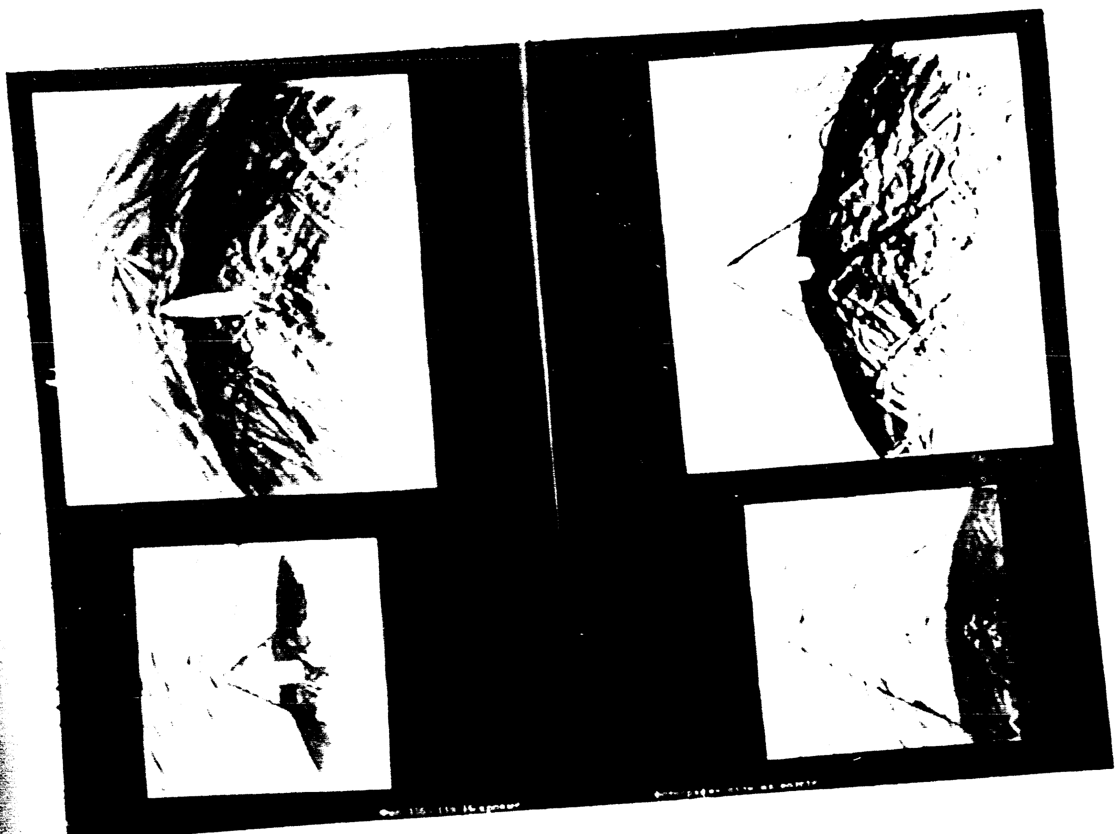
$$v_{np}^2 = \frac{2g}{\varphi}\frac{f}{\theta}\frac{\omega}{q}$$

We determine the coefficient of utilization of the unit weight of the charge:

$$\eta_\omega = \frac{mv_D^2}{2\omega} = \frac{v_D^2}{2g} : \frac{\omega}{q} .$$

The velocity is obtained very close to the predetermined velocity for the cylindrical bore at the same volumes W_D and W_O and under the same loading conditions except for I_K.

On the basis of the computed results obtained, there is, whenever necessary, applied a correction in order to obtain the required initial projectile velocity at the predetermined pressure p_m.



Figs. 116 - 119 - Spark Photographs of Bullet in Flight.

APPENDIX 1: TABLES FOR DETERMINING BURNT PART OF CHARGE ψ DURING BURNING OF POWDER IN CONSTANT VOLUME (BOMB)

(Compiled by M. E. Serebryakov)

On the basis of the general formula of pyrostatics, the part of the charge ψ burnt prior to any given moment can be computed with the aid of the following formula:

$$\psi = \frac{\dfrac{p - p_B}{p_m - p_B}}{\dfrac{p - p_B}{p_m - p_B}(1 - \partial) - \partial},$$

where:

$$\partial = \frac{1 - \alpha\Delta}{1 - \dfrac{\Delta}{\delta}}.$$

In these formulas, the following designations are used.

Δ is the loading density in the experiment.

δ is the density (specific gravity) of the powder.

α is the covolume of the powder gases.

p_m is the maximum pressure in the given experiment.

p_B is the pressure due to the igniter gases.

p is the pressure in a certain intermediate instant, which varies in the range of p_B-p_m.

Thus, the quantity ψ is a function of two parameters.

1) The parameter ∂, which is constant for the given experiment.

2) The variable ratio $\dfrac{p - p_B}{p_m - p_B}$, which varies from 0 to 1.

At α close to 1, δ close to 1.6, and Δ varying from 0.25 to 0, the quantity ∂, which depends upon the three quantities α, δ, and Δ,

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At α close to 1, δ close to 1.6, and Δ varying from 0.25 to 0, the quantity ∂, which depends upon the three quantities α, δ, and Δ,

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varies in the range of 0.86-1.00.

The tables are compiled for every value of ∂ from 0.86 to 0.97 at intervals of 0.01, as related to the ratio $\frac{p - p_B}{p_m - p_B}$ varied from 0 to 1 at intervals of 0.001.

The arrangement of the tables is analogous to the arrangement of four-place logarithms.

USE OF TABLES

To start with, the experimental data are used to compute the basic quantity ∂ in accordance with the following formula:

$$\partial = \frac{1 - \alpha\Delta}{1 - \frac{\Delta}{\delta}}.$$

If the quantities α and δ are not known in advance, it may be approximately assumed that $\delta = 1.6$ and $\alpha = 1$ for pyroxylin powders and 0.8 for nitroglycerol powders.

Having computed the basic quantity ∂, we find the corresponding table of ψ as a function of the ratio $\frac{p - p_B}{p_m - p_B}$, which, for brevity has been designated as β in the tables:

$$\frac{p - p_B}{p_m - p_B} = \beta.$$

Having found in the left-hand column of the page the first two digits of the quantity β (tenths and hundredths), and taking from the upper row of the table the number corresponding to the thousandths of β (from 0 to 9), we find the quantity ψ for the first three digits of β at the intersection of this column with the row corresponding to the first two digits of β.

The change in ψ corresponding to the fourth digit of β after the decimal point is found by interpolating values for ψ between the

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quantity found above and the neighboring quantity on the right.

Having obtained columns for the values of the time t and pressure p by treatment of the experimental p-t curve, and knowing the pressure p_B developed by the igniter and the maximum pressure p_m, we compute for every value of p a column of values of:

$$\zeta = \frac{p - p_B}{p_m - p_B},$$

where $p_m - p_B$ will be a constant for the given experiment.

Having computed the ratio ∂ for the given experiment, we immediately find in the corresponding table the column of values of ψ, whereupon we can set up a column for the values of $\Delta\psi$ and a column for the values of the ratio $\frac{\Delta\psi}{\Delta t}$, which expresses the rate of gas formation from the given powder under the given conditions, which enters into the expression for the experimental characteristic of the progressivity of burning:

$$\Gamma = \frac{1}{p}\frac{\Delta\psi}{\Delta t}.$$

<u>Example</u>. Let there be known from preliminary experiments the quantities $\alpha = 0.97$ and $\delta = 1.58$, as well as the loading density in the given experiment $\Delta = 0.20$. In this connection, there have been obtained $p_B = 40$ kg/cm^2, $p_m = 2170$ kg/cm^2.

Let us determine the basic quantity ∂:

$$\partial = \frac{1 - \alpha\Delta}{1 - \frac{\Delta}{\delta}} = \frac{1 - 0.97 \cdot 0.20}{1 - \frac{0.20}{1.58}} = \frac{1 - 0.194}{1 - 0.1266} = \frac{0.806}{0.8734} = 0.9235.$$

We shall make use of the table corresponding to the nearest basic quantity ∂ in the tables, i.e. $\partial = 0.92$ (p. 1044).

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Let the values for the pressure in any desired instants of time be:

$$p_1 = 204 \text{ and } p_2 = 1380.$$

We find the values:

$$\xi_1 = \frac{204 - 40}{2170 - 40} = \frac{164}{2130} = 0.0770;$$

$$\xi_2 = \frac{1380 - 40}{2170 - 40} = \frac{1340}{2170} = 0.6292.$$

To determine the value ψ_1, we find in the left-hand column of the table for $\partial = 0.92$ the ratio 0.07; in the upper row we find the number 7; and at the intersection of these horizontal and vertical lines we find the number 0.0834. Consequently, $\psi_1 = 0.0834$.

To determine ψ_2 corresponding to $\xi_2 = 0.6292$, we find the numbers 0.62 in the left-hand column and the number 9 in the upper row. The intersection of these will give the number 0.6483, which corresponds to the value of $\xi = 0.629$. The next larger value, $\xi = 0.630$, is associated with $\psi = 0.6493$. The difference between this and the first value equals 0.0010; it corresponds to 10 units of the forth digit of ξ; two units would correspond to $\Delta\psi = 0.0002$.

Thus, we shall have:

$$\begin{array}{ll} \xi = 0.629 & \left.\begin{array}{l}\psi = 0.6483 \\ \psi = 0.6493\end{array}\right\} \Delta\psi = 0.0010 \\ \xi = 0.630 & \\ \Delta\xi = 0.0002 & \Delta\psi = 0.0002 \\ \xi_2 = 0.6292 & \psi_2 = 0.6483 + 0.0002 = 0.6485 \end{array}$$

Since, as a rule, the differences $\Delta\psi$ between two neighboring columns differ very little from 10, the entire interpolation is easily carried out mentally.

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APPENDIX 1

TABLES COMPILED BY PROF. SEREBRIAKOV

$\partial = 0.86$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0.000	0.0012	0.0024	0.0036	0.0048	0.0060	0.0071	0.0083	0.0095	0.0107
0.01	0.0118	130	142	153	165	176	188	200	212	223
2	234	246	257	269	280	292	303	315	326	338
3	349	361	372	384	395	407	418	430	441	453
4	464	476	487	499	510	522	533	545	556	568
0.05	579	591	602	614	625	637	648	660	671	683
0.06	0694	706	717	729	740	751	762	774	785	797
7	0808	820	831	843	854	866	877	889	900	911
8	0922	933	944	956	967	978	989	1000	1012	1023
9	1034	1045	1056	1067	1078	1090	1101	1112	1124	1135
0.10	1146	1157	1168	1180	1191	1202	1213	1224	1235	1247
0.11	1258	1269	1280	1291	1303	1314	1325	1336	1348	1359
12	1370	1381	1392	1403	1415	1426	1437	1448	1460	1471
13	1482	1493	1504	1515	1527	1538	1549	1560	1572	1583
14	1594	1605	1616	1627	1638	1650	1661	1672	1683	1694
15	1705	1716	1727	1738	1749	1760	1771	1782	1793	1804
0.16	1815	1826	1837	1848	1859	1870	1881	1892	1903	1914
17	1925	1936	1947	1958	1969	1980	1991	2002	2013	2024
18	2035	2046	2057	2068	2079	2090	2100	2111	2122	2133
19	2144	2155	2165	2176	2187	2198	2209	2220	2230	2241
20	2252	2263	2273	2284	2295	2306	2317	2328	2338	2349
0.21	2360	2371	2381	2392	2403	2414	2425	2436	2446	2457
22	2468	2479	2489	2500	2511	2522	2533	2544	2554	2565
23	2576	2587	2597	2608	2619	2630	2641	2652	2662	2673
24	2684	2695	2705	2716	2727	2738	2749	2760	2770	2784
25	2792	2803	2813	2824	2835	2846	2857	2868	2878	2889
0.26	2900	2910	2921	2932	2943	2953	2964	2975	2985	2996
27	3007	3017	3028	3039	3050	3060	3071	3082	3092	3103
28	3114	3124	3135	3146	3156	3167	3177	3188	3199	3209
29	3220	3230	3241	3252	3262	3273	3283	3294	3305	3316
30	3326	3336	3347	3358	3368	3379	3389	3400	3411	3421
0.31	3432	3442	3453	3463	3474	3484	3495	3505	3516	3526
32	3537	3547	3558	3560	3579	3589	3600	3610	3621	3631
33	3642	3652	3663	3673	3684	3694	3705	3715	3726	3736
34	3747	3757	3768	3778	3789	3799	3810	3820	3831	3841
35	3851	3861	3872	3882	3893	3903	3914	3924	3935	3945
0.36	3955	3965	3976	3986	3997	4007	4018	4028	4039	4049
37	4059	4070	4080	4090	4101	4111	4121	4132	4142	4152
38	4162	4173	4183	4193	4204	4214	4224	4235	4245	4255
39	4265	4276	4286	4296	4307	4317	4327	4338	4348	4358
40	4368	4379	4389	4399	4410	4420	4430	4441	4451	4461
0.41	4471	4482	4492	4502	4512	4523	4533	4543	4553	4563
42	4573	4584	4594	4604	4614	4624	4634	4644	4654	4664
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4735	4745	4755	4765

0.05	579	591	602	614	625	637	648	660	672	[illegible]
0.06	0694	706	717	729	740	751	762	774	785	797
7	0808	820	831	843	854	866	877	889	900	911
8	0922	933	944	956	967	978	989	1000	1012	1023
9	1034	1045	1056	1067	1078	1090	1101	1112	1124	1135
0.10	1146	1157	1168	1180	1191	1202	1213	1224	1235	1247
0.11	1258	1269	1280	1291	1303	1314	1325	1336	1348	1359
12	1370	1381	1392	1403	1415	1426	1437	1448	1460	1471
13	1482	1493	1504	1515	1527	1538	1549	1560	1572	1583
14	1594	1605	1616	1627	1638	1650	1661	1672	1683	1694
15	1705	1716	1727	1738	1749	1760	1771	1782	1793	1804
0.16	1815	1826	1837	1848	1859	1870	1881	1892	1903	1914
17	1925	1936	1947	1958	1969	1980	1991	2002	2013	2024
18	2035	2046	2057	2068	2079	2090	2100	2111	2122	2133
19	2144	2155	2165	2176	2187	2198	2209	2220	2230	2241
20	2252	2263	2273	2284	2295	2306	2317	2328	2338	2349
0.21	2360	2371	2381	2392	2403	2414	2425	2436	2446	2457
22	2468	2479	2489	2500	2511	2522	2533	2544	2554	2565
23	2576	2587	2597	2608	2619	2630	2641	2652	2662	2673
24	2684	2695	2705	2716	2727	2738	2749	2760	2770	2784
25	2792	2803	2813	2824	2835	2846	2857	2868	2878	2889
0.26	2900	2910	2921	2932	2943	2953	2964	2975	2985	2996
27	3007	3017	3028	3039	3050	3060	3071	3082	3092	3103
28	3114	3124	3135	3146	3156	3167	3177	3188	3199	3209
29	3220	3230	3241	3252	3262	3273	3283	3294	3305	3316
30	3326	3336	3347	3358	3368	3379	3389	3400	3411	3421
0.31	3432	3442	3453	3463	3474	3484	3495	3505	3516	3526
32	3537	3547	3558	3560	3579	3589	3600	3610	3621	3631
33	3642	3652	3663	3673	3684	3694	3705	3715	3726	3736
34	3747	3757	3768	3778	3789	3799	3810	3820	3831	3841
35	3851	3861	3872	3882	3893	3903	3914	3924	3935	3945
0.36	3955	3965	3976	3986	3997	4007	4018	4028	4039	4049
37	4059	4070	4080	4090	4101	4111	4121	4132	4142	4152
38	4162	4173	4183	4193	4204	4214	4224	4235	4245	4255
39	4265	4276	4286	4296	4307	4317	4327	4338	4348	4358
40	4368	4379	4389	4399	4410	4420	4430	4441	4451	4461
0.41	4471	4482	4492	4502	4512	4523	4533	4543	4553	4563
42	4573	4584	4594	4604	4614	4624	4634	4644	4654	4664
43	4674	4685	4695	4705	4715	4725	4735	4745	4755	4765
44	4775	4786	4796	4806	4816	4826	4836	4846	4856	4866
45	4876	4887	4897	4907	4917	4927	4937	4947	4957	4967
0.46	4977	4988	4998	5008	5018	5028	5038	5048	5058	5068
47	5078	5089	5099	5109	5119	5129	5139	5149	5159	5169
48	5179	5189	5199	5209	5219	5229	5239	5249	5259	5269
49	5279	5289	5299	5309	5319	5329	5339	5349	5359	5369
0.50	5379	5389	5399	5409	5419	5429	5439	5449	5459	5469

STAT

(cont'd)

∂ = 0.86

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5478	0.5487	0.5497	0.5507	0.5517	0.5527	0.5537	0.5547	0.5557	0.5567
52	5577	5586	5596	5606	5616	5626	5636	5646	5656	5666
53	5676	5685	5695	5705	5715	5725	5734	5744	5754	5764
54	5774	5783	5793	5803	5813	5823	5832	5842	5852	5862
55	5872	5881	5891	5901	5910	5920	5930	5939	5949	5959
0.56	5969	5978	5988	5998	6007	6017	6027	6036	6046	6056
57	6066	6075	6085	6095	6104	6114	6124	6133	6143	6153
58	6163	6172	6182	6192	6201	6211	6221	6230	6240	6250
59	6260	6269	6279	6289	6298	6308	6317	6327	6336	6346
60	6356	6365	6375	6385	6394	6404	6413	6423	6432	6442
0.61	6452	6461	6471	6481	6490	6500	6510	6519	6529	6538
62	6548	6557	6567	6577	6586	6596	6606	6615	6625	6634
63	6644	6653	6663	6673	6682	6692	6702	6711	6721	6732
64	6740	6749	6759	6768	6778	6787	6797	6806	6816	6825
65	6835	6844	6854	6863	6873	6882	6892	6901	6911	6920
0.66	6930	6939	6949	6958	6968	6977	6986	6996	7005	7015
67	7024	7033	7043	7052	7062	7071	7080	7090	7099	7109
68	7118	7127	7137	7146	7156	7165	7174	7184	7193	7203
69	7212	7221	7231	7240	7250	7259	7268	7278	7287	7297
70	7306	7315	7325	7334	7344	7353	7362	7372	7381	7391
0.71	7400	7409	7419	7428	7438	7447	7456	7466	7475	7485
72	7494	7503	7513	7522	7532	7541	7550	7560	7569	7579
73	7588	7597	7607	7616	7626	7635	7644	7654	7663	7673
74	7682	7691	7701	7710	7719	7728	7738	7747	7756	7765
75	7775	7784	7794	7803	7812	7821	7831	7840	7849	7858
0.76	7868	7877	7886	7896	7905	7914	7923	7932	7941	7951
77	7960	7969	7978	7988	7997	8006	8015	8024	8033	8043
78	8052	8061	8070	8080	8089	8098	8107	8116	8126	8135
79	8144	8153	8162	8172	8181	8190	8199	8208	8218	8227
80	8236	8246	8255	8264	8273	8282	8291	8300	8309	8318
0.81	8327	8337	8346	8355	8364	8373	8382	8391	8400	8409
82	8418	8428	8437	8446	8355	8464	8473	8482	8491	8500
83	8509	8519	8528	8537	8546	8555	8564	8573	8582	8591
84	8600	8609	8618	8627	8636	8645	8654	8663	8672	8681
85	8690	8699	8708	8717	8726	8735	8744	8753	8762	8771
0.86	8780	8789	8798	8807	8816	8824	8833	8842	8851	8860
87	8869	8878	8887	8895	8904	8913	8922	8931	8938	8948
88	8957	8966	8975	8984	8992	9001	9010	9019	9027	9036
89	9045	9054	9062	9071	9080	9088	9097	9106	9114	9123
90	9132	9140	9149	9158	9167	9175	9184	9193	9201	9210
0.91	9219	9227	9236	9245	9254	9262	9271	9280	9289	9298
92	9306	9315	9323	9332	9341	9350	9359	9367	9376	9385
93	9394	9403	9411	9420	9429	9438	9447	9455	9464	9473
					9517	9525	9534	9543	9552	9560

55	5872	5881	5891	5901	5910	5920	5930	5939	5849	5959
0.56	5969	5978	5988	5998	6007	6017	6027	6036	6046	6056
57	6066	6075	6085	6095	6104	6114	6124	6133	6143	6153
58	6163	6172	6182	6192	6201	6211	6221	6230	6240	6250
59	6260	6269	6279	6289	6298	6308	6317	6327	6336	6346
60	6356	6365	6375	6385	6394	6404	6413	6423	6432	6442
0.61	6452	6461	6471	6481	6490	6500	6510	6519	6529	6538
62	6548	6557	6567	6577	6586	6596	6606	6615	6625	6634
63	6644	6653	6663	6673	6682	6692	6702	6711	6721	6732
64	6740	6749	6759	6768	6778	6787	6797	6806	6816	6825
65	6835	6844	6854	6863	6873	6882	6892	6901	6911	6920
0.66	6930	6939	6949	6958	6968	6977	6986	6996	7005	7015
67	7024	7033	7043	7052	7062	7071	7080	7090	7099	7109
68	7118	7127	7137	7146	7156	7165	7174	7184	7193	7203
69	7212	7221	7231	7240	7250	7259	7268	7278	7287	7297
70	7306	7315	7325	7334	7344	7353	7362	7372	7381	7391
0.71	7400	7409	7419	7428	7438	7447	7456	7466	7475	7485
72	7494	7503	7513	7522	7532	7541	7550	7560	7569	7579
73	7588	7597	7607	7616	7626	7635	7644	7654	7663	7673
74	7682	7691	7701	7710	7719	7728	7738	7747	7756	7765
75	7775	7784	7794	7803	7812	7821	7831	7840	7849	7858
0.76	7868	7877	7886	7896	7905	7914	7923	7932	7941	7951
77	7960	7969	7978	7988	7997	8006	8015	8024	8033	8043
78	8052	8061	8070	8080	8089	8098	8107	8116	8126	8135
79	8144	8153	8162	8172	8181	8190	8199	8208	8218	8227
80	8236	8246	8255	8264	8273	8282	8291	8300	8309	8318
0.81	8327	8337	8346	8355	8364	8373	8382	8391	8400	8409
82	8418	8428	8437	8446	8355	8464	8473	8482	8491	8500
83	8509	8519	8528	8537	8546	8555	8564	8573	8582	8591
84	8600	8609	8618	8627	8636	8645	8654	8663	8672	8681
85	8690	8699	8708	8717	8726	8735	8744	8753	8762	8771
0.86	8780	8789	8798	8807	8816	8824	8833	8842	8851	8860
87	8869	8878	8887	8895	8904	8913	8922	8931	8938	8948
88	8957	8966	8975	8984	8992	9001	9010	9019	9027	9036
89	9045	9054	9062	9071	9080	9088	9097	9106	9114	9123
90	9132	9140	9149	9158	9167	9175	9184	9193	9201	9210
0.91	9219	9227	9236	9245	9254	9262	9271	9280	9289	9298
92	9306	9315	9323	9332	9341	9350	9359	9367	9376	9385
93	9394	9403	9411	9420	9429	9438	9447	9455	9464	9473
94	9482	9490	9492	9508	9517	9525	9534	9543	9552	9560
95	9568	9577	9585	9594	9603	9611	9620	9628	9637	9645
0.96	9654	9663	9671	9680	9689	9697	9706	9714	9723	9731
97	9740	9749	9757	9766	9775	9783	9792	9800	9809	9817
98	9826	9835	9843	9851	9860	9868	9877	9885	9894	9903
99	9912	9920	9929	9938	9947	9955	9964	9973	9982	9991
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd)

$\partial = 0.87$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0.000	0.0012	0.0024	0.0035	0.0047	0.0058	0.0070	0.0082	0.0093	0.0105
0.01	0.0116	128	138	150	162	173	185	196	207	219
2	230	242	253	264	276	287	299	310	321	133
3	344	356	367	378	390	401	412	424	435	446
4	457	469	480	491	503	514	525	537	548	559
5	570	582	593	604	616	627	638	650	661	672
0.06	683	695	706	717	729	740	751	763	774	785
7	796	808	819	830	841	852	864	875	886	897
8	908	920	931	942	953	964	976	987	998	1009
9	0.1020	1032	1043	1054	1065	1076	1088	1099	1110	1121
0.10	1132	1144	1155	1166	1177	1188	1200	1211	1222	1233
0.11	1244	1256	1267	1278	1289	1300	1312	1323	1334	1345
12	1356	1367	1378	1389	1400	1411	1422	1433	1444	1455
13	1466	1476	1487	1498	1509	1520	1531	1542	1553	1564
14	1575	1585	1596	1607	1618	1629	1640	1651	1662	1673
15	1684	1694	1705	1716	1727	1738	1749	1760	1771	1782
0.16	1793	1803	1814	1825	1836	1847	1858	1869	1880	1891
17	1902	1912	1923	1934	1945	1956	1967	1978	1989	2000
18	2011	2021	2032	2043	2054	2065	2076	2087	2098	2109
19	2120	2130	2141	2152	2163	2174	2184	2195	2206	2217
20	2228	2238	2249	2260	2271	2282	2292	2303	2314	2325
0.21	2336	2346	2357	2368	2379	2390	2400	2411	2423	2433
22	2444	2454	2465	2476	2487	2498	2508	2519	2530	2541
23	2552	2562	2573	2584	2594	2605	2616	2626	2637	2648
24	2659	2669	2680	2691	2701	2712	2723	2733	2744	2755
25	2766	2776	2787	2798	2808	2819	2830	2848	2851	2862
0.26	2873	2883	2894	2905	2915	2926	2937	2947	2958	2969
27	2980	2990	3001	3012	3022	3033	3043	3054	2064	3075
28	3086	3096	3107	3118	3128	3139	3149	3160	3170	3181
29	3192	3202	3213	3224	3234	3245	3255	3266	3276	3287
30	3298	3308	3319	3330	3340	3351	3361	3372	3382	3393
0.31	3404	3414	3425	3435	3446	3456	3467	3477	3488	3498
32	3509	3519	3530	3540	3551	3561	3572	3582	3593	3603
33	3614	3624	3635	3645	3655	3666	3676	3687	3697	3708
34	3718	3728	3739	3749	3759	3770	3780	3791	3801	3812
35	3822	3832	3843	3853	3863	3874	3884	3895	3905	3916
0.36	3926	3936	3947	3957	3967	3978	3988	3999	4009	4020
37	4030	4040	4050	4061	4071	4081	4092	4102	4112	4123
38	4133	4143	4153	4164	4174	4184	4195	4205	4215	4226
39	4236	4246	4256	4266	4276	4287	4297	4307	4317	4328
40	4338	4348	4358	4368	4378	4389	4399	4409	4419	4429
0.41	4440	4450	4460	4470	4480	4491	4501	4511	4521	4531
42	4542	4552	4562	4572	4582	4592	4603	4613	4623	4633
43	4643	4653	4663	4673	4683	4693	4704	4714	4724	4734
44	4744	4754	4764	4774	4784	4794	4805	4815	4825	4835
45	4845	4855	4865	4875	4885	4895	4906	4916	4926	4936

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7	796	808	819	830	841	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	908	920	931	942	953	964	976	987	998	1009
9	0.1020	1032	1043	1054	1065	1076	1088	1099	1110	1121
0.10	1132	1144	1155	1166	1177	1188	1200	1211	1222	1233
0.11	1244	1256	1267	1278	1289	1300	1312	1323	1334	1345
12	1356	1367	1378	1389	1400	1411	1422	1433	1444	1455
13	1466	1476	1487	1498	1509	1520	1531	1542	1553	1564
14	1575	1585	1596	1607	1618	1629	1640	1651	1662	1673
15	1684	1694	1705	1716	1727	1738	1749	1760	1771	1782
0.16	1793	1803	1814	1825	1836	1847	1858	1869	1880	1891
17	1902	1912	1923	1934	1945	1956	1967	1978	1989	2000
18	2011	2021	2032	2043	2054	2065	2076	2087	2098	2109
19	2120	2130	2141	2152	2163	2174	2184	2195	2206	2217
20	2228	2238	2249	2260	2271	2282	2292	2303	2314	2325
0.21	2336	2346	2357	2368	2379	2390	2400	2411	2423	2433
22	2444	2454	2465	2476	2487	2498	2508	2519	2530	2541
23	2552	2562	2573	2584	2594	2605	2616	2626	2637	2648
24	2659	2669	2680	2691	2701	2712	2723	2733	2744	2755
25	2766	2776	2787	2798	2808	2819	2830	2848	2851	2862
0.26	2873	2883	2894	2905	2915	2926	2937	2947	2958	2969
27	2980	2990	3001	3012	3022	3033	3043	3054	2064	3075
28	3086	3096	3107	3118	3128	3139	3149	3160	3170	3181
29	3192	3202	3213	3224	3234	3245	3255	3266	3276	3287
30	3298	3308	3319	3330	3340	3351	3361	3372	3382	3393
0.31	3404	3414	3425	3435	3446	3456	3467	3477	3488	3498
32	3509	3519	3530	3540	3551	3561	3572	3582	3593	3603
33	3614	3624	3635	3645	3655	3666	3676	3687	3697	3708
34	3718	3728	3739	3749	3759	3770	3780	3791	3801	3812
35	3822	3832	3843	3853	3863	3874	3884	3895	3905	3916
0.36	3926	3936	3947	3957	3967	3978	3988	3999	4009	4020
37	4030	4040	4050	4061	4071	4081	4092	4102	4112	4123
38	4133	4143	4153	4164	4174	4184	4195	4205	4215	4226
39	4236	4246	4256	4266	4276	4287	4297	4307	4317	4328
40	4338	4348	4358	4368	4378	4389	4399	4409	4419	4429
0.41	4440	4450	4460	4470	4480	4491	4501	4511	4521	4531
42	4542	4552	4562	4572	4582	4592	4603	4613	4623	4633
43	4643	4653	4663	4673	4683	4693	4704	4714	4724	4734
44	4744	4754	4764	4774	4784	4794	4805	4815	4825	4835
45	4845	4855	4865	4875	4885	4895	4906	4916	4926	4936
0.46	4946	4956	4966	4976	4986	4996	5007	5017	5027	5037
47	5047	5057	5067	5077	5087	5097	5108	5118	5128	5138
48	5148	5158	5168	5178	5188	5198	5208	5218	5228	5238
49	5248	5258	5268	5278	5288	5298	5308	5318	5328	5338
50	5348	5358	5368	5378	5388	5398	5408	5418	5428	5438

(cont'd)

$\partial = 0.87$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5447	0.5457	0.5467	0.5477	0.5487	0.5497	0.5507	0.5517	0.5527	0.5537
52	5546	5556	5566	5576	5586	5596	5606	5616	5626	5636
53	5645	5655	5665	5675	5685	5695	5705	5715	5725	5735
54	5744	5754	5764	5774	5784	5793	5803	5812	5823	5833
55	5842	5852	5862	5872	5882	5891	5901	5911	5921	5931
0.56	5940	5950	5960	5970	5980	5989	5999	6009	6019	6029
57	6038	6048	6058	6068	6078	6087	6097	6107	6117	6127
58	6136	6146	6156	6166	6175	6185	6195	6204	6214	6224
59	6233	6243	6253	6263	6272	6282	6292	6301	6311	6321
60	6330	6340	6350	6360	6369	6379	6389	6398	6408	6418
0.61	6427	6437	6447	6457	6466	6476	6486	6495	6505	6515
62	6524	6534	6544	6553	6563	6573	6582	6592	6601	6611
63	6620	6630	6640	6649	6659	6668	6678	6687	6697	6707
64	6716	6726	6736	6745	6755	6764	6774	6783	6793	6803
65	6812	6822	6832	6841	6851	6860	6870	6879	6889	6899
0.66	6908	6918	6928	6937	6947	6956	6966	6975	6985	6995
67	7004	7014	7023	7033	7042	7052	7061	7071	7080	7090
68	7099	7109	7118	7128	7137	7147	7156	7166	7775	7185
69	7194	7204	7213	7223	7232	7242	7251	7260	7270	7279
70	7288	7298	7307	7317	7326	7336	7345	7354	7364	7373
0.71	7382	7392	7401	7411	7420	7430	7439	7448	7458	7467
72	7476	7486	7495	7505	7514	7524	7533	7542	7552	7561
73	7570	7580	7589	7599	7608	7618	7627	7636	7646	7655
74	7664	7673	7682	7691	7701	7710	7719	7729	7738	7747
75	7757	7766	7775	7784	7794	7803	7812	7822	7831	7840
0.76	7850	7859	7868	7877	7887	7896	7905	7915	7924	7933
77	7943	7952	7961	7970	7980	7989	7998	8008	8017	8026
78	8036	8045	8054	8063	8072	8082	8091	8100	8109	8118
79	8128	8137	8146	8155	8164	8174	8183	8192	8201	8210
80	8220	8229	8238	8247	8256	8265	8275	8284	8293	8302
0.81	8311	8320	8329	8338	8347	8356	8366	8375	8384	8393
82	8402	8411	8420	8429	8438	8447	8457	8466	8475	8484
83	8493	8502	8511	8520	8529	8538	8547	8556	8565	8574
84	8583	8592	8601	8610	8619	8628	8637	8646	8655	8664
85	8673	8682	8691	8700	8709	8718	8727	8736	8745	8754
0.86	8763	8772	8781	8790	8799	8808	8817	8826	8835	8844
87	8853	8862	8871	8880	8889	8898	8907	8916	8925	8934
88	8943	8952	8961	8970	8979	8988	8997	9006	9015	9024
89	9033	9042	9051	9060	9069	9078	9087	9095	9104	9113
90	9122	9131	9140	9149	9158	9167	9176	9184	9193	9202
0.91	9211	9220	9229	9238	9247	9256	9265	9273	9282	9291
92	9300	9309	9317	9326	9335	9344	9353	9361	9370	9379
93	9388	9397	9405	9414	9423	9432	9441	9449	9458	9467
94	9476	9485	9493	9502	9511	9520	9529	9537	9546	9555
95	9564	9573	9581	9590	9599	9608	9617	9625	9634	9643
0.96	9652	9660	9669	9678	9686	9695	9704	9712	9721	9730
97	9739	9747	9756	9765	9773	9782	9791	9799	9808	9817
	[illegible]	[illegible]	[illegible]	9852	9860	9869	9878	9886	9895	9904

60	6330	6340	6350	6360	6369	6379	6389	6398	6408	6418
0.61	6427	6437	6447	6457	6466	6476	6486	6495	6505	6515
62	6524	6534	6544	6553	6563	6573	6582	6592	6601	6611
63	6620	6630	6640	6649	6659	6668	6678	6687	6697	6707
64	6716	6726	6736	6745	6755	6764	6774	6783	6793	6803
65	6812	6822	6832	6841	6851	6860	6870	6879	6889	6899
0.66	6908	6918	6928	6937	6947	6956	6966	6975	6985	6995
67	7004	7014	7023	7033	7042	7052	7061	7071	7080	7090
68	7099	7109	7118	7128	7137	7147	7156	7166	7775	7185
69	7194	7204	7213	7223	7232	7242	7251	7260	7270	7279
70	7288	7298	7307	7317	7326	7336	7345	7354	7364	7373
0.71	7382	7392	7401	7411	7420	7430	7439	7448	7458	7467
72	7476	7486	7495	7505	7514	7524	7533	7542	7552	7561
73	7570	7580	7589	7599	7608	7618	7627	7636	7646	7655
74	7664	7673	7682	7691	7701	7710	7719	7729	7738	7747
75	7757	7766	7775	7784	7794	7803	7812	7822	7831	7840
0.76	7850	7859	7868	7877	7887	7896	7905	7915	7924	7933
77	7943	7952	7961	7970	7980	7989	7998	8008	8017	8026
78	8036	8045	8054	8063	8072	8082	8091	8100	8109	8118
79	8128	8137	8146	8155	8164	8174	8183	8192	8201	8210
80	8220	8229	8238	8247	8256	8265	8275	8284	8293	8302
0.81	8311	8320	8329	8338	8347	8356	8366	8375	8384	8393
82	8402	8411	8420	8429	8438	8447	8457	8466	8475	8484
83	8493	8502	8511	8520	8529	8538	8547	8556	8565	8574
84	8583	8592	8601	8610	8619	8628	8637	8646	8655	8664
85	8673	8682	8691	8700	8709	8718	8727	8736	8745	8754
0.86	8763	8772	8781	8790	8799	8808	8817	8826	8835	8844
87	8853	8862	8871	8880	8889	8898	8907	8916	8925	8934
88	8943	8952	8961	8970	8979	8988	8997	9006	9015	9024
89	9033	9042	9051	9060	9069	9078	9087	9095	9104	9113
90	9122	9131	9140	9149	9158	9167	9176	9184	9193	9202
0.91	9211	9220	9229	9238	9247	9256	9265	9273	9282	9291
92	9300	9309	9317	9326	9335	9344	9353	9361	9370	9379
93	9388	9397	9405	9414	9423	9432	9441	9449	9458	9467
94	9476	9485	9493	9502	9511	9520	9529	9537	9546	9555
95	9564	9573	9581	9590	9599	9608	9617	9625	9634	9643
0.96	9652	9660	9669	9678	9686	9695	9704	9712	9721	9730
97	9739	9747	9756	9765	9773	9782	9791	9799	9808	9817
98	9826	9834	9843	9852	9860	9869	9878	9886	9895	9904
99	9913	9921	9930	9939	9947	9956	9965	9973	9982	9991
1.00	1,000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)

$\partial = 0.88$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0.000	0.0012	0.0024	0.0035	0.0046	0.0058	0.0069	0.0081	0.0092	0.0103
1	0.0114	126	137	148	160	171	183	194	205	216
2	227	238	250	261	272	283	294	306	317	328
3	339	350	362	373	384	396	407	418	430	441
4	452	463	474	485	397	508	519	530	541	553
5	564	576	587	598	609	620	632	643	654	665
0.06	676	688	699	710	721	732	744	755	766	777
7	788	800	811	822	833	844	856	867	878	889
8	900	912	923	934	945	956	967	978	989	0.1000
9	0.1011	1023	1034	1045	1056	1067	1078	1089	1100	1111
0.10	1122	1133	1144	1155	1166	1177	1188	1199	1210	1221
0.11	1232	1243	1254	1265	1276	1287	1298	1309	1320	1331
12	1342	1353	1364	1375	1385	1396	1407	1418	1429	1440
13	1451	1462	1473	1484	1495	1505	1516	1527	1538	1549
14	1560	1571	1582	1593	1604	1615	1625	1636	1647	1658
15	1669	1680	1690	1701	1712	1723	1734	1745	1756	1767
0.16	1778	1789	1800	181[illegible]	1821	1832	1843	1854	1865	1876
17	1887	1898	1909	1920	1930	1941	1952	1963	1974	1985
18	1995	2006	2017	2027	2038	2049	2060	2070	2081	2092
19	2103	2114	2124	2135	2146	2156	2167	2178	2188	2199
20	2210	2221	2231	2242	2253	2263	2274	2285	2295	2306
0.21	2317	2328	2338	2349	2360	2370	2381	2392	2402	2413
22	2424	2435	2445	2456	2467	2477	2488	2499	2509	2520
23	2531	2542	2552	2563	2574	2584	2595	2606	2616	2627
24	2638	2648	2659	2670	2680	2691	2702	2712	2723	2733
25	2744	2754	2765	2776	2786	2797	2808	2818	2829	2839
0.26	2850	2860	2871	2882	2892	2903	2914	2924	2935	2940
27	2956	2966	2977	2988	2998	3009	3020	3030	3041	3051
28	3062	3072	3083	3094	3104	3115	3126	3136	3147	3157
29	3168	3178	3189	3199	3210	3220	3231	3241	3252	3262
30	3273	3283	3294	3304	3315	3325	3336	3346	3357	3367
0.31	3378	3388	3399	3409	3420	3430	3441	3451	3462	3472
32	3483	3493	3504	3514	3525	3535	3546	3556	3567	3577
33	3588	3598	3608	3619	3629	3640	3650	3661	3671	3682
34	3692	3703	3713	3723	3734	3744	3754	3765	3775	3785
35	3795	3806	3816	3826	3837	3847	3857	3868	3878	3888
0.36	3898	3908	3918	3929	3939	3950	3960	3971	3981	3992
37	4002	4013	4023	4033	4044	4054	4064	4075	4085	4095
38	4105	4116	4126	4136	4147	4157	4167	4178	4188	4198
39	4208	4219	4229	4239	4249	4259	4270	4280	4290	4300
40	4310	4321	4331	4341	4351	4361	4372	4382	4392	4402
0.41	4412	4423	4433	4443	4453	4463	4474	4484	4494	4504
42	4514	4525	4535	4545	4555	4565	4575	4585	4595	4605
43	4615	4626	4636	4646	4656	4666	4676	4686	4696	4706
44	4716	4727	4737	4747	4757	4767	4777	4787	4797	4807
45	4817	4828	4838	4848	4858	4868	4878	4888	4898	4908
0.46	4918	4928	4938	4948	4958	4968	4978	4988	4998	5008

3	339	350	362	373	384	396	407	418	430	441
4	452	463	474	485	397	508	519	530	541	553
5	564	576	587	598	609	620	632	643	654	665
0.06	676	688	699	710	721	732	744	755	766	777
7	788	800	811	822	833	844	856	867	878	889
8	900	912	923	934	945	956	967	978	989	0.1000
9	0.1011	1023	1034	1045	1056	1067	1078	1089	1100	1111
0.10	1122	1133	1144	1155	1166	1177	1188	1199	1210	1221
0.11	1232	1243	1254	1265	1276	1287	1298	1309	1320	1331
12	1342	1353	1364	1375	1385	1396	1407	1418	1429	1440
13	1451	1462	1473	1484	1495	1505	1516	1527	1538	1549
14	1560	1571	1582	1593	1604	1615	1625	1636	1647	1658
15	1669	1680	1690	1701	1712	1723	1734	1745	1756	1767
0.16	1778	1789	1800	1810	1821	1832	1843	1854	1865	1876
17	1887	1898	1909	1920	1930	1941	1952	1963	1974	1985
18	1995	2006	2017	2027	2038	2049	2060	2070	2081	2092
19	2103	2114	2124	2135	2146	2156	2167	2178	2188	2199
20	2210	2221	2231	2242	2253	2263	2274	2285	2295	2306
0.21	2317	2328	2338	2349	2360	2370	2381	2392	2402	2413
22	2424	2435	2445	2456	2467	2477	2488	2499	2509	2520
23	2531	2542	2552	2563	2574	2584	2595	2606	2616	2627
24	2638	2648	2659	2670	2680	2691	2702	2712	2723	2733
25	2744	2754	2765	2776	2786	2797	2808	2818	2829	2839
0.26	2850	2860	2871	2882	2892	2903	2914	2924	2935	2940
27	2956	2966	2977	2988	2998	3009	3020	3030	3041	3051
28	3062	3072	3083	3094	3104	3115	3126	3136	3147	3157
29	3168	3178	3189	3199	3210	3220	3231	3241	3252	3262
30	3273	3283	3294	3304	3315	3325	3336	3346	3357	3367
0.31	3378	3388	3399	3409	3420	3430	3441	3451	3462	3472
32	3483	3493	3504	3514	3525	3535	3546	3556	3567	3577
33	3588	3598	3608	3619	3629	3640	3650	3661	3671	3682
34	3692	3703	3713	3723	3734	3744	3754	3765	3775	3785
35	3795	3806	3816	3826	3837	3847	3857	3868	3878	3888
0.36	3898	3908	3918	3929	3939	3950	3960	3971	3981	3992
37	4002	4013	4023	4033	4044	4054	4064	4075	4085	4095
38	4105	4116	4126	4136	4147	4157	4167	4178	4188	4198
39	4208	4219	4229	4239	4249	4259	4270	4280	4290	4300
40	4310	4321	4331	4341	4351	4361	4372	4382	4392	4402
0.41	4412	4423	4433	4443	4453	4463	4474	4484	4494	4504
42	4514	4525	4535	4545	4555	4565	4575	4585	4595	4605
43	4615	4626	4636	4646	4656	4666	4676	4686	4696	4706
44	4716	4727	4737	4747	4757	4767	4777	4787	4797	4807
45	4817	4828	4838	4848	4858	4868	4878	4888	4898	4908
0.46	4918	4928	4938	4948	4958	4968	4978	4988	4998	5008
47	5018	5028	5038	5038	5058	5068	5078	5088	5098	5108
48	5119	5129	5139	5149	5159	5169	5179	5189	5199	5209
49	5219	5229	5239	5249	5259	5269	5279	5289	5299	5309
0.50	5319	5329	5339	5349	5369	5399	5379	5389	5399	5409

STAT

(cont'd.)

$\partial = 0.88$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5418	0.5428	0.5438	0.5448	0.5458	0.5468	0.5478	0.5488	0.5498	0.5508
52	5518	5527	5537	5547	5557	5667	5577	5587	5597	5607
53	5617	5626	5636	5646	5656	5666	5676	5686	5696	5706
54	5716	5725	5735	5745	5755	5765	5774	5784	5794	5804
55	5814	5823	5833	5843	5853	5863	5872	5882	5892	5902
0.56	5912	5921	5931	5941	5951	5961	5970	5980	5990	6000
57	6010	6019	6029	6039	6049	6059	6068	6078	6088	6098
58	6108	6117	6127	6137	6146	6156	6166	6175	6185	6195
59	6205	6214	6224	6234	6243	6253	6262	6272	6282	6292
60	6302	6311	6321	6331	6340	6350	6360	6369	6379	6389
0.61	6399	6408	6418	6428	6437	6447	6457	6466	6476	6486
62	6496	6505	6515	6525	6534	6544	6554	6563	6573	6583
63	6593	6602	6612	6622	6631	6641	6651	6660	6670	6680
64	6690	6699	6709	6719	6728	6738	6748	6757	6767	6777
65	6787	6796	6806	6816	6825	6835	6845	6854	6864	6874
0.66	6883	6892	6902	6912	6921	6931	6941	6950	6960	6970
67	6979	6988	6998	7008	7017	7027	7037	7046	7056	7066
68	7075	7084	7094	7103	7113	7122	7132	7141	7151	7160
69	7170	7179	7189	7198	7208	7217	7227	7236	7246	7255
70	7265	7274	7284	7293	7303	7312	7322	7331	7341	7350
0.71	7360	7369	7379	7388	7397	7407	7416	7426	7435	7445
72	7454	7463	7472	7482	7491	7501	7510	7520	7529	7539
73	7548	7557	7567	7576	7585	7595	7604	7614	7623	7633
74	7642	7651	7661	7670	7679	7689	7698	7708	7717	7727
75	7736	7745	7755	7764	7773	7783	7792	7802	7811	7821
0.76	7830	7839	7848	7858	7867	7876	7886	7895	7904	7914
77	7923	7932	7941	7951	7960	7969	7979	7988	7997	8007
78	8016	8025	8034	8044	8053	8062	8072	8081	8090	8100
79	8109	8118	8127	8137	8146	8155	8165	8174	8183	8193
80	8202	8211	8220	8229	8238	8248	8257	8266	8275	8284
0.81	8294	8303	8312	8321	8330	8340	8349	8358	8367	8376
82	8385	8394	8403	8412	8421	8431	8440	8449	8458	8467
83	8476	8485	8494	8503	8512	8522	8531	8540	8549	8558
84	8567	8576	8585	8594	8603	8613	8622	8631	8640	8649
85	8658	8667	8676	8685	8694	8704	8713	8722	8731	8740
0.86	8740	8758	8767	8776	8785	8795	8804	8813	8822	8831
87	8840	8849	8858	8867	8876	8886	8895	8904	8913	8922
88	8931	8940	8949	8958	8967	8977	8986	8995	9004	9013
89	9022	9031	9040	9049	9058	9067	9076	9085	9094	9103
90	9112	9121	9130	9139	9148	9157	9166	9175	9184	9193
0.91	9202	9211	9220	9229	9238	9247	9256	9265	9274	9283
92	9292	9300	9309	9318	9327	9336	9345	9354	9363	9372
93	9381	9389	9398	9407	9416	9425	9434	9443	9452	9461
94	9470	9478	9487	9496	9505	9514	9522	9531	9540	9549
95	9558	9566	9575	9584	9593	9602	9610	9619	9628	9637
0.96	9646	9654	9663	9672	9681	9690	9698	9707	9716	9725
97	9734	9743	9752	9760	9769	9778	9787	9796	9805	9814

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58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
59	6205	6214	6224	6234	6243	6253	6262	6272	6282	6292
60	6302	6311	6321	6331	6340	6350	6360	6369	6379	6389
0.61	6399	6408	6418	6428	6437	6447	6457	6466	6476	6486
62	6496	6505	6515	6525	6534	6544	6554	6563	6573	6583
63	6593	6602	6612	6622	6631	6641	6651	6660	6670	6680
64	6690	6699	6709	6719	6728	6738	6748	6757	6767	6777
65	6787	6796	6806	6816	6825	6835	6845	6854	6864	6874
0.66	6883	6892	6902	6912	6921	6931	6941	6950	6960	6970
67	6979	6988	6998	7008	7017	7027	7037	7046	7056	7066
68	7075	7084	7094	7103	7113	7122	7132	7141	7151	7160
69	7170	7179	7189	7198	7208	7217	7227	7236	7246	7255
70	7265	7274	7284	7293	7303	7312	7322	7331	7341	7350
0.71	7360	7369	7379	7388	7397	7407	7416	7426	7435	7445
72	7454	7463	7472	7482	7491	7501	7510	7520	7529	7539
73	7548	7557	7567	7576	7585	7595	7604	7614	7623	7633
74	7642	7651	7661	7670	7679	7689	7698	7708	7717	7727
75	7736	7745	7755	7764	7773	7783	7792	7802	7811	7821
0.76	7830	7839	7848	7858	7867	7876	7886	7895	7904	7914
77	7923	7932	7941	7951	7960	7969	7979	7988	7997	8007
78	8016	8025	8034	8044	8053	8062	8072	8081	8090	8100
79	8109	8118	8127	8137	8146	8155	8165	8174	8183	8193
80	8202	8211	8220	8229	8238	8248	8257	8266	8275	8284
0.81	8294	8303	8312	8321	8330	8340	8349	8358	8367	8376
82	8385	8394	8403	8412	8421	8431	8440	8449	8458	8467
83	8476	8485	8494	8503	8512	8522	8531	8540	8549	8558
84	8567	8576	8585	8594	8603	8613	8622	8631	8640	8649
85	8658	8667	8676	8685	8694	8704	8713	8722	8731	8740
0.86	8740	8758	8767	8776	8785	8795	8804	8813	8822	8831
87	8840	8849	8858	8867	8876	8886	8895	8904	8913	8922
88	8931	8940	8949	8958	8967	8977	8986	8995	9004	9013
89	9022	9031	9040	9049	9058	9067	9076	9085	9094	9103
90	9112	9121	9130	9139	9148	9157	9166	9175	9184	9193
0.91	9202	9211	9220	9229	9238	9247	9256	9265	9274	9283
92	9292	9300	9309	9318	9327	9336	9345	9354	9363	9372
93	9381	9389	9398	9407	9416	9425	9434	9443	9452	9461
94	9470	9478	9487	9496	9505	9514	9522	9531	9540	9549
95	9558	9566	9575	9584	9593	9602	9610	9619	9628	9637
0.96	9646	9654	9663	9672	9681	9690	9698	9707	9716	9725
97	9734	9743	9752	9760	9769	9778	9787	9796	9805	9814
98	9823	9832	9841	9849	9858	9867	9876	9885	9894	9903
99	9912	9920	9929	9938	9947	9956	9964	9973	9982	0.9991
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd)

∂=0.89

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0.000	0.0012	0.0023	0.0034	0.0045	0.0057	0.0068	0.0079	0.0090	0.0101
0.01	0.0112	124	135	146	157	169	180	191	202	213
2	224	236	247	258	269	281	292	303	314	325
3	336	348	359	370	381	392	403	414	425	436
4	447	459	470	481	492	503	514	525	536	547
5	558	570	571	592	603	614	625	636	647	658
0.06	669	681	692	703	714	725	736	747	758	769
7	780	791	802	813	824	835	846	857	868	879
8	890	901	912	923	934	945	956	967	978	989
9	0.1000	1011	1022	1033	1044	1055	1066	1077	1088	1099
10	1110	1121	1132	1143	1154	1165	1176	1187	1198	1209
0.11	1220	1231	1242	1253	1264	1275	1286	1297	1308	1319
12	1330	1341	1352	1363	1374	1385	1396	1407	1418	1429
13	1440	1451	1462	1473	1484	1495	1506	1517	1528	1539
14	1549	1560	1571	1582	1593	1604	1615	1626	1637	1648
15	1658	1669	1680	1691	1701	1712	1723	1734	1745	1756
0.16	1766	1777	1788	1799	1809	1820	1831	1842	1853	1864
17	1874	1885	1896	1907	1917	1928	1939	1950	1961	1972
18	1982	1993	2004	2015	2025	2036	2047	2058	2069	2079
19	2090	2100	2111	2122	2132	2143	2154	2164	2175	2186
20	2197	2207	2218	2229	2239	2250	2261	2271	2282	2293
0.21	2304	2314	2325	2335	2346	2357	2367	2378	2388	2399
22	2410	2420	2431	2441	2452	2463	2473	2484	2494	2505
23	2516	2526	2537	2547	2558	2569	2579	2590	2600	2611
24	2622	2632	2643	2653	2664	2675	2685	2696	2706	2717
25	2728	2738	2749	2759	2770	2781	2791	2802	2812	2823
0.26	2834	2844	2855	2865	2876	2887	2897	2908	2918	2929
27	2940	2950	2961	2971	2982	2992	3003	3013	3024	3034
28	3045	3055	3066	3076	3087	3097	3108	3118	3129	3139
29	3150	3160	3170	3181	3191	3202	3212	3223	3233	3244
30	3254	3264	3274	3285	3295	3306	3316	3327	3337	3348
0.31	3358	3368	3378	3389	3399	3410	3420	3431	3441	3452
32	3462	3472	3482	3493	3503	3514	3524	3535	3545	3556
33	3566	3577	3587	3597	3608	3618	3628	3639	3649	3659
34	3669	3680	3690	3700	3711	3721	3731	3742	3752	3762
35	3772	3783	3793	3803	3814	3824	3834	3845	3855	3865
0.36	3875	3886	3896	3906	3917	3927	3937	3948	3958	3968
37	3978	3989	3999	4009	4020	4030	4040	4051	4061	4071
38	4081	4092	4102	4112	4122	4132	4143	4153	4163	4173
39	4183	4194	4204	4214	4224	4234	4245	4255	4265	4275
40	4285	4296	4306	4316	4326	4336	4347	4357	4367	4377
0.41	4387	4398	4408	4418	4428	4438	4449	4459	4469	4479
42	4489	4500	4510	4520	4530	4540	4550	4560	4570	4580
43	4590	4601	4611	4621	4631	4641	4651	4661	4671	4681
44	4691	4702	4712	4722	4732	4742	4752	4762	4772	4782
45	4792	4803	4813	4823	4833	4843	4853	4863	4873	4883
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4954	4964	4974	4984

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	780	791	802	813	824	835	846	857	868	879
8	890	901	912	923	934	945	956	967	978	989
9	0.1000	1011	1022	1033	1044	1055	1066	1077	1088	1099
10	1110	1121	1132	1143	1154	1165	1176	1187	1198	1209
0.11	1220	1231	1242	1253	1264	1275	1286	1297	1308	1319
12	1330	1341	1352	1363	1374	1385	1396	1407	1418	1429
13	1440	1451	1462	1473	1484	1495	1506	1517	1528	1539
14	1549	1560	1571	1582	1593	1604	1615	1626	1637	1648
15	1658	1669	1680	1691	1701	1712	1723	1734	1745	1756
0.16	1766	1777	1788	1799	1809	1820	1831	1842	1853	1864
17	1874	1885	1896	1907	1917	1928	1939	1950	1961	1972
18	1982	1993	2004	2015	2025	2036	2047	2058	2069	2079
19	2090	2100	2111	2122	2132	2143	2154	2164	2175	2186
20	2197	2207	2218	2229	2239	2250	2261	2271	2282	2293
0.21	2304	2314	2325	2335	2346	2357	2367	2378	2388	2399
22	2410	2420	2431	2441	2452	2463	2473	2484	2494	2505
23	2516	2526	2537	2547	2558	2569	2579	2590	2600	2611
24	2622	2632	2643	2653	2664	2675	2685	2696	2706	2717
25	2728	2738	2749	2759	2770	2781	2791	2802	2812	2823
0.26	2834	2844	2855	2865	2876	2887	2897	2908	2918	2929
27	2940	2950	2961	2971	2982	2992	3003	3013	3024	3034
28	3045	3055	3066	3076	3087	3097	3108	3118	3129	3139
29	3150	3160	3170	3181	3191	3202	3212	3223	3233	3244
30	3254	3264	3274	3285	3295	3306	3316	3327	3337	3348
0.31	3358	3368	3378	3389	3399	3410	3420	3431	3441	3452
32	3462	3472	3482	3493	3503	3514	3524	3535	3545	3556
33	3566	3577	3587	3597	3608	3618	3628	3639	3649	3659
34	3669	3680	3690	3700	3711	3721	3731	3742	3752	3762
35	3772	3783	3793	3803	3814	3824	3834	3845	3855	3865
0.36	3875	3886	3896	3906	3917	3927	3937	3948	3958	3968
37	3978	3989	3999	4009	4020	4030	4040	4051	4061	4071
38	4081	4092	4102	4112	4122	4132	4143	4153	4163	4173
39	4183	4194	4204	4214	4224	4234	4245	4255	4265	4275
40	4285	4296	4306	4316	4326	4336	4347	4357	4367	4377
0.41	4387	4398	4408	4418	4428	4438	4449	4459	4469	4479
42	4489	4500	4510	4520	4530	4540	4550	4560	4570	4580
43	4590	4601	4611	4621	4631	4641	4651	4661	4671	4681
44	4691	4702	4712	4722	4732	4742	4752	4762	4772	4782
45	4792	4803	4813	4823	4833	4843	4853	4863	4873	4883
0.46	4893	4904	4914	4924	4934	4944	4954	4964	4974	4984
47	4994	5005	5015	5025	5035	5045	5055	5065	5075	5085
48	5095	5105	5115	5125	5135	5145	5155	5165	5175	5185
49	5195	5205	5215	5225	5235	5245	5255	5265	5275	5285
50	5295	5305	5315	5325	5335	5345	5355	5365	5375	5385

STAT

(cont'd.)

$\partial = 0.89$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5394	0.5404	0.5414	0.5424	0.5434	0.5444	0.5454	0.5464	0.5474	0.5484
52	5493	5503	5013	5523	5533	5543	5553	5563	5573	5583
53	5592	5602	5612	5622	5632	5642	5652	5662	5672	5682
54	5691	5701	5711	5721	5731	5741	5751	5761	5771	5781
55	5790	5800	5810	5820	5830	5840	5849	5859	5869	5879
0.56	5888	5898	5908	5918	5928	5938	5947	5957	5967	5977
57	5986	5996	6006	6016	6026	6036	6045	6055	6065	6075
58	6084	6094	6104	6114	6124	6134	6143	6153	6163	6173
59	6182	6192	6202	6212	6221	6231	6241	6250	6260	6270
60	6279	6289	6299	6309	6318	6328	6338	6347	6357	6367
0.61	6376	6386	6396	6406	6415	6425	6435	6444	6454	6464
62	6473	6483	6493	6503	6512	6522	6532	6541	6551	6561
63	6570	6580	6590	6599	6609	6618	6628	6637	6647	6657
64	6666	6676	6686	6695	6705	6714	6724	6733	6743	6753
65	6762	6772	6782	6791	6801	6810	6820	6829	6839	6849
0.66	6858	6868	6878	6887	6897	6906	6916	6925	6935	6945
67	6954	6964	6974	6983	6993	7002	7012	7021	7031	7041
68	7050	7060	7069	7079	7088	7098	7107	7117	7126	7136
69	7145	7155	7164	7174	7183	7193	7202	7212	7221	7231
70	7240	7250	7259	7269	7278	7288	7297	7307	7316	7326
0.71	7335	7344	7354	7363	7373	7382	7392	7401	7411	7420
72	7429	7438	7448	7457	7467	7476	7486	7495	7505	7514
73	7523	7582	7542	7551	7561	7570	7580	7589	7599	7608
74	7617	7626	7636	7645	7755	7664	7674	7683	7693	7702
75	7711	7721	7730	7740	7749	7758	7767	7777	7786	7796
0.76	7804	7814	7823	7832	7842	7851	7860	7870	7879	7888
77	7897	7907	7916	7925	7935	7944	7953	7963	7972	7981
78	7990	8000	8009	8018	8028	8037	8046	8056	8065	8074
79	8083	8093	8102	8118	8121	8130	8139	8149	8158	8167
80	8176	8186	8195	8204	8214	8223	8232	8242	8251	8280
0.81	8269	8279	8288	8297	8307	8316	8325	8835	8344	8353
82	8362	8372	8381	8390	8400	8409	8418	8428	8437	8446
83	8455	8465	8474	8483	8493	8502	8511	8521	8530	8539
84	8548	8558	8567	8576	8585	8594	8604	8613	8622	8631
85	8640	8650	8659	8668	8677	8686	8696	8705	8714	8723
0.86	8732	8742	8751	8760	8769	8778	8788	8797	8806	8815
87	8824	8834	8843	8852	8861	8870	8880	8889	8898	8907
88	8916	8926	8935	8944	8953	8962	8972	8981	8990	8999
89	9008	9017	9027	9036	9045	9054	9064	9073	9082	9091
90	9100	9109	9118	9127	9136	9146	9155	9164	9173	9182
0.91	9191	9200	9209	9218	9228	9237	9246	9255	9264	9273
92	9282	9291	9300	9310	9319	9328	9337	9346	9355	9364
93	9373	9382	9392	9401	9410	9419	9428	9437	9446	9455
94	9464	9473	9482	9491	9500	9509	9518	9527	9536	9545
95	9554	9563	9572	9581	9590	9599	9608	9617	9626	9635
0.96	9644	9653	9662	9671	9680	9689	9698	9707	9716	9725
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9770	9779	9788	9797	9806	9814

				[illegible]	[illegible]	[illegible]	6143	6153	6163	6173
59	6182	6192	6202	6212	6221	6231	6241	6250	6260	6270
60	6279	6289	6299	6309	6318	6328	6338	6347	6357	6367
0.61	6376	6386	6396	6406	6415	6425	6435	6444	6454	6464
62	6473	6483	6493	6503	6512	6522	6532	6541	6551	6561
63	6570	6580	6590	6599	6609	6618	6628	6637	6647	6657
64	6666	6676	6686	6695	6705	6714	6724	6733	6743	6753
65	6762	6772	6782	6791	6801	6810	6820	6829	6839	6849
0.66	6858	6868	6878	6887	6897	6906	6916	6925	6935	6945
67	6954	6964	6974	6983	6993	7002	7012	7021	7031	7041
68	7050	7060	7069	7079	7088	7098	7107	7117	7126	7136
69	7145	7155	7164	7174	7183	7193	7202	7212	7221	7231
70	7240	7250	7259	7269	7278	7288	7297	7307	7316	7326
0.71	7335	7344	7354	7363	7373	7382	7392	7401	7411	7420
72	7429	7438	7448	7457	7467	7476	7486	7495	7505	7514
73	7523	7582	7542	7551	7561	7570	7580	7589	7599	7608
74	7617	7626	7636	7645	7755	7664	7674	7683	7693	7702
75	7711	7721	7730	7740	7749	7758	7767	7777	7786	7796
0.76	7804	7814	7823	7832	7842	7851	7860	7870	7879	7888
77	7897	7907	7916	7925	7935	7944	7953	7963	7972	7981
78	7990	8000	8009	8018	8028	8037	8046	8056	8065	8074
79	8083	8093	8102	8118	8121	8130	8139	8149	8158	8167
80	8176	8186	8195	8204	8214	8223	8232	8242	8251	8280
0.81	8269	8279	8288	8297	8307	8316	8325	8835	8344	8353
82	8362	8372	8381	8390	8400	8409	8418	8428	8437	8446
83	8455	8465	8474	8483	8493	8502	8511	8521	8530	8539
84	8548	8558	8567	8576	8585	8594	8604	8613	8622	8631
85	8640	8650	8659	8668	8677	8686	8696	8705	8714	8723
0.86	8732	8742	8751	8760	8769	8778	8788	8797	8806	8815
87	8824	8834	8843	8852	8861	8870	8880	8889	8898	8907
88	8916	8926	8935	8944	8953	8962	8972	8981	8990	8999
89	9008	9017	9027	9036	9045	9054	9064	9073	9082	9091
90	9100	9109	9118	9127	9136	9146	9155	9164	9173	9182
0.91	9191	9200	9209	9218	9228	9237	9246	9255	9264	9273
92	9282	9291	9300	9310	9319	9328	9337	9346	9355	9364
93	9373	9382	9392	9401	9410	9419	9428	9437	9446	9455
94	9464	9473	9482	9491	9500	9509	9518	9527	9536	9545
95	9554	9563	9572	9581	9590	9599	9608	9617	9626	9635
0.96	9644	9653	9662	9671	9680	9689	9698	9707	9716	9725
97	9734	9743	9752	9761	9770	9779	9788	9797	9806	9814
98	9823	9832	9841	9850	9859	9868	9877	9886	9895	9904
99	9912	9921	9930	9939	9948	9957	9966	9975	9984	9992
1.00	1.000	-	-	-	-	-	-	-	-	-

Sanitized Copy Approved for Release 2010/10/29 : CIA-RDP81-01043R001100040001-0

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$\partial = 0.90$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0012	0.0023	0.0034	0.0045	0.0057	0.0068	0.0079	0.0090	0.0101
0.01	0.0112	123	134	145	157	168	179	190	201	212
2	223	234	245	256	267	278	289	300	311	322
3	333	344	355	366	377	388	399	410	421	432
4	443	454	465	476	487	498	509	520	531	542
0.05	553	564	575	586	597	608	619	630	641	652
0.06	663	674	685	695	706	717	728	739	750	761
7	772	783	794	804	815	826	837	848	859	870
8	881	892	903	913	924	935	946	957	968	979
9	990	1001	1012	1022	1033	1044	1055	1066	1077	0.1088
0.10	0.1099	1110	1121	1131	1142	1153	1164	1175	1186	1197
0.11	1208	1219	1230	1240	1251	1262	1273	1284	1295	1305
12	1316	1327	1338	1348	1359	1370	1381	1392	1403	1413
13	1424	1435	1446	1456	1467	1478	1489	1500	1511	1521
14	1532	1543	1554	1564	1575	1586	1597	1608	1618	1529
15	1640	1650	1661	1672	1682	1693	1704	1715	1725	1736
0.16	1747	1757	1768	1779	1789	1800	1811	1822	1832	1843
17	1854	1864	1875	1886	1896	1907	1918	1929	1939	1950
18	1961	1971	1982	1993	2003	2014	2025	2036	2046	2057
19	2068	2078	2089	2100	2110	2121	2131	2142	2152	2163
20	2174	2184	2195	2206	2216	2227	2238	2249	2259	2270
0.21	2280	2290	2301	2312	2322	2333	2344	2355	2365	2376
22	2386	2396	2407	2418	2428	2439	2450	2461	2471	2442
23	2492	2503	2513	2524	2534	2545	2555	2566	2576	2587
24	2597	2608	2618	2629	2639	2650	2660	2671	2681	2692
25	2702	2713	2723	2734	2744	2755	2765	2776	2786	2797
0.26	2807	2818	2828	2839	2849	2860	2870	2881	2891	2902
27	2912	2923	2933	2944	2954	2964	2975	2985	2996	3006
28	3016	3027	3037	3048	3058	3068	3079	3089	3100	3110
29	3120	3131	3141	3152	3162	3172	3183	3193	3204	3214
30	3224	3235	3245	3256	3266	3276	3287	3297	3308	3318
0.31	3328	3339	3349	3360	3370	3380	3391	3401	3412	3422
32	3432	3443	3453	3464	3474	3484	3495	3505	3516	3526
33	3536	3547	3557	3567	3578	3588	3598	3609	3619	3629
34	3639	3650	3660	3670	3681	3691	3701	3712	3722	3732
35	3742	3753	3763	3773	3784	3794	3804	3815	3825	3835
0.36	3845	3856	3866	3876	3887	3897	3907	3918	3928	3938
37	3948	3959	3969	3979	3990	4000	4010	4021	4031	4041
38	4051	4062	4072	4082	4093	4103	4113	4124	4134	4144
39	4154	4165	4175	4185	4195	4205	4216	4226	4236	4246
40	4256	4267	4277	4287	4297	4307	4318	4328	4338	4348
0.41	4358	4369	4379	4389	4399	4409	4420	4430	4440	4450
42	4460	4470	4480	4490	4500	4511	4521	4531	4541	4551
43	4561	4571	4581	4591	4601	4612	4622	4632	4642	4652
44	4662	4672	4682	4692	4702	4713	4723	4733	4743	4753
45	4763	4773	4783	4793	4803	4813	4823	4833	4843	4853
0.46	4863	4874	4884	4894	4904	4914	4924	4934	4944	4954
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5004	5014	5024	5034	5044	5054

	[illegible]	[illegible]	[illegible]	476	487	498	509	520	531	542
0.05	553	564	575	586	597	608	619	630	641	652
0.06	663	674	685	695	706	717	728	739	750	761
7	772	783	794	804	815	826	837	848	859	870
8	881	892	903	913	924	935	946	957	968	979
9	990	1001	1012	1022	1033	1044	1055	1066	1077	0.1088
0.10	0.1099	1110	1121	1131	1142	1153	1164	1175	1186	1197
0.11	1208	1219	1230	1240	1251	1262	1273	1284	1295	1305
12	1316	1327	1338	1348	1359	1370	1381	1392	1403	1413
13	1424	1435	1446	1456	1467	1478	1489	1500	1511	1521
14	1532	1543	1554	1564	1575	1586	1597	1608	1618	1529
15	1640	1650	1661	1672	1682	1693	1704	1715	1725	1736
0.16	1747	1757	1768	1779	1789	1800	1811	1822	1832	1843
17	1854	1864	1875	1886	1896	1907	1918	1929	1939	1950
18	1961	1971	1982	1993	2003	2014	2025	2036	2046	2057
19	2068	2078	2089	2100	2110	2121	2131	2142	2152	2163
20	2174	2184	2195	2206	2216	2227	2238	2249	2259	2270
0.21	2280	2290	2301	2312	2322	2333	2344	2355	2365	2376
22	2386	2396	2407	2418	2428	2439	2450	2461	2471	2442
23	2492	2503	2513	2524	2534	2545	2555	2566	2576	2587
24	2597	2608	2618	2629	2639	2650	2660	2671	2681	2692
25	2702	2713	2723	2734	2744	2755	2765	2776	2786	2797
0.26	2807	2818	2828	2839	2849	2860	2870	2881	2891	2902
27	2912	2923	2933	2944	2954	2964	2975	2985	2996	3006
28	3016	3027	3037	3048	3058	3068	3079	3089	3100	3110
29	3120	3131	3141	3152	3162	3172	3183	3193	3204	3214
30	3224	3235	3245	3256	3266	3276	3287	3297	3308	3318
0.31	3328	3339	3349	3360	3370	3380	3391	3401	3412	3422
32	3432	3443	3453	3464	3474	3484	3495	3505	3516	3526
33	3536	3547	3557	3567	3578	3588	3598	3609	3619	3629
34	3639	3650	3660	3670	3681	3691	3701	3712	3722	3732
35	3742	3753	3763	3773	3784	3794	3804	3815	3825	3835
0.36	3845	3856	3866	3876	3887	3897	3907	3918	3928	3938
37	3948	3959	3969	3979	3990	4000	4010	4021	4031	4041
38	4051	4062	4072	4082	4093	4103	4113	4124	4134	4144
39	4154	4165	4175	4185	4195	4205	4216	4226	4236	4246
40	4256	4267	4277	4287	4297	4307	4318	4328	4338	4348
0.41	4358	4369	4379	4389	4399	4409	4420	4430	4440	4450
42	4460	4470	4480	4490	4500	4511	4521	4531	4541	4551
43	4561	4571	4581	4591	4601	4612	4622	4632	4642	4652
44	4662	4672	4682	4692	4702	4713	4723	4733	4743	4753
45	4763	4773	4783	4793	4803	4813	4823	4833	4843	4853
0.46	4863	4874	4884	4894	4904	4914	4924	4934	4944	4954
47	4964	4974	4984	4994	5004	5014	5024	5034	5044	5054
48	5064	5075	5085	5095	5105	5115	5125	5135	5145	5155
49	5165	5175	5185	5195	5205	5215	5225	5235	5245	5255
0.50	5265	5275	5285	5295	5305	5315	5325	5335	5345	5355

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$\partial = 0.90$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5365	0.5375	0.5385	0.5395	0.5405	0.5415	0.5425	0.5435	0.5445	0.5455
52	5465	5474	5484	5494	5504	5514	5524	5534	5544	5554
53	5564	5574	5583	5593	5603	5613	5623	5633	5643	5653
54	5663	5673	5683	5693	5702	5712	5722	5732	5742	5752
55	5762	5772	5782	5792	5802	5812	5821	5831	5841	5851
0.56	5861	5871	5881	5891	5901	5911	5921	5930	5940	5950
57	5960	5970	5980	5990	6000	6009	6019	6029	6039	6049
58	6058	6068	6078	6088	6097	6107	6117	6127	6137	6146
59	6156	6166	6176	6185	6195	6205	6215	6225	6234	6244
60	6254	6264	6273	6283	6293	6302	6312	6322	6331	6341
0.61	6351	6361	6370	6380	6390	6399	6409	6419	6428	6438
62	6448	6458	6467	6477	6487	6496	6506	6516	6525	6535
63	6545	6555	6564	6574	6584	6593	6603	6613	6622	6632
64	6642	6652	6661	6671	6681	6690	6700	6710	6719	6729
65	6739	6748	6758	6767	6777	6787	6796	6806	6815	6825
0.66	6835	6844	6854	6863	6873	6883	6892	6908	6911	6921
67	6931	6940	6950	6959	6969	6979	6988	6998	7007	7017
68	7027	7036	7046	7055	7065	7075	7084	7094	7103	7113
69	7123	7132	7142	7151	7161	7170	7180	7189	7199	7208
70	7218	7227	7237	7246	7256	7265	7275	7284	7294	7303
0.71	7313	7322	7332	7341	7351	7360	7370	7379	7389	7398
72	7408	7417	7427	7436	7446	7455	7465	7474	7484	7493
73	7503	7514	7512	7531	7541	7550	7560	7569	7579	7588
74	7598	7607	7617	7626	7636	7645	7655	7664	7674	7683
75	7693	7702	7712	7721	7731	7740	7750	7759	7769	7778
0.76	7788	7797	7807	7816	7826	7835	7845	7854	7864	7873
77	7883	7892	7902	7911	7920	7929	7939	7948	7958	7967
78	7977	7986	7996	8005	8014	8023	8033	8042	8052	8061
79	8071	8080	8090	8099	8108	8117	8127	8136	8146	8155
80	8165	8174	8184	8193	8202	8211	8221	8230	8240	8249
0.81	8259	8268	8278	8287	8296	8305	8315	8324	8334	8343
82	8353	8362	8372	8381	8390	8400	8409	8418	8428	8437
83	8446	8455	8465	8474	8483	8493	8502	8511	8521	8530
84	8539	8548	8558	8567	8576	8586	8595	8604	8614	8623
85	8632	8641	8651	8660	8669	8679	8688	8697	8707	8716
0.86	8725	8735	8744	8753	8762	8771	8781	8790	8799	8808
87	8817	8827	8836	8845	8854	8863	8873	8882	8891	8900
88	8909	8919	8928	8937	8946	8955	8965	8974	8983	8992
89	9001	9011	9020	9029	9038	9047	9057	9066	9075	9084
90	9093	9103	9112	9121	9130	9139	9149	9158	9167	9176
0.91	9185	9195	9204	9213	9222	9231	9240	9249	9258	9267
92	9276	9286	9295	9304	9313	9322	9331	9340	9349	9358
93	9367	9377	9386	9395	9404	9413	9422	9431	9440	9449
94	9458	9468	9477	9486	9495	9504	9513	9522	9531	9540
95	9549	9559	9568	9577	9586	9595	9604	9613	9622	9631
0.96	9640	9649	9658	9667	9676	9685	9694	9703	9712	9721
97	9730	9739	9748	9757	9766	9775	9784	9793	9802	9811
98	9820	9829	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

58	6058	6068	6078	6088	6097	6107	6117	6127	6137	6146
59	6156	6166	6176	6185	6195	6205	6215	6225	6234	6244
60	6254	6264	6273	6283	6293	6302	6312	6322	6331	6341
0.61	6351	6361	6370	6380	6390	6399	6409	6419	6428	6438
62	6448	6458	6467	6477	6487	6496	6506	6516	6525	6535
63	6545	6555	6564	6574	6584	6593	6603	6613	6622	6632
64	6642	6652	6661	6671	6681	6690	6700	6710	6719	6729
65	6739	6748	6758	6767	6777	6787	6796	6806	6815	6825
0.66	6835	6844	6854	6863	6873	6883	6892	6908	6911	6921
67	6931	6940	6950	6959	6969	6979	6988	6998	7007	7017
68	7027	7036	7046	7055	7065	7075	7084	7094	7103	7113
69	7123	7132	7142	7151	7161	7170	7180	7189	7199	7208
70	7218	7227	7237	7246	7256	7265	7275	7284	7294	7303
0.71	7313	7322	7332	7341	7351	7360	7370	7379	7389	7398
72	7408	7417	7427	7436	7446	7455	7465	7474	7484	7493
73	7503	7514	7512	7531	7541	7550	7560	7569	7579	7588
74	7598	7607	7617	7626	7636	7645	7655	7664	7674	7683
75	7693	7702	7712	7721	7731	7740	7750	7759	7769	7778
0.76	7788	7797	7807	7816	7826	7835	7845	7854	7864	7873
77	7883	7892	7902	7911	7920	7929	7939	7948	7958	7967
78	7977	7986	7996	8005	8014	8023	8033	8042	8052	8061
79	8071	8080	8090	8099	8108	8117	8127	8136	8146	8155
80	8165	8174	8184	8193	8202	8211	8221	8230	8240	8249
0.81	8259	8268	8278	8287	8296	8305	8315	8324	8334	8343
82	8353	8362	8372	8381	8390	8400	8409	8418	8428	8437
83	8446	8455	8465	8474	8483	8493	8502	8511	8521	8530
84	8539	8548	8558	8567	8576	8586	8595	8604	8614	8623
85	8632	8641	8651	8660	8669	8679	8688	8697	8707	8716
0.86	8725	8735	8744	8753	8762	8771	8781	8790	8799	8808
87	8817	8827	8836	8845	8854	8863	8873	8882	8891	8900
88	8909	8919	8928	8937	8946	8955	8965	8974	8983	8992
89	9001	9011	9020	9029	9038	9047	9057	9066	9075	9084
90	9093	9103	9112	9121	9130	9139	9149	9158	9167	9176
0.91	9185	9195	9204	9213	9222	9231	9240	9249	9258	9267
92	9276	9286	9295	9304	9313	9322	9331	9340	9349	9358
93	9367	9377	9386	9395	9404	9413	9422	9431	9440	9449
94	9458	9468	9477	9486	9495	9504	9513	9522	9531	9540
95	9549	9559	9568	9577	9586	9595	9604	9613	9622	9631
0.96	9640	9649	9658	9667	9676	9685	9694	9703	9712	9721
97	9730	9739	9748	9757	9766	9775	9784	9793	9802	9811
98	9820	9829	9838	9847	9856	9865	9874	9883	9892	9901
99	9910	9919	9928	9937	9946	9955	9964	9973	9982	0.9991
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STAT

(cont'd.)

$\partial = 0.91$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0.0000	0.0011	0.0022	0.0033	0.0044	0.0055	0.0066	0.0077	0.0088	0.0099
0.01	0.0110	121	132	143	154	165	175	186	197	208
2	219	230	241	252	263	274	284	295	306	317
3	328	339	350	361	372	383	393	404	415	426
4	437	448	459	470	481	492	502	513	524	535
5	546	557	568	579	590	601	611	622	633	644
0.06	655	666	677	688	699	710	720	731	742	753
7	764	775	796	796	807	818	829	840	850	861
8	872	883	894	904	915	926	937	948	958	969
9	980	990	0.1001	1012	1022	1033	1044	1055	1065	1076
0.10	0.1087	1097	1108	1119	1129	1140	1151	1162	1172	1183
0.11	1194	1204	1215	1226	1236	1247	1258	1269	1279	1290
12	1301	1311	1322	1333	1343	1354	1365	1376	1386	1397
13	1408	1418	1429	1440	1450	1461	1472	1483	1493	1504
14	1515	1525	1536	1547	1557	1568	1579	1590	1600	1611
15	1622	1632	1643	1654	1664	1675	1686	1697	1707	1718
0.16	1729	1739	1750	1761	1771	1782	1792	1803	1814	1825
17	1835	1845	1856	1867	1877	1888	1898	1909	1920	1931
18	1941	1952	1962	1973	1983	1994	2004	2015	2026	2036
19	2047	2058	2068	2079	2089	2100	2110	2121	2132	2142
20	2153	2163	2174	2185	2195	2206	2216	2227	2238	2248
0.21	2259	2270	2280	2291	2301	2312	2322	2333	2343	2354
22	2364	2375	2385	2396	2406	2417	2428	2438	2448	2459
23	2469	2480	2490	2501	2511	2522	2532	2543	2553	2564
24	2574	2584	2595	2606	2616	2627	2637	2648	2658	2669
25	2679	2690	2700	2711	2721	2732	2742	2753	2763	2774
0.26	2784	2794	2804	2815	2825	2836	2846	2857	2867	2877
27	2888	2898	2908	2919	2929	2940	2950	2961	2971	2981
28	2992	3002	3012	3023	3033	3044	3054	3065	3075	3085
29	3096	3106	3116	3127	3137	3148	3158	3169	3179	3189
30	3200	3210	3220	3231	3241	3252	3262	3273	3283	3293
0.31	3304	3314	3324	3335	3345	3355	3366	3376	3386	3397
32	3407	3417	3427	3438	3448	3458	3469	3479	3489	3500
33	3510	3520	3530	3541	3551	3561	3572	3582	3592	3603
34	3613	3623	3633	3644	3654	3664	3675	3685	3695	3706
35	3716	3726	3736	3747	3757	3767	3778	3788	3798	3809
0.36	3819	3830	3840	3850	3860	3870	3881	3891	3901	3911
37	3921	3932	3942	3952	3962	3972	3983	3993	4003	4013
38	4023	4034	4044	4054	4064	4074	4085	4095	4105	4115
39	4125	4136	4146	4156	4166	4176	4187	4197	4207	4217
40	4227	4238	4248	4258	4268	4278	4289	4299	4309	4319
0.41	4329	4340	4350	4360	4370	4380	4390	4400	4410	4420
42	4430	4441	4451	4461	4471	4481	4491	4501	4511	4521
43	4531	4542	4552	4562	4572	4582	4592	4602	4612	4622
44	4632	4643	4653	4663	4673	4683	4693	4703	4713	4723
45	4733	4744	4754	4764	4774	4784	4794	4804	4814	4824

0.06	655	666	677	688	699	710	720	731	742	753
7	764	775	796	796	807	818	829	840	850	861
8	872	883	894	904	915	926	937	948	958	969
9	980	990	0.1001	1012	1022	1033	1044	1055	1065	1076
0.10	0.1087	1097	1108	1119	1129	1140	1151	1162	1172	1183
0.11	1194	1204	1215	1226	1236	1247	1258	1269	1279	1290
12	1301	1311	1322	1333	1343	1354	1365	1376	1386	1397
13	1408	1418	1429	1440	1450	1461	1472	1483	1493	1504
14	1515	1525	1536	1547	1557	1568	1579	1590	1600	1611
15	1622	1632	1643	1654	1664	1675	1686	1697	1707	1718
0.16	1729	1739	1750	1761	1771	1782	1792	1803	1814	1825
17	1835	1845	1856	1867	1877	1888	1898	1909	1920	1931
18	1941	1952	1962	1973	1983	1994	2004	2015	2026	2036
19	2047	2058	2068	2079	2089	2100	2110	2121	2132	2142
20	2153	2163	2174	2185	2195	2206	2216	2227	2238	2248
0.21	2259	2270	2280	2291	2301	2312	2322	2333	2343	2354
22	2364	2375	2385	2396	2406	2417	2428	2438	2448	2459
23	2469	2480	2490	2501	2511	2522	2532	2543	2553	2564
24	2574	2584	2595	2606	2616	2627	2637	2648	2658	2669
25	2679	2690	2700	2711	2721	2732	2742	2753	2763	2774
0.26	2784	2794	2804	2815	2825	2836	2846	2857	2867	2877
27	2888	2898	2908	2919	2929	2940	2950	2961	2971	2981
28	2992	3002	3012	3023	3033	3044	3054	3065	3075	3085
29	3096	3106	3116	3127	3137	3148	3158	3169	3179	3189
30	3200	3210	3220	3231	3241	3252	3262	3273	3283	3293
0.31	3304	3314	3324	3335	3345	3355	3366	3376	3386	3397
32	3407	3417	3427	3438	3448	3458	3469	3479	3489	3500
33	3510	3520	3530	3541	3551	3561	3572	3582	3592	3603
34	3613	3623	3633	3644	3654	3664	3675	3685	3695	3706
35	3716	3726	3736	3747	3757	3767	3778	3788	3798	3809
0.36	3819	3830	3840	3850	3860	3870	3881	3891	3901	3911
37	3921	3932	3942	3952	3962	3972	3983	3993	4003	4013
38	4023	4034	4044	4054	4064	4074	4085	4095	4105	4115
39	4125	4136	4146	4156	4166	4176	4187	4197	4207	4217
40	4227	4238	4248	4258	4268	4278	4289	4299	4309	4319
0.41	4329	4340	4350	4360	4370	4380	4390	4400	4410	4420
42	4430	4441	4451	4461	4471	4481	4491	4501	4511	4521
43	4531	4542	4552	4562	4572	4582	4592	4602	4612	4622
44	4632	4643	4653	4663	4673	4683	4693	4703	4713	4723
45	4733	4744	4754	4764	4774	4784	4794	4804	4814	4824
0.46	4834	4844	4854	4864	4874	4884	4894	4904	4914	4924
47	4934	4944	4954	4964	4974	4984	4994	5004	5014	5024
48	5034	5045	5055	5065	5075	4085	5095	5105	5115	5125
49	5135	5145	5155	5165	5175	5185	5195	5205	5215	5225
0.50	5235	5245	5255	5265	5275	5285	5295	5305	5315	5325

STAT

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(cont'd.)
∂ = 0.91

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5335	0.5345	0.5355	0.5365	0.5375	0.5385	0.5395	0.5405	0.5415	0.5425
52	5434	5444	5454	5464	5474	5484	5494	5504	5514	5524
53	5533	5543	5553	5563	5573	5583	5593	5603	5613	5623
54	5632	5642	5652	5662	5672	5682	5692	5702	5712	5722
55	5731	5741	5751	5761	5771	5781	5791	5801	5811	5821
0.56	5830	5840	5850	5860	5870	5880	5890	5900	5910	5920
57	5929	5939	5949	5959	5969	5978	5988	5998	6008	6018
58	6027	6037	6047	6057	6067	6076	6086	6096	6106	6116
59	6125	6135	6145	6155	6165	6174	6184	6194	6204	6214
60	6223	6233	6243	6253	6263	6272	6282	6292	6302	6312
0.61	6321	6331	6341	6351	6361	6370	6380	6390	6400	6410
62	6419	6429	6439	6449	6459	6468	6478	6488	6498	6508
63	6517	6527	6537	6546	6556	6566	6575	6585	6595	6605
64	6614	6624	6634	6643	6653	6663	6672	6682	6692	6702
65	6711	6721	6731	6740	6750	6760	6769	6779	6789	6799
0.66	6808	6818	6828	6837	6847	6857	6866	6876	6886	6896
67	6905	6915	6925	6934	6944	6954	6963	6973	6983	6993
68	7002	7012	7022	7031	7041	7051	7060	7070	7080	7090
69	7099	7109	7119	7128	7138	7147	7157	7167	7176	7186
70	7195	7205	7215	7224	7234	7243	7253	7263	7272	7282
0.71	7291	7301	7311	7320	7330	7339	7349	7359	7368	7378
72	7387	7397	7406	7416	7425	7435	7444	7454	7463	7473
73	7482	7492	7501	7511	7520	7530	7539	7549	7558	7568
74	7577	7587	7596	7606	7615	7625	7634	7644	7653	7663
75	7672	7682	7691	7701	7710	7720	7729	7739	7748	7758
0.76	7767	7777	7786	7796	7805	7815	7824	7834	7843	7853
77	7862	7872	7881	7891	7900	7910	7919	7929	7938	7948
78	7957	7967	7976	7986	7995	8005	8014	8024	8033	8043
79	8052	8062	8071	8081	8090	8100	8109	8119	8128	8138
80	8147	8157	8166	8176	8185	8195	8204	8214	8223	8233
0.81	8242	8252	8261	8271	8280	8289	8299	8308	8318	8327
82	8336	8346	8355	8365	8374	8383	8393	8402	8412	8421
83	8430	8440	8449	8458	8468	8477	8486	8496	8505	8514
84	8523	8533	8542	8551	8561	8570	8579	8589	8598	8607
85	8616	8626	8635	8644	8654	8663	8672	8682	8691	8700
0.86	8709	8719	8728	8737	8747	8756	8765	8775	8784	8793
87	8802	8812	8821	8830	8840	8849	8858	8868	8877	8886
88	8895	8905	8914	8923	8933	8942	8951	8961	8970	8979
89	8988	8998	9007	9016	9026	9035	9044	9054	9063	9072
90	9081	9091	9100	9109	9119	9128	9137	9147	9156	9165
0.91	9174	9184	9193	9202	9212	9221	9230	9240	9249	9258
92	9267	9277	9296	9295	9304	9313	9323	9332	9341	9350
93	9359	9369	9378	9387	9396	9405	9415	9424	9433	9442
94	9451	9461	9470	9479	9488	9497	9507	9516	9525	9534
95	9543	9553	9562	9571	9580	9589	9599	9608	9617	9625
0.96	9635	9645	9654	9663	9672	9681	9691	9700	9709	9718
97	9727	9737	9746	9755	9764	9773	9782	9791	9800	9809
98	9818	9828	9837	9846	9855	9864	9873	9882	9891	9900

57	5929	5939	5949	5959	5969	5978	5988	5998	6008	6018
58	6027	6037	6047	6057	6067	6076	6086	6096	6106	6116
59	6125	6135	6145	6155	6165	6174	6184	6194	6204	6214
60	6223	6233	6243	6253	6263	6272	6282	6292	6302	6312
0.61	6321	6331	6341	6351	6361	6370	6380	6390	6400	6410
62	6419	6429	6439	6449	6459	6468	6478	6488	6498	6508
63	6517	6527	6537	6546	6556	6566	6575	6585	6595	6605
64	6614	6624	6634	6643	6653	6663	6672	6682	6692	6702
65	6711	6721	6731	6740	6750	6760	6769	6779	6789	6799
0.66	6808	6818	6828	6837	6847	6857	6866	6876	6886	6896
67	6905	6915	6925	6934	6944	6954	6963	6973	6983	6993
68	7002	7012	7022	7031	7041	7051	7060	7070	7080	7090
69	7099	7109	7119	7128	7138	7147	7157	7167	7176	7186
70	7195	7205	7215	7224	7234	7243	7253	7263	7272	7282
0.71	7291	7301	7311	7320	7330	7339	7349	7359	7368	7378
72	7387	7397	7406	7416	7425	7435	7444	7454	7463	7473
73	7482	7492	7501	7511	7520	7530	7539	7549	7558	7568
74	7577	7587	7596	7606	7615	7625	7634	7644	7653	7663
75	7672	7682	7691	7701	7710	7720	7729	7739	7748	7758
0.76	7767	7777	7786	7796	7805	7815	7824	7834	7843	7853
77	7862	7872	7881	7891	7900	7910	7919	7929	7938	7948
78	7957	7967	7976	7986	7995	8005	8014	8024	8033	8043
79	8052	8062	8071	8081	8090	8100	8109	8119	8128	8138
80	8147	8157	8166	8176	8185	8195	8204	8214	8223	8233
0.81	8242	8252	8261	8271	8280	8289	8299	8308	8318	8327
82	8336	8346	8355	8365	8374	8383	8393	8402	8412	8421
83	8430	8440	8449	8458	8468	8477	8486	8496	8505	8514
84	8523	8533	8542	8551	8561	8570	8579	8589	8598	8607
85	8616	8626	8635	8644	8654	8663	8672	8682	8691	8700
0.86	8709	8719	8728	8737	8747	8756	8765	8775	8784	8793
87	8802	8812	8821	8830	8840	8849	8858	8868	8877	8886
88	8895	8905	8914	8923	8933	8942	8951	8961	8970	8979
89	8988	8998	9007	9016	9026	9035	9044	9054	9063	9072
90	9081	9091	9100	9109	9119	9128	9137	9147	9156	9165
0.91	9174	9184	9193	9202	9212	9221	9230	9240	9249	9258
92	9267	9277	9296	9295	9304	9313	9323	9332	9341	9350
93	9359	9369	9378	9387	9396	9405	9415	9424	9433	9442
94	9451	9461	9470	9479	9488	9497	9507	9516	9525	9534
95	9543	9553	9562	9571	9580	9589	9599	9608	9617	9625
0.96	9635	9645	9654	9663	9672	9681	9691	9700	9709	9718
97	9727	9737	9746	9755	9764	9773	9782	9791	9800	9809
98	9818	9828	9837	9846	9855	9864	9873	9882	9891	9900
99	9909	9919	9928	9937	9946	9955	9964	9973	9982	9991
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)
$\delta = 0.92$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0011	0.0022	0.0033	0.0044	0.0055	0.0065	0.0076	0.0087	0.0098
0.01	0.0109	120	131	142	153	164	175	185	196	0.0207
2	218	229	240	251	262	273	283	294	305	316
3	326	337	348	359	370	381	391	402	413	424
4	434	445	456	467	478	489	499	510	521	532
5	542	553	564	575	586	597	607	618	629	640
0.06	650	661	672	683	694	705	715	726	737	748
7	758	769	780	791	801	812	823	834	844	855
8	865	876	886	897	908	919	929	940	950	961
9	971	982	992	0.1003	1014	1025	1035	1046	1056	1067
0.10	0.1077	1088	1098	1109	1120	1131	1141	1152	1162	1173
0.11	1183	1194	1205	1216	1226	1237	1248	1258	1269	1280
12	1290	1301	1311	1322	1333	1343	1354	1364	1375	1386
13	1396	1407	1417	1428	1439	1449	1460	1470	1481	1492
14	1502	1513	1523	1534	1545	1555	1566	1576	1587	1598
15	1608	1619	1629	1640	1651	1661	1672	1682	1693	1703
0.16	1714	1725	1735	1746	1757	1767	1778	1788	1799	1810
17	1820	1831	1841	1852	1863	1873	1884	1894	1905	1916
18	1926	1937	1947	1958	1968	1979	1989	2000	2010	2021
19	2031	2042	2052	2063	2073	2084	2094	2105	2115	2126
20	2136	2147	2157	2168	2178	2189	2199	2210	2220	2231
0.21	2241	2252	2262	2273	2283	2294	2304	2315	2325	2336
22	2346	2357	2367	2378	2388	2399	2409	2420	2430	2441
23	2451	2462	2472	2483	2493	2504	2514	2525	2535	2546
24	2556	2566	2577	2587	2598	2608	2619	2629	2639	2650
25	2660	2670	2681	2691	2702	2712	2723	2733	2743	2754
0.26	2764	2774	2785	2795	2806	2816	2827	2837	2847	2858
27	2868	2878	2889	2899	2910	2920	2931	2941	2951	2962
28	2972	2982	2993	3003	3014	3024	3035	3045	3055	3066
29	3076	3086	3096	3107	3117	3127	3138	3149	3159	3169
30	3179	3189	3199	3210	3220	3230	3241	3251	3261	3271
0.31	3282	3292	3302	3313	3323	3333	3344	3354	3364	3374
32	3385	3395	3405	3416	3426	3436	3447	3457	3467	3477
33	3488	3498	3508	3519	3529	3539	3549	3559	3570	3580
34	3590	3600	3610	3621	3631	3641	3651	3661	3672	3682
35	3692	3702	3712	3723	3733	3743	3753	3763	3774	3784
0.36	3794	3804	3814	3825	3835	3845	3855	3865	3876	3886
37	3896	3906	3916	3927	3937	3947	3957	3967	3978	3988
38	3998	4008	4018	4029	4039	4049	4059	4069	4080	4090
39	4100	4110	4120	4131	4141	4151	4161	4171	4182	4192
40	4202	4212	4222	4232	4242	4252	4263	4273	4283	4293
0.41	4303	4313	4323	4333	4343	4353	4364	4374	4384	4394
42	4404	4414	4424	4434	4444	4454	4465	4475	4485	4495
43	4505	4515	4525	4535	4545	4555	4566	4576	4586	4596
44	4606	4616	4626	4636	4646	4656	4667	4677	4687	4697
45	4707	4717	4727	4737	4747	4757	4767	4777	4787	4797
0.46	4807	4817	4827	4837	4847	4857	4868	4878	4888	4898

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	758	769	780	791	801	812	823	834	844	855
8	865	876	886	897	908	919	929	940	950	961
9	971	982	992	0.1003	1014	1025	1035	1046	1056	1067
0.10	0.1077	1088	1098	1109	1120	1131	1141	1152	1162	1173
0.11	1183	1194	1205	1216	1226	1237	1248	1258	1269	1280
12	1290	1301	1311	1322	1333	1343	1354	1364	1375	1386
13	1396	1407	1417	1428	1439	1449	1460	1470	1481	1492
14	1502	1513	1523	1534	1545	1555	1566	1576	1587	1598
15	1608	1619	1629	1640	1651	1661	1672	1682	1693	1703
0.16	1714	1725	1735	1746	1757	1767	1778	1788	1799	1810
17	1820	1831	1841	1852	1863	1873	1884	1894	1905	1916
18	1926	1937	1947	1958	1968	1979	1989	2000	2010	2021
19	2031	2042	2052	2063	2073	2084	2094	2105	2115	2126
20	2136	2147	2157	2168	2178	2189	2199	2210	2220	2231
0.21	2241	2252	2262	2273	2283	2294	2304	2315	2325	2336
22	2346	2357	2367	2378	2388	2399	2409	2420	2430	2441
23	2451	2462	2472	2483	2493	2504	2514	2525	2535	2546
24	2556	2566	2577	2587	2598	2608	2619	2629	2639	2650
25	2660	2670	2681	2691	2702	2712	2723	2733	2743	2754
0.26	2764	2774	2785	2795	2806	2816	2827	2837	2847	2858
27	2868	2878	2889	2899	2910	2920	2931	2941	2951	2962
28	2972	2982	2993	3003	3014	3024	3035	3045	3055	3066
29	3076	3086	3096	3107	3117	3127	3138	3149	3159	3169
30	3179	3189	3199	3210	3220	3230	3241	3251	3261	3271
0.31	3282	3292	3302	3313	3323	3333	3344	3354	3364	3374
32	3385	3395	3405	3416	3426	3436	3447	3457	3467	3477
33	3488	3498	3508	3519	3529	3539	3549	3559	3570	3580
34	3590	3600	3610	3621	3631	3641	3651	3661	3672	3682
35	3692	3702	3712	3723	3733	3743	3753	3763	3774	3784
0.36	3794	3804	3814	3825	3835	3845	3855	3865	3876	3886
37	3896	3906	3916	3927	3937	3947	3957	3967	3978	3988
38	3998	4008	4018	4029	4039	4049	4059	4069	4080	4090
39	4100	4110	4120	4131	4141	4151	4161	4171	4182	4192
40	4202	4212	4222	4232	4242	4252	4263	4273	4283	4293
0.41	4303	4313	4323	4333	4343	4353	4364	4374	4384	4394
42	4404	4414	4424	4434	4444	4454	4465	4475	4485	4495
43	4505	4515	4525	4535	4545	4555	4566	4576	4586	4596
44	4606	4616	4626	4636	4646	4656	4667	4677	4687	4697
45	4707	4717	4727	4737	4747	4757	4767	4777	4787	4797
0.46	4807	4817	4827	4837	4847	4857	4868	4878	4888	4898
47	4908	4918	4928	4938	4948	4958	4968	4978	4988	4998
48	5008	5018	5028	5038	5048	5058	5069	5079	5089	5099
49	5109	5119	5129	5139	5149	5159	5169	5179	5189	5199
0.50	5209	5219	5229	5239	5249	5259	5269	5279	5289	5299

STAT

(cont'd.)

$\partial = 0.92$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5309	0.5318	0.5328	0.5338	0.5348	0.5358	0.5368	0.5378	0.5388	0.5398
52	5408	5418	5428	5438	5448	5458	5468	5478	5488	5498
53	5508	5517	5527	5537	5547	5557	5567	5577	5587	5597
54	5607	5616	5626	5636	5646	5656	5666	5676	5686	5696
55	5706	5715	5725	5735	5745	5755	5765	5775	5785	5795
0.56	5805	5814	5824	5834	5844	5854	5864	5874	5884	5894
57	5904	5913	5923	5933	5943	5953	5963	5973	5983	5993
58	6003	6012	6022	6032	6042	6052	6061	6071	6081	6091
59	6101	6110	6120	6130	6140	6150	6159	6169	6179	6189
60	6199	6208	6218	6228	6238	6248	6257	6267	6277	6287
0.61	6297	6306	6316	6326	6336	6346	6355	6365	6375	6385
62	6395	6404	6414	6424	6434	6444	6453	6463	6473	6483
63	6493	6502	6512	6522	6532	6542	6551	6561	6571	6581
64	6591	6600	6610	6620	6629	6639	6649	6658	6668	6678
65	6688	6697	6707	6717	6726	6736	6746	6755	6765	6775
0.66	6785	6794	6804	6814	6823	6833	6843	6852	6862	6872
67	6882	6891	6901	6911	6920	6930	6940	6949	6959	6969
68	6979	6988	6998	7008	7017	7027	7037	7046	7056	7066
69	7076	7085	7095	7105	7114	7124	7134	7143	7153	7163
70	7173	7181	7192	7201	7211	7220	7230	7240	7249	7259
0.71	7269	7278	7288	7297	7307	7317	7326	7336	7345	7355
72	7365	7374	7384	7393	7403	7413	7422	7432	7441	7451
73	7461	7470	7480	7489	7499	7509	7518	7528	7537	7547
74	7557	7566	7576	7585	7595	7605	7614	7624	7633	7643
75	7653	7662	7672	7681	7691	7701	7710	7720	7729	7739
0.76	7749	7758	7768	7777	7787	7797	7806	7816	7825	7835
77	7845	7854	7864	7873	7883	7893	7902	7912	7921	7931
78	7941	7950	7960	7969	7979	7988	7998	8007	8017	8026
79	8036	8045	8055	8064	8074	8083	8093	8102	8112	8121
80	8131	8140	8150	8159	8169	8178	8188	8197	8207	8216
0.81	8226	8235	8245	8254	8264	8273	8283	8292	8302	8311
82	8321	8330	8340	8349	8358	8367	8377	8386	8396	8405
83	8415	8424	8434	8443	8452	8461	8471	8480	8490	8499
84	8509	8518	8528	8537	8546	8555	8565	8574	8584	8598
85	8603	8612	8622	8631	8640	8649	8659	8668	8678	8687
0.86	8697	8706	8716	8725	8734	8743	8753	8762	8772	8781
87	8791	8800	8810	8819	8828	8837	8847	8856	8866	8875
88	8885	8894	8904	8913	8922	8931	8941	8950	8960	8969
89	8979	8988	8998	9007	9016	9025	9035	9044	9054	9063
90	9073	9082	9092	9101	9110	9119	9129	9138	9148	9157
0.91	9167	9176	9186	9195	9204	9213	9223	9232	9241	9250
92	9260	9269	9278	9288	9297	9306	9316	9325	9334	9343
93	9353	9362	9371	9381	9390	9399	9409	9418	9427	9436
94	9446	9455	9464	9474	9483	9492	9502	9511	9520	9529
95	9539	9548	9557	9567	9576	9585	9595	9604	9613	9622
0.96	9632	9641	9650	9659	9668	9678	9687	9696	9705	9714
97	9724	9733	9742	9751	9760	9770	9779	9788	9797	9806

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57	5904	5913	5923	5933	5943	5953	5963	5973	5983	5993
58	6003	6012	6022	6032	6042	6052	6061	6071	6081	6091
59	6101	6110	6120	6130	6140	6150	6159	6169	6179	6189
60	6199	6208	6218	6228	6238	6248	6257	6267	6277	6287
0.61	6297	6306	6316	6326	6336	6346	6355	6365	6375	6385
62	6395	6404	6414	6424	6434	6444	6453	6463	6473	6483
63	6493	6502	6512	6522	6532	6542	6551	6561	6571	6581
64	6591	6600	6610	6620	6629	6639	6649	6658	6668	6678
65	6688	6697	6707	6717	6726	6736	6746	6755	6765	6775
0.66	6785	6794	6804	6814	6823	6833	6843	6852	6862	6872
67	6882	6891	6901	6911	6920	6930	6940	6949	6959	6969
68	6979	6988	6998	7008	7017	7027	7037	7046	7056	7066
69	7076	7085	7095	7105	7114	7124	7134	7143	7153	7163
70	7173	7181	7192	7201	7211	7220	7230	7240	7249	7259
0.71	7269	7278	7288	7297	7307	7317	7326	7336	7345	7355
72	7365	7374	7384	7393	7403	7413	7422	7432	7441	7451
73	7461	7470	7480	7489	7499	7509	7518	7528	7537	7547
74	7557	7566	7576	7585	7595	7605	7614	7624	7633	7643
75	7653	7662	7672	7681	7691	7701	7710	7720	7729	7739
0.76	7749	7758	7768	7777	7787	7797	7806	7816	7825	7835
77	7845	7854	7864	7873	7883	7893	7902	7912	7921	7931
78	7941	7950	7960	7969	7979	7988	7998	8007	8017	8026
79	8036	8045	8055	8064	8074	8083	8093	8102	8112	8121
80	8131	8140	8150	8159	8169	8178	8188	8197	8207	8216
0.81	8226	8235	8245	8254	8264	8273	8283	8292	8302	8311
82	8321	8330	8340	8349	8358	8367	8377	8386	8396	8405
83	8415	8424	8434	8443	8452	8461	8471	8480	8490	8499
84	8509	8518	8528	8537	8546	8555	8565	8574	8584	8598
85	8603	8612	8622	8631	8640	8649	8659	8668	8678	8687
0.86	8697	8706	8716	8725	8734	8743	8753	8762	8772	8781
87	8791	8800	8810	8819	8828	8837	8847	8856	8866	8875
88	8885	8894	8904	8913	8922	8931	8941	8950	8960	8969
89	8979	8988	8998	9007	9016	9025	9035	9044	9054	9063
90	9073	9082	9092	9101	9110	9119	9129	9138	9148	9157
0.91	9167	9176	9186	9195	9204	9213	9223	9232	9241	9250
92	9260	9269	9278	9288	9297	9306	9316	9325	9334	9343
93	9353	9362	9371	9381	9390	9399	9409	9418	9427	9436
94	9446	9455	9464	9474	9483	9492	9502	9511	9520	9529
95	9539	9548	9557	9567	9576	9585	9595	9604	9613	9622
0.96	9632	9641	9650	9659	9668	9678	9687	9696	9705	9714
97	9724	9733	9742	9751	9760	9770	9779	9788	9797	9806
98	9816	9825	9834	9843	9852	9862	9871	9880	9889	9898
99	9908	9917	9926	9935	9944	9954	9963	9972	9981	9991
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)

$\partial = 0.93$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0011	0.0022	0.0033	0.0043	0.0054	0.0065	0.0075	0.0086	0.0097
0.01	0.0107	118	129	140	150	161	172	182	193	204
2	214	225	236	247	257	268	279	290	300	311
3	322	333	344	355	365	376	387	397	408	419
4	429	440	451	462	472	483	494	504	515	526
5	536	547	558	569	579	590	601	611	622	633
0.06	643	654	665	675	686	696	707	717	728	739
7	749	760	771	781	792	802	813	823	834	844
8	855	866	877	887	898	908	919	929	940	950
9	961	972	983	993	1004	1014	1025	1035	1046	1056
0.10	0.1067	1078	1089	1099	1110	1120	1131	1141	1152	1162
0.11	1173	1184	1194	1205	1215	1226	1236	1247	1257	1268
12	1278	1289	1300	1310	1321	1331	1342	1352	1363	1373
13	1384	1395	1406	1416	1427	1437	1448	1458	1469	1479
14	1490	1500	1511	1521	1532	1542	1553	1563	1574	1584
15	1595	1605	1616	1626	1637	1647	1658	1668	1679	1689
0.16	1700	1710	1721	1731	1742	1752	1763	1773	1784	1794
17	1805	1815	1826	1836	1847	1857	1868	1878	1889	1899
18	1910	1920	1931	1941	1952	1962	1973	1983	1994	2004
19	2015	2025	2035	2046	2056	2067	2077	2088	2098	2108
20	2119	2129	2139	2150	2160	2171	2181	2192	2202	2212
0.21	2223	2233	2243	2254	2264	2275	2285	2296	2306	2316
22	2327	2337	2347	2358	2368	2379	2389	2400	2410	2420
23	2431	2441	2451	2462	2472	2483	2493	2504	2514	2524
24	2535	2545	2555	2566	2576	2587	2597	2608	2618	2628
25	2639	2650	2660	2670	2681	2691	2701	2712	2722	2732
0.26	2742	2753	2763	2773	2784	2794	2804	2815	2825	2835
27	2845	2856	2866	2876	2887	2897	2907	2918	2928	2938
28	2948	2959	2969	2979	2990	3000	3010	3021	3031	3041
29	3051	3062	3072	3082	3093	3103	3113	3124	3134	3144
30	3154	3165	3175	3185	3196	3206	3216	3227	3237	3247
0.31	3257	3268	3278	3288	3299	3309	3319	3330	3340	3350
32	3360	3371	3381	3391	3401	3411	3422	3432	3442	3452
33	3462	3473	3483	3493	3503	3513	3524	3534	3544	3554
34	3564	3575	3585	3595	3605	3615	3626	3636	3646	3656
35	3666	3677	3687	3697	3707	3717	3728	3738	3748	3758
0.36	3768	3779	3789	3799	3809	3819	3830	3840	3850	3860
37	3870	3881	3891	3901	3911	3921	3932	3942	3952	3962
38	3972	3983	3993	4003	4013	4023	4034	4044	4054	4064
39	4074	4085	4095	4105	4115	4125	4136	4146	4156	4166
40	4176	4187	4197	4207	4217	4227	4237	4247	4257	4267
0.41	4277	4288	4298	4308	4318	4328	4338	4348	4358	4368
42	4378	4389	4399	4409	4419	4429	4439	4449	4459	4469
43	4479	4489	4499	4509	4519	4529	4539	4549	4559	4569
44	4579	4590	4600	4610	4620	4630	4640	4650	4660	4670
45	4680	4690	4700	4710	4720	4730	4740	4750	4760	4770
0.46	[illegible]	[illegible]	4801	4811	4821	4831	4841	4851	4861	4871

9	961	972	983	993	1004	1014	1025	1035	1046	1056
0.10	0.1067	1078	1089	1099	1110	1120	1131	1141	1152	1162
0.11	1173	1184	1194	1205	1215	1226	1236	1247	1257	1268
12	1278	1289	1300	1310	1321	1331	1342	1352	1363	1373
13	1384	1395	1406	1416	1427	1437	1448	1458	1469	1479
14	1490	1500	1511	1521	1532	1542	1553	1563	1574	1584
15	1595	1605	1616	1626	1637	1647	1658	1668	1679	1689
0.16	1700	1710	1721	1731	1742	1752	1763	1773	1784	1794
17	1805	1815	1826	1836	1847	1857	1868	1878	1889	1899
18	1910	1920	1931	1941	1952	1962	1973	1983	1994	2004
19	2015	2025	2035	2046	2056	2067	2077	2088	2098	2108
20	2119	2129	2139	2150	2160	2171	2181	2192	2202	2212
0.21	2223	2233	2243	2254	2264	2275	2285	2296	2306	2316
22	2327	2337	2347	2358	2368	2379	2389	2400	2410	2420
23	2431	2441	2451	2462	2472	2483	2493	2504	2514	2524
24	2535	2545	2555	2566	2576	2587	2597	2608	2618	2628
25	2639	2650	2660	2670	2681	2691	2701	2712	2722	2732
0.26	2742	2753	2763	2773	2784	2794	2804	2815	2825	2835
27	2845	2856	2866	2876	2887	2897	2907	2918	2928	2938
28	2948	2959	2969	2979	2990	3000	3010	3021	3031	3041
29	3051	3062	3072	3082	3093	3103	3113	3124	3134	3144
30	3154	3165	3175	3185	3196	3206	3216	3227	3237	3247
0.31	3257	3268	3278	3288	3299	3309	3319	3330	3340	3350
32	3360	3371	3381	3391	3401	3411	3422	3432	3442	3452
33	3462	3473	3483	3493	3503	3513	3524	3534	3544	3554
34	3564	3575	3585	3595	3605	3615	3626	3636	3646	3656
35	3666	3677	3687	3697	3707	3717	3728	3738	3748	3758
0.36	3768	3779	3789	3799	3809	3819	3830	3840	3850	3860
37	3870	3881	3891	3901	3911	3921	3932	3942	3952	3962
38	3972	3983	3993	4003	4013	4023	4034	4044	4054	4064
39	4074	4085	4095	4105	4115	4125	4136	4146	4156	4166
40	4176	4187	4197	4207	4217	4227	4237	4247	4257	4267
0.41	4277	4288	4298	4308	4318	4328	4338	4348	4358	4368
42	4378	4389	4399	4409	4419	4429	4439	4449	4459	4469
43	4479	4489	4499	4509	4519	4529	4539	4549	4559	4569
44	4579	4590	4600	4610	4620	4630	4640	4650	4660	4670
45	4680	4690	4700	4710	4720	4730	4740	4750	4760	4770
0.46	4781	4791	4801	4811	4821	4831	4841	4851	4861	4871
47	4881	4891	4901	4911	4921	4931	4941	4951	4961	4971
48	4981	4991	5001	5011	5021	5031	5041	5051	5061	5072
49	5082	5092	5102	5112	5122	5132	5142	5152	5162	5172
50	5182	5192	5202	5212	5222	5232	5242	5252	5262	5272

STAT

(cont'd.)
$\partial = 0.93$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5281	0.5291	0.5301	0.5311	0.5321	0.5331	0.5341	0.5351	0.5361	0.5371
52	5381	5391	5401	5411	5421	5431	5441	5451	5461	5471
53	5481	5490	5500	5510	5520	5530	5540	5550	5560	5570
54	5580	5589	5599	5609	5619	5629	5639	5649	5659	5669
55	5679	5688	5698	5708	5718	5728	5738	5748	5758	5768
0.56	5778	5787	5797	5807	5817	5827	5837	5847	5857	5867
57	5877	5886	5896	5906	5916	5926	5936	5946	5956	5966
58	5976	5985	5995	6005	6015	6025	6035	6045	6055	6065
59	6075	6084	6094	6104	6114	6124	6133	6143	6153	6163
60	6173	6182	6192	6202	6212	6222	6231	6241	6251	6261
0.61	6271	6280	6290	6300	6310	6320	6329	6339	6349	6359
62	6369	6378	6388	6398	6408	6418	6427	6437	6447	6457
63	6467	6476	6486	6496	6506	6516	6525	6535	6445	6555
64	6565	6574	6584	6594	6604	6614	6623	6633	6643	6653
65	6663	6672	6682	6692	6702	6712	6721	6731	6741	6751
0.66	6761	6770	6780	6790	6800	6810	6819	6829	6839	6849
67	6859	6868	6878	6888	6897	6907	6917	6926	6936	6946
68	6956	6965	6975	6985	6994	7004	7014	7023	7033	7043
69	7053	7062	7072	7082	7091	7101	7111	7120	7130	7140
70	7150	7159	7169	7179	7188	7198	7208	7217	7227	7237
0.71	7247	7256	7266	7276	7285	7295	7305	7314	7324	7334
72	7344	7353	7363	7373	7382	7392	7402	7411	7421	7431
73	7441	7450	7460	7469	7479	7489	7498	7508	7517	7527
74	7537	7546	7556	7565	7575	7585	7594	7604	7613	7623
75	7633	7642	7652	7661	7671	7681	7690	7700	7709	7719
0.76	7729	7738	7748	7757	7767	7777	7786	7796	7805	7815
77	7825	7834	7844	7853	7863	7873	7882	7892	7901	7911
78	7921	7930	7940	7949	7959	7969	7978	7988	7997	8007
79	8017	8026	8036	8045	8055	8065	8074	8084	8093	8103
80	8113	8122	8132	8141	8151	8161	8170	8180	8189	8199
0.81	8209	8218	8228	8237	8247	8257	8266	8276	8285	8295
82	8305	8314	8324	8333	8343	8352	8362	8371	8381	8390
83	8400	8409	8419	8428	8438	8447	8457	8466	8476	8485
84	8495	8504	8514	8523	8533	8542	8552	8561	8571	8580
85	8590	8599	8609	8618	8628	8637	8647	8656	8666	8675
0.86	8685	8694	8704	8713	8723	8732	8742	8751	8761	8770
87	8780	8789	8799	8808	8818	8827	8837	8846	8856	8865
88	8875	8884	8894	8903	8913	8922	8932	8941	8951	8960
89	8970	8979	8989	8998	9007	9016	9026	9035	9045	9054
90	9064	9073	9083	9092	9101	9110	9120	9129	9139	9148
0.91	9158	9167	9177	9186	9195	9204	9214	9223	9233	9242
92	9252	9261	9271	9280	9289	9298	9308	9317	9327	9336
93	9346	9355	9365	9374	9383	9392	9402	9411	9421	9430
94	9440	9449	9459	9468	9477	9486	9496	9505	9515	9524
95	9534	9543	9553	9562	9571	9580	9590	9599	9609	9618
0.96	9628	9637	9647	9656	9665	9674	9684	9693	9702	9711
97	9721	9730	9740	9749	9758	9767	9777	9786	9795	9804

58	5976	5985	5995	6005	6015	6025	6035	6045	6055	6065
59	6075	6084	6094	6104	6114	6124	6133	6143	6153	6163
60	6173	6182	6192	6202	6212	6222	6231	6241	6251	6261
0.61	6271	6280	6290	6300	6310	6320	6329	6339	6349	6359
62	6369	6378	6388	6398	6408	6418	6427	6437	6447	6457
63	6467	6476	6486	6496	6506	6516	6525	6535	6445	6555
64	6565	6574	6584	6594	6604	6614	6623	6633	6643	6653
65	6663	6672	6682	6692	6702	6712	6721	6731	6741	6751
0.66	6761	6770	6780	6790	6800	6810	6819	6829	6839	6849
67	6859	6868	6878	6888	6897	6907	6917	6926	6936	6946
68	6956	6965	6975	6985	6994	7004	7014	7023	7033	7043
69	7053	7062	7072	7082	7091	7101	7111	7120	7130	7140
70	7150	7159	7169	7179	7188	7198	7208	7217	7227	7237
0.71	7247	7256	7266	7276	7285	7295	7305	7314	7324	7334
72	7344	7353	7363	7373	7382	7392	7402	7411	7421	7431
73	7441	7450	7460	7469	7479	7489	7498	7508	7517	7527
74	7537	7546	7556	7565	7575	7585	7594	7604	7613	7623
75	7633	7642	7652	7661	7671	7681	7690	7700	7709	7719
0.76	7729	7738	7748	7757	7767	7777	7786	7796	7805	7815
77	7825	7834	7844	7853	7863	7873	7882	7892	7901	7911
78	7921	7930	7940	7949	7959	7969	7978	7988	7997	8007
79	8017	8026	8036	8045	8055	8065	8074	8084	8093	8103
80	8113	8122	8132	8141	8151	8161	8170	8180	8189	8199
0.81	8209	8218	8228	8237	8247	8257	8266	8276	8285	8295
82	8305	8314	8324	8333	8343	8352	8362	8371	8381	8390
83	8400	8409	8419	8428	8438	8447	8457	8466	8476	8485
84	8495	8504	8514	8523	8533	8542	8552	8561	8571	8580
85	8590	8599	8609	8618	8628	8637	8647	8656	8666	8675
0.86	8685	8694	8704	8713	8723	8732	8742	8751	8761	8770
87	8780	8789	8799	8808	8818	8827	8837	8846	8856	8865
88	8875	8884	8894	8903	8913	8922	8932	8941	8951	8960
89	8970	8979	8989	8998	9007	9016	9026	9035	9045	9054
90	9064	9073	9083	9092	9101	9110	9120	9129	9139	9148
0.91	9158	9167	9177	9186	9195	9204	9214	9223	9233	9242
92	9252	9261	9271	9280	9289	9298	9308	9317	9327	9336
93	9346	9355	9365	9374	9383	9392	9402	9411	9421	9430
94	9440	9449	9459	9468	9477	9486	9496	9505	9515	9524
95	9534	9543	9553	9562	9571	9580	9590	9599	9609	9618
0.96	9628	9637	9647	9656	9665	9674	9684	9693	9702	9711
97	9721	9730	9740	9749	9758	9767	9777	9786	9795	9804
98	9814	9823	9833	9842	9851	9860	9869	9878	9888	9897
99	9907	9916	9926	9935	9944	9954	9963	9972	9982	9991
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)

$\partial = 0.94$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0011	0.0022	0.0032	0.0043	0.0054	0.0065	0.0075	0.0086	0.0096
0.01	0.0107	118	129	139	150	160	171	181	192	202
2	213	224	235	245	256	266	277	287	298	308
3	319	330	340	351	361	372	382	393	403	414
4	425	436	447	457	468	478	489	499	510	520
5	531	541	552	562	573	583	594	604	615	625
0.06	636	647	658	668	679	689	700	710	721	731
7	742	752	763	773	784	794	805	815	826	836
8	847	858	869	879	890	900	911	921	932	942
9	952	963	974	984	995	0.1005	1016	1026	1037	1047
0.10	0.1057	1067	1078	1088	1099	1109	1120	1130	1141	1151
0.11	1162	1172	1183	1193	1204	1214	1225	1235	1246	1256
12	1267	1277	1288	1298	1309	1319	1330	1340	1351	1361
13	1372	1382	1393	1403	1413	1424	1434	1445	1455	1466
14	1476	1486	1497	1507	1517	1528	1538	1549	1559	1570
15	1580	1590	1601	1611	1621	1632	1642	1653	1663	1674
0.16	1684	1694	1705	1715	1725	1736	1746	1757	1767	1778
17	1788	1798	1809	1819	1829	1840	1850	1861	1871	1882
18	1892	1902	1913	1923	1933	1944	1954	1965	1975	1986
19	1996	2006	2017	2027	2037	2048	2058	2069	2079	2090
20	2100	2110	2120	2131	2141	2151	2162	2172	2182	2193
0.21	2203	2213	2223	2234	2244	2254	2265	2275	2285	2296
22	2306	2316	2326	2337	2347	2357	2368	2378	2388	2399
23	2409	2419	2429	2440	2450	2460	2471	2483	2491	2502
24	2512	2522	2532	2543	2553	2563	2574	2584	2594	2605
25	2615	2625	2635	2646	2656	2666	2677	2687	2697	2708
0.26	2718	2728	2738	2749	2759	2769	2780	2790	2800	2811
27	2821	2831	2841	2852	2862	2873	2883	2893	2903	2914
28	2924	2934	2944	2955	2965	2975	2986	2996	3006	3017
29	3027	3037	3047	3058	3068	3078	3089	3099	3109	3120
30	3130	3140	3150	3160	3171	3181	3191	3201	3211	3222
0.31	3232	3242	3252	3262	3273	3283	3293	3303	3313	3324
32	3334	3344	3354	3364	3375	3385	3395	3405	3415	3426
33	3436	3446	3456	3466	3477	3487	3497	3507	3517	3528
34	3538	3548	3558	3568	3579	3589	3599	3609	3619	3630
35	3640	3650	3660	3670	3681	3691	3701	3711	3721	3732
0.36	3742	3752	3762	3772	3783	3793	3803	3813	3823	3834
37	3844	3854	3864	3874	3884	3895	3905	3915	3925	3935
38	3945	3955	3965	3975	3985	3996	4006	4016	4026	4036
39	4046	4056	4066	4076	4086	4097	4107	4117	4127	4137
40	4147	4157	4167	4177	4187	4198	4208	4218	4228	4238
0.41	4248	4258	4268	4278	4288	4299	4309	4319	4329	4339
42	4349	4359	4369	4279	4389	4400	4410	4420	4430	4440
43	4450	4460	4470	4480	4490	4501	4511	4521	4531	4541
44	4551	4561	4571	4581	4591	4602	4612	4622	4632	4642
45	4652	4662	4672	4682	4692	4703	4713	4723	4733	4743
0.46	4753	4763	4773	4783	4793	4803	4813	4823	4833	4843
					4894	4904	4914	4924	4934	4944

5	531	541	552	562	573	583	594	604	615	625
0.06	636	647	658	668	679	689	700	710	721	731
7	742	752	763	773	784	794	805	815	826	836
8	847	858	869	879	890	900	911	921	932	942
9	952	963	974	984	995	0.1005	1016	1026	1037	1047
0.10	0.1057	1067	1078	1088	1099	1109	1120	1130	1141	1151
0.11	1162	1172	1183	1193	1204	1214	1225	1235	1246	1256
12	1267	1277	1288	1298	1309	1319	1330	1340	1351	1361
13	1372	1382	1393	1403	1413	1424	1434	1445	1455	1466
14	1476	1486	1497	1507	1517	1528	1538	1549	1559	1570
15	1580	1590	1601	1611	1621	1632	1642	1653	1663	1674
0.16	1684	1694	1705	1715	1725	1736	1746	1757	1767	1778
17	1788	1798	1809	1819	1829	1840	1850	1861	1871	1882
18	1892	1902	1913	1923	1933	1944	1954	1965	1975	1986
19	1996	2006	2017	2027	2037	2048	2058	2069	2079	2090
20	2100	2110	2120	2131	2141	2151	2162	2172	2182	2193
0.21	2203	2213	2223	2234	2244	2254	2265	2275	2285	2296
22	2306	2316	2326	2337	2347	2357	2368	2378	2388	2399
23	2409	2419	2429	2440	2450	2460	2471	2483	2491	2502
24	2512	2522	2532	2543	2553	2563	2574	2584	2594	2605
25	2615	2625	2635	2646	2656	2666	2677	2687	2697	2708
0.26	2718	2728	2738	2749	2759	2769	2780	2790	2800	2811
27	2821	2831	2841	2852	2862	2873	2883	2893	2903	2914
28	2924	2934	2944	2955	2965	2975	2986	2996	3006	3017
29	3027	3037	3047	3058	3068	3078	3089	3099	3109	3120
30	3130	3140	3150	3160	3171	3181	3191	3201	3211	3222
0.31	3232	3242	3252	3262	3273	3283	3293	3303	3313	3324
32	3334	3344	3354	3364	3375	3385	3395	3405	3415	3426
33	3436	3446	3456	3466	3477	3487	3497	3507	3517	3528
34	3538	3548	3558	3568	3579	3589	3599	3609	3619	3630
35	3640	3650	3660	3670	3681	3691	3701	3711	3721	3732
0.36	3742	3752	3762	3772	3783	3793	3803	3813	3823	3834
37	3844	3854	3864	3874	3884	3895	3905	3915	3925	3935
38	3945	3955	3965	3975	3985	3996	4006	4016	4026	4036
39	4046	4056	4066	4076	4086	4097	4107	4117	4127	4137
40	4147	4157	4167	4177	4187	4198	4208	4218	4228	4238
0.41	4248	4258	4268	4278	4288	4299	4309	4319	4329	4339
42	4349	4359	4369	4279	4389	4400	4410	4420	4430	4440
43	4450	4460	4470	4480	4490	4501	4511	4521	4531	4541
44	4551	4561	4571	4581	4591	4602	4612	4622	4632	4642
45	4652	4662	4672	4682	4692	4703	4713	4723	4733	4743
0.46	4753	4763	4773	4783	4793	4803	4813	4823	4833	4843
47	4853	4864	4874	4884	4894	4904	4914	4924	4934	4944
48	4954	4964	4974	4984	4994	5004	5014	5024	5034	5044
49	5054	5064	5074	5084	5094	5104	5114	5124	5134	5144
50	5154	5164	5174	5184	5194	5204	5214	5224	5234	5244

STAT

(cont'd.)
$\partial = 0.94$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5254	0.5264	0.5274	0.5284	0.5294	0.5304	0.5314	0.5324	0.5334	0.5344
52	5354	5364	5374	5384	5394	5404	5414	5424	5434	5444
53	5454	5464	5474	5484	5494	5504	5514	5524	5534	5544
54	5554	5564	5574	5584	5594	5604	5614	5624	5634	5644
55	5653	5663	5673	5683	5693	5703	5713	5723	5733	5743
0.56	5752	5762	5772	5782	5792	5802	5812	5822	5832	5842
57	5851	5861	5871	5881	5891	5901	5911	5921	5931	5941
58	5950	5960	5970	5980	5990	6000	6010	6020	6030	6040
59	6049	6059	6069	6079	6089	6098	6108	6118	6128	6138
60	6147	6157	6167	6177	6187	6196	6206	6216	6226	6236
0.61	6245	6255	6265	6275	6285	6294	6304	6314	6324	6334
62	6343	6353	6363	6373	6383	6392	6402	6412	6422	6432
63	6441	6451	6461	6471	6481	6490	6500	6510	6520	6530
64	6539	6549	6559	6569	6579	6588	6598	6608	6618	6628
65	6637	6647	6657	6667	6677	6686	6696	6706	6716	6726
0.66	6735	6745	6755	6765	6775	6784	6794	6804	6814	6824
67	6833	6843	6853	6863	6873	6882	6892	6902	6912	6922
68	6931	6941	6951	6961	6971	6980	6990	7000	7010	7020
69	7029	7039	7049	7059	7068	7078	7088	7097	7107	7117
70	7126	7136	7146	7156	7165	7175	7185	7194	7204	7214
0.71	7223	7233	7243	7253	7262	7272	7282	7291	7301	7311
72	7320	7330	7340	7350	7359	7369	7379	7388	7398	7408
73	7417	7427	7437	7447	7456	7466	7476	7485	7495	7505
74	7514	7524	7534	7544	7553	7563	7573	7582	7592	7602
75	7611	7621	7631	7641	7650	7660	7670	7679	7689	7699
0.76	7708	7718	7728	7738	7747	7757	7767	7776	7786	7796
77	7805	7815	7825	7835	7844	7854	7864	7873	7883	7893
78	7902	7912	7922	7932	7941	7951	7961	7970	7980	7990
79	7999	8009	8019	8029	8038	8048	8058	8067	8077	8087
80	8096	8106	8116	8126	8135	8145	8155	8164	8174	8184
0.81	8193	8203	8212	8222	8232	8241	8251	8260	8270	8280
82	8289	8299	8308	8318	8328	8337	8347	8356	8366	8376
83	8385	8395	8404	8414	8424	8433	8443	8452	8362	8472
84	8481	8491	8500	8510	8520	8529	8539	8548	8558	8568
85	8577	8587	8596	8606	8616	8625	8635	8644	8654	8664
0.86	8673	8683	8692	8702	8712	8721	8731	8740	8750	8760
87	8769	8779	8788	8798	8807	8817	8826	8836	8845	8855
88	8864	8874	8883	8893	8902	8912	8921	8931	8940	8950
89	8959	8969	8978	8988	8997	9007	9016	9026	9035	9045
90	9054	9064	9073	9083	9092	9102	9111	9121	9130	9140
0.91	9150	9160	9169	9179	9188	9198	9207	9217	9226	9236
92	9245	9255	9264	9274	9283	9293	9302	9312	9321	9331
93	9340	9350	9359	9369	9378	9388	9397	9407	9416	9426
94	9435	9445	9454	9464	9473	9483	9492	9502	9511	9521
95	9530	9540	9549	9559	9568	9577	9587	9596	9606	9615
0.96	9624	9634	9643	9653	9662	9671	9681	9690	9700	9709
97	9718	9728	9737	9747	9756	9865	9775	9784	9794	9803
98	9812	9822	9831	9841	9850	9859	9869	9878	9888	9897

0.61	6245	6255	6265	6275	6285	6294	6304	6314	6324	6334
62	6343	6353	6363	6373	6383	6392	6402	6412	6422	6432
63	6441	6451	6461	6471	6481	6490	6500	6510	6520	6530
64	6539	6549	6559	6569	6579	6588	6598	6608	6618	6628
65	6637	6647	6657	6667	6677	6686	6696	6706	6716	6726
0.66	6735	6745	6755	6765	6775	6784	6794	6804	6814	6824
67	6833	6843	6853	6863	6873	6882	6892	6902	6912	6922
68	6931	6941	6951	6961	6971	6980	6990	7000	7010	7020
69	7029	7039	7049	7059	7068	7078	7088	7097	7107	7117
70	7126	7136	7146	7156	7165	7175	7185	7194	7204	7214
0.71	7223	7233	7243	7253	7262	7272	7282	7291	7301	7311
72	7320	7330	7340	7350	7359	7369	7379	7388	7398	7408
73	7417	7427	7437	7447	7456	7466	7476	7485	7495	7505
74	7514	7524	7534	7544	7553	7563	7573	7582	7592	7602
75	7611	7621	7631	7641	7650	7660	7670	7679	7689	7699
0.76	7708	7718	7728	7738	7747	7757	7767	7776	7786	7796
77	7805	7815	7825	7835	7844	7854	7864	7873	7883	7893
78	7902	7912	7922	7932	7941	7951	7961	7970	7980	7990
79	7999	8009	8019	8029	8038	8048	8058	8067	8077	8087
80	8096	8106	8116	8126	8135	8145	8155	8164	8174	8184
0.81	8193	8203	8212	8222	8232	8241	8251	8260	8270	8280
82	8289	8299	8308	8318	8328	8337	8347	8356	8366	8376
83	8385	8395	8404	8414	8424	8433	8443	8452	8362	8472
84	8481	8491	8500	8510	8520	8529	8539	8548	8558	8568
85	8577	8587	8596	8606	8616	8625	8635	8644	8654	8664
0.86	8673	8683	8692	8702	8712	8721	8731	8740	8750	8760
87	8769	8779	8788	8798	8807	8817	8826	8836	8845	8855
88	8864	8874	8883	8893	8902	8912	8921	8931	8940	8950
89	8959	8969	8978	8988	8997	9007	9016	9026	9035	9045
90	9054	9064	9073	9083	9092	9102	9111	9121	9130	9140
0.91	9150	9160	9169	9179	9188	9198	9207	9217	9226	9236
92	9245	9255	9264	9274	9283	9293	9302	9312	9321	9331
93	9340	9350	9359	9369	9378	9388	9397	9407	9416	9426
94	9435	9445	9454	9464	9473	9483	9492	9502	9511	9521
95	9530	9540	9549	9559	9568	9577	9587	9596	9606	9615
0.96	9624	9634	9643	9653	9662	9671	9681	9690	9700	9709
97	9718	9728	9737	9747	9756	9865	9775	9784	9794	9803
98	9812	9822	9831	9841	9850	9859	9869	9878	9888	9897
99	9906	9916	9925	9935	9944	9953	9963	9972	9981	9991
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)

∂ = 0.95

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0011	0.0022	0.0032	0.0043	0.0053	0.0064	0.0074	0.0085	0.0096
0.01	0.0106	117	127	138	148	159	169	180	190	0.0201
2	211	222	232	243	253	264	274	285	295	306
3	316	327	337	348	358	369	379	390	400	411
4	421	432	442	453	463	474	484	495	505	516
5	526	537	547	558	568	579	589	600	610	621
0.06	631	642	652	663	673	684	694	705	715	726
7	736	747	757	768	778	789	799	810	820	831
8	841	851	862	872	883	893	904	914	925	935
9	945	955	966	976	987	997	1008	1018	1029	1039
0.10	1049	1059	1070	1080	1091	1101	1112	1122	1133	1143
0.11	1153	1163	1173	1184	1195	1205	1216	1226	1237	1247
12	1257	1267	1278	1288	1299	1309	1320	1330	1341	1351
13	1361	1371	1382	1392	1403	1413	1424	1434	1445	1455
14	1465	1475	1486	1496	1507	1517	1528	1538	1549	1559
15	1569	1579	1590	1600	1610	1621	1631	1641	1652	1662
0.16	1672	1682	1693	1703	1713	1724	1734	1744	1755	1765
17	1775	1785	1796	1806	1816	1827	1837	1847	1858	1868
18	1878	1888	1899	1909	1919	1930	1940	1950	1961	1971
19	1981	1991	2002	2012	2022	2033	2043	2053	2064	2074
20	2084	2094	2105	2115	2125	2136	2146	2156	2167	2177
0.21	2187	2197	2208	2218	2228	2239	2249	2259	2270	2280
22	2290	2300	2311	2321	2331	2342	2352	2362	2373	2383
23	2393	2403	2414	2424	2434	2445	2455	2465	2476	2486
24	2496	2506	2517	2527	2537	2648	2558	2568	2579	2589
25	2599	2610	2620	2630	2640	2650	2661	2671	2581	2691
0.26	2701	2712	2722	2732	2742	2752	2763	2773	2783	2793
27	2803	2814	2824	2834	2844	2854	2865	2875	2885	2895
28	2905	2916	2926	2936	2946	2956	2967	2977	2987	2997
29	3007	3018	3028	3038	3048	3058	3069	3079	3089	3099
30	3109	3120	3130	3140	3150	3160	3171	3181	3191	3201
0.31	3211	3222	3232	3242	3252	3262	3273	3283	3293	3303
32	3313	3324	3334	3344	3354	3364	3375	3384	3395	3405
33	3415	3426	3436	3446	3456	3466	3477	3487	3497	3507
34	3517	3528	3538	3548	3558	3568	3579	3589	3599	3609
35	3619	3630	3640	3650	3660	3670	3681	3691	3701	3711
0.36	3721	3732	3742	3752	3762	3772	3782	3792	3802	3812
37	3822	3833	3843	3853	3863	3873	3883	3893	3903	3913
38	3923	3943	3944	3954	3964	3974	3984	3994	4004	4014
39	4024	4035	4045	4055	4065	4075	4085	4095	4105	4115
40	4125	4136	4146	4156	4166	4176	4186	4196	4206	4216
0.41	4226	4237	4247	4257	4267	4277	4287	4297	4307	4317
42	4327	4338	4348	4358	4368	4378	4388	4398	4408	4418
43	4428	4439	4449	4459	4469	4479	4489	4499	4509	4519
44	4529	4540	4550	4560	4570	4580	4590	4600	4610	4620
45	4630	4640	4650	4660	4670	4680	4690	4700	4710	4720
0.46	4730	4740	4750	4760	4770	4780	4790	4800	4810	4820

0.06	631	642	652	663	673	684	694	705	715	726
7	736	747	757	768	778	789	799	810	820	831
8	841	851	862	872	883	893	904	914	925	935
9	945	955	966	976	987	997	1008	1018	1029	1039
0.10	1049	1059	1070	1080	1091	1101	1112	1122	1133	1143
0.11	1153	1163	1173	1184	1195	1205	1216	1226	1237	1247
12	1257	1267	1278	1288	1299	1309	1320	1330	1341	1351
13	1361	1371	1382	1392	1403	1413	1424	1434	1445	1455
14	1465	1475	1486	1496	1507	1517	1528	1538	1549	1559
15	1569	1579	1590	1600	1610	1621	1631	1641	1652	1662
0.16	1672	1682	1693	1703	1713	1724	1734	1744	1755	1765
17	1775	1785	1796	1806	1816	1827	1837	1847	1858	1868
18	1878	1888	1899	1909	1919	1930	1940	1950	1961	1971
19	1981	1991	2002	2012	2022	2033	2043	2053	2064	2074
20	2084	2094	2105	2115	2125	2136	2146	2156	2167	2177
0.21	2187	2197	2208	2218	2228	2239	2249	2259	2270	2280
22	2290	2300	2311	2321	2331	2342	2352	2362	2373	2383
23	2393	2403	2414	2424	2434	2445	2455	2465	2476	2486
24	2496	2506	2517	2527	2537	2648	2558	2568	2579	2589
25	2599	2610	2620	2630	2640	2650	2661	2671	2581	2691
0.26	2701	2712	2722	2732	2742	2752	2763	2773	2783	2793
27	2803	2814	2824	2834	2844	2854	2865	2875	2885	2895
28	2905	2916	2926	2936	2946	2956	2967	2977	2987	2997
29	3007	3018	3028	3038	3048	3058	3069	3079	3089	3099
30	3109	3120	3130	3140	3150	3160	3171	3181	3191	3201
0.31	3211	3222	3232	3242	3252	3262	3273	3283	3293	3303
32	3313	3324	3334	3344	3354	3364	3375	3384	3395	3405
33	3415	3426	3436	3446	3456	3466	3477	3487	3497	3507
34	3517	3528	3538	3548	3558	3568	3579	3589	3599	3609
35	3619	3630	3640	3650	3660	3670	3681	3691	3701	3711
0.36	3721	3732	3742	3752	3762	3772	3782	3792	3802	3812
37	3822	3833	3843	3853	3863	3873	3883	3893	3903	3913
38	3923	3943	3944	3954	3964	3974	3984	3994	4004	4014
39	4024	4035	4045	4055	4065	4075	4085	4095	4105	4115
40	4125	4136	4146	4156	4166	4176	4186	4196	4206	4216
0.41	4226	4237	4247	4257	4267	4277	4287	4297	4307	4317
42	4327	4338	4348	4358	4368	4378	4388	4398	4408	4418
43	4428	4439	4449	4459	4469	4479	4489	4499	4509	4519
44	4529	4540	4550	4560	4570	4580	4590	4600	4610	4620
45	4630	4640	4650	4660	4670	4680	4690	4700	4710	4720
0.46	4730	4740	4750	4760	4770	4780	4790	4800	4810	4820
47	4830	4840	4850	4860	4870	4880	4890	4900	4910	4920
48	4931	4941	4951	4961	4971	4981	4991	5001	5011	5021
49	5031	5041	5051	5061	5071	5081	5091	5101	5111	5121
50	5131	5141	5151	5161	5171	5181	5191	5201	5211	5221

STAT

Sanitized Copy Approved for Release 2010/10/29 : CIA-RDP81-01043R001100040001-0

(cont'd.)

$\partial = 0.95$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5231	0.5241	0.5251	0.5261	0.5271	0.5281	0.5291	0.5301	0.5311	0.5321
52	5331	5441	5351	5361	5371	5381	5391	5401	5411	5421
53	5431	5441	5451	5461	5471	5481	5491	5500	5510	5520
54	5530	5540	5550	5560	5570	5580	5590	5600	5610	5620
55	5630	5640	5650	5660	5670	5680	5690	5700	5710	5719
0.56	5729	5739	5749	5759	5769	5779	5789	5799	5809	5818
57	5828	5838	5848	5858	5868	5878	5888	5898	5908	5917
58	5927	5937	5947	5957	5967	5977	5987	5997	6007	6016
59	6026	6036	6046	6056	6066	6076	6086	6096	6106	6115
60	6125	6135	6145	6155	6165	6175	6185	6195	6205	6214
0.61	6224	6234	6244	6254	6264	6273	6283	6293	6303	6312
62	6322	6332	6342	6352	6362	6371	6381	6391	6401	6410
63	6420	6430	6440	6450	6460	6469	6479	6489	6499	6508
64	6518	6528	6538	6548	6558	6567	6577	6587	6597	6606
65	6616	6626	6636	6646	6656	6665	6675	6685	6695	6704
0.66	6714	6724	6734	6744	6754	6763	6773	6783	6793	6802
67	6812	6822	6832	6842	6852	6861	6871	6881	6891	6900
68	6910	6920	6930	6940	6950	6959	6969	6979	6989	6998
69	7008	7018	7028	7038	7048	7057	7067	7077	7087	7096
70	7106	7116	7126	7136	7146	7155	7165	7175	7185	7194
0.71	7204	7214	7224	7234	7244	7253	7263	7273	7283	7292
72	7302	7312	7322	7332	7342	7351	7361	7371	7381	7390
73	7400	7410	7420	7430	7440	7449	7459	7469	7479	7488
74	7498	7508	7518	7528	7538	7547	7557	7567	7577	7586
75	7596	7606	7616	7626	7635	7645	7655	7664	7674	7683
0.76	7693	7703	7713	7723	7732	7742	7752	7761	7771	7780
77	7790	7800	7810	7820	7829	7839	7849	7858	7868	7877
78	7887	7897	7907	7916	7926	7936	7945	7955	7965	7974
79	7984	7994	8004	8013	8023	8033	8042	8052	8062	8071
80	8081	8091	8101	8110	8120	8130	8139	8149	8159	8168
0.81	8178	8188	8198	8207	8217	8227	8236	8246	8256	8265
82	8275	8285	8295	8304	8314	8324	8333	8343	8353	8362
83	8372	8382	8392	8401	8411	8421	8430	8440	8450	8459
84	8469	8479	8489	8498	8508	8518	8527	8537	8547	8556
85	8566	8575	8585	8594	8604	8614	8623	8633	8643	8652
0.86	8662	8671	8681	8690	8700	8710	8719	8729	8739	8748
87	8758	8767	8777	8786	8796	8806	8815	8825	8835	8844
88	8854	8863	8873	8882	8892	8902	8911	8921	8931	8940
89	8950	8959	8969	8978	8988	8998	9007	9017	9027	9036
90	9046	9055	9065	9074	9084	9094	9103	9113	9123	9132
0.91	9142	9151	9161	9170	9180	9190	9199	9209	9219	9228
92	9238	9247	9257	9266	9276	9286	9295	9305	9315	9324
93	9334	9343	9253	9362	9372	9382	9391	9401	9411	9420
94	9430	9439	9449	9458	9468	9477	9487	9496	9506	9515
95	9525	9534	9544	9553	9563	9572	9582	9591	9601	9610
0.96	9620	9629	9639	9648	9658	9667	9677	9686	9696	9705
97	9715	9724	9734	9743	9753	9762	9772	9782	9791	9800
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9848	9857	9867	9876	9886	9895

60	6125	6135	6145	6155	6165	6175	6185	6195	6205	6214
0.61	6224	6234	6244	6254	6264	6273	6283	6293	6303	6312
62	6322	6332	6342	6352	6362	6371	6381	6391	6401	6410
63	6420	6430	6440	6450	6460	6469	6479	6489	6499	6508
64	6518	6528	6538	6548	6558	6567	6577	6587	6597	6606
65	6616	6626	6636	6646	6656	6665	6675	6685	6695	6704
0.66	6714	6724	6734	6744	6754	6763	6773	6783	6793	6802
67	6812	6822	6832	6842	6852	6861	6871	6881	6891	6900
68	6910	6920	6930	6940	6950	6959	6969	6979	6989	6998
69	7008	7018	7028	7038	7048	7057	7067	7077	7087	7096
70	7106	7116	7126	7136	7146	7155	7165	7175	7185	7194
0.71	7204	7214	7224	7234	7244	7253	7263	7273	7283	7292
72	7302	7312	7322	7332	7342	7351	7361	7371	7381	7390
73	7400	7410	7420	7430	7440	7449	7459	7469	7479	7488
74	7498	7508	7518	7528	7538	7547	7557	7567	7577	7586
75	7596	7606	7616	7626	7635	7645	7655	7664	7674	7683
0.76	7693	7703	7713	7723	7732	7742	7752	7761	7771	7780
77	7790	7800	7810	7820	7829	7839	7849	7858	7868	7877
78	7887	7897	7907	7916	7926	7936	7945	7955	7965	7974
79	7984	7994	8004	8013	8023	8033	8042	8052	8062	8071
80	8081	8091	8101	8110	8120	8130	8139	8149	8159	8168
0.81	8178	8188	8198	8207	8217	8227	8236	8246	8256	8265
82	8275	8285	8295	8304	8314	8324	8333	8343	8353	8362
83	8372	8382	8392	8401	8411	8421	8430	8440	8450	8459
84	8469	8479	8489	8498	8508	8518	8527	8537	8547	8556
85	8566	8575	8585	8594	8604	8614	8623	8633	8643	8652
0.86	8662	8671	8681	8690	8700	8710	8719	8729	8739	8748
87	8758	8767	8777	8786	8796	8806	8815	8825	8835	8844
88	8854	8863	8873	8882	8892	8902	8911	8921	8931	8940
89	8950	8959	8969	8978	8988	8998	9007	9017	9027	9036
90	9046	9055	9065	9074	9084	9094	9103	9113	9123	9132
0.91	9142	9151	9161	9170	9180	9190	9199	9209	9219	9228
92	9238	9247	9257	9266	9276	9286	9295	9305	9315	9324
93	9334	9343	9253	9362	9372	9382	9391	9401	9411	9420
94	9430	9439	9449	9458	9468	9477	9487	9496	9506	9515
95	9525	9534	9544	9553	9563	9572	9582	9591	9601	9610
0.96	9620	9629	9639	9648	9658	9667	9677	9686	9696	9705
97	9715	9724	9734	9743	9753	9762	9772	9782	9791	9800
98	9810	9819	9829	9838	9848	9857	9867	9876	9886	9895
99	9905	9914	9924	9933	9943	9952	9962	9971	9981	0.9990
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)

∂ = 0.96

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0011	0.0022	0.0032	0.0042	0.0053	0.0063	0.0073	0.0084	0.0094
0.01	0.0104	115	125	136	146	156	167	177	188	198
2	208	219	229	240	250	260	271	281	292	302
3	312	323	333	344	354	364	375	385	396	406
4	416	427	337	448	458	468	479	489	500	510
5	520	531	541	552	562	572	583	593	604	614
0.06	624	634	644	655	665	675	686	696	707	717
7	727	737	747	758	768	778	789	799	810	820
8	830	841	851	862	872	882	893	903	914	924
9	934	944	954	965	975	985	996	1006	1017	1027
0.10	0.1037	1047	1057	1068	1078	1088	1099	1109	1120	1130
0.11	1140	1151	1161	1172	1182	1192	1203	1213	1224	1234
12	1244	1254	1264	1275	1285	1295	1306	1316	1327	1337
13	1347	1357	1367	1378	1388	1398	1409	1419	1430	1440
14	1450	1460	1470	1481	1491	1501	1512	1522	1533	1543
15	1553	1563	1573	1584	1594	1604	1615	1625	1636	1646
0.16	1656	1666	1676	1686	1697	1707	1717	1727	1738	1748
17	1758	1768	1778	1788	1799	1809	1819	1829	1840	1850
18	1860	1871	1881	1891	1902	1912	1922	1932	1943	1954
19	1963	1974	1984	1994	2005	2015	2025	2035	2046	2057
20	2066	2076	2086	2096	2107	2117	2127	2137	2148	2158
0.21	2168	2178	2188	2198	2209	2219	2229	2239	2250	2260
22	2270	2280	2290	2300	2311	2321	2331	2341	2352	2362
23	2372	2382	2392	2402	2413	2423	2433	2443	2454	2464
24	2474	2484	2494	2504	2515	2525	2535	2545	2556	2566
25	2576	2586	2596	2606	2617	2627	2637	2647	2658	2668
0.26	2678	2688	2698	2708	2719	2729	2739	2749	2760	2770
27	2780	2790	2800	2810	2821	2831	2841	2851	2862	2872
28	2882	2892	2902	2912	2923	2933	2943	2953	2964	2974
29	2984	2994	3004	3014	3025	3035	3045	3055	3066	3076
30	3086	3096	3106	3116	3127	3137	3147	3157	3168	3178
0.31	3187	3197	3207	3217	3228	3238	3248	3258	3268	3278
32	3288	3299	3309	3319	3330	3340	3350	3360	3370	3380
33	3390	3400	3410	3420	3431	3441	3451	3461	3471	3481
34	3491	3502	3512	3522	3532	3543	3553	3563	3573	3583
35	3593	3603	3613	3623	3633	3644	3654	3664	3674	3684
0.36	3694	3704	3714	3724	3734	3745	3755	3765	3775	3785
37	3795	3805	3815	3825	3835	3846	3856	3866	3876	3886
38	3896	3906	3916	3926	3936	3947	3957	3967	3977	3987
39	3997	4007	4017	4027	4037	4048	4058	4068	4078	4088
40	4098	4108	4118	4128	4138	4149	4159	4169	4179	4189
0.41	4199	4209	4219	4229	4239	4250	4260	4270	4280	4290
42	4300	4310	4320	4330	4340	4350	4360	4370	4380	4390
43	4400	4410	4420	4430	4440	4450	4460	4470	4480	4490
44	4500	4511	4521	4531	4541	4551	4561	4571	4581	4591
45	4601	4611	4621	4631	4641	4651	4661	4671	4681	4691
0.46	4701	4711	4721	4731	4741	4751	4761	4771	4781	4791

0.06	624	634	644	655	665	675	686	696	707	717
7	727	737	747	758	768	778	789	799	810	820
8	830	841	851	862	872	882	893	903	914	924
9	934	944	954	965	975	985	996	1006	1017	1027
0.10	0.1037	1047	1057	1068	1078	1088	1099	1109	1120	1130
0.11	1140	1151	1161	1172	1182	1192	1203	1213	1224	1234
12	1244	1254	1264	1275	1285	1295	1306	1316	1327	1337
13	1347	1357	1367	1378	1388	1398	1409	1419	1430	1440
14	1450	1460	1470	1481	1491	1501	1512	1522	1533	1543
15	1553	1563	1573	1584	1594	1604	1615	1625	1636	1646
0.16	1656	1666	1676	1686	1697	1707	1717	1727	1738	1748
17	1758	1768	1778	1788	1799	1809	1819	1829	1840	1850
18	1860	1871	1881	1891	1902	1912	1922	1932	1943	1954
19	1963	1974	1984	1994	2005	2015	2025	2035	2046	2057
20	2066	2076	2086	2096	2107	2117	2127	2137	2148	2158
0.21	2168	2178	2188	2198	2209	2219	2229	2239	2250	2260
22	2270	2280	2290	2300	2311	2321	2331	2341	2352	2362
23	2372	2382	2392	2402	2413	2423	2433	2443	2454	2464
24	2474	2484	2494	2504	2515	2525	2535	2545	2556	2566
25	2576	2586	2596	2606	2617	2627	2637	2647	2658	2668
0.26	2678	2688	2698	2708	2719	2729	2739	2749	2760	2770
27	2780	2790	2800	2810	2821	2831	2841	2851	2862	2872
28	2882	2892	2902	2912	2923	2933	2943	2953	2964	2974
29	2984	2994	3004	3014	3025	3035	3045	3055	3066	3076
30	3086	3096	3106	3116	3127	3137	3147	3157	3168	3178
0.31	3187	3197	3207	3217	3228	3238	3248	3258	3268	3278
32	3288	3299	3309	3319	3330	3340	3350	3360	3370	3380
33	3390	3400	3410	3420	3431	3441	3451	3461	3471	3481
34	3491	3502	3512	3522	3532	3543	3553	3563	3573	3583
35	3593	3603	3613	3623	3633	3644	3654	3664	3674	3684
0.36	3694	3704	3714	3724	3734	3745	3755	3765	3775	3785
37	3795	3805	3815	3825	3835	3846	3856	3866	3876	3886
38	3896	3906	3916	3926	3936	3947	3957	3967	3977	3987
39	3997	4007	4017	4027	4037	4048	4058	4068	4078	4088
40	4098	4108	4118	4128	4138	4149	4159	4169	4179	4189
0.41	4199	4209	4219	4229	4239	4250	4260	4270	4280	4290
42	4300	4310	4320	4330	4340	4350	4360	4370	4380	4390
43	4400	4410	4420	4430	4440	4450	4460	4470	4480	4490
44	4500	4511	4521	4531	4541	4551	4561	4571	4581	4591
45	4601	4611	4621	4631	4641	4651	4661	4671	4681	4691
0.46	4701	4711	4721	4731	4741	4751	4761	4771	4781	4791
47	4802	4812	4822	4832	4842	4852	4862	4872	4882	4892
48	4902	4912	4922	4932	4942	4952	4962	4972	4982	4992
49	5002	5012	5022	5032	5042	5052	5062	5072	5082	5092
50	5102	5112	5122	5132	5142	5152	5162	5172	5182	5192

STAT

(cont'd.)

$\partial = 0.96$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5202	0.5212	0.5222	0.5232	0.5242	0.5252	0.5262	0.5272	0.5282	0.5292
52	5302	5311	5321	5331	5341	5351	5361	5371	5381	5391
53	5401	5411	5421	5431	5441	5451	5461	5471	5481	5491
54	5501	5510	5520	5530	5540	5550	5560	5570	5580	5590
55	5600	5610	5620	5630	5640	5650	5660	5670	5680	5690
0.56	5700	5709	5719	5729	5739	5749	5759	5769	5779	5789
57	5799	5809	5819	5829	5839	5849	5859	5869	5879	5889
58	5899	5908	5918	5928	5938	5948	5958	5968	5978	5988
59	5998	6007	6017	6027	6037	6047	6057	6067	6077	6087
60	6097	6106	6116	6126	6136	6146	6156	6166	6176	6186
0.61	6196	6205	6215	6225	6235	6245	6255	6265	6275	6285
62	6295	6304	6314	6324	6334	6344	6354	6364	6374	6384
63	6394	6403	6413	6423	6433	6443	6453	6463	6473	6483
64	6493	6502	6512	6522	6532	6542	6552	6562	6572	6582
65	6592	6601	6611	6621	6631	6640	6650	6660	6670	6680
0.66	6690	6700	6709	6719	6729	6739	6749	6759	6769	6779
67	6789	6799	6808	6818	6828	6838	6848	6858	6868	6878
68	6888	6898	6907	6917	6927	6937	6946	6956	6966	6976
69	6986	6996	7006	7015	7025	7035	7045	7055	7065	7075
70	7085	7095	7104	7114	7124	7134	7143	7153	7163	7173
0.71	7183	7193	7202	7212	7222	7232	7241	7251	7261	7271
72	7281	7291	7300	7310	7320	7330	7339	7349	7359	7369
73	7379	7389	7398	7408	7418	7428	7437	7447	7457	7467
74	7477	7487	7496	7506	7516	7526	7535	7545	7555	7565
75	7575	7585	7594	7604	7614	7624	7633	7643	7653	7663
0.76	7673	7683	7692	7702	7712	7722	7731	7741	7751	7761
77	7771	7781	7790	7800	7810	7820	7829	7839	7849	7859
78	7869	7878	7880	7898	7907	7917	7927	7937	7946	7956
79	7966	7976	7985	7995	8005	8015	8024	8034	8044	8054
80	8064	8074	8083	8093	8103	8113	8122	8132	8142	8152
0.81	8162	8171	8181	8191	8200	8210	8220	8229	8239	8249
82	8259	8268	8278	8288	8297	8307	8317	8326	8336	8346
83	8356	8365	8375	8385	8394	8404	8414	8423	8433	8443
84	8453	8462	8472	8482	8491	8501	8511	8520	8530	8540
85	8550	8559	8569	8579	8588	8598	8608	8617	8627	8637
0.86	8647	8656	8666	8676	8685	8695	8705	8714	8724	8734
87	8744	8753	8763	8773	8782	8792	8802	8811	8821	8831
88	8841	8850	8860	8870	8879	8889	8899	8908	8918	8928
89	8938	8947	8957	8967	8976	8986	8996	9005	9015	9025
90	9035	9044	9054	9064	9073	9083	9093	9102	9112	9122
0.91	9132	9141	9151	9161	9170	9180	9190	9199	9209	6219
92	9229	9238	9248	9258	9267	9277	9287	9296	9306	9316
93	9326	9335	9345	9355	9364	9374	9384	9393	9403	9413
94	9423	9432	9442	9452	9461	9471	9481	9490	9500	9510
95	9520	9529	9539	9548	9558	9567	9577	9587	9596	9606
0.96	9616	9625	9635	9644	9654	9663	9673	9683	9692	9702
97	9712	9721	9731	9740	9750	9759	9769	9779	9788	9798
98	9809	9817	9827	9836	9846	9855	9865	9875	9884	[illegible]

62	6295	6304	6314	6324	6334	6344	6354	6364	6374	6384
63	6394	6403	6413	6423	6433	6443	6453	6463	6473	6483
64	6493	6502	6512	6522	6532	6542	6552	6562	6572	6582
65	6592	6601	6611	6621	6631	6640	6650	6660	6670	6680
0.66	6690	6700	6709	6719	6729	6739	6749	6759	6769	6779
67	6789	6799	6808	6818	6828	6838	6848	6858	6868	6878
68	6888	6898	6907	6917	6927	6937	6946	6956	6966	6976
69	6986	6996	7006	7015	7025	7035	7045	7055	7065	7075
70	7085	7095	7104	7114	7124	7134	7143	7153	7163	7173
0.71	7183	7193	7202	7212	7222	7232	7241	7251	7261	7271
72	7281	7291	7300	7310	7320	7330	7339	7349	7359	7369
73	7379	7389	7398	7408	7418	7428	7437	7447	7457	7467
74	7477	7487	7496	7506	7516	7526	7535	7545	7555	7565
75	7575	7585	7594	7604	7614	7624	7633	7643	7653	7663
0.76	7673	7683	7692	7702	7712	7722	7731	7741	7751	7761
77	7771	7781	7790	7800	7810	7820	7829	7839	7849	7859
78	7869	7878	7880	7898	7907	7917	7927	7937	7946	7956
79	7966	7976	7985	7995	8005	8015	8024	8034	8044	8054
80	8064	8074	8083	8093	8103	8113	8122	8132	8142	8152
0.81	8162	8171	8181	8191	8200	8210	8220	8229	8239	8249
82	8259	8268	8278	8288	8297	8307	8317	8326	8336	8346
83	8356	8365	8375	8385	8394	8404	8414	8423	8433	8443
84	8453	8462	8472	8482	8491	8501	8511	8520	8530	8540
85	8550	8559	8569	8579	8588	8598	8608	8617	8627	8637
0.86	8647	8656	8666	8676	8685	8695	8705	8714	8724	8734
87	8744	8753	8763	8773	8782	8792	8802	8811	8821	8831
88	8841	8850	8860	8870	8879	8889	8899	8908	8918	8928
89	8938	8947	8957	8967	8976	8986	8996	9005	9015	9025
90	9035	9044	9054	9064	9073	9083	9093	9102	9112	9122
0.91	9132	9141	9151	9161	9170	9180	9190	9199	9209	6219
92	9229	9238	9248	9258	9267	9277	9287	9296	9306	9316
93	9326	9335	9345	9355	9364	9374	9384	9393	9403	9413
94	9423	9432	9442	9452	9461	9471	9481	9490	9500	9510
95	9520	9529	9539	9548	9558	9567	9577	9587	9596	9606
0.96	9616	9625	9635	9644	9654	9663	9673	9683	9692	9702
97	9712	9721	9731	9740	9750	9759	9769	9779	9788	9798
98	9809	9817	9827	9836	9846	9855	9865	9875	9884	9894
99	9904	9913	9923	9932	9942	9951	9961	9971	9980	0.9990
1.00	1,000	-	-	-	-	-	-	-	-	-

STAT

(cont'd.)

$\partial = 0.97$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00	0	0.0011	0.0021	0.0031	0.0042	0.0052	0.0062	0.0073	0.0083	0.0093
0.01	0.0103	114	124	134	145	155	165	176	186	196
2	206	217	227	237	248	258	268	279	289	299
3	309	320	330	340	350	360	371	381	391	401
4	411	422	432	442	453	463	473	484	494	504
5	514	524	534	545	555	565	575	586	596	606
0.06	616	626	636	647	657	667	677	688	698	708
7	718	728	738	749	759	769	779	760	800	810
8	820	830	840	851	861	871	881	892	902	912
9	922	933	943	953	964	974	984	995	1005	1015
0.10	0.1025	1035	1045	1056	1066	1076	1086	1097	1107	1117
0.11	1127	1137	1147	1158	1168	1178	1188	1199	1209	1219
12	1229	1239	1249	1260	1270	1280	1290	1301	1311	1321
13	1331	1342	1352	1362	1373	1383	1393	1404	1414	1424
14	1434	1445	1455	1465	1475	1485	1496	1506	1516	1526
15	1536	1547	1557	1567	1577	1587	1598	1608	1618	1628
0.16	1638	1649	1659	1669	1679	1689	1700	1710	1720	1730
17	1740	1751	1761	1771	1881	1791	1802	1812	1822	1832
18	1842	1853	1863	1873	1883	1893	1904	1914	1924	1934
19	1944	1954	1964	1974	1985	1995	2005	2015	2025	2035
20	2045	2056	2066	2076	2087	2097	2107	2117	2127	2137
0.21	2147	2158	2168	2178	2188	2198	2209	2219	2229	2239
22	2249	2260	2270	2280	2290	2300	2311	2321	2331	2341
23	2351	2361	2371	2381	2392	2402	2412	2422	2432	2442
24	2452	2463	2473	2483	2493	2503	2514	2524	2534	2544
25	2554	2565	2575	2585	2595	2605	2616	2626	2636	2646
0.26	2656	2666	2676	2686	2697	2707	2717	2727	2737	2747
27	2757	2767	2777	2787	2798	2808	2818	2828	2838	2848
28	2858	2868	2878	2888	2899	2909	2919	2929	2939	2949
29	2959	2969	2979	2989	3000	3010	3020	3030	3040	3050
30	3060	3070	3080	3090	3101	3111	3121	3131	3141	3151
0.31	3161	3171	3181	3191	3201	3212	3222	3232	3242	3252
32	3262	3273	3283	3293	3303	3313	3324	3334	3344	3354
33	3364	3374	3384	3394	3404	3415	3425	3435	3445	3455
34	3465	3475	3485	3495	3505	3516	3526	3536	3546	3556
35	3566	3576	3586	3596	3606	3617	3627	3637	3647	3657
0.36	3667	3677	3687	3697	3707	3718	3728	3738	3748	3758
37	3768	3778	3788	3798	3808	3818	3828	3838	3848	3858
38	3868	3878	3888	3898	3908	3919	3929	3939	3949	3959
39	3969	3979	3989	3999	4009	4020	4030	4040	4050	4060
40	4070	4080	4090	4100	4110	4120	4130	4140	4150	4160
0.41	4170	4180	4190	4200	4210	4220	4231	4241	4251	4261
42	4271	4281	4291	4301	4311	4321	4332	4342	4352	4362
43	4372	4382	4392	4402	4412	4422	4432	4442	4452	4462
44	4472	4482	4492	4502	4512	4523	4533	4543	4553	4563
45	4573	4583	4593	4603	4613	4623	4633	4643	4653	4663
0.46	4673	4683	4693	4703	4713	4723	4734	4744	4754	4764
47	4774	4784	4794	4804	4814	4824	4835	4845	4855	4865
[illegible]	4875	4885	4895	4905	4915	4925	4935	4945	4955	4965
					5015	5025	5035	5045	5055	5065

						1280	1290	1301	1311	1321
		1342	1352	1362	1373	1383	1393	1404	1414	1424
14	1434	1445	1455	1465	1475	1485	1496	1506	1516	1526
15	1536	1547	1557	1567	1577	1587	1598	1608	1618	1628
0.16	1638	1649	1659	1669	1679	1689	1700	1710	1720	1730
17	1740	1751	1761	1771	1881	1791	1802	1812	1822	1832
18	1842	1853	1863	1873	1883	1893	1904	1914	1924	1934
19	1944	1954	1964	1974	1985	1995	2005	2015	2025	2035
20	2045	2056	2066	2076	2087	2097	2107	2117	2127	2137
0.21	2147	2158	2168	2178	2188	2198	2209	2219	2229	2239
22	2249	2260	2270	2280	2290	2300	2311	2321	2331	2341
23	2351	2361	2371	2381	2392	2402	2412	2422	2432	2442
24	2452	2463	2473	2483	2493	2503	2514	2524	2534	2544
25	2554	2565	2575	2585	2595	2605	2616	2626	2636	2646
0.26	2656	2666	2676	2686	2697	2707	2717	2727	2737	2747
27	2757	2767	2777	2787	2798	2808	2818	2828	2838	2848
28	2858	2868	2878	2888	2899	2909	2919	2929	2939	2949
29	2959	2969	2979	2989	3000	3010	3020	3030	3040	3050
30	3060	3070	3080	3090	3101	3111	3121	3131	3141	3151
0.31	3161	3171	3181	3191	3201	3212	3222	3232	3242	3252
32	3262	3273	3283	3293	3303	3313	3324	3334	3344	3354
33	3364	3374	3384	3394	3404	3415	3425	3435	3445	3455
34	3465	3475	3485	3495	3505	3516	3526	3536	3546	3556
35	3566	3576	3586	3596	3606	3617	3627	3637	3647	3657
0.36	3667	3677	3687	3697	3707	3718	3728	3738	3748	3758
37	3768	3778	3788	3798	3808	3818	3828	3838	3848	3858
38	3868	3878	3888	3898	3908	3919	3929	3939	3949	3959
39	3969	3979	3989	3999	4009	4020	4030	4040	4050	4060
40	4070	4080	4090	4100	4110	4120	4130	4140	4150	4160
0.41	4170	4180	4190	4200	4210	4220	4231	4241	4251	4261
42	4271	4281	4291	4301	4311	4321	4332	4342	4352	4362
43	4372	4382	4392	4402	4412	4422	4432	4442	4452	4462
44	4472	4482	4492	4502	4512	4523	4533	4543	4553	4563
45	4573	4583	4593	4603	4613	4623	4633	4643	4653	4663
0.46	4673	4683	4693	4703	4713	4723	4734	4744	4754	4764
47	4774	4784	4794	4804	4814	4824	4835	4845	4855	4865
48	4875	4885	4895	4905	4915	4925	4935	4945	4955	4965
49	4975	4985	4995	5005	5015	5025	5035	5045	5055	5065
50	0.5075	5085	5095	5105	5115	5125	5135	5145	5155	5165

STAT

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(cont'd.)

$\partial = 0.97$

β	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.51	0.5175	0.5185	0.5195	0.5205	0.5215	0.5225	0.5235	0.5245	0.5255	0.5265
52	5275	5285	5295	5305	5315	5325	5335	5345	5355	5365
53	5375	5385	5395	5405	5415	5425	5435	5445	5455	5465
54	5475	5485	5495	5505	5515	5525	5535	5545	5555	5565
55	5575	5584	5594	5604	5614	5624	5634	5644	5654	5664
0.56	5674	5683	5693	5703	5713	5723	5733	5743	5753	5763
57	5773	5782	5792	5802	5812	5822	5832	5842	5852	5862
58	5872	5882	5892	5902	5912	5922	5932	5942	5952	5962
59	5972	5981	5991	6001	6011	6021	6031	6041	6051	6061
60	6071	6080	6090	6100	6110	6120	6130	6140	6150	6160
0.61	6170	6180	6190	6200	6210	6220	6230	6240	6250	6260
62	6270	6279	6289	6299	6309	6319	6329	6339	6349	6359
63	6369	6378	6388	6398	6408	6418	6428	6438	6448	6458
64	6468	6477	6487	6497	6507	9517	6527	6537	6547	6557
65	6567	6576	6586	6596	6606	6616	6626	6637	6647	6657
0.66	6666	6675	6685	6695	6705	6715	6725	6735	6745	6755
67	6765	6774	6784	6794	6804	6814	6824	6834	6844	6854
68	6864	6873	6883	6893	6903	6913	6923	6933	6943	6953
69	6963	6972	6982	6992	7002	7012	7022	7032	7042	7052
70	7062	7071	7081	7091	7101	7111	7121	7131	7141	7151
0.71	7161	7170	7180	7190	7200	7210	7220	7230	7240	7250
72	7260	7269	7279	7289	7300	7309	7319	7329	7339	7349
73	7359	7368	7378	7388	7398	7408	7418	7428	7438	7448
74	7458	7467	7377	7487	7497	7507	7517	7527	7537	7547
75	7557	7566	7576	7586	7596	7606	7615	7625	7635	7645
0.76	7655	7665	7675	7685	7695	7705	7714	7724	7734	7744
77	7754	7764	7774	7784	7794	7804	7813	7823	7833	7843
78	7853	7863	7873	7883	7893	7903	7912	7922	7932	7942
79	7952	7962	7971	7981	7991	8001	8010	8020	8030	8040
80	8050	8060	8069	8079	8089	8099	8109	8118	8128	8138
0.81	8148	8158	8167	8177	8187	8197	8207	8216	8226	8236
82	8246	8256	8265	8275	8285	8295	8305	8314	8324	8334
83	8344	8354	8363	8373	8383	8393	8403	8413	8422	8432
84	8442	8452	8461	8471	8481	8491	8501	8510	8520	8530
85	8540	8550	8559	8569	8579	8589	8599	8608	8618	8628
0.86	8638	8648	8657	8667	8677	8687	8697	8706	8716	8726
87	8736	8745	8755	8765	8774	8784	8794	8803	8813	8823
88	8833	8843	8853	8863	8872	8882	8892	8902	8912	8921
89	8931	8941	8951	8961	8970	8980	8990	9000	9010	9019
90	9029	9039	9049	9058	9068	9078	9087	9097	9107	9116
0.91	9126	9136	9146	9155	9165	9175	9184	9194	9204	9213
92	9223	9233	9243	9252	9262	9272	9281	9291	9301	9310
93	9320	9330	9340	9350	9360	9369	9379	9389	9398	9408
94	9418	9428	9437	9447	9457	9466	9476	9486	9495	9505
95	9515	9525	9534	9544	9554	9563	9573	9583	9592	9602
0.96	9612	9622	9631	9641	9651	9660	9670	9680	9689	9699
97	9709	9719	9728	9738	9748	9757	9767	9777	9786	9796
98	9806	9816	9825	9335	9845	9854	9864	9874	9883	9893

0.61	6170	6180	6190	6200	6210	6220	6230	6240	6250	6260
62	6270	6279	6289	6299	6309	6319	6329	6339	6349	6359
63	6369	6378	6388	6398	6408	6418	6428	6438	6448	6458
64	6468	6477	6487	6497	6507	9517	6527	6537	6547	6557
65	6567	6576	6586	6596	6606	6616	6626	6637	6647	6657
0.66	6666	6675	6685	6695	6705	6715	6725	6735	6745	6755
67	6765	6774	6784	6794	6804	6814	6824	6834	6844	6854
68	6864	6873	6883	6893	6903	6913	6923	6933	6943	6953
69	6963	6972	6982	6992	7002	7012	7022	7032	7042	7052
70	7062	7071	7081	7091	7101	7111	7121	7131	7141	7151
0.71	7161	7170	7180	7190	7200	7210	7220	7230	7240	7250
72	7260	7269	7279	7289	7300	7309	7319	7329	7339	7349
73	7359	7368	7378	7388	7398	7408	7418	7428	7438	7448
74	7458	7467	7377	7487	7497	7507	7517	7527	7537	7547
75	7557	7566	7576	7586	7596	7606	7615	7625	7635	7645
0.76	7655	7665	7675	7685	7695	7705	7714	7724	7734	7744
77	7754	7764	7774	7784	7794	7804	7813	7823	7833	7843
78	7853	7863	7873	7883	7893	7903	7912	7922	7932	7942
79	7952	7962	7971	7981	7991	8001	8010	8020	8030	8040
80	8050	8060	8069	8079	8089	8099	8109	8118	8128	8138
0.81	8148	8158	8167	8177	8187	8197	8207	8216	8226	8236
82	8246	8256	8265	8275	8285	8295	8305	8314	8324	8334
83	8344	8354	8363	8373	8383	8393	8403	8413	8422	8432
84	8442	8452	8461	8471	8481	8491	8501	8510	8520	8530
85	8540	8550	8559	8569	8579	8589	8599	8608	8618	8628
0.86	8638	8648	8657	8667	8677	8687	8697	8706	8716	8726
87	8736	8745	8755	8765	8774	8784	8794	8803	8813	8823
88	8833	8843	8853	8863	8872	8882	8892	8902	8912	8921
89	8931	8941	8951	8961	8970	8980	8990	9000	9010	9019
90	9029	9039	9049	9058	9068	9078	9087	9097	9107	9116
0.91	9126	9136	9146	9155	9165	9175	9184	9194	9204	9213
92	9223	9233	9243	9252	9262	9272	9281	9291	9301	9310
93	9320	9330	9340	9350	9360	9369	9379	9389	9398	9408
94	9418	9428	9437	9447	9457	9466	9476	9486	9495	9505
95	9515	9525	9534	9544	9554	9563	9573	9583	9592	9602
0.96	9612	9622	9631	9641	9651	9660	9670	9680	9689	9699
97	9709	9719	9728	9738	9748	9757	9767	9777	9786	9796
98	9806	9816	9825	9335	9845	9854	9864	9874	9883	9893
99	9903	9913	9922	9932	9942	9951	9971	9961	9980	9990
1.00	1.000	-	-	-	-	-	-	-	-	-

STAT

APPENDIX 2

PROF. DROZDOV'S TABLES FOR SOLVING PROBLEMS OF INTERNAL BALLISTICS

Table I - Maximum Values of Pressure p_m

B \ Δ	0.07	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.70	595	844	900	965	1036	1100	1165	1228	1300	1370	1439
0.75	587	826	880	940	1006	1065	1125	1190	1255	1320	1389
0.80	579	809	860	915	977	1030	1090	1152	1210	1275	1339
0.85	572	792	840	890	949	1000	1060	1115	1170	1230	1289
0.90	565	775	820	870	922	975	1030	1079	1130	1185	1243
0.95	558	758	800	850	898	950	1000	1045	1095	1145	1199
1.00	551	741	780	830	874	925	970	1013	1060	1105	1157
1.05	544	724	760	810	850	900	940	982	1025	1070	1116
1.10	537	708	743	790	827	875	910	951	995	1035	1075
1.15	530	692	725	770	802	850	880	926	965	1005	1048
1.20	523	676	710	750	784	820	855	903	940	980	1018
1.25	516	661	695	730	764	800	835	880	915	955	990
1.30	509	646	680	710	746	780	820	859	890	930	967
1.35	502	634	665	700	730	765	805	840	870	910	946
1.40	495	623	650	685	715	750	790	822	845	890	925
1.45	488	612	640	675	702	745	775	805	830	870	906
1.50	481	601	630	660	690	730	760	788	818	850	887
1.55	475	592	620	650	679	715	745	775	803	830	870
1.60	469	584	610	640	669	700	730	762	788	815	853
1.65	464	577	600	630	659	690	715	749	773	800	838
1.70	459	570	590	620	649	680	705	735	760	785	823
1.75	454	563	585	610	641	670	695	725	750	775	809
1.80	449	556	580	605	633	660	685	715	740	765	795
1.85	445	549	575	600	625	650	675	704	730	750	781
1.90	442	543	570	595	617	640	665	694	720	745	768
1.95	439	537	565	590	610	630	650	684	710	735	757
2.00	435	532	560	585	603	620	645	674	700	725	747
2.05	433	527	555	580	596	610	635	664	690	715	737
2.10	430	522	550	574	590	600	625	654	680	705	727
2.15	427	517	545	568	584	595	615	645	670	695	718
2.20	424	512	540	562	578	590	610	636	660	685	709
2.25	421	507	535	556	572	585	605	628	650	675	700
2.30	419	503	530	550	566	580	600	620	640	665	691
2.35	417	499	525	545	560	575	595	615	635	655	683
2.40	415	495	520	540	554	570	590	610	630	650	675
2.45	413	492	515	535	548	565	585	605	625	645	667
2.50	411	489	510	530	542	560	580	600	620	640	659
2.55	409	486	505	525	538	555	575	595	615	635	652
2.60	407	483	500	520	533	550	570	590	610	630	646
2.65	405	480	495	515	528	545	565	585	605	624	640
2.70	403	477	490	510	523	540	560	580	600	618	634
2.75	402	474	485	505	520	535	555	575	595	612	628
2.80	401	471	482	500	517	530	550	570	590	606	622
2.85	399	468	479	497	514	527	547	566	585	600	616
[illegible]	[illegible]	465	476	494	511	524	544	562	580	595	611

1.05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.10	537	708	743	790	827	875	910	951	995	1035	1075
1.15	530	692	725	770	802	850	880	926	965	1005	1048
1.20	523	676	710	750	784	820	855	903	940	980	1018
1.25	516	661	695	730	764	800	835	880	915	955	990
1.30	509	646	680	710	746	780	820	859	890	930	967
1.35	502	634	665	700	730	765	805	840	870	910	946
1.40	495	623	650	685	715	750	790	822	845	890	925
1.45	488	612	640	675	702	745	775	805	830	870	906
1.50	481	601	630	660	690	730	760	788	818	850	887
1.55	475	592	620	650	679	715	745	775	803	830	870
1.60	469	584	610	640	669	700	730	762	788	815	853
1.65	464	577	600	630	659	690	715	749	773	800	838
1.70	459	570	590	620	649	680	705	735	760	785	823
1.75	454	563	585	610	641	670	695	725	750	775	809
1.80	449	556	580	605	633	660	685	715	740	765	795
1.85	445	549	575	600	625	650	675	704	730	750	781
1.90	442	543	570	595	617	640	665	694	720	745	768
1.95	439	537	565	590	610	630	650	684	710	735	757
2.00	435	532	560	585	603	620	645	674	700	725	747
2.05	433	527	555	580	596	610	635	664	690	715	737
2.10	430	522	550	574	590	600	625	654	680	705	727
2.15	427	517	545	568	584	595	615	645	670	695	718
2.20	424	512	540	562	578	590	610	636	660	685	709
2.25	421	507	535	556	572	585	605	628	650	675	700
2.30	419	503	530	550	566	580	600	620	640	665	691
2.35	417	499	525	545	560	575	595	615	635	655	683
2.40	415	495	520	540	554	570	590	610	630	650	675
2.45	413	492	515	535	548	565	585	605	625	645	667
2.50	411	489	510	530	542	560	580	600	620	640	659
2.55	409	486	505	525	538	555	575	595	615	635	652
2.60	407	483	500	520	533	550	570	590	610	630	646
2.65	405	480	495	515	528	545	565	585	605	624	640
2.70	403	477	490	510	523	540	560	580	600	618	634
2.75	402	474	485	505	520	535	555	575	595	612	628
2.80	401	471	482	500	517	530	550	570	590	606	622
2.85	399	468	479	497	514	527	547	566	585	600	616
2.90	398	465	476	494	511	524	544	562	580	595	611
2.95	397	462	473	492	508	522	541	558	575	590	606
3.00	396	459	470	490	506	520	538	555	570	580	601
$(1 - z_0)^2$	.3453	.5549	.5900	.6214	.6487	.6727	.6926	.7111	.7280	.7425	.7571

Table I

p_m

B \ $\triangle$	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30
0.70	1439	1520	1595	1670	1745	1817	1900	1975	2060	2145	2230
0.75	1389	1460	1525	1595	1670	1741	1820	1895	1975	2055	2140
0.80	1339	1405	1470	1535	1600	1666	1740	1815	1890	1965	2050
0.85	1289	1350	1410	1475	1535	1598	1665	1735	1805	1880	1960
0.90	1243	1295	1350	1415	1470	1534	1595	1660	1720	1800	1880
0.95	1199	1250	1305	1365	1420	1480	1540	1600	1660	1725	1800
1.00	1157	1210	1265	1320	1370	1429	1490	1540	1600	1660	1725
1.05	1116	1170	1220	1275	1325	1383	1435	1485	1545	1605	1665
1.10	1075	1130	1180	1230	1280	1335	1390	1440	1495	1550	1610
1.15	1048	1100	1145	1195	1245	1300	1350	1395	1445	1500	1560
1.20	1018	1065	1110	1160	1210	1260	1305	1350	1400	1450	1515
1.25	990	1035	1080	1125	1175	1225	1270	1315	1365	1415	1475
1.30	967	1010	1055	1095	1140	1185	1230	1280	1330	1380	1435
1.35	946	990	1030	1070	1110	1157	1200	1245	1295	1345	1395
1.40	925	965	1005	1045	1085	1126	1170	1210	1260	1310	1355
1.45	906	950	985	1020	1060	1097	1140	1180	1230	1275	1315
1.50	887	930	965	1000	1035	1070	1110	1150	1200	1240	1285
1.55	870	910	945	980	1010	1046	1085	1125	1170	1210	1250
1.60	853	890	925	955	988	1023	1060	1100	1140	1180	1220
1.65	838	870	900	930	965	1000	1035	1075	1110	1150	1195
1.70	823	850	880	915	945	980	1015	1050	1085	1120	1170
1.75	809	830	865	900	935	965	995	1030	1060	1100	1150
1.80	795	820	850	880	915	943	975	1010	1040	1080	1125
1.85	781	805	835	865	900	930	960	990	1020	1060	1105
1.90	768	795	820	850	885	915	945	975	1005	1040	1080
1.95	757	782	810	835	870	900	930	960	990	1020	1060
2.00	747	772	797	825	857	887	915	945	975	1005	1040
2.05	737	762	787	815	845	875	900	930	960	990	1020
2.10	727	750	775	800	830	855	885	915	945	975	1000
2.15	718	740	765	790	815	840	870	900	925	955	980
2.20	709	730	750	775	800	825	855	885	910	940	965
2.25	700	722	740	765	790	815	842	870	895	920	950
2.30	691	713	730	756	779	804	829	854	879	905	935
2.35	683	703	723	747	767	793	816	840	865	890	920
2.40	675	695	715	735	757	783	804	829	850	875	905
2.45	667	687	707	728	743	772	794	816	840	865	890
2.50	659	680	700	719	737	764	786	811	831	855	880
2.55	652	672	692	712	731	756	778	800	822	845	870
2.60	646	665	685	705	725	748	770	790	813	835	860
2.65	640	658	678	698	719	740	762	782	804	826	850
2.70	634	652	672	691	713	732	754	776	796	817	840
2.75	628	646	666	686	706	726	747	768	788	808	830
2.80	622	640	660	680	700	720	740	760	780	800	820
2.85	616	635	654	647	694	714	733	754	772	794	815
2.90	611	629	648	667	686	706	726	746	766	787	810
2.95	606	624	642	660	678	698	718	739	760	781	805
3.00	601	620	638	656	674	694	714	734	755	776	800

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1.30	967	1010	1055	1095	1140	1185	1230	1280	1330	1380	1435
1.35	946	990	1030	1070	1110	1157	1200	1245	1295	1345	1395
1.40	925	965	1005	1045	1085	1126	1170	1210	1260	1310	1355
1.45	906	950	985	1020	1060	1097	1140	1180	1230	1275	1315
1.50	887	930	965	1000	1035	1070	1110	1150	1200	1240	1285
1.55	870	910	945	980	1010	1046	1085	1125	1170	1210	1250
1.60	853	890	925	955	988	1023	1060	1100	1140	1180	1220
1.65	838	870	900	930	965	1000	1035	1075	1110	1150	1195
1.70	823	850	880	915	945	980	1015	1050	1085	1120	1170
1.75	809	830	865	900	935	965	995	1030	1060	1100	1150
1.80	795	820	850	880	915	943	975	1010	1040	1080	1125
1.85	781	805	835	865	900	930	960	990	1020	1060	1105
1.90	768	795	820	850	885	915	945	975	1005	1040	1080
1.95	757	782	810	835	870	900	930	960	990	1020	1060
2.00	747	772	797	825	857	887	915	945	975	1005	1040
2.05	737	762	787	815	845	875	900	930	960	990	1020
2.10	727	750	775	800	830	855	885	915	945	975	1000
2.15	718	740	765	790	815	840	870	900	925	955	980
2.20	709	730	750	775	80	825	855	885	910	940	965
2.25	700	722	740	765	790	815	842	870	895	920	950
2.30	691	713	730	756	779	804	829	854	879	905	935
2.35	683	703	723	747	767	793	816	840	865	890	920
2.40	675	695	715	735	757	783	804	829	850	875	905
2.45	667	687	707	728	743	772	794	816	840	865	890
2.50	659	680	700	719	737	764	786	811	831	855	880
2.55	652	672	692	712	731	756	778	800	822	845	870
2.60	646	665	685	705	725	748	770	790	813	835	860
2.65	640	658	678	698	719	740	762	782	804	826	850
2.70	634	652	672	691	713	732	754	776	796	817	840
2.75	628	646	666	686	706	726	747	768	788	808	830
2.80	622	640	660	680	700	720	740	760	780	800	820
2.85	616	635	654	647	694	714	733	754	772	794	815
2.90	611	629	648	667	686	706	726	746	766	787	810
2.95	606	624	642	660	678	698	718	739	760	781	805
3.00	601	620	638	656	674	694	714	734	755	776	800
$(1 - z_0)^2$	.7571	.7688	.7799	.7903	.8003	.8100	.8184	.8260	.8330	.8391	.8446

STAT

Table I

P_m

B \ Δ	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
0.70	2230	2320	2415	2510	2605	2700	2800	2900	3005	3110	3210
0.75	2140	2225	2315	2400	2485	2545	2655	2765	2865	2965	3060
0.80	2050	2130	2210	2295	2375	2460	2545	2635	2725	2820	2915
0.85	1960	2040	2120	2195	2275	2355	2435	2515	2600	2685	2775
0.90	1880	1955	2030	2100	2170	2240	2310	2390	2470	2550	2640
0.95	1800	1870	1940	2005	2070	2135	2200	2270	2350	2430	2515
1.00	1725	1790	1850	1910	1970	2040	2110	2180	2250	2320	2400
1.05	1665	1720	1770	1830	1890	1960	2030	2100	2165	2230	2300
1.10	1610	1665	1706	1760	1825	1890	1955	2020	2086	2150	2215
1.15	1560	1610	1616	1710	1770	1830	1890	1950	2010	2080	2145
1.20	1515	1565	1615	1665	1720	1770	1830	1885	1940	2010	2060
1.25	1475	1520	1570	1620	1670	1720	1770	1825	1875	1930	2000
1.30	1435	1475	1525	1570	1615	1670	1725	1765	1815	1870	1930
1.35	1395	1435	1480	1525	1570	1620	1665	1715	1760	1810	1865
1.40	1355	1400	1445	1490	1530	1580	1620	1670	1715	1760	1810
1.45	1315	1365	1405	1450	1485	1540	1580	1625	1670	1710	1750
1.50	1285	1330	1370	1410	1450	1495	1540	1580	1620	1660	1700
1.55	1250	1295	1335	1375	1415	1460	1500	1540	1580	1620	1660
1.60	1220	1260	1300	1340	1380	1420	1460	1500	1540	1580	1620
1.65	1195	1225	1255	1305	1340	1380	1420	1460	1500	1545	1580
1.70	1170	1195	1230	1270	1305	1345	1385	1425	1465	1510	1545
1.75	1150	1165	1200	1240	1270	1310	1350	1390	1420	1480	1510
1.80	1125	1145	1170	1205	1235	1275	1315	1355	1385	1445	1475
1.85	1105	1125	1145	1180	1205	1240	1280	1320	1350	1410	1445
1.90	1080	1100	1120	1150	1185	1215	1250	1290	1320	1375	1415
1.95	1060	1080	1100	1130	1165	1190	1230	1265	1300	1340	1390
2.00	1040	1060	1080	1110	1145	1175	1210	1250	1280	1320	1360
2.05	1020	1040	1060	1090	1125	1155	1190	1230	1260	1290	1330
2.10	1000	1020	1040	1070	1105	1135	1170	1210	1240	1270	1310
2.15	980	1000	1020	1050	1085	1115	1150	1190	1220	1250	1285
2.20	965	985	1005	1035	1065	1100	1130	1165	1200	1230	1265
2.25	950	970	990	1020	1050	1080	1110	1140	1175	1210	1240
2.30	935	955	975	1000	1030	1060	1090	1120	1155	1190	1220
2.35	920	940	960	980	1010	1040	1070	1100	1135	1170	1200
2.40	905	925	945	965	995	1030	1050	1080	1115	1150	1180
2.45	890	910	930	950	980	1005	1030	1060	1095	1130	1160
2.50	880	900	920	940	965	990	1015	1045	1080	1110	1140
2.55	870	890	910	930	955	980	1005	1030	1065	1095	1125
2.60	860	880	900	920	945	970	995	1020	1050	1080	1110
2.65	850	870	890	910	935	960	985	1010	1035	1065	1095
2.70	840	860	880	900	925	950	975	1000	1025	1050	1080
2.75	830	850	870	890	915	940	965	990	1015	1040	1070
2.80	820	840	860	880	905	930	950	980	1005	1030	1060
2.85	815	830	850	870	895	920	945	970	995	1020	1050
2.90	810	825	845	865	885	910	935	960	985	1010	1040
2.95	805	820	840	860	880	905	930	955	980	1005	1030
3.00	800	815	835	855	875	900	925	950	975	1000	1020

1.15	1560	1610	1616	1710	1770	1830	1890	1950	2010	2080	2145
1.20	1515	1565	1615	1665	1720	1770	1830	1885	1940	2010	2060
1.25	1475	1520	1570	1620	1670	1720	1770	1825	1875	1930	2000
1.30	1435	1475	1525	1570	1615	1670	1725	1765	1815	1870	1930
1.35	1395	1435	1480	1525	1570	1620	1665	1715	1760	1810	1865
1.40	1355	1400	1445	1490	1530	1580	1620	1670	1715	1760	1810
1.45	1315	1365	1405	1450	1485	1540	1580	1625	1670	1710	1750
1.50	1285	1330	1370	1410	1450	1495	1540	1580	1620	1660	1700
1.55	1250	1295	1335	1375	1415	1460	1500	1540	1580	1620	1660
1.60	1220	1260	1300	1340	1380	1420	1460	1500	1540	1580	1620
1.65	1195	1225	1255	1305	1340	1380	1420	1460	1500	1545	1580
1.70	1170	1195	1230	1270	1305	1345	1385	1425	1465	1510	1545
1.75	1150	1165	1200	1240	1270	1310	1350	1390	1420	1480	1510
1.80	1125	1145	1170	1205	1235	1275	1315	1355	1385	1445	1475
1.85	1105	1125	1145	1180	1205	1240	1280	1320	1350	1410	1445
1.90	1080	1100	1120	1150	1185	1215	1250	1290	1320	1375	1415
1.95	1060	1080	1100	1130	1165	1190	1230	1265	1300	1340	1390
2.00	1040	1060	1080	1110	1145	1175	1210	1250	1280	1320	1360
2.05	1020	1040	1060	1090	1125	1155	1190	1230	1260	1290	1330
2.10	1000	1020	1040	1070	1105	1135	1170	1210	1240	1270	1310
2.15	980	1000	1020	1050	1085	1115	1150	1190	1220	1250	1285
2.20	965	985	1005	1035	1065	1100	1130	1165	1200	1230	1265
2.25	950	970	990	1020	1050	1080	1110	1140	1175	1210	1240
2.30	935	955	975	1000	1030	1060	1090	1120	1155	1190	1220
2.35	920	940	960	980	1010	1040	1070	1100	1135	1170	1200
2.40	905	925	945	965	995	1030	1050	1080	1115	1150	1180
2.45	890	910	930	950	980	1005	1030	1060	1095	1130	1160
2.50	880	900	920	940	965	990	1015	1045	1080	1110	1140
2.55	870	890	910	930	955	980	1005	1030	1065	1095	1125
2.60	860	880	900	920	945	970	995	1020	1050	1080	1110
2.65	850	870	890	910	935	960	985	1010	1035	1065	1095
2.70	840	860	880	900	925	950	975	1000	1025	1050	1080
2.75	830	850	870	890	915	940	965	990	1015	1040	1070
2.80	820	840	860	880	905	930	950	980	1005	1030	1060
2.85	815	830	850	870	895	920	945	970	995	1020	1050
2.90	810	825	845	865	885	910	935	960	985	1010	1040
2.95	805	820	840	860	880	905	930	955	980	1005	1030
3.00	800	815	835	855	875	900	925	950	975	1000	1020
$(1 - z_0)^2$	.8446	.8501	.8556	.8611	.8663	.8720	.8766	.8807	.8847	.8865	.8920

STAT

Table I

p_m

B \ Δ	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
0.70	3210	3325	3445	3560	3675	3802	3945	4080	4225	4365	4520
0.75	3060	3160	3270	3380	3495	3610	3730	3850	3980	4110	4240
0.80	2915	3005	3110	3215	3320	3425	3535	3650	3770	3890	4020
0.85	2775	2860	2960	3050	3150	3250	3350	3460	3570	3690	3815
0.90	2640	2725	2810	2900	2990	3080	3180	3280	3390	3510	3515
0.95	2515	2595	2680	2765	2850	2940	3035	3130	3230	3335	3440
1.00	2400	2485	2570	2645	2730	2820	2900	2995	3085	3180	3285
1.05	2300	2380	2465	2536	2615	2700	2775	2860	2950	3030	3140
1.10	2215	2285	2360	2430	2500	2515	2615	2715	2815	2900	3000
1.15	2145	2205	2275	2335	2400	2470	2545	2625	2710	2785	2880
1.20	2060	2130	2190	2250	2315	2380	2455	2530	2610	2680	2760
1.25	2000	2055	2115	2175	2235	2300	2370	2440	2515	2585	2660
1.30	1930	1985	2045	2100	2160	2220	2285	2350	2420	2490	2560
1.35	1865	1920	1975	2030	2090	2150	2215	2275	2340	2400	2470
1.40	1810	1860	1910	1965	2020	2080	2145	2100	2260	2320	2380
1.45	1750	1800	1850	1905	1965	2020	2075	2130	2190	2245	2300
1.50	1700	1750	1800	1850	1900	1950	2010	2065	2120	2175	2235
1.55	1660	1707	1750	1800	1850	1900	1950	2005	2060	2115	2175
1.60	1620	1660	1705	1755	1800	1850	1900	1950	2000	2055	2115
1.65	1580	1625	1670	1715	1760	1805	1850	1900	1950	2000	2065
1.70	1545	1585	1630	1670	1715	1760	1805	1855	1900	1955	2015
1.75	1510	1550	1590	1630	1675	1720	1760	1805	1855	1905	1965
1.80	1475	1515	1550	1590	1635	1680	1720	1765	1810	1860	1910
1.85	1445	1480	1515	1555	1600	1640	1680	1725	1770	1810	1870
1.90	1415	1450	1485	1525	1570	1615	1655	1695	1735	1780	1830
1.95	1390	1420	1460	1500	1540	1580	1620	1665	1705	1745	1790
2.00	1360	1395	1435	1475	1515	1550	1590	1635	1670	1710	1750
2.05	1330	1370	1405	1450	1490	1520	1560	1605	1635	1675	1710
2.10	1310	1345	1380	1425	1465	1495	1530	1570	1605	1640	1675
2.15	1285	1320	1355	1400	1440	1470	1500	1540	1575	1610	1640
2.20	1265	1225	1330	1375	1410	1445	1475	1520	1550	1580	1610
2.25	1240	1270	1305	1350	1385	1420	1450	1495	1525	1555	1585
2.30	1220	1250	1280	1325	1360	1395	1430	1470	1500	1530	1560
2.35	1200	1230	1260	1300	1335	1375	1405	1445	1475	1505	1535
2.40	1180	1210	1240	1280	1315	1345	1380	1420	1450	1480	1510
2.45	1160	1190	1220	1260	1295	1320	1355	1395	1425	1455	1485
2.50	1140	1170	1200	1240	1275	1300	1330	1370	1400	1430	1460
2.55	1125	1150	1185	1220	1255	1280	1310	1345	1375	1405	1435
2.60	1110	1135	1170	1200	1235	1260	1290	1320	1350	1380	1410
2.65	1095	1120	1155	1185	1215	1240	1270	1300	1325	1355	1385
2.70	1080	1105	1140	1170	1200	1225	1250	1280	1305	1330	1360
2.75	1070	1095	1125	1155	1285	1210	1235	1260	1285	1310	1340
2.80	1060	1085	1110	1140	1170	1195	1220	1245	1270	1295	1325
2.85	1050	1075	1095	1125	1155	1180	1205	1230	1255	1280	1310
2.90	1040	1065	1085	1110	1140	1165	1190	1215	1240	1265	1300
2.95	1030	1055	1075	1100	1125	1150	1175	1200	1225	1255	1290
3.00	1020	1045	1065	1090	1110	1035	1160	1185	1215	1245	1280
[illegible]	[illegible]	[illegible]	[illegible]	9016	9047	9076	9105	9131	9157	9179	9201

1.00	2400	2485	2570	2645	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.05	2300	2380	2465	2536	2615	2700	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.10	2215	2285	2360	2430	2500	[illegible]	[illegible]	[illegible]	2815	2900	3000
1.15	2145	2205	2275	2335	2400	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.20	2060	2130	2190	2250	2315	2380	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.25	2000	2055	2115	2175	2235	2300	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.30	1930	1985	2045	2100	2160	2220	[illegible]	2350	2420	[illegible]	[illegible]
1.35	1865	1920	1975	2030	2090	2150	[illegible]	2275	2340	2400	2470
1.40	1810	1860	1910	1965	2020	2080	[illegible]	2200	2260	2320	2380
1.45	1750	1800	1850	1905	1965	2020	2075	2130	2190	2245	2300
1.50	1700	1750	1800	1850	1900	1950	2010	2065	2120	2175	2235
1.55	1660	1707	1750	1800	1850	1900	[illegible]	2005	2060	2115	2175
1.60	1620	1660	1705	1755	1800	1850	[illegible]	1950	2000	2055	2115
1.65	1580	1625	1670	1715	1760	1805	[illegible]	1900	1950	2000	2065
1.70	1545	1585	1630	1670	1715	1760	[illegible]	1855	1900	1955	2015
1.75	1510	1550	1590	1630	1675	1720	[illegible]	1805	1855	1905	1965
1.80	1475	1515	1550	1590	1635	1680	[illegible]	1765	1810	1860	1910
1.85	1445	1480	1515	1555	1600	1640	1680	1725	1770	1810	1870
1.90	1415	1450	1485	1525	1570	1615	1655	1695	1735	1780	1830
1.95	1390	1420	1460	1500	1540	1580	1620	1665	1705	1745	1790
2.00	1360	1395	1435	1475	1515	1550	1590	1635	1670	1710	1750
2.05	1330	1370	1405	1450	1490	1520	1560	1605	1635	1675	1710
2.10	1310	1345	1380	1425	1465	1495	1530	1570	1605	1640	1675
2.15	1285	1320	1355	1400	1440	1470	1500	1540	1575	1610	1640
2.20	1265	1225	1330	1375	1410	1445	1475	1520	1550	1580	1610
2.25	1240	1270	1305	1350	1385	1420	1450	1495	1525	1555	1585
2.30	1220	1250	1280	1325	1360	1395	1430	1470	1500	1530	1560
2.35	1200	1230	1260	1300	1335	1375	1405	1445	1475	1505	1535
2.40	1180	1210	1240	1280	1315	1345	1380	1420	1450	1480	1510
2.45	1160	1190	1220	1260	1295	1320	1355	1395	1425	1455	1485
2.50	1140	1170	1200	1240	1275	1300	1330	1370	1400	1430	1460
2.55	1125	1150	1185	1220	1255	1280	1310	1345	1375	1405	1435
2.60	1110	1135	1170	1200	1235	1260	1290	1320	1350	1380	1410
2.65	1095	1120	1155	1185	1215	1240	1270	1300	1325	1355	1385
2.70	1080	1105	1140	1170	1200	1225	1250	1280	1305	1330	1360
2.75	1070	1095	1125	1155	1285	1210	1235	1260	1285	1310	1340
2.80	1060	1085	1110	1140	1170	1195	1220	1245	1270	1295	1325
2.85	1050	1075	1095	1125	1155	1180	1205	1230	1255	1280	1310
2.90	1040	1065	1085	1110	1140	1165	1190	1215	1240	1265	1300
2.95	1030	1055	1075	1100	1125	1150	1175	1200	1225	1255	1290
3.00	1020	1045	1065	1090	1110	1035	1160	1185	1215	1245	1280
$(1 - z_0)^2$	.8920	.8952	.8985	.9016	.9047	.9076	.9105	.9131	.9157	.9179	.9201

STAT

Table I

P_m

B \ A	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60
0.70	4520	4665	4815	4970	5125	5280					
0.75	4240	4385	4530	4680	4830	4980	5140				
0.80	4020	4155	4290	4430	4570	4710	4860	5010			
0.85	3815	3945	4075	4205	4335	4470	4615	4760	4910	5065	
0.90	3615	3740	3865	3985	4105	4235	4375	4515	4660	4805	4950
0.95	3440	3550	3665	3775	3885	4000	4135	4270	4410	4545	4680
1.00	3285	3390	3495	3595	3695	3800	3925	4055	4130	4320	4450
1.05	3140	3240	3340	3435	3525	3620	3735	3855	3980	4100	4220
1.10	3000	3090	3185	3275	3360	3450	3555	3665	3780	3890	4000
1.15	2880	2970	3055	3140	3220	3310	3400	3500	3610	3715	3820
1.20	2760	2850	2940	3030	3120	3210	3300	3390	3480	3570	3660
1.25	2660	2745	2830	2915	3000	3085	3170	3260	3345	3430	3520
1.30	2560	2645	2730	2815	2900	2985	3070	3150	3230	3310	3390
1.35	2470	2550	2630	2710	2790	2870	2950	3030	3110	3190	3270
1.40	2380	2460	2535	2615	2690	2775	2845	2925	3000	3080	3160
1.45	2300	2375	2455	2530	2610	2685	2760	2835	2910	2985	3060
1.50	2235	2310	2380	2455	2525	2595	2665	2740	2810	2885	2960
1.55	2175	2240	2310	2380	2450	2520	2590	2660	2730	2800	2870
1.60	2115	2180	2250	2315	2380	2445	2515	2580	2650	2710	2780
1.65	2065	2125	2185	2250	2315	2380	2445	2510	2575	2640	2700
1.70	2015	2070	2135	2195	2255	2315	2375	2435	2495	2555	2620
1.75	1965	2020	2080	2140	2195	2255	2315	2370	2430	2490	2550
1.80	1910	1965	2025	2080	2135	2190	2250	2300	2365	2420	2480
1.85	1870	1925	1980	2035	2090	2145	2200	2255	2310	2360	2420
1.90	1830	1885	1940	1990	2045	2100	2150	2205	2260	2310	2365
1.95	1790	1845	1900	1950	2000	2055	2105	2155	2210	2255	2310
2.00	1750	1805	1860	1910	1960	2010	2060	2110	2160	2205	2255
2.05	1710	1765	1820	1870	1920	1970	2015	2065	2115	2155	2205
2.10	1675	1730	1785	1835	1885	1930	1975	2025	2070	2110	2155
2.15	1640	1700	1755	1805	1850	1895	1935	1985	2030	2065	2105
2.20	1610	1675	1725	1770	1815	1860	1900	1945	1990	2025	2060
2.25	1585	1650	1700	1740	1785	1830	1865	1910	1955	1985	2020
2.30	1560	1625	1675	1710	1755	1800	1830	1875	1920	1950	1980
2.35	1535	1600	1650	1680	1725	1770	1800	1840	1885	1915	1945
2.40	1510	1575	1625	1655	1695	1740	1770	1810	1850	1880	1910
2.45	1485	1550	1600	1630	1665	1710	1740	1780	1820	1845	1875
2.50	1460	1525	1575	1605	1640	1680	1710	1750	1790	1815	1840
2.55	1435	1505	1550	1580	1615	1645	1680	1720	1760	1785	1810
2.60	1410	1475	1525	1555	1590	1620	1655	1690	1730	1755	1780
2.65	1385	1430	1485	1515	1565	1595	1630	1665	1700	1725	1750
2.70	1360	1420	1450	1480	1520	1550	1585	1615	1650	1680	1720
2.75	1340	1395	1425	1455	1495	1525	1555	1585	1625	1650	1695
2.80	1325	1370	1400	1430	1470	1500	1535	1560	1600	1625	1670
2.85	1310	1350	1380	1410	1445	1475	1505	1535	1575	1600	1645
2.90	1300	1330	1360	1390	1425	1455	1480	1515	1550	1575	1620
2.95	1290	1320	1350	1380	1410	1440	1470	1500	1530	1560	1600
3.00	1280	1310	1340	1370	1400	1430	1460	1490	[illegible]	[illegible]	[illegible]

0.95	3440	3550	3665	3775	3885	4000	4135	4270	4410	4545	4680
1.00	3285	3390	3495	3595	3695	3800	3925	4055	4130	4320	4450
1.05	3140	3240	3340	3435	3525	3620	3735	3855	3980	4100	4220
1.10	3000	3090	3185	3275	3360	3450	3555	3665	3780	3890	4000
1.15	2880	2970	3055	3140	3220	3310	3400	3500	3610	3715	3820
1.20	2760	2850	2940	3030	3120	3210	3300	3390	3480	3570	3660
1.25	2660	2745	2830	2915	3000	3085	3170	3260	3345	3430	3520
1.30	2560	2645	2730	2815	2900	2985	3070	3150	3230	3310	3390
1.35	2470	2550	2630	2710	2790	2870	2950	3030	3110	3190	3270
1.40	2380	2460	2535	2615	2690	2775	2845	2925	3000	3080	3160
1.45	2300	2375	2455	2530	2610	2685	2760	2835	2910	2985	3060
1.50	2235	2310	2380	2455	2525	2595	2665	2740	2810	2885	2960
1.55	2175	2240	2310	2380	2450	2520	2590	2660	2730	2800	2870
1.60	2115	2180	2250	2315	2380	2445	2515	2580	2650	2710	2780
1.65	2065	2125	2185	2250	2315	2380	2445	2510	2575	2640	2700
1.70	2015	2070	2135	2195	2255	2315	2375	2435	2495	2555	2620
1.75	1965	2020	2080	2140	2195	2255	2315	2370	2430	2490	2550
1.80	1910	1965	2025	2080	2135	2190	2250	2300	2365	2420	2480
1.85	1870	1925	1980	2035	2090	2145	2200	2255	2310	2360	2420
1.90	1830	1885	1940	1990	2045	2100	2150	2205	2260	2310	2365
1.95	1790	1845	1900	1950	2000	2055	2105	2155	2210	2255	2310
2.00	1750	1805	1860	1910	1960	2010	2060	2110	2160	2205	2255
2.05	1710	1765	1820	1870	1920	1970	2015	2065	2115	2155	2205
2.10	1675	1730	1785	1835	1885	1930	1975	2025	2070	2110	2155
2.15	1640	1700	1755	1805	1850	1895	1935	1985	2030	2065	2105
2.20	1610	1675	1725	1770	1815	1860	1900	1945	1990	2025	2060
2.25	1585	1650	1700	1740	1785	1830	1865	1910	1955	1985	2020
2.30	1560	1625	1675	1710	1755	1800	1830	1875	1920	1950	1980
2.35	1535	1600	1650	1680	1725	1770	1800	1840	1885	1915	1945
2.40	1510	1575	1625	1655	1695	1740	1770	1810	1850	1880	1910
2.45	1485	1550	1600	1630	1665	1710	1740	1780	1820	1845	1875
2.50	1460	1525	1575	1605	1640	1680	1710	1750	1790	1815	1840
2.55	1435	1505	1550	1580	1615	1645	1680	1720	1760	1785	1810
2.60	1410	1475	1525	1555	1590	1620	1655	1690	1730	1755	1780
2.65	1385	1430	1485	1515	1565	1595	1630	1665	1700	1725	1750
2.70	1360	1420	1450	1480	1520	1550	1585	1615	1650	1680	1720
2.75	1340	1395	1425	1455	1495	1525	1555	1585	1625	1650	1695
2.80	1325	1370	1400	1430	1470	1500	1535	1560	1600	1625	1670
2.85	1310	1350	1380	1410	1445	1475	1505	1535	1575	1600	1645
2.90	1300	1330	1360	1390	1425	1455	1480	1515	1550	1575	1620
2.95	1290	1320	1350	1380	1410	1440	1470	1500	1530	1560	1600
3.00	1280	1310	1340	1370	1400	1430	1460	1490	1520	1550	1580
$(1 - z_0)^2$	.9201	.9222	.9243	.9264	.9285	.9305	.9324	.9342	.9359	.9376	.9392

STAT

Table I

p_m

B \ Δ	0.60	0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.70
0.70											
0.75											
0.80											
0.85											
0.90	4950	5095									
0.95	4680	4825	4970	5120							
1.00	4450	4590	4735	4885	5040						
1.05	4220	4355	4495	4640	4790	4945	5105				
1.10	4000	4130	4265	4405	4550	4700	4855	5015			
1.15	3820	3940	4065	4195	4330	4470	4615	4765	4920	5080	
1.20	3660	3770	3885	4005	4130	4260	4395	4540	4690	4845	5000
1.25	3520	3620	3725	3835	3940	4070	4195	4330	4470	4615	4760
1.30	3390	3480	3575	3675	3780	3890	4005	4130	4260	4400	4530
1.35	3270	3375	3480	3585	3690	3795	3900	4005	4110	4220	4330
1.40	3160	3255	3350	3450	3550	3650	3750	3850	3950	4050	4150
1.45	3060	3150	3240	3330	3420	3510	3600	3695	3790	3885	3980
1.50	2960	3045	3130	3215	3300	3385	3470	3560	3650	3740	3830
1.55	2870	2950	3030	3116	3200	3285	3370	3455	3540	3625	3710
1.60	2780	2860	2940	3020	3100	3180	3260	3345	3430	3515	3600
1.65	2700	2775	2850	2930	3010	3090	3170	3250	3330	3410	3495
1.70	2620	2695	2770	2845	2920	2995	3070	3150	3230	3310	3390
1.75	2550	2620	2690	2765	2840	2915	2990	3065	3140	3215	3290
1.80	2480	2550	2620	2690	2760	2830	2900	2975	3050	3125	3200
1.85	2420	2485	2550	2620	2690	2760	2830	2900	2970	3040	3110
1.90	2365	2430	2495	2560	2625	2690	2755	2820	2890	2960	3030
1.95	2310	2370	2430	2495	2560	2625	2690	2755	2820	2885	2950
2.00	2255	2315	2375	2435	2495	2555	2620	2685	2750	2815	2880
2.05	2205	2265	2325	2385	2445	2505	2565	2625	2685	2745	2810
2.10	2155	2210	2265	2325	2385	2445	2505	2565	2625	2685	2745
2.15	2105	2160	2215	2270	2325	2380	2440	2500	2560	2620	2680
2.20	2060	2115	2170	2225	2280	2335	2390	2445	2500	2560	2620
2.25	2020	2070	2120	2175	2230	2285	2340	2395	2450	2505	2560
2.30	1980	2030	2080	2130	2180	2230	2285	2340	2395	2450	2505
2.35	1945	1995	2045	2095	2145	2195	2245	2295	2345	2395	2450
2.40	1910	1955	2000	2050	2100	2150	2200	2250	2300	2350	2400
2.45	1875	1920	1965	2010	2055	2100	2150	2200	2250	2300	2350
2.50	1840	1885	1935	1975	2020	2065	2110	2155	2205	2255	2305
2.55	1810	1855	1900	1945	1990	2035	2080	2125	2170	2215	2260
2.60	1780	1820	1860	1905	1950	1995	2040	2085	2130	2175	2220
2.65	1750	1790	1830	1870	1910	1955	2000	2045	2090	2135	2180
2.70	1720	1760	1800	1840	1880	1920	1965	2010	2055	2100	2145
2.75	1695	1735	1775	1815	1855	1895	1935	1975	2020	2065	2110
2.80	1670	1710	1750	1790	1830	1870	1910	1950	1990	2035	2080
2.85	1645	1685	1725	1765	1805	1845	1885	1925	1965	2005	2050
2.90	1620	1660	1700	1740	1780	1820	1860	1900	1940	1980	2025
2.95	1600	1640	1680	1720	1760	1800	1840	1880	1920	1960	2000
3.00	1580	1620	1660	1700	1740	1780	1820	1860	1900	1940	1980

1.20	3660	3770	3885	4005	4130	4260	4395	4540	4690	4845	[illegible]
1.25	3520	3620	3725	3835	3940	4070	4195	4330	4470	4615	4760
1.30	3390	3480	3575	3675	3780	3890	4005	4130	4260	4400	4530
1.35	3270	3375	3480	3585	3690	3795	3900	4005	4110	4220	4330
1.40	3160	3255	3350	3450	3550	3650	3750	3850	3950	4050	4150
1.45	3060	3150	3240	3330	3420	3510	3600	3695	3790	3885	3980
1.50	2960	3045	3130	3215	3300	3385	3470	3560	3650	3740	3830
1.55	2870	2950	3030	3116	3200	3285	3370	3455	3540	3625	3710
1.60	2780	2860	2940	3020	3100	3180	3260	3345	3430	3515	3600
1.65	2700	2775	2850	2930	3010	3090	3170	3250	3330	3410	3495
1.70	2620	2695	2770	2845	2920	2995	3070	3150	3230	3310	3390
1.75	2550	2620	2690	2765	2840	2915	2990	3065	3140	3215	3290
1.80	2480	2550	2620	2690	2760	2830	2900	2975	3050	3125	3200
1.85	2420	2485	2550	2620	2690	2760	2830	2900	2970	3040	3110
1.90	2365	2430	2495	2560	2625	2690	2755	2820	2890	2960	3030
1.95	2310	2370	2430	2495	2560	2625	2690	2755	2820	2885	2950
2.00	2255	2315	2375	2435	2495	2555	2625	2685	2750	2815	2880
2.05	2205	2265	2325	2385	2445	2505	2565	2625	2685	2745	2810
2.10	2155	2210	2265	2325	2385	2445	2505	2565	2625	2685	2745
2.15	2105	2160	2215	2270	2325	2380	2440	2500	2560	2620	2680
2.20	2060	2115	2170	2225	2280	2335	2390	2445	2500	2560	2620
2.25	2020	2070	2120	2175	2230	2285	2340	2395	2450	2505	2560
2.30	1980	2030	2080	2130	2180	2230	2285	2340	2395	2450	2505
2.35	1945	1995	2045	2095	2145	2195	2245	2295	2345	2395	2450
2.40	1910	1955	2000	2050	2100	2150	2200	2250	2300	2350	2400
2.45	1875	1920	1965	2010	2055	2100	2150	2200	2250	2300	2350
2.50	1840	1885	1935	1975	2020	2065	2110	2155	2205	2255	2305
2.55	1810	1855	1900	1945	1990	2035	2080	2125	2170	2215	2260
2.60	1780	1820	1860	1905	1950	1995	2040	2085	2130	2175	2220
2.65	1750	1790	1830	1870	1910	1955	2000	2045	2090	2135	2180
2.70	1720	1760	1800	1840	1880	1920	1965	2010	2055	2100	2145
2.75	1695	1735	1775	1815	1855	1895	1935	1975	2020	2065	2110
2.80	1670	1710	1750	1790	1830	1870	1910	1950	1990	2035	2080
2.85	1645	1685	1725	1765	1805	1845	1885	1925	1965	2005	2050
2.90	1620	1660	1700	1740	1780	1820	1860	1900	1940	1980	2025
2.95	1600	1640	1680	1720	1760	1800	1840	1880	1920	1960	2000
3.00	1580	1620	1660	1700	1740	1780	1820	1860	1900	1940	1980
$(1 - z_0)^2$	.9392	.9408	.9423	.9438	.9452	.9466	.9480	.9492	.9506	.9517	.9530

STAT

Table 1

P_m

B \ Δ	0.70	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80
0.70											
0.75											
0.80											
0.85											
0.90											
0.95											
1.00											
1.05											
1.10											
1.15											
1.20	5000										
1.25	4750	4910	5060								
1.30	4530	4670	4810	4950	5090						
1.35	4330	4455	4580	4710	4840	4970	5110				
1.40	4150	4260	4370	4485	4600	4720	[illegible]	4990	5140		
1.45	3980	4080	4190	4305	4420	4540	4670	4810	4960	5120	
1.50	3830	3925	4025	4135	4255	4385	4525	4680	4845	5020	5200
1.55	3710	3800	3895	4000	4115	4240	4375	4520	4675	4840	5010
1.60	3600	3685	3775	3875	3985	4105	4230	4365	4510	4665	4825
1.65	3495	3580	3665	3760	3865	3980	4100	4230	4365	4510	4660
1.70	3390	3475	3555	3645	3745	3855	3970	4090	4220	4360	4505
1.75	3290	3365	3440	3525	3620	3725	3835	3950	4080	4220	4365
1.80	3200	3275	3345	3425	3515	3615	3720	3830	3955	4090	4235
1.85	3110	3180	3255	3335	3425	3525	3630	3735	3850	3975	4110
1.90	3030	3100	3175	3255	3345	3445	3545	3645	3755	3870	3995
1.95	2950	3015	3085	3160	3245	3340	3435	3535	3645	3760	3885
2.00	2880	2945	3015	3090	3175	3270	3365	3460	3560	3665	3780
2.05	2810	2875	2940	3010	3090	3180	3270	3365	3465	3565	3685
2.10	2745	2805	2865	2930	3010	3100	3190	3285	3385	3485	3595
2.15	2680	2740	2800	2865	2940	3025	3110	3200	3300	3400	3510
2.20	2620	2680	2735	2795	2865	2945	3025	3115	3215	3315	3425
2.25	2560	2615	2670	2730	2800	2880	2960	3045	3140	3235	3345
2.30	2505	2560	2610	2665	2730	2805	2885	2970	3065	3160	3265
2.35	2450	2505	2555	2610	2675	2750	2825	2905	2995	3085	3190
2.40	2400	2450	2495	2545	2605	2675	2750	2830	2920	3010	3115
2.45	2350	2400	2440	2490	2545	2610	2685	2785	2855	2945	3045
2.50	2305	2355	2395	2445	2495	2555	2625	2700	2790	2880	2980
2.55	2260	2305	2345	2395	2445	2505	2575	2650	2740	2825	2920
2.60	2220	2265	2305	2355	2405	2465	2535	2605	2690	2770	2860
2.65	2180	2225	2265	2310	2355	2410	2475	2545	2630	2710	2800
2.70	2145	2190	2230	2275	2320	2375	2440	2505	2585	2660	2745
2.75	2110	2155	2195	2240	2285	2340	2405	2465	2535	2610	2690
2.80	2080	2125	2165	2205	2245	2295	2355	2415	2485	2560	2640
2.85	2050	2095	2135	2175	2215	2260	2315	2370	2435	2510	2590
2.90	2025	2070	2110	2150	2190	2230	2280	2330	2390	2465	2545
2.95	2000	2040	2085	2125	2165	2205	2250	2300	2360	2430	2505

1.05											
1.10											
1.15											
1.20	5000										
1.25	4750	4910	5060								
1.30	4530	4670	4810	4950	5090						
1.35	4330	4455	4580	4710	4840	4970	5110				
1.40	4150	4260	4370	4485	4600	4720	4850	4990	5140		
1.45	3980	4080	4190	4305	4420	4540	4670	4810	4960	5120	
1.50	3830	3925	4025	4135	4255	4385	4525	4680	4845	5020	5200
1.55	3710	3800	3895	4000	4115	4240	4375	4520	4675	4840	5010
1.60	3600	3685	3775	3875	3985	4105	4230	4365	4510	4665	4825
1.65	3495	3580	3665	3760	3865	3980	4100	4230	4365	4510	4660
1.70	3390	3475	3555	3645	3745	[illegible]	3970	[illegible]	4220	4360	4505
1.75	3290	3365	3440	3525	3620	3725	3835	[illegible]	4080	4220	4365
1.80	3200	3275	3345	3425	3515	[illegible]	3720	[illegible]	[illegible]	4095	4235
1.85	3110	3180	3255	3335	3425	[illegible]	3630	3735	3850	3975	4110
1.90	3030	3100	3175	3255	3345	3445	3545	3645	3775	3870	3995
1.95	2950	3015	3085	3160	3245	3340	3435	3535	3645	3760	3885
2.00	2880	2945	3015	3090	3175	3270	3365	3460	3560	3665	3780
2.05	2810	2875	2940	3010	3090	3180	3270	3365	3465	3565	3685
2.10	2745	2805	2865	2930	3010	3100	3190	3285	3385	3485	3595
2.15	2680	2740	2800	2865	2940	3025	3110	3200	3300	3400	3510
2.20	2620	2680	2735	2795	2865	2945	3025	3115	3215	3315	3425
2.25	2560	2615	2670	2730	2800	2880	2960	3045	3140	3235	3345
2.30	2505	2560	2610	2665	2730	2805	2885	2970	3065	3160	3265
2.35	2450	2505	2555	2610	2675	2730	2825	2905	2995	3085	3190
2.40	2400	2450	2495	2545	2605	2675	2750	2830	2920	3010	3115
2.45	2350	2400	2440	2490	2545	2610	[illegible]	[illegible]	[illegible]	2945	3045
2.50	2305	2355	2395	2445	2495	2555	2625	2700	2790	2880	2980
2.55	2260	2305	2345	2395	2445	2505	2575	2650	2740	2825	2920
2.60	2220	2265	2305	2355	2405	2465	2535	2605	2690	2770	2860
2.65	2180	2225	2265	2310	2355	2410	2475	2545	2630	2710	2800
2.70	2145	2190	2230	2275	2320	2375	2440	2505	2585	2660	2745
2.75	2110	2155	2195	2240	2285	2340	2405	2465	2535	2610	2690
2.80	2080	2125	2165	2205	2245	2295	2355	2415	2485	2560	2640
2.85	2050	2095	2135	2175	2215	2260	2315	2370	2435	2510	2590
2.90	2025	2070	2110	2150	2190	2230	2280	2330	2390	2465	2545
2.95	2000	2040	2085	2125	2165	2205	2250	2300	2360	2430	2505
3.00	1980	2020	2065	2105	2145	2185	2230	2275	2335	2400	2470
$(1 - z_0)^2$	.9530	.9541	.9552	.9563	.9573	.9585	.9595	.9606	.9615	.9624	.9633

STAT

Table II - Condition at the End of Burning $\frac{t_k}{t_0}$

B \ Δ	0.07	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.70	0.16	0.30	0.33	0.35	0.37	0.39	0.41	0.43	0.44	0.45	0.46
0.75	17	33	36	38	41	43	45	47	49	51	52
0.80	18	36	39	42	44	47	49	51	54	56	58
0.85	20	39	42	45	47	50	52	55	57	59	61
0.90	21	42	45	48	50	53	56	57	59	61	63
0.95	23	45	48	50	53	56	58	60	63	65	67
1.00	24	47	50	53	57	60	63	65	68	70	72
1.05	26	48	53	57	61	64	67	69	72	75	77
1.10	27	51	56	60	65	68	72	74	77	80	82
1.15	28	54	59	64	69	73	76	79	82	85	87
1.20	30	57	63	68	73	77	81	84	88	91	93
1.25	31	60	66	72	77	82	[illegible]	90	93	96	98
1.30	32	63	69	75	81	86	91	96	99	1.02	1.04
1.35	33	67	74	80	86	91	96	1.01	1.04	07	10
1.40	35	70	77	84	91	97	1.02	07	11	14	17
1.45	36	74	81	88	95	1.02	[illegible]	13	17	20	22
1.50	38	77	85	93	1.00	07	[illegible]	19	23	27	31
1.55	39	81	89	97	05	12	19	25	30	34	38
1.60	40	84	93	1.02	11	17	24	31	36	41	46
1.65	42	88	98	07	16	23	30	37	42	49	54
1.70	44	92	1.02	12	22	30	37	44	[illegible]	57	63
1.75	45	96	07	17	27	36	44	52	59	66	72
1.80	47	1.00	11	22	33	42	[illegible]	58	67	74	81
1.85	49	04	16	28	39	49	58	67	75	83	91
1.90	51	09	22	34	46	57	66	75	84	93	2.01
1.95	52	13	26	39	52	65	74	84	93	2.02	11
2.00	54	17	32	46	59	71	82	92	2.03	12	21
2.05	56	22	37	51	65	78	90	2.02	12	22	31
2.10	58	27	43	58	73	86	99	12	22	32	42
2.15	60	32	48	64	80	94	2.08	22	33	44	54
2.20	62	37	54	71	87	2.02	17	32	44	56	67
2.25	63	42	60	73	95	11	28	43	56	68	80
2.30	65	48	66	84	2.02	20	37	54	68	81	94
2.35	67	53	73	92	11	30	48	66	80	94	3.08
2.40	69	59	80	2.00	20	39	58	77	93	3.08	23
2.45	71	65	87	08	29	49	69	88	3.05	22	38
2.50	73	71	94	16	39	60	80	3.00	18	36	53
2.55	75	76	2.01	26	49	70	91	12	31	50	68
2.60	77	83	09	35	60	82	3.04	26	46	66	85
2.65	79	89	17	14	71	95	18	41	62	83	4.03
2.70	81	96	25	54	82	3.07	32	57	79	4.00	21
2.75	83	2.03	33	63	93	20	46	72	95	18	41
2.80	85	09	41	73	3.04	32	60	88	4.13	38	63
2.85	88	17	50	83	15	45	75	4.04	31	58	85
2.90	90	24	58	92	26	58	90	21	50	79	5.08
2.95	92	32	68	3.03	38	72	4.05	38	70	5.01	32
3.00	94	39	76	13	50	86	21	56	90	24	57

	[illegible]	[illegible]	[illegible]	[illegible]	69	73	76	79	82	85	87
1.20	30	57	63	68	73	77	81	84	88	91	[illegible]
1.25	31	60	66	72	77	82	85	90	93	96	98
1.30	32	63	69	75	81	86	91	96	99	1.02	1.04
1.35	33	67	74	80	86	91	96	1.01	1.04	07	10
1.40	35	70	77	84	91	97	1.02	07	11	14	17
1.45	36	74	81	88	95	1.01	07	13	17	20	23
1.50	38	77	85	93	1.00	07	13	19	23	27	31
1.55	39	81	89	97	05	12	19	25	29	34	38
1.60	40	84	93	1.02	11	[illegible]	24	31	35	41	46
1.65	42	88	98	07	[illegible]	23	30	37	42	49	54
1.70	44	92	1.02	[illegible]	[illegible]	[illegible]	[illegible]	43	[illegible]	[illegible]	63
1.75	45	96	[illegible]	[illegible]	[illegible]	[illegible]	43	[illegible]	[illegible]	[illegible]	72
1.80	47	1.00	12	22	[illegible]	41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.85	49	04	[illegible]	28	39	[illegible]	[illegible]	[illegible]	[illegible]	64	[illegible]
1.90	51	09	[illegible]	34	[illegible]	[illegible]	[illegible]	[illegible]	84	[illegible]	[illegible]
1.95	52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54	[illegible]	[illegible]	[illegible]
2.00	54	17	[illegible]	40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.05	56	22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21
2.10	58	27	4[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	32	31
2.15	60	32	48	64	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	42
2.20	62	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54
2.25	63	42	60	72	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.30	65	48	[illegible]	84	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.35	67	53	[illegible]	92	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	94	94
2.40	69	[illegible]	[illegible]	2.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.0[illegible]
2.45	71	[illegible]	[illegible]	[illegible]	29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	23
2.50	73	[illegible]	64	[illegible]	37	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	55
2.55	[illegible]	[illegible]	[illegible]	[illegible]	49	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.60	77	83	[illegible]	[illegible]	[illegible]	82	[illegible]	[illegible]	40	[illegible]	[illegible]
2.65	79	89	[illegible]	44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.05
2.70	81	96	[illegible]	54	82	3.07	[illegible]	[illegible]	[illegible]	4.00	21
2.75	83	2.03	33	63	93	2[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	41
2.80	85	09	41	73	3.04	32	[illegible]	[illegible]	4.1[illegible]	38	63
2.85	88	17	[illegible]	83	15	45	[illegible]	4.[illegible]	[illegible]	58	85
2.90	90	24	[illegible]	92	26	58	[illegible]	[illegible]	[illegible]	7[illegible]	5.08
2.95	92	32	68	3.03	38	72	4.[illegible]	[illegible]	7[illegible]	5.01	32
3.00	94	39	76	13	50	86	21	[illegible]	9[illegible]	24	57

STAT

Table 11

$$\frac{l_k}{l_0}$$

B \ Δ	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30
0.70	0.46	0.46	0.46	0.47	0.48	0.49	0.49	0.49	0.49	0.51	0.51
0.75	52	53	53	54	55	55	56	56	56	57	57
0.80	58	58	59	60	60	61	61	62	62	63	63
0.85	61	62	63	64	64	65	65	66	67	67	67
0.90	63	64	66	66	68	68	69	69	70	70	71
0.95	67	68	70	71	72	73	74	74	75	75	76
1.00	72	74	75	77	77	79	79	80	81	81	82
1.05	77	78	80	82	83	84	85	85	87	87	88
1.10	82	84	86	87	89	90	92	92	93	94	95
1.15	87	89	91	93	94	96	97	98	99	1.00	1.01
1.20	93	95	97	99	1.01	1.02	1.04	1.05	1.06	07	09
1.25	98	1.01	1.03	1.05	07	09	11	12	14	15	16
1.30	1.04	07	10	12	14	16	18	20	21	22	23
1.35	10	13	16	19	21	24	26	27	29	30	31
1.40	17	21	24	27	29	31	33	34	37	38	40
1.45	23	27	31	34	37	39	42	44	45	47	49
1.50	31	34	39	42	49	47	50	52	53	55	58
1.55	38	43	47	50	54	57	59	61	63	65	67
1.60	46	51	55	59	63	66	69	71	73	75	77
1.65	54	59	64	68	72	76	78	81	83	86	88
1.70	63	68	73	78	82	86	89	92	95	97	99
1.75	72	78	83	88	92	96	2.00	2.03	2.06	2.09	2.11
1.80	81	87	93	98	2.02	2.07	11	14	17	20	23
1.85	91	98	2.04	2.09	14	18	22	26	30	33	36
1.90	2.01	2.08	15	20	26	30	34	39	42	46	50
1.95	11	18	29	31	37	42	47	52	56	60	64
2.00	21	29	36	43	49	55	60	65	70	75	79
2.05	31	39	47	55	62	68	73	79	84	89	94
2.10	42	51	60	67	75	81	88	93	99	3.04	3.10
2.15	54	64	73	81	90	97	3.03	3.10	3.16	22	27
2.20	67	78	88	97	3.05	3.13	20	27	34	40	46
2.25	80	92	3.02	3.12	22	30	38	45	52	58	65
2.30	94	3.06	18	29	39	48	56	63	71	78	83
2.35	3.08	21	34	45	55	65	75	84	92	99	4.06
2.40	23	36	49	61	73	84	95	4.04	4.14	4.22	30
2.45	38	52	66	80	92	4.04	4.16	27	37	41	54
2.50	53	68	83	97	4.11	24	37	49	60	70	80
2.55	68	84	4.00	4.16	30	44	59	72	85	96	5.06
2.60	85	4.01	18	34	51	66	83	98	5.12	5.23	34
2.65	4.03	22	40	57	73	89	5.06	5.22	37	49	61
2.70	21	42	61	80	97	5.13	30	46	62	75	89
2.75	41	63	84	5.03	5.21	38	57	74	91	6.07	6.21
2.80	63	86	5.08	28	47	66	85	6.05	6.23	40	56
2.85	85	5.10	34	55	75	95	6.15	36	56	75	92
		[illegible]	[illegible]	[illegible]	6.07	6.27	48	69	90	7.09	7.26

1.15	87	89	91	93	94	96	97	98	99	1.00	1.01
1.20	93	95	97	99	1.01	1.02	1.04	1.05	1.06	07	09
1.25	98	1.01	1.03	1.05	07	09	11	12	14	15	16
1.30	1.04	07	10	12	14	16	18	20	21	22	23
1.35	10	13	16	19	21	24	26	27	29	30	31
1.40	17	21	24	27	29	31	33	34	37	38	40
1.45	23	27	31	34	37	39	42	44	45	47	49
1.50	31	34	39	42	49	47	50	52	53	55	58
1.55	38	43	47	50	54	57	59	61	63	65	67
1.60	46	51	55	59	63	66	69	71	73	75	77
1.65	54	59	64	68	72	76	78	81	83	86	88
1.70	63	68	73	78	82	86	89	92	95	97	99
1.75	72	78	83	88	92	96	2.00	2.03	2.06	2.09	2.11
1.80	81	87	93	98	2.02	2.07	11	14	17	20	23
1.85	91	98	2.04	2.09	14	18	22	26	30	33	36
1.90	2.01	2.08	15	20	26	30	34	39	42	46	50
1.95	11	18	29	31	37	42	47	52	56	60	64
2.00	21	29	36	43	49	55	60	65	70	75	79
2.05	31	39	47	55	62	68	73	79	84	89	94
2.10	42	51	60	67	75	81	88	93	99	3.04	3.10
2.15	54	64	73	81	90	97	3.03	3.10	3.16	22	27
2.20	67	78	88	97	3.05	3.13	20	27	34	40	46
2.25	80	92	3.02	3.12	22	30	38	45	52	58	65
2.30	94	3.06	18	29	39	48	56	63	71	78	83
2.35	3.08	21	34	45	55	65	75	84	92	99	4.06
2.40	23	36	49	61	73	84	95	4.04	4.14	4.22	30
2.45	38	52	66	80	92	4.04	4.16	27	37	41	54
2.50	53	68	83	97	4.11	24	37	49	60	70	80
2.55	68	84	4.00	4.16	30	44	59	72	85	96	5.06
2.60	85	4.01	18	34	51	66	83	98	5.12	5.23	34
2.65	4.03	22	40	57	73	89	5.06	5.22	37	49	61
2.70	21	42	61	80	97	5.13	30	46	62	75	89
2.75	41	63	84	5.03	5.21	38	57	74	91	6.07	6.21
2.80	63	86	5.08	28	47	66	85	6.05	6.23	40	56
2.85	85	5.10	34	55	75	95	6.15	36	56	75	92
2.90	5.08	36	62	86	6.07	6.27	48	69	90	7.09	7.26
2.95	32	63	90	6.14	37	59	79	7.01	7.23	44	65
3.00	57	91	6.21	48	71	93	7.14	36	58	80	8.01

STAT

Table II

$$\frac{l_k}{l_0}$$

B \ Δ	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
0.70	0.51	0.51	0.51	0.51	0.52	0.52	0.52	0.52	0.52	0.51	0.51
0.75	57	57	58	58	58	58	57	57	57	57	57
0.80	63	64	63	63	63	63	63	63	63	63	63
0.85	67	68	68	69	69	69	68	69	69	69	69
0.90	71	72	72	72	73	73	73	73	73	73	73
0.95	76	76	77	77	77	77	77	78	78	78	78
1.00	82	82	82	83	83	84	84	85	85	85	85
1.05	88	88	89	89	89	90	90	91	92	92	92
1.10	95	95	96	96	97	98	98	98	99	99	1.00
1.15	1.01	1.02	1.03	1.03	1.04	1.04	1.05	1.06	1.06	1.07	07
1.20	09	09	10	11	11	12	12	13	13	14	14
1.25	16	17	18	18	19	20	20	21	21	22	22
1.30	23	25	26	26	27	27	28	29	29	29	29
1.35	31	33	34	35	35	37	37	37	40	38	38
1.40	40	41	43	43	45	46	46	47	47	47	47
1.45	49	50	52	53	54	55	56	56	57	57	57
1.50	58	60	61	63	64	65	66	67	67	70	67
1.55	67	69	71	73	75	76	77	78	78	78	78
1.60	77	80	82	83	85	86	89	88	89	89	89
1.65	88	90	92	94	96	97	99	2.00	2.01	2.01	2.02
1.70	99	2.02	2.04	2.05	2.07	2.08	2.10	11	12	13	14
1.75	2.11	13	16	18	20	21	23	24	26	27	27
1.80	23	26	28	31	33	35	36	38	40	41	42
1.85	36	39	42	45	47	49	51	53	54	56	57
1.90	50	53	56	59	62	64	66	68	70	71	72
1.95	64	68	71	75	78	80	83	85	87	88	89
2.00	79	83	87	91	95	97	3.00	3.02	3.04	3.05	3.07
2.05	94	99	3.03	3.08	3.11	3.15	17	19	21	22	24
2.10	3.10	3.14	20	24	29	32	35	37	39	40	42
2.15	27	32	38	43	48	53	55	58	60	62	64
2.20	46	52	58	63	68	72	76	79	81	84	86
2.25	65	71	77	82	87	92	96	99	4.03	4.06	4.09
2.30	83	90	96	4.02	4.07	4.12	4.16	4.21	25	28	32
2.35	4.06	4.13	4.19	25	31	36	42	46	51	55	59
2.40	30	36	43	49	56	61	67	73	78	83	87
2.45	54	63	70	5.77	83	89	95	5.01	5.06	5.11	5.15
2.50	80	88	96	03	5.10	5.17	5.23	29	34	39	43
2.55	5.06	5.15	5.24	32	40	47	54	60	66	72	76
2.60	34	43	52	61	69	77	85	92	99	6.05	6.10
2.65	61	70	79	89	98	6.06	6.15	6.24	6.31	38	44
2.70	89	98	6.09	6.19	6.28	37	46	55	63	70	77
2.75	6.21	6.33	45	56	67	77	87	97	7.05	7.13	7.20
2.80	56	70	84	97	7.09	7.19	7.29	7.38	47	55	62
2.85	92	7.07	7.21	7.34	47	59	69	79	88	97	8.05
2.90	7.26	40	55	69	82	95	8.08	8.19	8.30	8.40	49

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B											
0.70	0.51	0.51	0.51	0.51	0.52	0.52	0.52	[illegible]	[illegible]	[illegible]	[illegible]
0.75	57	57	58	58	58	58	57	57	57	[illegible]	[illegible]
0.80	63	64	63	63	63	63	63	63	63	63	63
0.85	67	68	68	69	69	69	68	69	69	69	69
0.90	71	72	72	72	73	73	73	73	73	73	73
0.95	76	76	77	77	77	77	77	78	78	78	78
1.00	82	82	82	83	83	84	84	85	85	85	85
1.05	88	88	89	89	89	90	90	91	92	92	92
1.10	95	95	96	96	97	98	98	98	99	99	1.00
1.15	1.01	1.02	1.03	1.03	1.04	1.04	1.05	1.06	1.06	1.07	07
1.20	09	09	10	11	11	12	12	13	13	14	14
1.25	16	17	18	18	19	20	20	21	21	22	22
1.30	23	25	26	26	27	27	28	29	29	29	29
1.35	31	33	34	35	35	37	37	37	40	38	38
1.40	40	41	43	43	45	46	46	47	47	47	47
1.45	49	50	52	53	54	55	56	56	57	57	57
1.50	58	60	61	63	64	65	66	67	67	70	67
1.55	67	69	71	73	75	76	77	78	78	78	78
1.60	77	80	82	83	85	86	89	88	89	89	89
1.65	88	90	92	94	96	97	99	2.00	2.01	2.01	2.02
1.70	99	2.02	2.04	2.05	2.07	2.08	2.10	11	12	13	14
1.75	2.11	13	16	18	20	21	23	24	26	27	27
1.80	23	26	28	31	33	35	36	38	40	41	42
1.85	36	39	42	45	47	49	51	53	54	56	57
1.90	50	53	56	59	62	64	66	68	70	71	72
1.95	64	68	71	75	78	80	83	85	87	88	89
2.00	79	83	87	91	95	97	3.00	3.02	3.04	3.05	3.07
2.05	94	99	3.03	3.08	3.11	3.15	17	19	21	22	24
2.10	3.10	3.14	20	24	29	32	35	37	39	40	42
2.15	27	32	38	43	48	53	55	58	60	62	64
2.20	46	52	58	63	68	72	76	79	81	84	86
2.25	65	71	77	82	87	92	96	99	4.03	4.06	4.09
2.30	83	90	96	4.02	4.07	4.12	4.16	4.21	25	28	32
2.35	4.06	4.13	4.19	25	31	36	42	46	51	55	59
2.40	30	36	43	49	56	61	67	73	78	83	87
2.45	54	63	70	5.77	83	89	95	5.01	5.06	5.11	5.15
2.50	80	88	96	03	5.10	5.17	5.23	29	34	39	43
2.55	5.06	5.15	5.24	32	40	47	54	60	66	72	76
2.60	34	43	52	61	69	77	85	92	99	6.05	6.10
2.65	61	70	79	89	98	6.06	6.15	6.24	6.31	38	44
2.70	89	98	6.09	6.19	6.28	37	46	55	63	70	77
2.75	6.21	6.33	45	56	67	77	87	97	7.05	7.13	7.20
2.80	56	70	84	97	7.09	7.19	7.29	7.38	47	55	62
2.85	92	7.07	7.21	7.34	47	59	69	79	88	97	8.05
2.90	7.26	40	55	69	82	95	8.08	8.19	8.30	8.40	49
2.95	65	83	99	8.15	8.30	8.44	57	69	80	91	9.00
3.00	8.01	8.19	8.38	56	74	90	9.06	9.20	9.33	9.45	55

STAT

Table 11

$$\frac{l_k}{l_0}$$

Б \ Δ	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
0.70	0.51	0.51	0.51	0.50	0.50	0.50	0.50	0.49	0.49	0.49	0.49
0.75	57	57	57	56	56	56	56	55	55	55	55
0.80	63	63	62	62	62	62	62	61	61	61	60
0.85	69	69	68	68	68	68	67	67	67	66	66
0.90	73	73	72	72	72	71	71	71	70	70	70
0.95	78	78	78	78	78	77	77	77	76	76	76
1.00	85	85	85	84	84	84	83	83	83	83	82
1.05	92	92	92	91	91	90	90	90	89	89	88
1.10	1.00	99	99	99	98	97	97	96	96	96	95
1.15	07	1.07	1.06	1.06	1.05	1.05	1.04	1.03	1.03	1.03	1.02
1.20	14	14	14	14	14	13	13	12	12	11	10
1.25	22	22	21	21	21	20	20	20	19	19	18
1.30	29	29	29	29	29	28	28	28	27	27	26
1.35	38	38	38	38	38	37	37	37	37	36	35
1.40	47	47	47	47	47	46	46	46	45	45	[illegible]
1.45	57	57	57	57	57	56	56	[illegible]	55	55	54
1.50	67	68	68	68	67	67	66	66	65	65	65
1.55	78	78	78	78	78	78	77	77	76	76	76
1.60	89	90	90	90	90	90	89	89	89	88	88
1.65	2.02	2.02	2.02	2.03	2.03	2.02	2.02	2.02	2.01	2.01	2.00
1.70	14	14	14	15	15	15	15	15	14	14	13
1.75	27	28	29	29	29	29	29	29	29	29	28
1.80	42	42	43	44	44	44	44	44	43	43	42
1.85	57	57	58	58	59	59	59	59	59	58	58
1.90	72	73	73	73	74	74	74	74	74	74	74
1.95	89	90	91	91	92	92	92	92	92	91	91
2.00	3.07	3.07	3.08	3.09	3.09	3.09	3.10	3.10	3.09	3.09	3.08
2.05	24	25	26	27	28	28	28	28	28	27	27
2.10	42	44	45	46	47	48	49	49	48	48	47
2.15	64	66	68	69	70	70	71	71	71	70	69
2.20	86	88	90	92	93	94	95	95	95	94	93
2.25	4.09	4.11	4.14	4.15	4.17	4.18	4.18	4.19	4.19	4.18	4.17
2.30	32	36	39	41	42	43	44	44	44	43	43
2.35	59	63	67	69	70	71	72	72	73	72	71
2.40	87	90	93	96	99	5.00	5.00	5.01	5.00	5.00	5.00
2.45	5.15	5.19	5.23	5.26	5.28	29	30	30	30	30	30
2.50	43	46	49	52	54	55	57	58	59	60	60
2.55	76	80	83	87	89	91	92	93	94	95	95
2.60	6.10	6.15	6.20	6.23	6.25	6.27	6.29	6.31	6.32	6.33	6.33
2.65	44	49	53	57	60	63	65	67	69	70	71
2.70	77	83	88	92	96	99	7.02	7.04	7.06	7.09	7.10
2.75	7.20	7.26	7.31	7.36	7.41	7.45	48	50	52	4	54
2.80	62	69	76	82	87	91	95	98	99	8.00	8.01
2.85	8.05	8.13	8.20	8.26	8.32	8.37	8.41	8.44	8.46	48	50
2.90	49	57	64	70	76	81	86	90	94	97	9.00

0.70	0.51	0.51	0.51	0.50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.75	[illegible]	57	57	56	56	56	56	55	[illegible]	[illegible]	[illegible]
0.80	63	63	62	62	62	62	62	61	61	61	60
0.85	69	69	68	68	68	68	67	[illegible]	67	66	66
0.90	73	73	72	72	72	71	71	71	70	70	70
0.95	78	78	78	78	78	77	77	77	76	76	76
1.00	85	85	85	84	84	84	83	83	83	83	82
1.05	92	92	92	91	91	90	90	90	89	89	88
1.10	1.00	99	99	99	98	97	97	96	96	96	95
1.15	07	1.07	1.06	1.06	1.05	1.05	1.04	1.03	1.03	1.03	1.02
1.20	14	14	14	14	14	13	13	12	12	11	10
1.25	22	22	21	21	21	20	20	20	19	19	18
1.30	29	29	29	29	29	28	28	28	27	27	26
1.35	38	38	38	38	38	37	37	37	37	36	35
1.40	47	47	47	47	47	46	46	46	45	45	[illegible]
1.45	57	57	57	57	57	[illegible]	[illegible]	[illegible]	[illegible]	55	54
1.50	67	68	68	68	67	67	[illegible]	66	[illegible]	[illegible]	65
1.55	78	78	78	78	78	78	77	77	[illegible]	[illegible]	76
1.60	89	90	90	90	90	90	[illegible]	89	[illegible]	[illegible]	[illegible]
1.65	2.02	2.02	2.02	2.03	2.03	2.02	2.02	2.02	2.01	2.01	2.09
1.70	14	14	14	15	15	15	[illegible]	15	14	14	13
1.75	27	28	29	29	29	29	29	29	29	29	28
1.80	42	42	43	44	44	44	44	44	43	43	42
1.85	57	57	58	58	59	59	59	59	[illegible]	58	58
1.90	72	73	73	73	74	[illegible]	74	74	74	74	74
1.95	89	90	91	91	92	92	92	92	92	91	91
2.00	3.07	3.07	3.08	3.09	3.09	3.09	[illegible]	3.10	3.09	3.09	[illegible]
2.05	24	25	26	27	[illegible]	28	28	28	28	27	27
2.10	42	44	[illegible]	46	47	48	49	49	47	48	47
2.15	64	[illegible]	68	[illegible]	70	[illegible]	[illegible]	[illegible]	[illegible]	70	69
2.20	86	88	90	92	93	94	[illegible]	[illegible]	[illegible]	94	93
2.25	4.09	4.11	4.14	[illegible]	4.17	[illegible]	4.18	4.19	4.19	4.18	4.17
2.30	32	[illegible]	39	[illegible]	42	43	44	44	44	43	43
2.35	59	63	67	[illegible]	70	[illegible]	72	72	73	72	[illegible]
2.40	87	90	93	96	99	[illegible]	[illegible]	[illegible]	[illegible]	5.00	5.00
2.45	5.15	5.19	5.23	[illegible]	5.28	29	30	30	[illegible]	30	30
2.50	43	46	49	52	54	[illegible]	57	58	[illegible]	60	64
2.55	76	80	83	87	89	[illegible]	92	93	94	95	95
2.60	6.10	6.15	6.20	6.23	6.25	6.27	6.29	6.31	6.32	6.33	6.33
2.65	44	49	53	57	60	63	65	67	[illegible]	70	71
2.70	77	83	88	92	96	99	7.02	7.04	7.06	7.09	[illegible]
2.75	7.20	7.26	7.31	7.36	7.41	7.45	48	50	52	4	54
2.80	62	69	76	82	87	91	95	98	99	8.00	8.01
2.85	8.05	8.13	8.20	8.26	8.32	8.37	8.41	8.44	8.46	48	50
2.90	49	57	64	70	76	81	86	90	94	97	9.00
2.95	9.00	9.08	9.14	9.20	9.26	9.31	9.36	9.40	9.44	9.47	50
3.00	55	63	70	78	85	92	98	10.04	10.09	10.15	10.19

STAT

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Table 11

$$\frac{l_k}{l_0}$$

B \ Δ	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60
0.70	0.49	0.48	0.48	0.48	0.48	0.47	0.47	0.46	0.46	0.46	0.45
0.75	55	54	54	54	53	53	53	52	52	51	51
0.80	60	60	60	59	59	58	58	58	58	57	57
0.85	66	66	65	65	65	64	64	63	63	62	62
0.90	70	69	69	68	68	67	67	66	66	66	65
0.95	76	74	74	73	72	72	71	71	70	70	70
1.00	82	81	80	80	79	78	77	77	77	76	75
1.05	88	87	87	86	85	85	84	84	83	82	82
1.10	95	94	94	93	92	92	91	90	90	89	89
1.15	1.02	1.01	1.01	1.00	99	98	98	97	97	96	96
1.20	10	10	09	08	1.07	1.06	1.06	0.05	1.04	1.03	1.03
1.25	18	17	16	15	14	13	13	12	12	11	10
1.30	26	25	24	23	22	20	20	20	20	19	18
1.35	35	34	33	32	31	30	29	29	28	28	27
1.40	45	43	43	42	41	40	39	39	38	37	36
1.45	54	54	53	52	51	50	49	48	47	46	45
1.50	65	64	63	62	61	61	60	59	58	57	56
1.55	76	75	74	74	73	72	71	70	69	68	66
1.60	88	87	86	86	84	84	82	81	80	79	77
1.65	2.00	2.00	99	98	97	96	94	93	92	91	89
1.70	13	12	2.12	2.11	2.10	2.09	2.08	2.07	2.05	2.04	2.03
1.75	28	27	26	25	23	22	21	20	18	17	16
1.80	42	42	41	39	38	37	35	34	33	31	30
1.85	58	57	56	55	54	53	51	50	48	46	45
1.90	74	73	72	72	70	69	68	66	64	63	60
1.95	91	90	89	88	87	86	84	82	81	79	77
2.00	3.08	3.08	3.07	3.07	3.06	3.04	3.03	3.10	3.00	98	96
2.05	27	26	26	25	24	23	22	21	19	3.17	3.15
2.10	47	47	47	46	45	43	42	41	39	37	35
2.15	69	68	67	66	65	64	63	62	60	59	57
2.20	93	92	91	90	88	87	86	85	84	82	80
2.25	4.17	4.16	4.15	4.13	4.12	4.11	4.10	4.09	4.08	4.06	4.04
2.30	43	42	41	40	39	37	36	35	33	31	29
2.35	71	71	70	68	67	65	63	61	59	57	55
2.40	5.00	99	99	98	97	95	93	91	88	86	84
2.45	30	5.30	5.29	5.29	5.28	5.27	5.25	5.23	5.20	5.18	5.15
2.50	60	61	61	61	61	60	59	57	55	53	50
2.55	95	96	96	96	96	96	95	93	91	89	87
2.60	6.33	6.34	6.34	6.34	6.33	6.32	6.31	6.29	6.28	6.26	6.23
2.65	71	72	73	73	72	72	71	69	67	65	62
2.70	7.10	7.11	7.12	7.12	7.12	7.11	7.10	7.09	7.07	7.05	7.03
2.75	54	55	55	55	55	54	54	52	51	49	48
2.80	8.01	8.02	8.02	8.03	8.03	8.02	8.02	8.01	8.00	98	97
2.85	50	51	53	54	54	54	54	53	53	8.51	8.49
2190	9.00	9.04	9.06	9.08	9.08	9.09	9.09	9.08	9.07	9.06	9.05

0.95	76	74	74	73	72	72	71	71	[illegible]	[illegible]	[illegible]
1.00	82	81	80	80	79	78	77	77	77	76	75
1.05	88	87	87	86	[illegible]	85	84	84	[illegible]	[illegible]	82
1.10	95	94	94	93	92	92	91	90	[illegible]	89	89
1.15	1.02	1.01	1.01	1.00	99	98	98	97	97	[illegible]	96
1.20	10	10	09	08	1.07	1.06	1.06	[illegible]	1.04	[illegible]	1.03
1.25	18	17	16	15	14	13	13	[illegible]	12	11	10
1.30	26	25	24	23	22	20	20	[illegible]	20	19	18
1.35	35	34	33	32	[illegible]	30	29	29	28	28	27
1.40	45	43	43	42	[illegible]	40	39	39	38	37	36
1.45	54	54	53	52	[illegible]	50	49	[illegible]	47	46	45
1.50	65	64	63	62	[illegible]	61	60	[illegible]	58	57	56
1.55	76	75	74	74	[illegible]	72	71	[illegible]	69	68	[illegible]
1.60	88	87	86	86	84	84	[illegible]	[illegible]	[illegible]	79	77
1.65	2.00	2.00	99	98	97	[illegible]	94	[illegible]	92	[illegible]	89
1.70	13	12	2.12	2.11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.04	2.03
1.75	28	27	26	25	23	22	21	[illegible]	[illegible]	[illegible]	[illegible]
1.80	42	42	[illegible]	39	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	30
1.85	58	57	[illegible]	[illegible]	54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	45
1.90	74	73	72	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.95	91	90	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	77
2.00	3.08	3.08	3.07	3.07	3.06	3.04	[illegible]	[illegible]	[illegible]	[illegible]	96
2.05	27	26	26	[illegible]	24	23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.10	47	47	[illegible]	[illegible]	[illegible]	[illegible]	42	[illegible]	[illegible]	[illegible]	[illegible]
2.15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.20	93	92	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	80
2.25	4.17	4.16	[illegible]	[illegible]	[illegible]	4.11	[illegible]	[illegible]	[illegible]	[illegible]	4.04
2.30	43	42	41	40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	29
2.35	71	71	70	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.40	[illegible]	99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.45	30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.50	60	61	61	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	50
2.55	95	96	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	93	[illegible]	[illegible]	87
2.60	[illegible]	[illegible]	[illegible]	6.34	[illegible]	6.32	[illegible]	6.29	[illegible]	[illegible]	[illegible]
2.65	71	72	[illegible]	73	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	62
2.70	[illegible]	[illegible]	[illegible]	7.12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7.03
2.75	54	55	55	55	[illegible]	54	54	[illegible]	[illegible]	49	45
2.80	8.01	8.02	8.02	8.03	[illegible]	8.02	8.02	[illegible]	[illegible]	95	97
2.85	50	[illegible]	53	54	54	54	54	[illegible]	53	[illegible]	8.49
2190	9.00	9.04	9.06	9.08	[illegible]	9.09	9.09	[illegible]	9.07	[illegible]	9.05
2.95	50	53	56	58	[illegible]	61	62	62	62	62	[illegible]
3.00	10.19	10.21	10.24	10.25	10.26	10.26	10.27	10.26	10.26	10.25	10.24

STAT

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Table II

$$\frac{l_k}{l_0}$$

B \ Δ	0.60	0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.70
0.70	0.45	0.45	0.44	0.44	0.44	0.44	0.43	0.42	0.42	0.41	0.41
0.75	51	50	50	49	49	48	48	47	47	46	45
0.80	57	56	56	55	54	54	53	53	52	51	50
0.85	62	61	61	60	60	59	58	58	57	56	55
0.90	65	64	64	63	62	62	61	61	60	59	59
0.95	70	69	69	68	67	67	66	65	64	63	63
1.00	76	75	74	74	73	72	71	70	69	68	68
1.05	82	81	80	79	78	78	77	76	75	74	74
1.10	89	88	87	86	85	84	83	82	81	80	80
1.15	95	94	93	92	91	90	89	88	87	86	86
1.20	1.03	1.02	1.00	99	98	97	97	96	95	94	93
1.25	10	10	09	1.08	1.07	1.05	1.04	1.03	1.02	1.01	99
1.30	18	18	17	16	15	13	12	11	09	08	1.07
1.35	27	26	25	24	22	21	20	19	17	16	15
1.40	36	35	34	33	31	30	29	28	26	25	23
1.45	45	45	43	42	41	40	38	36	35	33	32
1.50	56	55	53	52	51	49	48	46	45	43	42
1.55	66	65	64	62	61	60	58	57	55	53	51
1.60	77	76	75	74	72	71	69	67	66	64	62
1.65	89	88	87	86	84	83	81	79	77	75	73
1.70	2.03	2.02	2.00	99	97	95	94	92	89	87	85
1.75	16	15	14	2.13	2.11	2.09	2.07	2.05	2.03	2.01	98
1.80	30	28	27	26	24	23	21	19	17	14	2.12
1.85	45	44	43	41	40	38	36	34	31	29	26
1.90	60	59	58	56	54	53	51	48	46	44	42
1.95	77	77	75	74	72	70	68	66	63	61	58
2.00	96	95	93	92	90	88	86	84	81	78	75
2.05	3.15	3.14	3.12	3.11	3.09	3.06	3.04	3.02	99	96	94
2.10	35	34	32	30	28	25	23	21	3.18	3.15	3.12
2.15	57	55	53	51	49	46	44	41	39	36	32
2.20	80	78	76	74	71	69	66	63	60	57	54
2.25	4.04	4.02	4.00	97	95	92	89	86	83	80	77
2.30	29	27	25	4.23	4.20	4.17	4.13	4.10	4.06	4.03	4.00
2.35	55	52	49	47	44	41	39	36	33	30	27
2.40	84	81	78	75	72	69	67	64	61	58	55
2.45	5.15	5.12	5.09	5.07	5.04	5.01	98	95	92	89	86
2.50	50	47	44	41	38	35	5.32	5.28	5.25	5.22	5.18
2.55	87	83	80	77	74	71	67	64	60	57	53
2.60	6.23	6.21	6.18	6.15	6.12	6.08	6.05	6.02	98	94	90
2.65	62	60	57	54	50	47	44	40	6.36	6.32	6.28
2.70	7.03	7.01	99	97	94	91	87	83	79	74	68
2.75	48	46	7.44	7.41	7.39	7.36	7.32	7.27	7.23	7.18	7.12
2.80	97	95	93	91	88	85	81	76	72	66	61
2.85	8.49	8.46	8.44	8.42	8.38	8.35	8.31	8.27	8.22	8.17	8.11
[illegible]				[illegible]	[illegible]	[illegible]	86	81	76	71	66

							61	61	60	59	59
0.95	70	69	69	68	67	67	66	65	64	63	63
1.00	76	75	74	74	73	72	71	70	69	68	68
1.05	82	81	80	79	78	78	77	76	75	74	74
1.10	89	88	87	86	85	84	83	82	81	80	80
1.15	95	94	93	92	91	90	89	88	87	86	86
1.20	1.03	1.02	1.00	99	98	97	97	96	95	94	93
1.25	10	10	09	1.08	1.07	1.05	1.04	1.03	1.02	1.01	99
1.30	18	18	17	16	15	13	12	11	09	08	1.07
1.35	27	26	25	24	22	21	20	19	17	16	15
1.40	36	35	34	33	31	30	29	28	26	25	23
1.45	45	45	43	42	41	40	38	36	35	33	32
1.50	56	55	53	52	51	49	48	46	45	43	42
1.55	66	65	64	62	61	60	58	57	55	53	51
1.60	77	76	75	74	72	71	69	67	66	64	62
1.65	89	88	87	86	84	83	81	79	77	75	73
1.70	2.03	2.02	2.00	99	97	95	94	92	89	87	85
1.75	16	15	14	2.13	2.11	2.09	2.07	2.05	2.03	2.01	98
1.80	30	28	27	26	24	23	21	19	17	14	2.12
1.85	45	44	43	41	40	38	36	34	31	29	26
1.90	60	59	58	56	54	53	51	48	46	44	42
1.95	77	77	75	74	72	70	68	66	63	61	58
2.00	96	95	93	92	90	88	86	84	81	78	75
2.05	3.15	3.14	3.12	3.11	3.09	3.06	3.04	3.02	99	96	94
2.10	35	34	32	30	28	25	23	21	3.18	3.15	3.12
2.15	57	55	53	51	49	46	44	41	39	36	32
2.20	80	78	76	74	71	69	66	63	60	57	54
2.25	4.04	4.02	4.00	97	95	92	89	86	83	80	77
2.30	29	27	25	4.23	4.20	4.17	4.13	4.10	4.06	4.03	4.00
2.35	55	52	49	47	44	41	39	36	33	30	27
2.40	84	81	78	75	72	69	67	64	61	58	55
2.45	5.15	5.12	5.09	5.07	5.04	5.01	98	95	92	89	86
2.50	50	47	44	41	38	35	5.32	5.28	5.25	5.22	5.18
2.55	87	83	80	77	74	71	67	64	60	57	53
2.60	6.23	6.21	6.18	6.15	6.12	6.08	6.05	6.02	98	94	90
2.65	62	60	57	54	50	47	44	40	6.36	6.32	6.28
2.70	7.03	7.01	99	97	94	91	87	83	79	74	68
2.75	48	46	7.44	7.41	7.39	7.36	7.32	7.27	7.23	7.18	7.12
2.80	97	95	93	91	88	85	81	76	72	66	61
2.85	8.49	8.46	8.44	8.42	8.38	8.35	8.31	8.27	8.22	8.17	8.11
2.90	9.05	9.02	9.00	97	93	89	86	81	76	71	66
2.95	61	61	59	9.56	9.53	9.49	9.44	9.39	9.34	9.28	9.23
3.00	10.24	10.23	10.21	10.18	10.15	10.12	10.08	10.04	9.99	94	89

STAT

Table II

$$\frac{l_k}{l_0}$$

B \ Δ	0.70	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80
0.70	0.41	0.40	0.40	0.39	0.38	0.38	0.37	0.37	0.36	0.35	0.35
0.75	45	45	44	44	43	42	42	41	41	40	39
0.80	50	50	49	48	48	47	46	45	44	44	43
0.85	52	55	54	53	52	51	50	49	48	48	47
0.90	59	58	57	57	56	55	54	53	52	51	50
0.95	63	62	62	61	60	59	58	57	56	55	54
1.00	68	67	66	65	65	64	63	62	61	60	59
1.05	74	73	72	71	70	69	68	67	66	65	64
1.10	80	78	77	76	75	74	73	72	71	70	69
1.15	86	84	83	82	81	80	79	78	77	76	75
1.20	93	92	91	90	88	87	86	84	83	81	80
1.25	99	98	97	96	94	93	92	91	89	88	87
1.30	1.07	1.06	1.04	1.03	1.02	1.00	99	97	96	95	94
1.35	15	14	12	11	09	08	1.06	1.05	1.03	1.02	1.00
1.40	23	22	20	19	17	16	14	13	11	09	08
1.45	32	30	29	28	26	24	23	22	20	18	16
1.50	42	40	39	37	35	33	31	30	28	26	24
1.55	51	50	48	47	45	43	41	39	37	35	33
1.60	62	61	59	58	56	54	52	49	47	45	43
1.65	73	72	70	68	66	64	62	60	57	55	53
1.70	85	83	82	80	78	75	73	71	68	66	63
1.75	98	97	95	93	91	88	86	83	81	78	75
1.80	2.12	2.10	2.08	2.06	2.03	2.01	98	96	93	90	88
1.85	26	24	22	20	17	14	2.12	2.09	2.06	2.04	2.01
1.90	42	40	37	34	32	29	26	24	21	18	15
1.95	58	55	51	48	46	43	40	37	35	32	30
2.00	75	72	68	65	61	58	55	53	50	47	45
2.05	94	90	86	83	79	76	73	70	67	64	61
2.10	3.12	3.09	3.06	3.02	99	95	92	88	85	82	78
2.15	32	29	26	23	3.19	3.16	3.12	3.09	3.05	3.01	97
2.20	54	51	47	44	41	38	34	30	26	22	3.19
2.25	77	74	71	67	64	60	57	53	49	46	42
2.30	4.00	97	93	90	87	83	80	77	73	70	66
2.35	27	4.24	4.20	4.17	4.14	4.10	4.07	4.03	99	96	93
2.40	55	52	49	45	42	39	34	30	4.26	4.22	4.18
2.45	86	83	79	76	72	67	63	59	54	49	43
2.50	5.18	5.15	5.11	5.07	5.02	97	92	87	82	76	70
2.55	53	48	45	40	35	5.30	5.25	5.19	5.12	5.05	98
2.60	90	86	82	78	73	69	63	57	50	43	5.35
2.65	6.28	6.24	6.20	6.15	6.10	6.05	99	92	84	76	67
2.70	68	64	59	59	49	44	6.38	6.31	6.23	6.15	6.06
2.75	7.12	7.07	7.02	98	93	88	81	73	65	56	46
2.80	61	56	51	7.45	7.40	7.34	7.27	7.20	7.11	7.02	93
2.85	8.11	8.06	8.01	95	89	83	77	70	62	54	7.45
2.90	66	60	54	8.48	8.42	8.35	8.28	8.21	8.14	8.06	8.00

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	65	64	63	62	61	60	59
1.05	74	73	72	71	70	69	68	67	66	65	64
1.10	80	78	77	76	75	74	73	72	71	70	69
1.15	86	84	83	82	81	80	79	78	77	76	75
1.20	93	92	91	90	88	87	86	84	83	81	80
1.25	99	98	97	96	94	93	92	91	89	88	87
1.30	1.07	1.06	1.04	1.03	1.02	1.00	99	97	96	95	94
1.35	15	14	12	11	09	08	1.06	1.05	1.03	1.02	1.00
1.40	23	22	20	19	17	16	14	13	11	09	08
1.45	32	30	29	28	26	24	23	22	20	18	16
1.50	42	40	39	37	35	33	31	30	28	26	24
1.55	51	50	48	47	45	43	41	39	37	35	33
1.60	62	61	59	58	56	54	52	49	47	45	43
1.65	73	72	70	68	66	64	62	60	57	55	53
1.70	85	83	82	80	78	75	73	71	68	66	63
1.75	98	97	95	93	91	88	86	83	81	78	75
1.80	2.12	2.10	2.08	2.06	2.03	2.01	98	96	93	90	88
1.85	26	24	22	20	17	14	2.12	2.09	2.06	2.04	2.01
1.90	42	40	37	34	32	29	26	24	21	18	15
1.95	58	55	51	48	46	43	40	37	35	32	30
2.00	75	72	68	65	61	58	55	53	50	47	45
2.05	94	90	86	83	79	76	73	70	67	64	61
2.10	3.12	3.09	3.06	3.02	99	95	92	88	85	82	78
2.15	32	29	26	23	3.19	3.16	3.12	3.09	3.05	3.01	97
2.20	54	51	47	44	41	38	34	30	26	22	3.19
2.25	77	74	71	67	64	60	57	53	49	46	42
2.30	4.00	97	93	90	87	83	80	77	73	70	66
2.35	27	4.24	4.20	4.17	4.14	4.10	4.07	4.03	99	96	93
2.40	55	52	49	45	42	39	34	30	4.26	4.22	4.18
2.45	86	83	79	76	72	67	63	59	54	49	43
2.50	5.18	5.15	5.11	5.07	5.02	97	92	87	82	76	70
2.55	53	48	45	40	35	5.30	5.25	5.19	5.12	5.05	98
2.60	90	86	82	78	73	69	63	57	50	43	5.35
2.65	6.28	6.24	6.20	6.15	6.10	6.05	99	92	84	76	67
2.70	68	64	59	59	49	44	6.38	6.31	6.23	6.15	6.06
2.75	7.12	7.07	7.02	98	93	88	81	73	65	56	46
2.80	61	56	51	7.45	7.40	7.34	7.27	7.20	7.11	7.02	93
2.85	8.11	8.06	8.01	95	89	83	77	70	62	54	7.45
2.90	66	60	54	8.48	8.42	8.35	8.28	8.21	8.14	8.06	8.00
2.95	9.23	9.16	9.10	9.04	98	91	86	79	73	67	62
3.00	89	84	78	72	9.65	9.58	51	9.44	9.37	9.30	9.22

STAT

Table III - Condition of Maximum Pressure

$$\frac{l_m}{l_0}$$

B \ Δ	0.07	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.70	0.151	0.298	0.321	0.343	0.363	0.381	0.401	0.422	0.437	0.441	0.446
0.75	158	329	354	378	401	424	447	469	485	501	516
0.80	175	360	386	411	436	461	485	509	530	551	571
0.85	191	390	417	443	468	493	517	541	564	586	606
0.90	208	418	446	472	497	521	544	567	586	604	621
0.95	223	443	474	500	523	544	563	581	598	614	628
1.00	238	462	493	517	537	555	572	588	603	618	632
1.05	252	473	505	527	545	561	571	592	606	620	633
1.10	264	478	514	539	548	564	579	593	607	620	633
1.15	280	478	514	539	548	564	579	593	607	620	632
1.20	293	476	512	537	546	562	577	591	604	617	629
1.25	305	474	503	525	543	559	574	588	601	613	625
1.30	317	470	497	519	538	554	569	583	596	608	619
1.35	324	463	487	509	529	547	563	577	590	602	612
1.40	328	456	479	500	520	538	554	569	582	593	603
1.45	329	449	470	491	511	529	545	559	572	583	594
1.50	329	441	462	482	501	518	534	548	561	574	586
1.55	327	433	454	474	492	508	524	539	553	566	578
1.60	322	424	445	465	483	500	516	531	545	558	570
1.65	315	416	437	457	475	492	508	523	537	550	562
1.70	308	408	430	450	468	485	501	516	530	543	555
1.75	302	400	424	444	462	478	493	508	522	535	548
1.80	296	393	415	435	453	470	486	501	515	528	541
1.85	290	386	408	428	446	463	479	494	508	521	534
1.90	284	380	402	422	440	457	473	488	502	515	528
1.95	278	375	397	417	434	450	466	481	495	509	522
2.00	273	370	392	411	428	444	460	475	489	503	516
2.05	268	365	386	405	422	438	454	469	484	498	511
2.10	263	360	380	399	416	432	448	463	478	492	505
2.15	258	355	374	392	410	427	443	458	473	487	500
2.20	254	350	369	387	405	422	438	453	468	482	496
2.25	249	346	365	383	400	417	433	448	463	477	490
2.30	245	341	360	378	395	412	428	443	458	472	486
2.35	241	336	355	373	390	407	423	438	453	467	481
2.40	238	332	350	368	385	402	418	434	449	463	477
2.45	234	328	346	364	381	398	414	430	445	459	473
2.50	231	324	342	360	377	394	410	426	441	455	469
2.55	228	321	339	357	374	391	407	422	437	451	465
2.60	224	318	335	352	369	385	401	417	432	447	461
2.65	221	314	331	348	365	382	398	414	429	443	457
2.70	219	311	328	345	362	378	394	410	425	439	453
2.75	216	308	325	342	358	374	390	406	421	435	449
2.80	213	304	321	338	355	371	387	403	418	432	445
2.85	210	301	318	335	352	368	384	399	414	428	441
2.90	208	298	315	332	349	365	380	395	410	424	437
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	345	361	376	391	406	420	433

0.80	175	360	386	411	436	461	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.85	191	390	417	443	468	493	517	541	564	586	606
0.90	208	418	446	472	497	521	544	567	586	604	621
0.95	223	443	474	500	523	544	563	581	598	614	628
1.00	238	462	493	517	537	555	572	588	603	618	632
1.05	252	473	505	527	545	561	571	592	606	620	633
1.10	264	478	514	539	548	564	579	593	607	620	633
1.15	280	478	514	539	548	564	579	593	607	620	632
1.20	293	476	512	537	546	562	577	591	604	617	629
1.25	305	474	503	525	543	559	574	588	601	613	625
1.30	317	470	497	519	538	554	569	583	596	608	619
1.35	324	463	487	509	529	547	563	577	590	602	612
1.40	328	456	479	500	520	538	554	569	582	593	603
1.45	329	449	470	491	511	529	545	559	572	583	594
1.50	329	441	462	482	501	518	534	548	561	574	586
1.55	327	433	454	474	492	508	524	539	553	566	578
1.60	322	424	445	465	483	500	516	531	545	558	570
1.65	315	416	437	457	475	492	508	523	537	550	562
1.70	308	408	430	450	468	485	501	516	530	543	555
1.75	302	400	424	444	462	478	493	508	522	535	548
1.80	296	393	415	435	453	470	486	501	515	528	541
1.85	290	386	408	428	446	463	479	494	508	521	534
1.90	284	380	402	422	440	457	473	488	502	515	528
1.95	278	375	397	417	434	450	466	481	495	509	522
2.00	273	370	392	411	428	444	460	475	489	503	516
2.05	268	365	386	405	422	438	454	469	484	498	511
2.10	263	360	380	399	416	432	448	463	478	492	505
2.15	258	355	374	392	410	427	443	458	473	487	500
2.20	254	350	369	387	405	422	438	453	468	482	496
2.25	249	346	365	383	400	417	433	448	463	477	490
2.30	245	341	360	378	395	412	428	443	458	472	486
2.35	241	336	355	373	390	407	423	438	453	467	481
2.40	238	332	350	368	385	402	418	434	449	463	477
2.45	234	328	346	364	381	398	414	430	445	459	473
2.50	231	324	342	360	377	394	410	426	441	455	469
2.55	228	321	339	357	374	391	407	422	437	451	465
2.60	224	318	335	352	369	385	401	417	432	447	461
2.65	221	314	331	348	365	382	398	414	429	443	457
2.70	219	311	328	345	362	378	394	410	425	439	453
2.75	216	308	325	342	358	374	390	406	421	435	449
2.80	213	304	321	338	355	371	387	403	418	432	445
2.85	210	301	318	335	352	368	384	399	414	428	441
2.90	208	298	315	332	349	365	380	395	410	424	437
2.95	205	294	311	328	345	361	376	391	406	420	433
3.00	0.203	0.291	0.308	0.325	0.342	0.358	0.373	0.388	0.402	0.416	0.429

STAT

Table III (cont'd.)

$\frac{l_m}{l_0}$

B \ Δ	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30
0.70	0.446	0.452	0.457	0.464	0.471	0.480	0.481	0.490	0.490	0.503	0.505
0.75	516	523	530	537	543	548	553	557	561	565	568
0.80	571	579	586	592	598	603	608	613	617	621	625
0.85	606	615	623	631	638	644	650	656	661	665	670
0.90	621	630	640	650	659	668	675	684	692	699	705
0.95	628	637	647	657	667	676	683	692	699	706	712
1.00	632	641	651	661	671	680	687	696	703	710	716
1.05	633	642	652	662	672	681	688	697	704	711	717
1.10	633	642	652	662	672	681	688	697	703	710	716
1.15	632	641	651	660	670	679	686	692	700	707	713
1.20	629	638	648	657	667	676	683	689	696	702	708
1.25	626	634	643	652	662	671	678	684	690	696	702
1.30	619	628	637	646	656	665	672	678	684	690	696
1.35	612	621	630	638	647	656	663	669	675	681	687
1.40	603	612	621	629	638	647	654	660	666	672	678
1.45	594	603	612	620	629	638	645	651	657	663	669
1.50	586	595	604	612	621	630	637	643	649	655	661
1.55	578	587	596	604	612	621	628	633	638	649	650
1.60	570	579	588	596	604	613	620	625	630	636	642
1.65	562	571	580	588	596	605	612	617	622	628	634
1.70	555	564	573	581	589	599	605	609	615	621	627
1.75	548	557	566	574	582	592	598	602	608	614	620
1.80	541	550	559	567	575	585	591	595	601	607	613
1.85	534	543	552	560	568	578	584	588	594	600	606
1.90	528	537	546	554	562	572	578	582	588	594	600
1.95	522	531	540	548	556	577	572	576	582	588	594
2.00	516	525	534	542	550	560	566	570	576	582	588
2.05	511	520	529	537	545	555	561	565	571	577	583
2.10	505	515	524	532	540	550	556	560	566	572	578
2.15	500	510	519	527	535	545	551	555	561	567	573
2.20	496	505	514	522	530	540	546	550	556	562	568
2.25	490	500	509	517	525	535	541	545	551	557	564
2.30	486	495	504	512	521	531	537	541	547	553	560
2.35	481	490	499	507	517	527	533	537	543	549	556
2.40	477	486	495	503	513	523	529	533	539	545	552
2.45	473	482	491	494	509	519	525	529	535	541	548
2.50	469	478	487	493	505	515	521	525	531	538	545
2.55	465	474	483	491	501	511	517	521	527	534	542
2.60	461	470	479	487	497	507	513	517	523	531	539
2.65	457	466	475	483	493	503	509	513	520	528	536
2.70	453	462	471	479	489	499	505	511	517	525	533
2.75	449	458	467	475	485	495	501	507	513	521	530
[illegible]	445	454	463	471	481	491	497	503	510	518	527

0.80	571	579	586	592	598	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.85	606	615	623	631	638	644	650	656	661	665	670
0.90	621	630	640	650	659	668	675	684	692	699	705
0.95	628	637	647	657	667	676	683	692	699	706	712
1.00	632	641	651	661	671	680	687	696	703	710	716
1.05	633	642	652	662	672	681	688	697	704	711	717
1.10	633	642	652	662	672	681	688	697	703	710	716
1.15	632	641	651	660	670	679	686	692	700	707	713
1.20	629	638	648	657	667	676	683	689	696	702	708
1.25	626	634	643	652	662	671	678	684	690	696	702
1.30	619	628	637	646	656	665	672	678	684	690	696
1.35	612	621	630	638	647	656	663	669	675	681	687
1.40	603	612	621	629	638	647	654	660	666	672	678
1.45	594	603	612	620	629	638	645	651	657	663	669
1.50	586	595	604	612	621	630	637	643	649	655	661
1.55	578	587	596	604	612	621	628	633	638	649	650
1.60	570	579	588	596	604	613	620	625	630	636	642
1.65	562	571	580	588	596	605	612	617	622	628	634
1.70	555	564	573	581	589	599	605	609	615	621	627
1.75	548	557	566	574	582	592	598	602	608	614	620
1.80	541	550	559	567	575	585	591	595	601	607	613
1.85	534	543	552	560	568	578	584	588	594	600	606
1.90	528	537	546	554	562	572	578	582	588	594	600
1.95	522	531	540	548	556	577	572	576	582	588	594
2.00	516	525	534	542	550	560	566	570	576	582	588
2.05	511	520	529	537	545	555	561	565	571	577	583
2.10	505	515	524	532	540	550	556	560	566	572	578
2.15	500	510	519	527	535	545	551	555	561	567	573
2.20	496	505	514	522	530	540	546	550	556	562	568
2.25	490	500	509	517	525	535	541	545	551	557	564
2.30	486	495	504	512	521	531	537	541	547	553	560
2.35	481	490	499	507	517	527	533	537	543	549	556
2.40	477	486	495	503	513	523	529	533	539	545	552
2.45	473	482	491	494	509	519	525	529	535	541	548
2.50	469	478	487	493	505	515	521	525	531	538	545
2.55	465	474	483	491	501	511	517	521	527	534	542
2.60	461	470	479	487	497	507	513	517	523	531	539
2.65	457	466	475	483	493	503	509	513	520	528	536
2.70	453	462	471	479	489	499	505	511	517	525	533
2.75	449	458	467	475	485	495	501	507	513	521	530
2.80	445	454	463	471	481	491	497	503	510	518	527
2.85	441	450	459	467	477	487	493	500	507	515	524
2.90	437	446	455	463	473	484	490	497	504	512	521
2.95	433	442	451	459	470	481	487	494	501	509	518
3.00	0.429	0.438	0.447	0.456	0.467	0.478	0.484	0.491	0.498	0.506	0.515

STAT

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Table III

$$\frac{l_m}{l_0}$$

B \ Δ	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
0.70	0.497	0.502	0.505	0.510	0.513	0.514	0.514	0.513	0.511	0.509	0.507
0.75	568	570	571	571	571	571	570	568	566	564	565
0.80	625	638	629	630	629	629	628	626	625	623	623
0.85	670	675	679	683	686	690	690	688	687	685	683
0.90	705	711	715	720	723	727	730	726	726	724	723
0.95	712	718	723	728	732	736	739	739	738	737	735
1.00	716	721	725	731	735	739	742	742	742	741	740
1.05	717	722	726	731	735	739	742	744	745	744	743
1.10	716	721	725	730	734	738	741	743	744	745	745
1.15	713	718	722	727	731	735	738	740	741	743	744
1.20	708	713	718	723	727	731	734	736	737	738	739
1.25	702	707	712	717	721	725	728	730	731	732	733
1.30	696	701	706	711	715	719	722	724	725	726	727
1.35	687	692	697	702	706	710	713	715	716	717	718
1.40	678	683	688	693	697	701	704	706	707	708	709
1.45	669	674	679	684	688	692	695	697	698	699	700
1.50	661	666	671	676	680	684	687	689	691	692	693
1.55	650	655	661	666	670	675	678	680	682	683	685
1.60	642	647	653	658	662	667	670	672	674	675	677
1.65	634	639	645	650	654	659	662	664	666	667	670
1.70	627	632	638	643	647	652	655	657	659	660	663
1.75	620	625	631	636	640	645	648	650	652	653	656
1.80	613	619	625	630	634	639	642	644	646	648	650
1.85	606	613	619	624	628	633	636	638	640	642	644
1.90	600	607	613	618	622	627	630	632	634	636	638
1.95	594	601	607	612	616	621	624	626	628	630	633
2.00	588	595	601	606	610	615	618	620	622	624	627
2.05	583	590	596	601	605	610	613	615	617	619	622
2.10	578	585	591	596	600	605	608	610	612	614	617
2.15	573	580	586	591	595	600	603	605	607	609	613
2.20	568	575	581	586	590	595	598	600	602	604	609
2.25	564	571	577	582	586	591	594	596	598	600	605
2.30	560	567	573	578	582	587	590	592	594	596	601
2.35	556	563	569	574	578	583	586	588	590	592	597
2.40	552	559	565	570	574	579	582	584	586	588	593
2.45	548	555	561	566	570	575	578	580	582	584	589
2.50	545	552	558	563	567	572	575	577	579	581	586
2.55	542	549	555	560	564	569	572	574	576	578	583
2.60	539	546	552	557	561	566	569	571	573	575	580
2.65	536	543	549	554	558	563	566	568	570	572	577
2.70	533	540	546	551	555	560	563	565	567	570	574
2.75	530	537	543	548	552	557	560	562	565	568	571
2.80	527	534	540	545	549	554	557	560	563	566	569
2.85	524	531	537	542	546	551	555	558	561	564	567
	[illegible]	[illegible]	[illegible]	539	543	549	553	556	559	562	565

0.95	712	718	723	728	732	736	739	739	738	737	735
1.00	716	721	725	731	735	739	742	742	742	741	740
1.05	717	722	726	731	735	739	742	744	745	744	743
1.10	716	721	725	730	734	738	741	743	744	745	745
1.15	713	718	722	727	731	735	738	740	741	743	744
1.20	708	713	718	723	727	731	734	736	737	738	739
1.25	702	707	712	717	721	725	728	730	731	732	733
1.30	696	701	706	711	715	719	722	724	725	726	727
1.35	687	692	697	702	706	710	713	715	716	717	718
1.40	678	683	688	693	697	701	704	706	707	708	709
1.45	669	674	679	684	688	692	695	697	698	699	700
1.50	661	666	671	676	680	684	687	689	691	692	693
1.55	650	655	661	666	670	675	678	680	682	683	685
1.60	642	647	653	658	662	667	670	672	674	675	677
1.65	634	639	645	650	654	659	662	664	666	667	670
1.70	627	632	638	643	647	652	655	657	659	660	663
1.75	620	625	631	636	640	645	648	650	652	653	656
1.80	613	619	625	630	634	639	642	644	646	648	650
1.85	606	613	619	624	628	633	636	638	640	642	644
1.90	600	607	613	618	622	627	630	632	634	636	638
1.95	594	601	607	612	616	621	624	626	628	630	633
2.00	588	595	601	606	610	615	618	620	622	624	627
2.05	583	590	596	601	605	610	613	615	617	619	622
2.10	578	585	591	596	600	605	608	610	612	614	617
2.15	573	580	586	591	595	600	603	605	607	609	613
2.20	568	575	581	586	590	595	598	600	602	604	609
2.25	564	571	577	582	586	591	594	596	598	600	605
2.30	560	567	573	578	582	587	590	592	594	596	601
2.35	556	563	569	574	578	583	586	588	590	592	597
2.40	552	559	565	570	574	579	582	584	586	588	593
2.45	548	555	561	566	570	575	578	580	582	584	589
2.50	545	552	558	563	567	572	575	577	579	581	586
2.55	542	549	555	560	564	569	572	574	576	578	583
2.60	539	546	552	557	561	566	569	571	573	575	580
2.65	536	543	549	554	558	563	566	568	570	572	577
2.70	533	540	546	551	555	560	563	565	567	570	574
2.75	530	537	543	548	552	557	560	562	565	568	571
2.80	527	534	540	545	549	554	557	560	563	566	569
2.85	524	531	537	542	546	551	555	558	561	564	567
2.90	521	528	534	539	543	549	553	556	559	562	565
2.95	518	525	531	536	541	547	551	554	557	560	563
3.00	0.515	0.522	0.528	0.534	0.539	0.545	0.549	0.552	0.555	0.558	0.561

STAT

Table III

$$\frac{l_m}{l_0}$$

B \ Δ	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
0.70	0.507	0.505	0.503	0.500	0.500	0.495	0.492	0.490	0.490	0.485	0.482
0.75	565	563	562	560	558	555	553	550	548	545	542
0.80	623	621	620	618	615	613	611	608	605	602	599
0.85	683	681	679	677	675	672	669	666	663	660	656
0.90	723	721	719	716	713	710	707	703	700	697	692
0.95	735	732	730	727	724	722	718	715	712	709	705
1.00	740	738	736	734	731	729	725	722	718	715	711
1.05	743	741	739	737	735	732	729	727	724	721	718
1.10	745	744	740	739	737	734	731	729	726	723	720
1.15	744	743	740	738	736	733	730	728	725	723	720
1.20	739	738	736	735	733	731	728	726	723	721	719
1.25	733	732	731	730	729	727	724	722	720	718	716
1.30	727	726	725	724	723	720	719	718	716	714	712
1.35	718	717	716	715	714	713	711	710	709	707	705
1.40	709	708	707	706	705	704	703	702	701	700	699
1.45	700	699	698	697	696	696	695	694	693	692	692
1.50	693	692	691	690	689	689	689	689	688	687	686
1.55	685	684	683	683	682	682	682	682	681	681	680
1.60	677	677	676	676	676	676	676	676	675	675	675
1.65	670	670	669	669	669	669	669	669	669	669	669
1.70	663	663	663	664	664	664	664	664	664	664	664
1.75	656	656	656	657	657	658	658	658	659	659	659
1.80	650	651	651	562	652	563	653	653	654	654	654
1.85	644	645	645	646	646	647	647	648	649	649	650
1.90	638	639	639	640	641	642	642	643	644	645	646
1.95	633	634	634	635	636	637	638	639	640	641	642
2.00	627	628	629	630	631	632	633	634	635	636	637
2.05	622	623	624	625	626	627	628	629	630	631	632
2.10	617	618	619	620	621	622	623	624	625	626	627
2.15	613	614	616	617	618	619	620	621	622	623	624
2.20	609	610	612	613	614	615	616	617	618	619	620
2.25	605	606	608	609	610	611	612	613	614	615	616
2.30	601	602	604	606	607	608	609	610	611	612	613
2.35	597	598	600	602	603	604	605	606	607	608	609
2.40	593	594	596	598	600	601	602	603	604	605	606
2.45	589	591	593	595	597	598	599	600	601	602	603
2.50	586	588	590	592	594	595	596	597	598	599	600
2.55	583	585	587	589	591	592	593	594	595	596	597
2.60	580	582	584	586	588	589	590	591	592	593	594
2.65	577	579	581	583	585	586	587	589	590	591	592
2.70	574	576	578	580	582	584	585	587	588	589	590
2.75	517	573	575	577	579	581	583	585	586	587	588
2.80	569	571	573	575	577	579	581	583	584	585	586
2.85	567	569	571	573	575	577	579	581	582	583	584
2.90	565	567	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

0.95	735	732	730	727	724	722	718	715	712	709	705
1.00	740	738	736	734	731	729	725	722	718	715	711
1.05	743	741	739	737	735	732	729	727	724	721	718
1.10	745	744	740	739	737	734	731	729	726	723	720
1.15	744	743	740	738	736	733	730	728	725	723	720
1.20	739	738	736	735	733	731	728	726	723	721	719
1.25	733	732	731	730	729	727	724	722	720	718	716
1.30	727	726	725	724	723	720	719	718	716	714	712
1.35	718	717	716	715	714	713	711	710	709	707	705
1.40	709	708	707	706	705	704	703	702	701	700	699
1.45	700	699	698	697	696	696	695	694	693	692	692
1.50	693	692	691	690	689	689	689	689	688	687	686
1.55	685	684	683	683	682	682	682	682	681	681	680
1.60	677	677	676	676	676	676	676	676	675	675	675
1.65	670	670	669	669	669	669	669	669	669	669	669
1.70	663	663	663	664	664	664	664	664	664	664	664
1.75	656	656	656	657	657	658	658	658	659	659	659
1.80	650	651	651	562	652	563	653	653	654	654	654
1.85	644	645	645	646	646	647	647	648	649	649	650
1.90	638	639	639	640	641	642	642	643	644	645	646
1.95	633	634	634	635	636	637	638	639	640	641	642
2.00	627	628	629	630	631	632	633	634	635	636	637
2.05	622	623	624	625	626	627	628	629	630	631	632
2.10	617	618	619	620	621	622	623	624	625	626	627
2.15	613	614	616	617	618	619	620	621	622	623	624
2.20	609	610	612	613	614	615	616	617	618	619	620
2.25	605	606	608	609	610	611	612	613	614	615	616
2.30	601	602	604	606	607	608	609	610	611	612	613
2.35	597	598	600	602	603	604	605	606	607	608	609
2.40	593	594	596	598	600	601	602	603	604	605	606
2.45	589	591	593	595	597	598	599	600	601	602	603
2.50	586	588	590	592	594	595	596	597	598	599	600
2.55	583	585	587	589	591	592	593	594	595	596	597
2.60	580	582	584	586	588	589	590	591	592	593	594
2.65	577	579	581	583	585	586	587	589	590	591	592
2.70	574	576	578	580	582	584	585	587	588	589	590
2.75	517	573	575	577	579	581	583	585	586	587	588
2.80	569	571	573	575	577	579	581	583	584	585	586
2.85	567	569	571	573	575	577	579	581	582	583	584
2.90	565	567	569	571	573	575	577	579	580	581	582
2.95	563	565	567	569	571	573	575	577	578	579	580
3.00	0.561	0.563	0.565	0.567	0.569	0.571	0.573	0.575	0.576	0.577	0.578

STAT

Table III

$\frac{l_m}{l_0}$

B \ Δ	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60
0.70	0.482	0.480	0.477	0.474	0.471	0.470	0.464	0.460	0.457	0.453	0.449
0.75	542	538	535	532	528	524	521	516	512	508	503
0.80	599	596	593	590	586	583	579	575	571	568	564
0.85	656	653	649	645	641	637	633	628	623	619	614
0.90	692	688	683	679	674	670	665	659	654	648	643
0.95	705	701	697	693	688	683	678	673	668	663	658
1.00	711	707	703	690	694	690	685	680	676	671	666
1.05	718	714	710	706	702	697	692	688	683	678	674
1.10	720	716	713	709	705	701	697	693	688	684	680
1.15	720	716	714	710	707	703	699	696	691	688	684
1.20	719	715	714	710	706	704	701	698	694	691	687
1.25	716	713	711	708	705	703	700	698	695	692	689
1.30	712	709	708	706	703	700	697	695	693	690	687
1.35	705	703	702	700	698	695	692	690	688	685	682
1.40	699	698	697	695	693	690	687	685	683	680	677
1.45	692	691	690	698	688	686	682	680	678	675	672
1.50	688	685	684	683	682	681	679	676	673	670	667
1.55	680	680	679	678	677	676	674	671	668	665	662
1.60	675	675	674	673	672	671	669	666	663	660	558
1.65	669	668	668	667	667	666	665	662	659	656	654
1.70	664	663	663	662	662	661	660	658	655	653	650
1.75	659	658	658	658	658	657	657	655	652	650	647
1.80	654	654	654	654	654	653	651	649	647	645	643
1.85	650	650	650	650	650	649	648	646	644	642	640
1.90	646	646	646	646	645	644	643	642	640	638	636
1.95	642	642	642	642	641	640	639	638	637	635	633
2.00	637	637	637	637	636	635	634	633	632	631	630
2.05	632	632	632	632	631	631	631	630	629	628	627
2.10	627	628	629	628	628	628	627	626	625	624	623
2.15	624	625	625	625	624	624	624	623	622	621	620
2.20	620	621	622	622	621	620	620	619	619	618	618
2.25	616	617	618	618	617	617	617	616	615	614	613
2.30	613	614	615	615	614	613	613	612	612	611	610
2.35	609	610	611	612	611	610	610	609	609	608	607
2.40	606	607	608	608	607	606	606	605	605	604	604
2.45	603	604	605	605	604	603	603	602	602	601	601
2.50	600	601	602	602	601	600	600	599	598	598	598
2.55	597	598	599	599	598	597	597	596	596	595	595
2.60	594	595	596	598	595	594	594	593	593	592	592
2.65	592	593	594	594	593	592	592	591	591	590	590
2.70	598	591	592	592	591	590	590	589	589	588	588
2.75	588	589	590	590	589	588	588	587	587	586	586
2.80	585	587	588	588	587	586	586	585	585	584	584
2.85	584	585	586	585	585	584	584	583	583	582	582
2.90	582	583	584	584	583	582	582	581	581	580	580

								9789	9776	9763	9754
0.80	599	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.85	656	653	649	645	641	637	633	628	[illegible]	[illegible]	[illegible]
0.90	692	688	683	679	674	670	665	659	654	648	643
0.95	705	701	697	693	688	683	678	673	668	663	658
1.00	711	707	703	690	694	690	685	680	676	671	666
1.05	718	714	710	706	702	697	692	688	683	678	674
1.10	720	716	713	709	705	701	697	693	688	684	680
1.15	720	716	714	710	707	703	699	696	691	688	684
1.20	719	715	714	710	706	704	701	698	694	691	687
1.25	716	713	711	708	705	703	700	698	695	692	689
1.30	712	709	708	706	703	700	697	695	693	690	687
1.35	705	703	702	700	698	695	692	690	688	685	682
1.40	699	698	697	695	693	690	687	685	683	680	677
1.45	692	691	690	698	688	686	682	680	678	675	672
1.50	688	685	684	683	682	681	679	676	673	670	667
1.55	680	680	679	678	677	676	674	671	668	665	662
1.60	675	675	674	673	672	671	669	666	663	660	658
1.65	669	668	668	667	667	666	665	662	659	656	654
1.70	664	663	663	662	662	661	660	658	655	653	650
1.75	659	658	658	658	658	657	657	655	652	650	647
1.80	654	654	654	654	654	653	651	649	647	645	643
1.85	650	650	650	650	650	649	648	646	644	642	640
1.90	646	646	646	646	645	644	643	642	640	638	636
1.95	642	642	642	642	641	640	639	638	637	635	633
2.00	637	637	637	637	636	635	634	633	632	631	630
2.05	632	632	632	632	631	631	631	630	629	628	627
2.10	627	628	629	628	628	628	627	626	625	624	623
2.15	624	625	625	625	624	624	624	623	622	621	620
2.20	620	621	622	622	621	620	620	619	619	618	618
2.25	616	617	618	618	617	617	617	616	615	614	613
2.30	613	614	615	615	614	613	613	612	612	611	610
2.35	609	610	611	612	611	610	610	609	609	608	607
2.40	606	607	608	608	607	606	606	605	605	604	604
2.45	603	604	605	605	604	603	603	602	602	601	601
2.50	600	601	602	602	601	600	600	599	598	598	598
2.55	597	598	599	599	598	597	597	596	596	595	595
2.60	594	595	596	598	595	594	594	593	593	592	592
2.65	592	593	594	594	593	592	592	591	591	590	590
2.70	598	591	592	592	591	590	590	589	589	588	588
2.75	588	589	590	590	589	588	588	587	587	586	586
2.80	585	587	588	588	587	586	586	585	585	584	584
2.85	584	585	586	585	585	584	584	583	583	582	582
2.90	582	583	584	584	583	582	582	581	581	580	580
2.95	580	581	582	582	581	580	580	579	579	578	578
3.00	0.579	0.579	0.580	0.580	0.579	0.578	0.578	0.577	0.577	0.576	0.576

STAT

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Table III

$$\frac{l_m}{l_0}$$

B \ Δ	0.60	0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.70
0.70	0.449	0.445	0.440	0.436	0.431	0.426	0.421	0.416	0.411	0.406	0.401
0.75	503	498	494	489	484	479	473	468	462	456	450
0.80	563	558	552	546	540	534	528	521	515	507	500
0.85	614	609	603	697	591	585	578	571	564	556	549
0.90	643	637	631	626	620	614	609	602	596	589	581
0.95	658	653	648	642	637	631	626	619	613	607	600
1.00	666	661	657	652	647	642	637	631	625	619	613
1.05	674	669	665	660	656	652	647	642	637	631	626
1.10	680	676	672	668	663	659	654	650	645	640	634
1.15	684	680	676	672	667	663	658	654	650	645	639
1.20	687	683	679	675	670	666	661	657	653	642	643
1.25	689	685	681	676	671	667	662	658	654	649	644
1.30	687	683	679	674	669	665	660	656	652	648	643
1.35	683	678	674	670	665	661	656	652	648	644	639
1.40	677	673	669	665	660	656	652	648	644	640	635
1.45	672	668	664	660	656	652	648	644	640	636	631
1.50	667	664	660	656	652	648	644	640	636	632	628
1.55	662	659	656	652	648	644	640	636	632	628	624
1.60	658	655	562	648	645	641	637	633	629	625	621
1.65	654	651	648	644	641	637	633	629	625	629	617
1.70	650	647	644	641	638	634	630	626	622	618	614
1.75	647	644	641	638	635	631	627	623	619	615	611
1.80	643	640	637	634	631	628	624	620	616	612	608
1.85	640	637	634	631	628	625	621	617	613	609	605
1.90	636	634	631	628	625	622	618	614	610	605	602
1.95	633	631	629	626	623	620	616	612	608	604	600
2.00	630	628	626	624	621	618	614	610	606	602	598
2.05	627	625	623	621	618	615	611	607	603	599	595
2.10	623	621	619	617	615	612	608	604	600	596	593
2.15	620	618	616	614	612	609	606	602	598	594	591
2.20	617	616	614	612	610	607	604	600	596	592	589
2.25	613	612	611	609	607	604	601	597	593	590	587
2.30	610	609	608	607	605	602	599	595	592	588	585
2.35	607	606	605	604	603	600	597	593	590	587	584
2.40	604	603	602	601	600	598	595	591	588	585	582
2.45	601	600	599	598	597	596	593	589	586	583	580
2.50	598	597	596	595	594	593	590	587	584	581	578
2.55	595	594	593	592	591	590	588	585	582	579	576
2.60	592	591	590	589	588	587	585	582	579	576	573
2.65	590	589	588	587	586	585	583	580	577	574	571
2.70	588	587	586	585	584	583	581	578	575	572	569
2.75	586	585	584	583	582	581	579	576	573	570	567
2.80	584	583	582	581	580	579	577	575	572	569	566
2.85	582	581	580	579	578	577	575	573	570	567	564
2.90	580	579	578	577	576	575	573	571	568	565	562

0.70	0.449	0.445	0.440	0.436	0.431	0.426	0.421	0.416	0.411	0.406	0.401
0.75	503	498	494	489	484	479	473	468	462	456	450
0.80	563	558	552	546	540	534	528	521	515	507	500
0.85	614	609	603	697	591	585	578	571	564	556	549
0.90	643	637	631	626	620	614	609	602	596	589	581
0.95	658	653	648	642	637	631	626	619	613	607	600
1.00	666	661	657	652	647	642	637	631	625	619	613
1.05	674	669	665	660	656	652	647	642	637	631	626
1.10	680	676	672	668	663	659	654	650	645	640	634
1.15	684	680	676	672	667	663	658	654	650	645	639
1.20	687	683	679	675	670	666	661	657	653	642	643
1.25	689	685	681	676	671	667	662	658	654	649	644
1.30	687	683	679	674	669	665	660	656	652	648	643
1.35	683	678	674	670	665	661	656	652	648	644	639
1.40	677	673	669	665	660	656	652	648	644	640	635
1.45	672	668	664	660	656	652	648	644	640	636	631
1.50	667	664	660	656	652	648	644	640	636	632	628
1.55	662	659	656	652	648	644	640	636	632	628	624
1.60	658	655	562	648	645	641	637	633	629	625	621
1.65	654	651	648	644	641	637	633	629	625	629	617
1.70	650	647	644	641	638	634	630	626	622	618	614
1.75	647	644	641	638	635	631	627	623	619	615	611
1.80	643	640	637	634	631	628	624	620	616	612	608
1.85	640	637	634	631	628	625	621	617	613	609	605
1.90	636	634	631	628	625	622	618	614	610	605	602
1.95	633	631	629	626	623	620	616	612	608	604	600
2.00	630	628	626	624	621	618	614	610	606	602	598
2.05	627	625	623	621	618	615	611	607	603	599	595
2.10	623	621	619	617	615	612	608	604	600	596	593
2.15	620	618	616	614	612	609	606	602	598	594	591
2.20	617	616	614	612	610	607	604	600	596	592	589
2.25	613	612	611	609	607	604	601	597	593	590	587
2.30	610	609	608	607	605	602	599	595	592	588	585
2.35	607	606	605	604	603	600	597	593	590	587	584
2.40	604	603	602	601	600	598	595	591	588	585	582
2.45	601	600	599	598	597	596	593	589	586	583	580
2.50	598	597	596	595	594	593	590	587	584	581	578
2.55	595	594	593	592	591	590	588	585	582	579	576
2.60	592	591	590	589	588	587	585	582	579	576	573
2.65	590	589	588	587	586	585	583	580	577	574	571
2.70	588	587	586	585	584	583	581	578	575	572	569
2.75	586	585	584	583	582	581	579	576	573	570	567
2.80	584	583	582	581	580	579	577	575	572	569	566
2.85	582	581	580	579	578	577	575	573	570	567	564
2.90	580	579	578	577	576	575	573	571	568	565	562
2.95	578	577	576	575	574	573	571	569	567	564	561
3.00	0.576	0.575	0.574	0.573	0.572	0.571	0.569	0.567	0.565	0.562	0.559

STAT

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Table III

$$\frac{l_m}{l_0}$$

B \ Δ	0.70	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80
0.70	0.401	0.396	0.391	0.386	0.380	0.374	0.368	0.362	0.356	0.349	0.342
0.75	450	444	438	432	420	426	414	408	401	393	383
0.80	500	493	485	478	462	462	454	446	438	430	423
0.85	549	541	533	524	516	506	497	488	479	471	462
0.90	581	574	567	559	551	543	534	525	516	507	499
0.95	600	593	586	579	572	564	557	548	540	531	522
1.00	613	606	600	593	586	578	571	563	555	547	540
1.05	626	620	614	607	600	594	587	579	572	564	556
1.10	634	628	622	616	610	603	596	590	582	575	567
1.15	639	635	631	627	623	617	609	602	593	585	576
1.20	643	639	635	631	627	622	614	607	599	591	582
1.25	644	640	636	632	628	623	616	609	602	595	587
1.30	643	639	635	631	627	622	616	610	604	598	591
1.35	639	635	631	627	623	619	613	608	603	598	592
1.40	635	631	627	623	619	615	609	604	600	596	591
1.45	631	627	623	619	615	611	606	601	597	593	589
1.50	628	624	619	615	611	607	602	598	594	590	586
1.55	624	620	615	611	607	603	598	594	591	587	584
1.60	621	617	612	608	604	600	596	592	589	585	582
1.65	617	613	609	605	601	597	593	589	586	583	580
1.70	614	610	606	602	598	594	590	587	584	581	578
1.75	611	607	603	599	595	592	588	585	582	579	576
1.80	608	604	600	596	593	590	586	583	580	577	574
1.85	605	601	597	594	591	588	584	581	578	575	572
1.90	602	598	595	592	589	586	582	579	576	573	570
1.95	600	596	593	590	587	584	580	577	574	571	568
2.00	598	594	591	588	585	582	578	575	572	569	565
2.05	595	592	589	586	583	580	577	574	571	568	565
2.10	593	590	587	584	581	578	575	572	569	566	563
2.15	591	588	585	582	579	576	573	570	567	564	561
2.20	589	586	583	580	577	574	571	568	565	562	559
2.25	587	584	581	578	575	573	570	567	564	561	558
2.30	585	582	579	576	573	571	568	565	562	559	556
2.35	584	581	578	575	572	570	567	564	561	558	555
2.40	582	579	576	573	570	568	565	562	559	556	553
2.45	580	577	574	571	568	566	563	560	557	554	551
2.50	578	575	572	569	566	564	561	558	555	552	548
2.55	576	573	570	568	565	563	560	557	554	551	548
2.60	573	571	568	566	563	561	558	555	552	549	546
2.65	571	569	567	565	562	560	557	554	551	548	545
2.70	569	567	565	563	561	559	556	553	550	547	544
2.75	567	565	563	561	559	557	554	551	548	545	542
2.80	566	564	562	560	558	556	553	550	547	544	541
2.85	564	562	560	558	556	554	551	548	545	542	539
2.90	562	560	558	556	554	552	549	546	543	540	537

0.85	549	541	533	524	516	506	497	488	479	471	462
0.90	581	574	567	559	551	543	534	525	516	507	499
0.95	600	593	586	579	572	564	557	548	540	531	522
1.00	613	606	600	593	586	578	571	563	555	547	540
1.05	626	620	614	607	600	594	587	579	572	564	556
1.10	634	628	622	616	610	603	596	590	582	575	567
1.15	639	635	631	627	623	617	609	602	593	585	576
1.20	643	639	635	631	627	622	614	607	599	591	582
1.25	644	640	636	632	628	623	616	609	602	595	587
1.30	643	639	635	631	627	622	616	610	604	598	591
1.35	639	635	631	627	623	619	613	608	603	598	592
1.40	635	631	627	623	619	615	609	604	600	596	591
1.45	631	627	623	619	615	611	606	601	597	593	589
1.50	628	624	619	615	611	607	602	598	594	590	586
1.55	624	620	615	611	607	603	598	594	591	587	584
1.60	621	617	612	608	604	600	596	592	589	585	582
1.65	617	613	609	605	601	597	593	589	586	583	580
1.70	614	610	606	602	598	594	590	587	584	581	578
1.75	611	607	603	599	595	592	588	585	582	579	576
1.80	608	604	600	596	593	590	586	583	580	577	574
1.85	605	601	597	594	591	588	584	581	578	575	572
1.90	602	598	595	592	589	586	582	579	576	573	570
1.95	600	596	593	590	587	584	580	577	574	571	568
2.00	598	594	591	588	585	582	578	575	572	569	565
2.05	595	592	589	586	583	580	577	574	571	568	565
2.10	593	590	587	584	581	578	575	572	569	566	563
2.15	591	588	585	582	579	576	573	570	567	564	561
2.20	589	586	583	580	577	574	571	568	565	562	559
2.25	587	584	581	578	575	573	570	567	564	561	558
2.30	585	582	579	576	573	571	568	565	562	559	556
2.35	584	581	578	575	572	570	567	564	561	558	555
2.40	582	579	576	573	570	568	565	562	559	556	552
2.45	580	577	574	571	568	566	563	560	557	554	551
2.50	578	575	572	569	566	564	561	558	555	552	548
2.55	576	573	570	568	565	563	560	557	554	551	548
2.60	573	571	568	566	563	561	558	555	552	549	546
2.65	571	569	567	565	562	560	557	554	551	548	545
2.70	569	567	565	563	561	559	556	553	550	547	544
2.75	567	565	563	561	559	557	554	551	548	545	542
2.80	566	564	562	560	558	556	553	550	547	544	541
2.85	564	562	560	558	556	554	551	548	545	542	539
2.90	562	560	558	556	554	552	549	546	543	540	537
2.95	561	559	557	555	553	551	548	545	542	539	536
3.00	0.559	0.557	0.555	0.553	0.551	0.549	0.546	0.543	0.540	0.537	0.534

STAT

Table IV

Values of $\log \left[1 - \frac{B\Theta}{2} (1 - Z_0)^2 \right]$

B \ Δ	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
0.70	$\bar{1}.9894$	$\bar{1}.9872$	$\bar{1}.9854$	$\bar{1}.9840$	$\bar{1}.9827$	$\bar{1}.9816$	$\bar{1}.9807$	$\bar{1}.9798$
0.75	9886	9863	9844	9828	9815	9802	9792	9783
0.80	9879	9854	9834	9817	9802	9780	9778	9768
0.85	9871	9845	9823	9805	9789	9776	9763	9754
0.90	9863	9836	9812	9793	9777	9762	9749	9739
0.95	9855	9827	9802	9782	9765	9748	9735	9723
1.00	9847	9817	9791	9769	9751	9734	9720	9708
1.05	9840	9808	9780	9757	9738	9721	9706	9693
1.10	9832	9798	9769	9745	9725	9707	9691	9683
1.15	9824	9788	9750	9733	9712	9689	9677	9662
1.20	9817	9779	9738	9721	9699	9679	9682	9647
1.25	9809	9770	9737	9709	9686	9665	9647	9632
1.30	9800	9760	9726	9698	9673	9652	9633	9617
1.35	9793	9751	9716	9686	9660	9638	9618	9602
1.40	9785	9742	9704	9674	9647	9624	9604	9526
1.45	9777	9731	9693	9661	9633	9609	9589	9570
1.50	9769	9722	9682	9648	9620	9595	9574	9555
1.55	9761	9713	8672	9637	9608	9581	9559	9539
1.60	9754	9703	9661	9625	9594	9567	9543	9528
1.65	9745	9693	9649	9612	9581	9553	9529	9507
1.70	9737	9684	9638	9600	9566	9539	9513	9487
1.75	9730	9674	9628	9588	9554	9524	9499	9481
1.80	9721	9664	9616	9575	9541	9510	9483	9461
1.85	9713	9655	9605	9563	9527	9496	9468	9444
1.90	9705	9645	9593	9550	9513	9480	9453	9427
1.95	9697	9635	9583	9539	9500	9467	9438	9412
2.00	9689	9626	9571	9526	9486	9452	9422	9396
2.05	9680	9615	9560	9513	9472	9437	9406	9380
2.10	9673	9606	9548	9501	9459	9422	9390	9364
2.15	9665	9598	9540	9490	9448	9410	9379	9347
2.20	9652	9586	9523	9475	9431	9393	9365	9336
2.25	9648	9577	9514	9463	9417	9378	9344	9314
2.30	9641	9566	9503	9450	9404	9364	9329	9298
2.35	9632	9556	9490	9437	9390	9349	9312	9281
2.40	9625	9546	9480	9429	9376	9334	9297	9266
2.45	9616	9537	9469	9411	9362	9319	9278	9248
2.50	9608	9524	9453	9398	9348	9304	9260	9236
2.55	9600	9516	9446	9386	9334	9289	9249	9215
2.60	9591	9507	9434	9373	5321	9274	9233	9199
2.65	9582	9495	9420	9357	9320	9255	9214	9180
2.70	9575	9486	9410	9349	9285	9243	9200	9163
2.75	9569	9476	9399	9334	9276	9228	9184	9145
2.80	9558	9466	9387	6321	9262	9212	9168	9128
2.85	9552	9458	9376	9316	9265	9200	9155	9112
2.90	9542	9446	9366	9294	9234	9181	9135	9094

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0.85	9871	9845	9823	9805	9789	9776	9763	9754
0.90	9863	9836	9812	9793	9777	9762	9749	9739
0.95	9855	9827	9802	9782	9765	9748	9735	9723
1.00	9847	9817	9791	9769	9751	9734	9720	9708
1.05	9840	9808	9780	9757	9738	9721	9706	9693
1.10	9832	9798	9769	9745	9725	9707	9691	9683
1.15	9824	9788	9750	9733	9712	9689	9677	9662
1.20	9817	9779	9738	9721	9699	9679	9682	9647
1.25	9809	9770	9737	9709	9686	9665	9647	9632
1.30	9800	9760	9726	9698	9673	9652	9633	9617
1.35	9793	9751	9716	9686	9660	9638	9618	9602
1.40	9785	9742	9704	9674	9647	9624	9604	9526
1.45	9777	9731	9693	9661	9633	9609	9589	9570
1.50	9769	9722	9682	9648	9620	9595	9574	9555
1.55	9761	9713	8672	9637	9608	9581	9559	9539
1.60	9754	9703	9661	9625	9594	9567	9543	9528
1.65	9745	9693	9649	9612	9581	9553	9529	9507
1.70	9737	9684	9638	9600	9566	9539	9513	9487
1.75	9730	9674	9628	9588	9554	9524	9499	9481
1.80	9721	9664	9616	9575	9541	9510	9483	9461
1.85	9713	9655	9605	9563	9527	9496	9468	9444
1.90	9705	9645	9593	9550	9513	9480	9453	9427
1.95	9697	9635	9583	9539	9500	9467	9438	9412
2.00	9689	9626	9571	9526	9486	9452	9422	9396
2.05	9680	9615	9560	9513	9472	9437	9406	9380
2.10	9673	9606	9548	9501	9459	9422	9390	9364
2.15	9665	9598	9540	9490	9448	9410	9379	9347
2.20	9652	9586	9523	9475	9431	9393	9365	9336
2.25	9648	9577	9514	9463	9417	9378	9344	9314
2.30	9641	9566	9503	9450	9404	9364	9329	9298
2.35	9632	9556	9490	9437	9390	9349	9312	9281
2.40	9625	9546	9480	9429	9376	9334	9297	9266
2.45	9616	9537	9469	9411	9362	9319	9278	9248
2.50	9608	9524	9453	9398	9348	9304	9260	9236
2.55	9600	9516	9446	9386	9334	9289	9249	9215
2.60	9591	9507	9434	9373	5321	9274	9233	9199
2.65	9582	9495	9420	9357	9320	9255	9214	9180
2.70	9575	9486	9410	9349	9285	9243	9200	9163
2.75	9569	9476	9399	9334	9276	9228	9184	9145
2.80	9558	9466	9387	6321	9262	9212	9168	9128
2.85	9552	9458	9376	9316	9265	9200	9155	9112
2.90	9542	9446	9366	9294	9234	9181	9135	9094
2.95	9534	9433	9359	9282	9221	9171	9118	9076
3.00	9525	9425	9341	9262	9205	9150	9104	9060

STAT

Table IV

$$\log \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right]$$

B \ Δ	0.15	0.16	0.17	0.18	0.19	0.20	0.25	0.30	0.35
0.70	Ī.9790	Ī.9784	Ī.9778	Ī.9773	Ī.9768	Ī.9763	Ī.9748	Ī.9735	Ī.9727
0.75	9775	9768	9762	9756	9751	9746	9730	9716	9706
0.80	9760	9752	9745	9739	9733	9729	9711	9695	9686
0.85	9745	9736	9729	9722	9717	9711	9692	9765	9665
0.90	9729	9720	9712	9705	9699	9693	9674	9656	9645
0.95	9713	9711	9696	9689	9682	9675	9654	9635	9624
1.00	9697	9691	9679	9672	9664	9658	9635	9616	9604
1.05	9682	9672	9663	9654	9647	9640	9616	9596	9583
1.10	9662	9655	9646	9634	9629	9621	9597	9576	9562
1.15	9650	9638	9628	9621	9611	9600	9578	9555	9541
1.20	9639	9618	9611	9608	9594	9587	9562	9535	9519
1.25	9618	9607	9590	9585	9576	9568	9539	9514	9490
1.30	9611	9590	9578	9568	9559	9550	9519	9494	9477
1.35	9587	9573	9561	9550	9521	9532	9501	9473	9456
1.40	9575	9557	9543	9533	9511	9513	9480	9453	9434
1.45	9554	9540	9531	9515	9505	9495	9461	9431	9412
1.50	9549	9518	9510	9498	9486	9476	9441	9410	9391
1.55	9526	9507	9492	9472	9469	9458	9421	9390	9369
1.60	9507	9489	9479	9462	9450	9439	9401	9360	9348
1.65	9501	9472	9458	9447	9431	9421	9381	9356	9334
1.70	9474	9455	9440	9426	9413	9402	9361	9327	9303
1.75	9459	9438	9422	9407	9395	9383	9341	9304	9281
1.80	9439	9421	9404	9389	9375	9364	9321	9282	9258
1.85	9422	9402	9402	9387	9371	9345	9300	9261	9235
1.90	9406	9388	9369	9353	9341	9326	9279	9240	9213
1.95	9390	9370	9351	9335	9320	9306	9258	9218	9191
2.00	9373	9352	9334	9317	9305	9287	9238	9196	9168
2.05	9355	9335	9319	9297	9284	9268	9217	9173	9144
2.10	9339	9318	9397	9279	9253	9248	9196	9151	9122
2.15	9322	9299	9279	9258	9244	9228	9175	9129	9099
2.20	9304	9282	9260	9242	9225	9209	9154	9106	9075
2.25	9289	9263	9242	9223	9205	9190	9133	9074	9051
2.30	9271	9246	9224	9206	9186	9170	9111	9061	9028
2.35	9253	9226	9205	9184	9167	9150	9089	9038	9006
2.40	9235	9210	9187	9166	9146	9130	9067	9015	8980
2.45	9218	9191	9168	9141	9126	9109	9046	8991	8956
2.50	9200	9174	9150	9127	9107	9089	9024	8969	8932
2.55	9185	9156	9139	9102	9088	9069	9002	8945	8907
2.60	9166	9137	9110	9088	9067	9048	8980	8922	8883
2.65	9147	9120	9092	9068	9047	9028	8958	8898	8859
2.70	9130	9100	9073	9049	9027	9007	8935	8875	8834
2.75	9113	9082	9055	9029	9007	8986	8913	8850	8809
2.80	9093	9063	9034	9009	8986	8966	8890	8839	8784
2.85	9076	9045	9016	8989	8966	8951	8868	8802	8759
2.90	9058	9025	8996	8968	8945	8924	8845	8781	8734

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.90	9729	9720	9712	9705	9699	9693	9674	9656	9645
0.95	9713	9711	9696	9689	9682	9675	9654	9635	9624
1.00	9697	9691	9679	9672	9664	9658	9635	9616	9604
1.05	9682	9672	9663	9654	9647	9640	9616	9596	9583
1.10	9662	9655	9646	9634	9629	9621	9597	9576	9562
1.15	9650	9638	9628	9621	9611	9600	9578	9555	9541
1.20	9639	9618	9611	9608	9594	9587	9562	9535	9519
1.25	9618	9607	9590	9585	9576	9568	9539	9514	9490
1.30	9611	9590	9578	9568	9559	9550	9519	9494	9477
1.35	9587	9573	9561	9550	9521	9532	9501	9473	9456
1.40	9575	9557	9543	9533	9511	9513	9480	9453	9434
1.45	9554	9540	9531	9515	9505	9495	9461	9431	9412
1.50	9549	9518	9510	9498	9486	9476	9441	9410	9391
1.55	9526	9507	9492	9472	9469	9458	9421	9390	9369
1.60	9507	9489	9479	9462	9450	9439	9401	9360	9348
1.65	9501	9472	9458	9447	9431	9421	9381	9356	9334
1.70	9474	9455	9440	9426	9413	9402	9361	9327	9303
1.75	9459	9438	9422	9407	9395	9383	9341	9304	9281
1.80	9439	9421	9404	9389	9375	9364	9321	9282	9258
1.85	9422	9402	9402	9387	9371	9345	9300	9261	9235
1.90	9406	9388	9369	9353	9341	9326	9279	9240	9213
1.95	9390	9370	9351	9335	9320	9306	9258	9218	9191
2.00	9373	9352	9334	9317	9305	9287	9238	9196	9168
2.05	9355	9335	9319	9297	9284	9268	9217	9173	9144
2.10	9339	9318	9397	9279	9253	9248	9196	9151	9122
2.15	9322	9299	9279	9258	9244	9228	9175	9129	9099
2.20	9304	9282	9260	9242	9225	9209	9154	9106	9075
2.25	9289	9263	9242	9223	9205	9190	9133	9074	9051
2.30	9271	9246	9224	9206	9186	9170	9111	9061	9028
2.35	9253	9226	9205	9184	9167	9150	9089	9038	9006
2.40	9235	9210	9187	9166	9146	9130	9067	9015	8980
2.45	9218	9191	9168	9141	9126	9109	9046	8991	8956
2.50	9200	9174	9150	9127	9107	9089	9024	8969	8932
2.55	9185	9156	9139	9102	9088	9069	9002	8945	8907
2.60	9166	9137	9110	9088	9067	9048	8980	8922	8883
2.65	9147	9120	9092	9068	9047	9028	8958	8898	8859
2.70	9130	9100	9073	9049	9027	9007	8935	8875	8834
2.75	9113	9082	9055	9029	9007	8986	8913	8850	8809
2.80	9093	9063	9034	9009	8986	8966	8890	8839	8784
2.85	9076	9045	9016	8989	8966	8951	8868	8802	8759
2.90	9058	9025	8996	8968	8945	8924	8845	8781	8734
2.95	9039	9007	8976	8950	8924	8904	8823	8754	8709
3.00	9021	8988	8957	8930	8904	8881	8800	8725	8693

STAT

Table IV

$$\log \left[1 - \frac{B\theta}{2} (1 - z_0)^2\right]$$

B \ Δ	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
0.70	$\bar{1}$.9720	$\bar{1}$.9715	$\bar{1}$.9711	$\bar{1}$.9707	$\bar{1}$.9704	$\bar{1}$.9702	$\bar{1}$.9700	$\bar{1}$.9698	$\bar{1}$.9697
0.75	9699	9693	9689	9686	9683	9680	9670	9675	9674
0.80	9678	9673	9668	9663	9661	9658	9656	9653	9651
0.85	9658	9651	9647	9642	9639	9635	9633	9630	9628
0.90	9636	9629	9625	9620	9616	9613	9610	9608	9606
0.95	9615	9609	9603	9598	9594	9590	9588	9585	9583
1.00	9594	9587	9581	9575	9571	9567	9565	9562	9560
1.05	9572	9565	9559	9553	9549	9545	9542	9539	9537
1.10	9551	9543	9537	9531	9526	9522	9519	9516	9513
1.15	9530	9521	9514	9509	9504	9499	9496	9492	9490
1.20	9509	9499	9492	9485	9480	9476	9472	9469	9467
1.25	9486	9477	9469	9463	9458	9453	9449	9446	9443
1.30	9465	9450	9447	9440	9435	9430	9425	9422	9419
1.35	9421	9432	9424	9417	9411	9406	9402	9398	9395
1.40	9413	9410	9401	9394	9393	9383	9378	9374	9371
1.45	9398	9387	9378	9360	9365	9359	9355	9350	9348
1.50	9376	9362	9355	9347	9341	9335	9330	9327	9323
1.55	9354	9342	9332	9324	9318	9310	9306	9301	9298
1.60	9333	9319	9308	9300	9293	9287	9282	9278	9274
1.65	9312	9304	9293	9285	9279	9270	9266	9262	9258
1.70	9286	9273	9261	9252	9245	9239	9233	9228	9223
1.75	9263	9249	9238	9228	9221	9214	9208	9203	9198
1.80	9242	9216	9214	9204	9196	9189	9183	9177	9173
1.85	9217	9202	9190	9179	9172	9163	9158	9153	9147
1.90	9194	9178	9166	9155	9146	9139	9133	9126	9121
1.95	9170	9155	9141	9131	9122	9114	9107	9102	9097
2.00	9146	9131	9117	9106	9097	9089	9082	9076	9071
2.05	9123	9106	9092	9080	9072	9063	9056	9050	9045
2.10	9099	9073	9067	9056	9046	9037	9030	9024	9019
2.15	9075	9058	9043	9029	9021	9012	9005	8998	8992
2.20	9051	9033	9018	9006	8996	8986	8978	8972	8966
2.25	9027	9008	8993	8979	8969	8960	8952	8945	8939
2.30	9003	8984	8968	8954	8944	8934	8928	8918	8912
2.35	8978	8958	8942	8929	8917	8907	8899	8891	8885
2.40	8954	8933	8916	8902	8891	8880	8875	8865	8859
2.45	8929	8908	8890	8876	8865	8855	8845	8843	8831
2.50	8904	8883	8865	8860	8839	8827	8819	8810	8803
2.55	8879	8857	8839	8824	8811	8800	8798	8782	8776
2.60	8855	8832	8813	8798	8784	8773	8764	8756	8746
2.65	8829	8805	8786	8770	8757	8746	8737	8727	8720
2.70	8803	8782	8759	8743	8730	8718	8709	8699	8691
2.75	8778	8753	8733	8716	8703	8690	8681	8671	8663
2.80	8752	8727	8706	8688	8676	8662	8652	8643	8633
2.85	8727	8700	8680	8661	8648	8634	8624	8614	8605
2.90	8700	8674	8652	8634	8619	8605	8595	8585	8577
2.95	8674	8647	8625	8605	8591	8577	8567	8556	8548

1.05	9572	9565	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.10	9551	9543	9537	9531	9526	9522	9519	9516	[illegible]
1.15	9530	9521	9514	9509	9504	9499	9496	9492	9490
1.20	9509	9499	9492	9485	9480	9476	9472	9469	9467
1.25	9486	9477	9469	9463	9458	9453	9449	9446	9443
1.30	9465	9450	9447	9440	9435	9430	9425	9422	9419
1.35	9421	9432	9424	9417	9411	9406	9402	9398	9395
1.40	9413	9410	9401	9394	9393	9383	9378	9374	9371
1.45	9398	9387	9378	9360	9365	9359	9355	9350	9348
1.50	9376	9362	9355	9347	9341	9335	9330	9327	9323
1.55	9354	9342	9332	9324	9318	9310	9306	9301	9298
1.60	9333	9319	9308	9300	9293	9287	9282	9278	9274
1.65	9312	9304	9293	9285	9279	9270	9266	9262	9258
1.70	9286	9273	9261	9252	9245	9239	9233	9228	9223
1.75	9263	9249	9238	9228	9221	9214	9208	9203	9198
1.80	9242	9216	9214	9204	9196	9189	9183	9177	9173
1.85	9217	9202	9190	9179	9172	9163	9158	9153	9147
1.90	9194	9178	9166	9155	9146	9139	9133	9126	9121
1.95	9170	9155	9141	9131	9122	9114	9107	9102	9097
2.00	9146	9131	9117	9106	9097	9089	9082	9076	9071
2.05	9123	9106	9092	9080	9072	9063	9056	9050	9045
2.10	9099	9073	9067	9056	9046	9037	9030	9024	9019
2.15	9075	9058	9043	9029	9021	9012	9005	8998	8992
2.20	9051	9033	9018	9006	8996	8986	8978	8972	8966
2.25	9027	9008	8993	8979	8969	8960	8952	8945	8939
2.30	9003	8984	8968	8954	8944	8934	8928	8918	8912
2.35	8978	8958	8942	8929	8917	8907	8899	8891	8885
2.40	8954	8933	8916	8902	8891	8880	8875	8865	8859
2.45	8929	8908	8890	8876	8865	8855	8845	8843	8831
2.50	8904	8883	8865	8860	8839	8827	8819	8810	8803
2.55	8879	8857	8839	8824	8811	8800	8798	8782	8776
2.60	8855	8832	8813	8798	8784	8773	8764	8756	8746
2.65	8829	8805	8786	8770	8757	8746	8737	8727	8720
2.70	8803	8782	8759	8743	8730	8718	8709	8699	8691
2.75	8778	8753	8733	8716	8703	8690	8681	8671	8663
2.80	8752	8727	8706	8688	8676	8662	8652	8643	8633
2.85	8727	8700	8680	8661	8648	8634	8624	8614	8605
2.90	8700	8674	8652	8634	8619	8605	8595	8585	8577
2.95	8674	8647	8625	8605	8591	8577	8567	8556	8548
3.00	8647	8620	8597	8578	8563	8549	8538	8527	8519

STAT

APPENDIX 3

VALUES OF B AS A FUNCTION OF p_m AND Δ

(Compiled by M.S. Gorokhov)

…2	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84
									1.071	1.134	1.200	1.270	1.342	1.417	1.495	1.557	1.664	1.755	1.850	1.950	2.056
									1.096	1.161	1.229	1.299	1.372	1.448	1.529	1.614	1.702	1.795	1.892	1.994	2.102
									1.122	1.189	1.258	1.330	1.405	1.483	1.565	1.651	1.742	1.837	1.936	2.041	2.151
							1.022	1.085	1.150	1.217	1.288	1.363	1.440	1.520	1.604	1.692	1.784	1.881	1.983	2.090	2.202
							1.049	1.113	1.179	1.248	1.321	1.397	1.476	1.558	1.644	1.734	1.829	1.928	2.032	2.142	2.257
							1.077	1.142	1.210	1.282	1.356	1.433	1.514	1.598	1.687	1.780	1.877	1.979	2.085	2.197	2.315
							1.107	1.172	1.243	1.317	1.393	1.472	1.555	1.641	1.732	1.827	1.927	2.031	2.140	2.255	2.376
							1.138	1.207	1.279	1.354	1.432	1.514	1.599	1.688	1.781	1.878	1.980	2.087	3.179	2.317	2.441
					1.087	1.103	1.172	1.243	1.317	1.395	1.475	1.558	1.645	1.736	1.832	1.932	2.037	2.147	2.262	2.383	2.511
					1.070	1.138	1.208	1.281	1.357	1.437	1.520	1.606	1.695	1.789	1.887	1.990	2.098	2.211	2.330	2.455	2.585
					1.105	1.175	1.247	1.322	1.401	1.483	1.568	1.657	1.749	1.845	1.946	2.052	2.163	2.279	2.401	2.529	2.665
					1.142	1.213	1.288	1.366	1.447	1.531	1.619	1.711	1.806	1.906	2.010	2.119	2.233	2.358	2.478	2.611	2.751
			1.041	1.111	1.182	1.256	1.383	1.413	1.497	1.584	1.674	1.768	1.867	1.971	2.079	2.191	2.309	2.432	2.562	2.699	2.843
			1.079	1.151	1.225	1.301	1.381	1.465	1.551	1.640	1.734	1.332	1.934	2.041	2.152	2.269	2.390	2.518	2.652	2.794	2.943
			1.121	1.195	1.272	1.251	1.483	1.519	1.609	1.702	1.799	1.901	2.006	2.116	2.231	2.352	2.478	2.611	2.750		
	1.020	1.092	1.166	1.243	1.322	1.404	1.490	1.580	1.673	1.770	1.870	1.975	2.084	2.198	2.318	2.443	2.574	2.712	2.856		
	1.064	1.138	1.215	1.295	1.377	1.463	1.552	1.645	1.741	1.841	1.946	2.055	2.170	2.288	2.412	2.542	2.678	2.821	2.971		
	1.111	1.189	1.269	1.351	1.437	1.527	1.620	1.716	1.816	1.920	2.029	2.144	2.263	2.386	2.515	2.550	2.793				
…85	1.163	1.244	1.328	1.414	1.504	1.597	1.694	1.795	1.899	2.007	2.121	2.241	2.356	2.494	2.628	2.769	2.918				
…39	1.221	1.306	1.393	1.488	1.577	1.674	1.775	1.880	1.990	2.105	2.224	2.348	2.477	2.612	2.753						
…98	1.284	1.372	1.464	1.559	1.658	1.760	1.866	1.977	2.092	2.212	2.337	2.467	2.601	2.742	2.890						
…265	1.355	1.448	1.544	1.644	1.748	1.856	1.967	2.084	2.205	2.331	2.462	2.599	2.741								
…340	1.435	1.533	1.634	1.739	1.849	1.963	2.081	2.204	2.332	2.465	2.503	2.747	2.897								
…425	1.525	1.628	1.736	1.848	1.964	2.086	2.209	2.340	2.476	2.616	2.751										
…520	1.627	1.731	1.852	1.972	2.096	2.222	2.355	2.494	2.638	2.786	2.940										
…629	1.743	1.862	1.985	2.112	2.244	2.381	2.522	2.669	2.822												
…756	1.879	2.006	2.138	2.275	2.416	2.562	2.714														
…906	2.039	2.177	2.319	2.466	2.619																

APPENDIX 5

VALUES OF B AS A FUNCTION OF P_m AND Δ

(Compiled by M.S. Gorokhov)

[illegible]	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84
									1.071	1.134	1.200	1.270	1.342	1.417	1.495	[illegible]	[illegible]	1.755	1.850	1.950	2.056
									1.096	1.161	1.229	1.299	1.374	1.448	1.529	1.614	1.702	1.795	1.892	1.994	2.102
									1.122	1.189	1.258	1.330	1.405	1.483	1.565	1.651	1.742	1.837	1.936	2.041	2.151
							1.022	1.085	1.150	1.217	1.288	1.363	1.440	1.520	1.604	1.692	1.784	1.881	1.983	2.090	2.202
							1.049	1.113	1.179	1.248	1.321	1.397	1.476	1.558	[illegible]	[illegible]	1.829	1.928	2.032	2.142	2.257
							1.077	1.142	1.210	1.282	1.356	1.433	1.514	1.598	1.687	1.780	1.877	1.979	2.085	2.197	2.315
							1.107	1.172	1.243	1.317	1.393	1.474	1.555	1.641	1.732	1.827	1.927	2.031	2.140	2.255	2.376
							1.138	1.207	1.279	1.354	1.432	1.514	[illegible]	1.688	1.781	1.878	1.980	2.087	[illegible]	2.317	[illegible]
					1.087	1.103	1.172	[illegible]	1.317	1.395	1.475	1.558	[illegible]	1.736	[illegible]	[illegible]	2.037	[illegible]	2.262	2.383	[illegible]
					1.070	1.138	1.208	1.281	1.357	1.437	[illegible]	[illegible]	1.695	1.784	[illegible]	[illegible]	[illegible]	2.211	2.330	2.455	2.585
					1.105	1.175	1.247	1.322	1.401	1.483	1.506	1.657	[illegible]	[illegible]	[illegible]	[illegible]	2.163	[illegible]	[illegible]	[illegible]	[illegible]
					1.142	1.213	1.288	1.360	1.447	1.531	1.619	1.711	1.806	[illegible]	[illegible]	2.119	2.233	2.358	2.478	2.611	2.751
			1.041	1.111	1.182	1.256	1.383	1.413	1.497	1.584	1.674	1.768	1.867	1.971	[illegible]	[illegible]	[illegible]	2.432	2.562	2.699	[illegible]
			1.079	1.151	1.225	1.301	1.381	1.465	1.551	1.640	1.734	[illegible]	[illegible]	2.041	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			1.121	1.195	1.272	1.251	1.483	1.519	[illegible]	1.702	[illegible]	1.901	[illegible]	2.116	[illegible]	[illegible]	[illegible]	2.611	[illegible]		
	1.020	1.092	1.166	1.243	1.322	1.404	1.490	1.580	1.673	1.770	1.870	1.975	2.084	2.148	[illegible]	2.443	[illegible]	2.712	2.850		
	1.064	1.138	1.215	1.295	1.377	1.463	1.552	1.645	[illegible]	1.842	[illegible]	2.055	2.170	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
	1.111	1.189	1.269	1.351	1.437	1.527	1.620	1.716	1.816	[illegible]	[illegible]	[illegible]	2.263	[illegible]	[illegible]	[illegible]	[illegible]				
[illegible]	1.163	1.244	1.328	1.414	1.504	1.597	1.694	1.795	[illegible]	[illegible]	2.121	2.241	2.356	[illegible]	[illegible]	[illegible]	[illegible]				
[illegible]	1.221	1.306	1.393	1.488	1.577	1.674	1.775	1.880	1.990	2.105	2.224	2.348	[illegible]	[illegible]	[illegible]						
[illegible]	1.284	1.372	1.464	1.559	1.658	1.760	1.866	1.977	2.092	2.212	2.337	2.467	[illegible]	[illegible]	[illegible]						
[illegible]	1.355	1.448	1.544	1.644	1.748	1.856	1.967	2.084	2.205	2.331	[illegible]	[illegible]	[illegible]								
[illegible]	1.435	1.533	1.634	1.739	1.849	1.963	2.081	2.204	2.332	2.465	[illegible]	[illegible]	[illegible]								
[illegible]	1.525	1.628	1.736	1.848	1.964	2.086	[illegible]	2.340	2.476	2.616	2.751										
[illegible]	1.627	1.731	1.852	1.972	2.096	2.222	2.355	2.494	2.638	[illegible]	[illegible]										
[illegible]	1.743	1.862	1.985	2.112	2.244	2.381	2.522	[illegible]	2.842												
[illegible]	1.879	2.006	2.136	2.275	2.416	2.562	2.714														
[illegible]	[illegible]	2.177	2.319	2.466	2.619																

APPENDIX 2

VALUES OF B AS A FUNCTION OF P_m AND Δ

(Compiled by [illegible] Sorokhov)

	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74
4200											1.071	1.134	1.200	1.270	1.342	1.417	1.495	1.557
4100											1.096	1.161	1.229	1.299	1.37?	1.448	1.5?9	1.61?
4000											1.1?2	1.189	1.258	1.330	1.405	1.483	1.565	1.65?
3900									1.042	1.085	1.15?	[illegible]	1.2??	1.363	1.44?	1.52?	1.6?4	1.6??
3800									1.049	1.113	1.1??	1.?48	1.3?1	1.3?7	1.476	1.558	1.6??	1.73?
3700									1.077	1.1??	1.2??	1.2??	1.356	1.433	1.514	1.5?6	1.687	1.7??
3600									1.107	1.172	1.2??	1.317	1.3?3	1.472	1.555	1.641	1.73?	1.?27
3500									1.138	1.2?7	1.?7?	1.35?	1.?32	1.514	1.59?	1.688	1.73?	1.?7?
3400							1.087	1.103	1.17?	1.2??	1.3?7	1.3??	1.?75	1.558	1.6?5	1.730	1.83?	1.?3?
3300							1.070	1.138	1.2?8	1.2?1	1.35?	1.?37	1.5??	[illegible]	1.?95	1.789	1.?8?	[illegible]
3200							1.105	1.175	1.2?7	1.3??	1.??1	1.?83	1.5?8	1.?57	1.74?	1.8?5	1.?4?	[illegible]
3100							1.14?	1.213	1.2??	1.30?	1.4??	[illegible]	1.?1?	1.711	1.800	1.???	[illegible]	[illegible]
3000					1.041	1.111	1.18?	1.256	1.?83	1.?13	1.4?7	1.?8?	1.?74	1.768	1.8?7	1.9?1	[illegible]	[illegible]
2900					1.074	1.151	1.225	1.301	1.381	1.?65	1.551	[illegible]	1.?3?	1.83?	[illegible]	2.041	[illegible]	[illegible]
2800					1.121	1.195	1.272	1.?51	1.?83	1.5??	[illegible]	[illegible]	1.7??	?.?01	2.?00	2.?16	[illegible]	[illegible]
2700			1.020	1.092	1.166	1.243	1.3?2	1.404	1.4??	[illegible]	1.?73	[illegible]	[illegible]	[illegible]	2.?84	2.148	2.3??	[illegible]
2600			1.064	1.138	1.215	1.295	1.377	1.4?3	1.?5?	1.?45	[illegible]	[illegible]	[illegible]	2.?55	2.170	2.2?8	[illegible]	[illegible]
2500			1.111	1.189	1.264	1.351	1.437	1.527	1.6??	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.203	2.38?	[illegible]	[illegible]
2400	1.009	1.085	1.163	1.2?4	1.328	1.4??	1.50?	1.5?7	1.6??	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.356	2.4??	[illegible]	
2300	1.059	1.13?	1.221	1.3?6	1.393	1.4??	1.577	[illegible]	1.?75	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
2200	1.115	1.198	1.284	1.37?	1.40?	1.5??	1.6??	[illegible]	1.?8?	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
2100	1.177	1.265	1.355	1.??8	1.5??	1.5??	1.748	1.?56	1.?7?	2.?8?	[illegible]	[illegible]	[illegible]	[illegible]				
2000	1.247	1.340	1.435	1.533	1.634	1.739	1.???	1.?63	2.0??	[illegible]	[illegible]	[illegible]	[illegible]					
1900	1.326	1.445	1.525	1.628	1.730	1.8??	1.9??	2.??6	2.2??	[illegible]	[illegible]	[illegible]	[illegible]					
1800	1.416	1.520	1.627	1.731	1.85?	1.?72	2.0??	2.??2	2.3?5	[illegible]	[illegible]	[illegible]						
1700	1.519	1.62?	1.743	1.?62	1.985	2.112	2.2??	2.3??	2.5?2	[illegible]	[illegible]							
1600	1.638	1.756	1.879	2.?06	2.13?	2.275	2.410	2.5??	2.71?									
1500	1.778	1.906	2.03?	2.177	2.31?	2.466	2.619											

APPENDIX 4

VALUES OF Λ_K AS A FUNCTION OF ξ_m AND [illegible]

(Compiled by M.S. Gorokhov)

0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84
									0.882	0.887	0.959	1.039	1.126	1.222	1.330	1.451	1.558	1.744	1.944	2.133	2.376
									0.854	0.923	1.000	1.085	1.177	1.279	1.393	1.543	1.671	1.840	2.035	2.261	2.526
									0.890	0.963	1.043	1.133	1.232	1.341	1.464	1.603	1.763	1.947	2.159	2.406	2.695
							0.789	0.856	0.928	1.005	1.091	1.187	1.292	1.419	1.542	1.693	1.866	2.065	2.290	2.567	2.886
							0.823	0.893	0.969	1.054	1.143	1.243	1.356	1.484	1.62[illegible]	1.7[illegible]	1.932	2.199	2.151	2.744	3.105
							0.860	0.934	1.015	1.103	1.201	1.310	1.432	1.56[illegible]	1.72[illegible]	1.905	2.1[illegible]	2.34[illegible]	2.620	2.955	3.35[illegible]
							0.900	0.977	1.064	1.160	1.266	1.383	1.515	1.664	1.834	2.029	2.255	2.517	2.825	3.633	3.633
							0.944	1.027	1.12[illegible]	1.223	1.337	1.464	1.607	1.769	1.955	2.1[illegible]	2.4[illegible]	2.70[illegible]	3.054	3.456	3.962
					0.833	0.910	0.993	1.084	1.183	1.293	1.416	1.554	1.71[illegible]	1.868	2.0[illegible]3	2.330	[illegible]	2.931	3.317	3.781	4.347
					0.876	0.958	1.046	1.142	1.251	1.372	1.5[illegible]	1.65[illegible]	1.827	2.[illegible]3	[illegible]	[illegible]13	[illegible].823	3.13[illegible]	3.644	[illegible].152	4.8[illegible]
					0.923	1.010	1.105	1.210	1.327	1.458	1.606	1.773	1.9[illegible]	2.1[illegible]	[illegible]	2.7[illegible]	[illegible]	3.4[illegible]	3.882	4.5[illegible]	5.3[illegible]
					0.976	1.069	1.173	1.28[illegible]	1.426	1.559	1.720	1.9[illegible]3	2.113	2.356	2.63[illegible]	2.9[illegible]	3.303	3.834	4.407	4.114	5.993
			0.857	0.943	1.035	1.136	1.249	1.375	1.515	1.672	1.85[illegible]	2.054	2.2[illegible]	2.56[illegible]	2.[illegible]5	3.[illegible]1	3.7[illegible]	[illegible].251	4.918	5.747	6.778
			0.909	1.002	1.102	1.212	1.335	1.473	1.627	1.800	1.9[illegible]	2.22[illegible]	[illegible]	2.8[illegible]5	3.2[illegible]	3.6[illegible]	[illegible].123	4.756	5.538	6.5[illegible]	7.767
			0.968	1.069	1.178	1.298	1.433	1.585	1.756	1.951	2.175	2.43[illegible]	[illegible].737	3.[illegible]	3.511	[illegible]	[illegible]	5.37[illegible]	[illegible].303		
	0.845	0.935	1.034	1.143	1.263	1.396	1.546	1.715	1.907	2.128	2.383	2.6[illegible]	3.[illegible]	3.[illegible]5	[illegible]	[illegible]14	[illegible]	6.132	7.255		
	0.903	1.001	1.109	1.228	1.361	1.521	1.679	1.86[illegible]	2.066	2.337	2.62[illegible]	[illegible]	3.373	3.851	[illegible]	[illegible].127	[illegible]48	[illegible]	[illegible].471		
	0.969	1.078	1.197	1.329	1.477	1.644	1.834	2.05	2.24[illegible]	2.54[illegible]	2.942	3.340	3.74[illegible]	[illegible].36[illegible]	[illegible].93	[illegible]5	[illegible]9				
[illegible].937	1.046	1.166	1.299	1.447	1.613	1.802	2.019	2.267	2.553	2.887	3.283	3.756	4.3[illegible]	[illegible]5	[illegible].643	6.[illegible]	[illegible].2[illegible]5				
[illegible]15	1.135	1.266	1.417	1.584	1.774	1.991	2.240	2.526	2.[illegible]4	3.260	3.731	4.[illegible]5	[illegible]	5.82	[illegible].86[illegible]						
[illegible]5	1.239	1.388	1.558	1.750	1.969	2.219	2.511	2.851	3.25[illegible]	3.745	4.2[illegible]7	4.9[illegible]	[illegible].834	[illegible].8[illegible]5	[illegible]						
[illegible].211	1.364	1.536	1.731	1.953	2.208	2.502	2.848	3.256	3.741	4.344	5.033	5.[illegible]3	6.[illegible]75								
[illegible].34[illegible]	1.515	1.714	1.942	2.204	2.567	2.862	3.282	3.782	4.383	5.112	6.006	[illegible].119	[illegible].528								
[illegible]8	1.702	1.935	2.203	2.515	2.882	3.316	3.837	4.467	5.232	[illegible].167	7.321										
[illegible]96	1.936	2.213	2.539	2.925	3.382	3.925	4.579	5.386	6.395	7.65[illegible]	[illegible].205										
[illegible]	2.236	2.579	2.985	3.469	4.051	4.755	5.618	6.691	8.043												
[illegible].270	2.635	3.068	3.587	4.216	4.986	5.935	7.110														

APPENDIX [illegible]

VALUES OF Λ_K AS A FUNCTION OF F_m AND λ

(Compiled by M.S. Gorokhov)

	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74
4200											0.882	[illegible]	[illegible]	[illegible]	1.126	[illegible]	[illegible]	[illegible]
4100											[illegible]	[illegible]	1.000	[illegible]	1.177	[illegible]	[illegible]	[illegible]
4000											[illegible]	[illegible]	1.043	1.133	1.232	1.341	[illegible]	[illegible]
3900									[illegible]	[illegible]	[illegible]	1.005	[illegible]	1.187	1.292	[illegible]	[illegible]	[illegible]
3800									[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.243	1.356	1.484	[illegible]	[illegible]
3700									[illegible]	[illegible]	[illegible]	[illegible]	1.201	[illegible]	1.432	[illegible]	[illegible]	[illegible]
3600									[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.383	1.515	[illegible]	1.834	2.029
3500									[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.464	[illegible]	1.769	1.955	2.169
3400							[illegible]	[illegible]	[illegible]	[illegible]	1.183	[illegible]	[illegible]	1.554	[illegible]	1.888	[illegible]	[illegible]
3300							.876	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.656	1.827	2.023	[illegible]	2.513
3200							[illegible]	[illegible]	1.105	[illegible]	[illegible]	[illegible]	[illegible]	1.773	[illegible]	[illegible]	[illegible]	[illegible]
3100							.976	[illegible]	1.173	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.356	[illegible]	[illegible]
3000					0.857	[illegible]	1.035	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2900					[illegible]	[illegible]	[illegible]	[illegible]	1.335	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2800					[illegible]	[illegible]	1.178	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2700			0.845	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2600			[illegible]	[illegible]	[illegible]	1.228	1.361	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2500			[illegible]	[illegible]	1.197	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2400	0.841	0.937	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2300	[illegible]	1.015	1.135	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2200	0.985	[illegible]	1.239	1.388	1.558	1.750	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
2100	1.076	1.211	1.364	1.536	1.731	1.953	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
2000	1.186	[illegible]	1.515	1.714	1.942	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
1900	1.320	[illegible]	1.702	[illegible]	2.203	2.515	2.882	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]				
1800	1.486	1.696	1.936	2.213	[illegible]	[illegible]	3.382	[illegible]	4.579	[illegible]	[illegible]	[illegible]	[illegible]					
1700	1.693	1.944	2.236	[illegible]	2.985	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]							
1600	1.961	2.270	2.635	3.068	3.587	[illegible]	[illegible]	5.935	7.110									

STAT

APPENDIX 4

TABLES OF FUNCTION $\int_0^{\xi} z^{B\ B_1} d\xi$

$\frac{B}{B_1} = 5$

0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
0	0	0	0	0	0	0	0	0	0	0	0	0	0
.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
37	37	37	37	38	38	38	39	39	39	39	40	40	40
52	53	53	53	54	55	55	56	57	58	58	59	59	59
66	67	67	68	69	70	71	72	74	76	77	77	78	78
79	80	80	82	83	84	85	87	90	92	94	94	96	97
90	91	92	94	95	97	98	0.101	0.105	0.108	0.110	0.111	0.113	0.115
.100	0.101	0.103	0.104	0.106	0.108	0.109	114	119	123	125	127	130	132
109	110	112	114	116	118	119	125	132	136	139	142	146	148
117	118	120	123	125	127	128	135	143	148	152	155	160	164
124	125	127	130	133	135	137	144	153	160	164	168	174	179
130	132	134	137	140	142	144	152	162	170	176	180	188	193
136	138	140	143	146	148	150	159	171	179	186	191	200	206
141	143	145	148	151	154	156	166	179	188	195	201	211	218
145	147	149	153	156	159	161	172	186	196	204	210	222	230
149	151	153	157	160	163	165	177	192	203	212	218	232	241
152	154	156	160	164	167	169	181	197	209	219	226	241	253
155	157	159	163	167	170	173	185	202	214	225	233	249	260
157	159	162	166	169	173	176	188	206	219	230	239	256	269
159	161	164	168	171	175	178	191	210	223	235	244	263	277
161	163	166	170	173	177	180	194	213	227	240	249	269	284
163	164	157	172	175	179	182	196	216	231	244	254	275	291
164	165	168	173	177	180	183	198	218	234	247	258	280	297
165	166	169	174	178	182	185	199	220	236	250	261	284	302
166	167	170	175	179	183	186	200	222	238	252	264	288	307
166	168	171	176	180	184	187	201	223	240	255	267	292	312
167	169	172	176	180	184	187	202	225	242	257	269	295	316
167	169	172	177	181	185	188	203	226	243	258	271	298	319
168	170	173	177	181	185	189	204	226	244	260	273	300	322
168	170	173	178	182	186	189	204	227	245	261	274	302	325
169	170	173	178	182	186	189	205	228	246	262	275	304	328

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APPENDIX 4

TABLES OF FUNCTION $\int_0 z^{B} B_1 d\beta$

$\frac{B}{B_1} = 5$

β \ γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060
0	0	0	0	0	0	0	0	0	0	0	0
0.020	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020
0.040	36	37	37	37	37	38	38	38	39	39	39
0.060	52	52	53	53	53	54	55	55	56	57	58
0.080	66	66	67	67	68	69	70	71	72	74	76
0.100	78	79	80	80	82	83	84	85	87	90	92
0.120	89	90	91	92	94	95	97	98	0.101	0.105	0.108
0.140	99	0.100	0.101	0.103	0.104	0.106	0.108	0.109	114	119	123
0.160	0.108	109	110	112	114	116	118	119	125	132	135
0.180	116	117	118	120	123	125	127	128	135	143	14[illegible]
0.200	123	124	125	127	130	133	135	137	144	153	160
0.220	129	130	132	134	137	140	142	144	152	162	[illegible]
0.240	134	136	138	140	143	146	148	150	159	[illegible]	[illegible]
0.260	139	141	143	145	148	151	154	156	166	179	188
0.280	143	145	147	149	153	156	159	161	172	186	[illegible]
0.300	147	149	151	153	157	160	162	165	[illegible]	192	203
0.320	150	152	154	156	160	164	167	169	181	197	209
0.340	153	155	157	159	163	167	170	173	185	202	214
0.360	155	157	159	162	166	169	173	176	188	206	[illegible]
0.380	157	159	161	164	168	171	175	178	191	210	223
0.400	159	161	163	166	170	173	177	180	194	[illegible]	227
0.420	160	163	164	167	172	175	179	182	196	216	231
0.440	161	164	165	168	173	177	181	183	198	218	234
0.460	162	165	166	169	174	178	182	185	199	220	236
0.480	163	166	167	170	175	179	183	186	200	222	238
0.500	164	166	168	171	176	180	184	187	201	223	240
0.520	165	167	169	172	176	180	184	187	202	225	242
0.540	165	167	169	172	177	181	185	188	203	226	243
0.560	166	168	170	173	177	181	185	189	204	226	244
0.580	166	168	170	173	178	182	186	189	204	227	245
0.600	166	169	170	173	178	182	186	189	205	228	24[illegible]

STAT

$\frac{B}{B_1}$ - 6

.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
0	0	0	0	0	0	0	0	0	0	0	0	0	0
.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
36	36	36	37	37	38	38	38	39	39	39	39	40	40
51	51	51	53	53	54	54	55	57	58	58	58	59	59
64	64	65	67	67	68	69	71	74	75	76	76	77	78
75	76	77	79	80	81	82	85	89	91	92	93	95	96
85	86	88	90	91	92	94	98	0.103	0.106	0.108	0.109	0.112	0.113
94	95	97	99	0.101	0.102	0.104	0.109	116	120	122	124	128	130
.102	0.103	0.105	0.107	110	111	113	119	127	132	135	138	143	146
109	110	112	115	117	119	121	128	137	143	147	151	157	161
115	116	118	121	123	126	128	136	146	153	158	163	170	175
120	121	123	126	129	132	134	143	154	162	168	173	182	188
124	125	127	131	134	137	139	149	162	171	177	183	193	200
127	129	131	135	138	141	143	154	168	178	185	192	203	211
130	132	134	138	142	145	147	158	173	184	192	200	212	222
133	135	137	141	145	148	151	162	178	190	199	207	221	232
135	137	139	144	147	151	154	165	182	195	204	213	229	240
137	139	141	146	149	153	156	168	186	199	209	218	236	248
139	140	143	148	151	155	158	170	189	203	213	223	242	255
140	141	144	149	153	156	159	172	192	206	217	227	247	262
141	142	145	150	154	158	161	174	194	209	221	231	252	268
142	143	146	151	155	159	162	175	196	211	224	234	256	273
142	144	147	152	156	160	163	176	197	213	226	237	260	278
143	145	148	152	156	160	164	177	198	215	228	240	263	282
144	145	148	153	157	161	164	178	199	216	230	242	266	286
144	146	148	153	157	161	165	179	200	217	231	243	269	289
144	146	149	154	158	162	165	179	201	218	232	245	271	292
144	146	149	154	158	162	165	180	202	219	233	246	273	295
144	146	149	154	158	162	166	180	202	220	234	247	275	297
145	146	149	154	158	162	166	180	202	220	235	248	276	299
145	146	150	154	158	162	166	180	203	221	235	249	277	300

$\frac{B}{B_1} = 6$

β \ γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080
0	0	0	0	0	0	0	0	0	0	0	0	0
0.020	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020
0.040	35	36	36	36	37	37	38	38	38	39	39	39
0.060	50	51	51	51	53	53	54	54	55	57	58	58
0.080	63	64	64	65	67	67	68	69	71	74	75	76
0.100	74	75	76	77	79	80	81	82	85	89	91	92
0.120	84	85	86	88	90	91	92	94	98	0.103	0.106	0.108
0.140	93	94	95	97	99	0.101	0.102	0.104	0.109	116	120	122
0.160	0.101	0.102	0.103	0.105	0.107	110	111	113	119	127	132	135
0.180	107	109	110	112	115	117	119	121	128	137	143	147
0.200	113	115	116	118	121	123	126	128	136	146	153	158
0.220	118	120	121	123	126	129	132	134	143	154	162	168
0.240	122	124	125	127	131	134	137	139	149	162	171	177
0.260	125	127	129	131	135	138	141	143	154	168	178	185
0.280	128	130	132	134	138	142	145	147	158	173	184	192
0.300	131	133	135	137	141	145	148	151	162	178	190	199
0.320	133	135	137	139	144	147	151	154	165	182	195	204
0.340	134	137	139	141	146	149	153	156	168	186	199	209
0.360	136	139	140	143	148	151	155	158	170	189	203	213
0.380	137	140	141	144	149	153	156	159	[illegible]	192	206	217
0.400	138	141	142	145	150	154	158	161	174	194	209	221
0.420	139	142	143	146	151	155	159	162	175	196	211	224
0.440	139	142	144	147	152	156	160	163	176	197	213	226
0.460	140	143	145	148	152	156	160	164	177	198	215	228
0.480	140	144	145	148	153	157	161	164	178	199	216	230
0.500	141	144	146	148	153	157	161	165	179	200	217	231
0.520	141	144	146	149	154	158	162	165	179	201	218	232
0.540	141	144	146	149	154	158	162	165	180	202	219	233
0.560	142	144	146	149	154	158	162	166	180	202	220	234
0.580	142	145	146	149	154	158	162	166	180	202	220	235
0.600	142	145	146	150	154	158	162	166	180	203	221	235

$$\frac{B}{B_1} = 7$$

0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.019	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
35	35	36	36	36	37	37	37	38	39	39	39	39	39	40
49	49	50	51	51	52	53	53	55	56	57	58	58	58	59
61	62	62	63	65	66	66	67	70	72	74	75	76	77	77
71	72	73	74	76	77	78	79	83	87	90	91	93	94	95
80	81	82	84	86	87	89	90	95	0.100	0.104	0.106	0.108	0.110	0.112
88	89	90	92	94	96	98	99	0.105	112	117	120	122	126	128
94	95	97	98	0.101	0.104	0.106	0.107	114	123	128	132	135	140	144
99	0.101	0.103	0.104	107	110	112	114	122	132	138	143	147	153	158
.104	106	107	109	113	115	118	120	128	140	147	153	157	166	171
108	110	111	113	117	120	123	125	134	147	155	[illegible]	167	177	183
111	113	114	117	121	124	127	129	139	153	162	170	176	187	195
114	116	117	120	124	127	130	132	143	158	169	177	183	196	205
116	118	120	122	126	130	133	135	146	162	174	183	190	204	214
118	120	122	124	128	132	135	138	149	[illegible]	178	188	196	211	222
119	121	123	126	130	134	137	140	152	169	182	193	201	218	230
120	123	124	127	131	135	138	141	154	172	185	197	206	224	237
121	124	125	128	132	136	140	143	155	174	188	200	210	229	243
122	124	126	129	133	137	141	144	157	176	191	203	213	233	249
123	125	127	130	134	138	142	145	158	177	193	205	216	237	254
123	126	128	130	135	139	142	145	159	179	194	207	218	241	258
124	126	128	131	135	139	143	146	159	180	196	209	220	244	262
124	126	128	131	136	140	143	146	160	180	197	210	222	246	265
124	127	128	131	136	140	143	147	160	181	198	211	223	248	268
125	127	129	131	136	140	144	147	161	182	198	212	224	250	270
125	127	129	132	136	140	144	147	161	182	199	213	225	251	272
125	127	129	132	136	140	144	147	161	182	199	214	226	253	274
125	127	129	132	136	140	144	148	161	182	200	214	227	254	276
125	127	129	132	137	140	144	148	161	183	200	214	227	254	277
125	127	129	132	136	140	144	148	162	183	200	[illegible]	228	255	278

STAT

$$\frac{B}{B_1} = 7$$

ε \ γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.
0	0	0	0	0	0	0	0	0	0	0	0	
0.020	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.
0.040	35	35	36	36	36	37	37	37	38	39	39	
0.060	49	49	50	51	51	52	53	53	55	56	57	
0.080	61	62	62	63	65	66	66	67	70	72	74	
0.100	71	72	73	74	76	77	78	79	83	87	90	
0.120	80	81	82	84	86	87	89	90	95	0.100	0.104	0.
0.140	88	89	90	92	94	96	98	99	0.105	112	117	
0.160	94	95	97	98	0.101	0.104	0.106	0.107	114	123	128	
0.180	99	0.101	0.103	0.104	107	110	112	114	122	132	138	
0.200	0.104	106	107	109	113	115	118	120	128	140	147	
0.220	108	110	111	113	117	120	[illegible]	[illegible]	134	147	155	
0.240	111	113	114	117	121	124	[illegible]	129	139	155	162	
0.260	114	116	117	120	124	127	130	132	143	158	169	
0.280	116	118	120	122	126	130	[illegible]	[illegible]	146	162	174	
0.300	118	120	122	124	128	132	[illegible]	138	149	166	178	
0.320	119	121	123	126	130	134	[illegible]	140	152	169	182	
0.340	120	123	124	127	131	[illegible]	[illegible]	141	154	172	[illegible]	
0.360	121	124	125	128	[illegible]	136	140	143	155	174	188	
0.380	122	124	126	129	133	[illegible]	141	144	157	176	191	
0.400	123	125	127	130	134	138	142	[illegible]	158	177	193	
0.420	123	126	128	130	[illegible]	139	142	[illegible]	159	179	194	
0.440	124	126	128	131	135	139	143	146	159	180	196	
0.460	124	126	128	131	136	140	143	146	160	180	197	
0.480	124	127	128	131	136	140	[illegible]	147	160	181	198	
0.500	125	127	129	131	136	140	144	147	161	182	198	
0.520	125	127	129	132	136	140	144	147	161	182	[illegible]	
0.540	125	127	129	132	136	140	144	147	161	182	199	
0.560	125	127	129	132	136	140	144	148	161	182	200	
0.580	125	127	129	132	137	140	144	148	161	183	200	
0.600	125	127	129	132	136	140	144	148	162	183	200	

STAT

$$\frac{B}{B_1} = 8$$

0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.018	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020
34	35	35	35	36	36	37	37	38	39	39	39	39	39	40
48	48	48	49	50	51	52	53	54	56	57	57	58	58	59
59	59	60	61	62	64	65	66	69	72	73	74	75	76	77
68	69	70	71	73	75	76	77	81	85	88	90	91	93	95
76	77	78	80	82	84	85	87	92	98	0.102	104	0.106	0.109	0.111
83	84	85	87	89	92	93	95	0.101	0.109	114	117	120	124	127
88	89	90	93	95	98	0.100	0.102	109	119	12[illegible]	129	132	138	142
92	94	95	98	0.101	0.103	106	108	116	127	134	[illegible]	143	150	155
96	98	99	0.102	105	108	110	113	122	134	142	148	[illegible]	162	167
0.99	0.101	102	105	108	112	114	117	127	140	[illegible]	[illegible]	[illegible]	172	179
102	103	105	108	111	115	117	120	131	[illegible]	[illegible]	[illegible]	169	181	190
104	105	107	110	114	117	120	123	134	149	[illegible]	[illegible]	176	189	199
105	107	109	111	116	119	122	125	136	152	[illegible]	174	182	195	207
107	108	110	113	117	121	124	127	138	[illegible]	[illegible]	[illegible]	186	202	214
108	109	111	114	118	122	126	128	140	158	[illegible]	182	190	208	221
108	110	112	115	119	123	127	129	142	160	174	[illegible]	194	213	227
109	111	113	116	120	124	127	130	143	[illegible]	[illegible]	187	197	217	232
110	112	113	116	120	124	128	131	144	163	178	190	200	221	237
110	112	114	116	121	125	128	132	145	164	174	[illegible]	202	224	241
110	113	114	117	121	125	129	132	[illegible]	165	180	[illegible]	204	226	244
111	113	114	117	122	126	129	132	146	165	181	194	[illegible]	228	247
111	113	114	117	122	126	129	133	146	166	182	[illegible]	206	230	250
111	113	114	117	122	126	130	133	146	166	182	196	207	232	252
111	113	115	118	122	126	130	133	146	166	183	196	208	233	254
111	113	115	118	122	126	130	133	147	167	183	[illegible]	209	234	255
111	113	115	118	122	126	130	133	147	167	184	197	209	235	256
111	113	115	118	122	126	130	133	147	167	184	198	210	236	257
111	113	115	118	122	126	130	133	147	167	184	198	210	236	258
111	113	115	118	122	126	130	133	147	167	184	198	210	237	259

$$\frac{B}{B_1} = \delta$$

β \ γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.08
0	0	0	0	0	0	0	0	0	0	0	0	0
0.020	0.018	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.02
0.040	34	35	35	35	36	36	37	37	38	39	39	3
0.060	48	48	48	49	50	51	52	53	54	56	57	5
0.080	59	59	60	61	62	64	65	66	69	72	73	7
0.100	68	69	70	71	73	75	76	77	81	85	88	9
0.120	76	77	78	80	82	84	85	87	92	98	0.102	10
0.140	83	84	85	87	89	92	93	95	0.101	0.109	114	11
0.160	88	89	90	93	95	98	0.100	0.102	109	119	125	12
0.180	92	94	95	98	0.101	0.103	106	108	116	127	134	13
0.200	96	98	99	0.102	105	108	110	113	122	134	142	14
0.220	0.99	0.101	102	105	108	112	114	117	127	140	149	15
0.240	102	103	105	108	111	115	117	120	131	145	155	16
0.260	104	105	107	110	114	117	120	123	134	149	160	16
0.280	105	107	109	111	116	119	122	[illegible]	136	152	164	17
0.300	107	108	110	113	117	121	124	[illegible]	138	155	168	.7
0.320	108	109	111	114	118	122	126	128	140	158	171	18
0.340	108	110	112	115	119	123	127	129	142	160	174	18
0.360	109	111	113	116	120	124	127	130	143	161	176	18
0.380	110	112	113	116	120	124	128	131	144	163	178	19
0.400	110	112	114	116	[illegible]	[illegible]	128	132	145	164	179	19
0.420	110	113	114	117	121	125	129	132	[illegible]	165	180	19
0.440	111	113	114	117	122	126	129	132	146	165	181	19
0.460	111	113	114	117	122	126	129	133	146	166	182	19
0.480	111	113	114	117	122	126	130	133	146	166	182	19
0.500	111	113	115	118	122	126	130	133	146	166	183	19
0.520	111	113	115	118	122	126	130	133	147	167	183	19
0.540	111	113	115	118	122	126	130	133	147	167	184	19
0.560	111	113	115	118	122	126	130	133	147	167	184	19
0.580	111	113	115	118	122	126	130	133	147	167	184	19
0.600	111	113	115	118	122	126	130	133	147	167	184	19

$$\frac{B}{B_1} = 9$$

	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020
3	34	34	35	35	36	36	36	38	38	39	39	39	39	40
6	47	47	48	49	50	51	51	54	55	56	57	57	58	59
6	57	58	59	61	62	63	64	68	70	72	74	74	76	77
5	66	67	69	71	72	74	75	79	84	87	89	90	93	94
2	73	74	77	79	80	83	84	89	96	0.100	0.103	0.105	0.108	0.110
8	79	80	83	85	87	90	91	98	0.106	111	115	118	122	125
2	84	85	88	90	93	95	97	0.105	114	121	126	129	135	139
6	88	89	92	95	98	0.100	0.102	111	122	129	135	139	147	152
9	91	92	95	99	0.101	104	106	116	128	136	143	148	158	164
92	94	95	98	0.101	104	107	109	120	133	142	150	156	167	175
94	96	97	0.100	103	107	110	112	123	137	148	156	163	175	185
95	97	99	102	105	109	112	114	126	141	152	161	169	182	194
96	98	0.100	103	107	110	113	116	128	144	156	165	174	189	201
97	99	101	104	108	111	114	117	129	146	159	169	178	195	208
08	0.100	102	105	109	112	115	118	131	148	161	172	181	200	214
98	101	102	105	109	113	116	119	132	149	163	174	184	203	219
99	101	102	106	110	113	117	120	132	150	165	176	187	206	223
99	101	103	106	110	114	117	120	133	151	165	178	189	209	227
99	102	103	106	110	114	118	121	134	152	167	179	190	212	230
00	102	103	106	111	114	118	121	134	153	168	180	191	214	233
00	102	103	107	111	114	118	121	134	153	168	181	192	216	235
00	102	104	107	111	115	118	121	134	154	169	182	193	217	237
00	102	104	107	111	115	118	121	134	154	169	182	194	218	239
00	102	104	107	111	115	118	121	135	154	170	183	194	219	240
00	102	104	107	111	115	118	122	135	154	170	183	195	220	241
00	102	104	107	111	115	118	122	135	154	170	183	195	220	242
00	102	104	107	111	115	118	122	135	154	170	183	195	221	243
00	102	104	107	111	115	119	122	135	154	170	184	196	221	243
00	102	104	107	111	115	119	122	135	154	170	184	196	221	244

-RE

$$\frac{B}{B_1} = 9$$

β \ γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.1[illegible]
0	0	0	0	0	0	0	0	0	0	0	0	0	0.[illegible]
0.020	0.018	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	
0.040	33	34	34	35	35	36	36	36	38	38	39	39	
0.060	46	47	47	48	49	50	51	51	54	55	56	57	
0.080	56	57	58	59	61	62	63	64	68	70	72	74	
0.100	65	66	67	69	71	72	74	75	79	84	87	89	
0.120	72	73	74	77	79	80	83	84	89	96	0.100	0.103	0.[illegible]
0.140	78	79	80	83	85	87	90	91	98	0.106	111	115	
0.160	82	84	85	88	90	93	95	97	0.105	114	121	126	
0.180	86	88	89	92	95	98	0.100	0.102	111	122	129	135	
0.200	89	91	92	95	99	0.101	104	106	116	128	136	143	
0.220	92	94	95	98	0.101	104	107	109	120	133	142	150	
0.240	94	96	97	0.100	103	107	110	112	123	137	148	156	
0.260	95	97	99	102	105	109	112	114	126	141	152	161	
0.280	96	98	0.100	103	107	110	113	116	128	144	156	165	
0.300	97	99	101	104	108	111	114	117	129	146	159	169	
0.320	98	0.100	102	105	109	112	115	118	131	148	161	172	
0.340	98	101	102	105	109	113	116	119	132	149	163	174	
0.360	99	101	102	106	110	113	117	120	132	150	165	176	
0.380	99	101	103	106	110	114	117	120	133	151	165	178	
0.400	99	102	103	106	110	114	118	121	134	152	167	179	
0.420	0.100	102	103	106	111	114	118	121	134	153	168	180	
0.440	100	102	103	107	111	114	118	121	134	153	168	181	
0.460	100	102	104	107	111	115	118	121	134	154	169	182	
0.480	100	102	104	107	111	115	118	121	134	154	169	182	
0.500	100	102	104	107	111	115	118	121	135	154	170	183	
0.520	100	102	104	107	111	115	118	122	135	154	170	183	
0.540	100	102	104	107	111	115	118	122	135	154	170	183	
0.560	100	102	104	107	111	115	118	122	135	154	170	183	
0.580	100	102	104	107	111	115	119	122	135	154	170	184	
0.600	100	102	104	107	111	115	119	122	135	154	170	184	

$$\frac{B}{B_1} = 10$$

0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.018	0.018	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020
33	33	34	34	35	35	36	36	37	38	39	39	39	39	40
45	46	46	47	48	49	50	51	53	55	56	57	57	58	59
55	56	56	57	59	61	62	63	66	70	72	73	74	76	77
63	64	64	66	68	70	71	73	77	83	86	88	89	92	94
69	70	71	73	75	77	79	81	87	94	98	0.101	0.103	0.107	110
74	75	76	78	81	83	85	87	95	0.103	0.108	113	115	121	124
78	79	80	82	85	88	90	92	0.101	111	117	123	126	133	137
81	82	84	86	89	92	94	97	106	117	125	131	136	144	150
83	85	87	89	92	95	98	0.100	110	122	132	139	144	154	161
85	87	89	91	94	98	0.101	103	113	127	137	145	151	163	171
87	89	90	92	96	0.100	103	105	116	131	141	150	157	170	180
88	90	91	93	98	101	104	107	118	134	145	154	162	176	188
89	91	92	94	99	102	105	108	120	136	148	158	166	182	194
89	91	93	95	99	103	106	109	121	138	150	161	170	187	200
90	92	93	96	0.100	104	107	110	122	139	152	164	173	191	205
90	92	94	96	100	104	107	110	122	140	154	166	175	194	210
90	92	94	96	101	105	108	111	123	141	155	167	177	197	214
90	93	94	97	101	105	108	111	123	142	156	168	178	199	217
90	93	94	97	101	105	108	111	124	142	157	169	180	201	219
91	93	94	97	101	105	108	111	124	142	157	170	181	203	221
91	93	94	97	101	105	108	112	124	143	158	170	182	204	223
91	93	94	97	102	105	109	112	124	143	159	171	182	205	225
91	93	94	97	102	106	109	112	124	143	159	171	183	206	226
91	93	94	97	102	106	109	113	124	143	159	172	183	207	227
91	93	94	97	102	106	109	112	124	143	159	172	183	207	228
91	93	95	97	102	106	109	112	125	143	159	172	183	208	228
91	93	95	97	102	106	109	112	125	143	159	172	183	208	229
91	93	95	97	102	106	109	112	125	143	159	172	184	208	229
91	93	95	97	102	106	109	112	125	144	159	172	184	208	229

$$\frac{B}{B_1} = 10$$

γ / β	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080
0	0	0	0	0	0	0	0	0	0	0	0	0
0.020	0.018	0.018	0.019	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020
0.040	33	33	34	34	35	35	36	36	37	38	39	39
0.060	45	46	46	47	48	49	50	51	53	55	56	57
0.080	55	56	56	57	59	61	62	63	66	70	72	73
0.100	63	64	64	66	68	70	71	73	77	83	86	88
0.120	69	70	71	73	75	77	79	81	87	94	98	0.101
0.140	74	75	76	78	81	83	85	87	95	0.103	0.108	113
0.160	78	79	80	82	85	88	90	92	0.101	111	117	123
0.180	81	82	84	86	89	92	94	97	106	117	125	131
0.200	83	85	87	89	92	95	98	0.100	110	122	132	139
0.220	85	87	89	91	94	98	0.101	103	113	127	137	145
0.240	87	89	90	92	96	0.100	103	105	116	131	141	150
0.260	88	90	91	93	98	101	104	107	118	134	145	154
0.280	89	91	92	94	99	102	105	108	120	136	148	158
0.300	89	91	93	95	99	103	106	109	121	138	150	161
0.320	90	92	93	96	0.100	104	107	110	122	139	152	164
0.340	90	92	94	96	100	104	107	110	122	140	154	166
0.360	90	92	94	96	101	105	108	111	123	141	155	167
0.380	90	93	94	97	101	105	108	111	123	142	156	168
0.400	90	93	94	97	101	105	108	111	124	142	157	169
0.420	91	93	94	97	101	105	108	111	124	142	157	170
0.440	91	93	94	97	101	105	108	112	124	143	158	170
0.460	91	93	94	97	102	105	109	112	124	143	159	171
0.480	91	93	94	97	102	106	109	112	124	143	159	171
0.500	91	93	94	97	102	106	109	112	124	143	159	172
0.520	91	93	94	97	102	106	109	112	124	143	159	172
0.540	91	93	95	97	102	106	109	112	125	143	159	172
0.560	91	93	95	97	102	106	109	112	125	143	159	172
0.580	91	93	95	97	102	106	109	112	125	143	159	172
0.600	91	93	95	97	102	106	109	112	125	144	159	172

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